A Prescription for Endpoint Divergences and Renormalization in Higgs Decay Induced by a b Quark Loop

Xing Wang

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

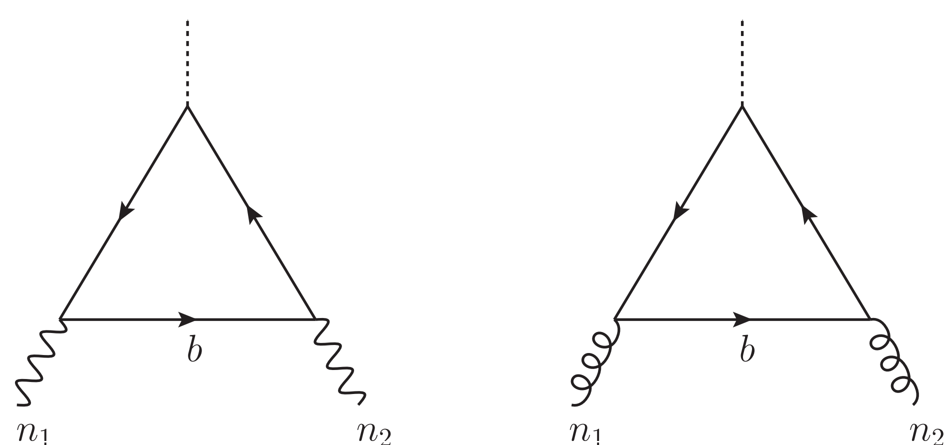
With Z. Liu, B. Mecaj, M. Neubert and M. Schnubel

2009.04456, 2009.06779 and to appear

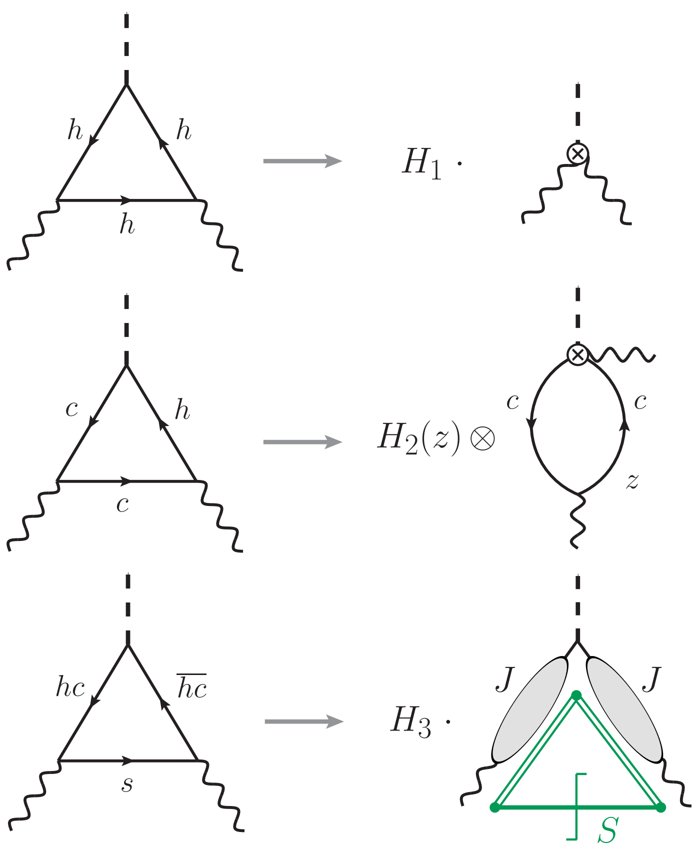
World SCET 2021, April 22

Motivation

- Scale hierarchy $M_h^2 \gg m_b^2$ in $H \to \gamma\gamma(gg)$ induced by a b quark loop indicates factorization, and is relevant in precision studies.
- This is a NLP problem (SCET2), and is sufficiently complicated but simple enough (e.g., the operator basis is small) to investigate NLP SCET.
- Despite of some consensus of several generic features of NLP SCET, establishing a renormalized factorization and dealing with endpoint divergences are not fully understood yet. [Beneke et al., Moult et al., 2016-2020]
- I will briefly sketch how we renormalize and use "plus-type subtraction" to deal with endpoint divergences.



Bare Factorization: Plus-Type Subtraction and Emergence of Cutoff



$$\mathcal{M}_{\gamma\gamma}$$

$$= H_1^{(0)}\langle O_1^{(0)}\rangle$$

$$+$$

$$= 2\int_0^1 dz H_2^{(0)}(z)\langle O_2^{(0)}(z)\rangle \qquad \left(H_2^{(0)}(z) = \frac{\bar{H}_2(z)}{z(1-z)}\right)$$

$$+$$

$$= H_3\langle O_3^{(0)}\rangle = H_3^{(0)}\int_0^\infty \frac{d\ell_+}{\ell_+}\int_0^\infty \frac{d\ell_-}{\ell_-} J^{(0)}\left(-M_h\ell_+\right) J^{(0)}\left(M_h\ell_-\right) S^{(0)}\left(\ell_+\ell_-\right)$$

- Endpoint divergences occur when $z \to 0,1$ and $\ell_\pm \to \infty$.
- Some are regularized by DR, while others are rapidity divergences.
- Rapidity divergences are cancelled additively, not like LP!

[Becher, Neubert, '10]
[Chiu, Jain, Neil, Rothstein, '11]

Cancellation of rapidity divergences indicates close relation between the two integrands in the endpoint region (next slide)

infinity bin

"plus-type" subtraction

[Liu, Neubert, 1912.08818]

$$\mathcal{M}_{\gamma\gamma} = \left(H_1^{(0)} + \Delta H_1^{(0)}\right)\langle O_1^{(0)}\rangle + 2\int_0^1 dz\left[H_2^{(0)}(z)\langle O_2^{(0)}(z)\rangle - [\![H_2^{(0)}(z)]\!][\![\langle O_2^{(0)}(z)\rangle]\!] - [\![H_2^{(0)}(\bar{z})]\!][\![\langle O_2^{(0)}(\bar{z})\rangle]\!]\right]$$

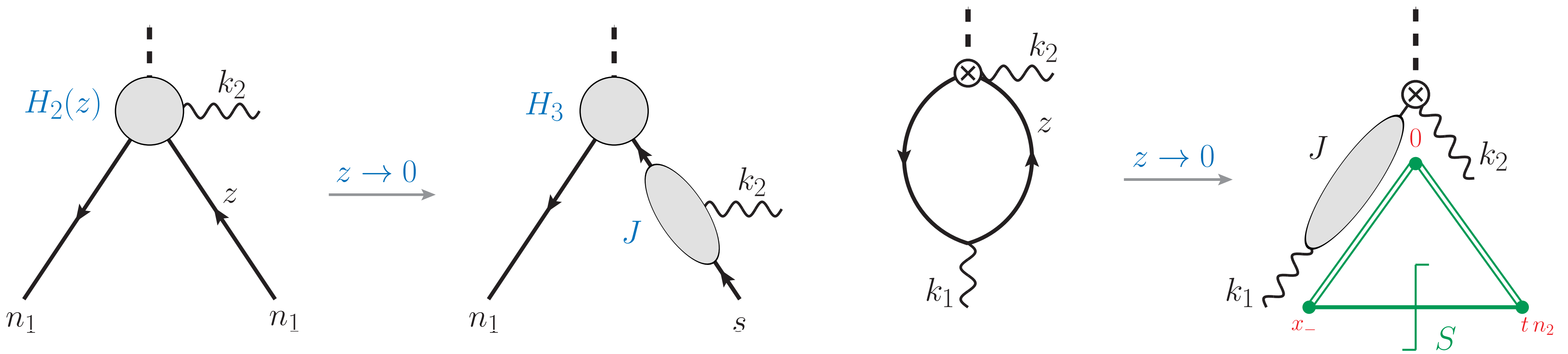
infinity bin

$$+ \lim_{\sigma\to -1} H_3^{(0)}\int_0^{m_H}\frac{d\ell_-}{\ell_-}\int_0^{\sigma m_H}\frac{d\ell_+}{\ell_+} J^{(0)}\left(m_H\ell_-\right) J^{(0)}\left(-m_H\ell_+\right) S^{(0)}\left(\ell_+\ell_-\right)\Big|_{\text{leading power}}$$

- $[\![f(z)]\!]$ means that one retains only the leading terms of the function $f(z)$.
- Cutoffs are emergent after adding back the subtraction and double counting is removed, which is $\Delta H_1^{(0)}$.
- Rapidity regulator is no longer needed due to plus-type subtraction, but cutoffs don't commute with renormalization.
- The factorization formula for $gg \to h$ is very similar to its abelian cousin (to appear).

Re-factorization conditions

- Re-factorization conditions relate the integrands in the endpoint region, but they only make sense in D dimension.
- These can also be used to obtain relations among renormalization factors, e.g., Z_J and $[\![Z_{22}]\!]$
- They also ensure all order relations between "left-over" terms due to cutoffs when renormalizing operators.



$$[\![H_2^{(0)}(z)]\!] = \frac{[\![\bar{H}_2^{(0)}(z)]\!]}{z} = -\frac{H_3^{(0)}}{z} J\left(zM_h^2\right)$$

$$[\![\langle\gamma\gamma|O_2^{(0)}(z)|h\rangle]\!] = -\frac{1}{2}\varepsilon_\perp^*(k_1)\cdot\varepsilon_\perp^*(k_2)\int_0^\infty \frac{d\ell_+}{\ell_+} J^{(0)}\left(-M_h\ell_+\right) S^{(0)}\left(zM_h\ell_+\right)$$

- These conditions also hold in $gg \to h$ amplitude (to appear).
- Re-factorization should be generic to deal with endpoint divergences, including SCET1. See the following two talks.

Renormalization ($h \to \gamma\gamma$ & $gg \to h$)

- There are operator mixings when renormalizing them, please refer to our papers.

- The renormalization for the non-abelian case is slightly different, since the amplitude itself is not IR safe. Extra divergences can be accounted for by a global renormalization:

$$\mathcal{M}_{gg}(\mu) = Z_{gg}^{-1}(\mu)\mathcal{M}_{gg}^{(0)}, \quad \text{with} \quad Z_{gg}^{-1} = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[\frac{2C_A}{\epsilon^2} + \frac{-2C_A \ln(-M_h^2/\mu^2) + \beta_0}{\epsilon}\right] + \mathcal{O}(\alpha_s^2)$$

- This global renormalization factor changes the renormalization factors for the operators, and therefore the anomalous dimensions. Here are Z factors of the soft function at NLO as a comparison:

$$Z_S^{gg}(w,w';\mu) = \delta(w-w') + \frac{\alpha_s(\mu)}{4\pi}\left\{\left[(C_F - C_A)\left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\omega}{\mu^2}\right) - \frac{3C_F - \beta_0}{\epsilon}\right]\delta(w-w') - \frac{4(C_F - C_A/2)}{\epsilon}w\Gamma(w,w')\right\}$$

$$Z_S^{\gamma\gamma}(w,w';\mu) = \delta(w-w') + \frac{\alpha_s(\mu)}{4\pi}\left\{\left[C_F\left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\omega}{\mu^2}\right) - \frac{3C_F}{\epsilon}\right]\delta(w-w') - \frac{4C_F}{\epsilon}w\Gamma(w,w')\right\}$$

- We derived the non-abelian renormalization factors, not only from consistency condition, but also using the method in [Bodwin, et al., 2101.04872]. See details in our coming papers.

Renormalized Factorization: Plus-Type Subtraction and Cutoff

$$\mathcal{M}_{\gamma\gamma} = H_1(\mu)\langle O_1(\mu)\rangle + 2\int_0^1 dz \Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!]\Big]$$
$$+ \lim_{\sigma\to -1} H_3(\mu)\int_0^{M_h}\frac{d\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{d\ell_+}{\ell_+} J(M_h\ell_-,\mu)\, J(-M_h\ell_+,\mu)\, S(\ell_+\ell_-,\mu)\Big|_{\text{leading power}}$$

- This master formula is free of any divergences for $H \to \gamma\gamma$. Its non-abelian cousin is similar, but needs Z_{gg}^{-1}.
- To establish such a renormalized formula is not so straightforward:
  - With cutoffs in the convolution, exchanging integration limits doesn't commute with renormalization, e.g.,

$$S(w,\mu) = \int_0^{\infty} dw' Z_S(w,w';\mu)\, S^{(0)}(w') \quad \text{v.s} \quad \int_0^{\sigma M_h^2}\frac{dw'}{w'} S^{(0)}(w') \times \cdots$$

- After exchanging the integration limits when expressing everything in terms of renormalized ones, there are some "left-over" terms. We proved to all orders that the sum of these terms is purely **hard**, and it can be absorbed into H_1. The same procedure also applies to $gg \to H$.

infinity bin — left-over — mixing

$$H_1(\mu) = \Big(H_1^{(0)} + \Delta H_1^{(0)} - \delta H_1^{(0)} - \delta' H_1^{(0)}\Big) Z_{11}^{-1}$$
$$+ 2\int_0^1 dz \Big[H_2^{(0)}(z) Z_{21}^{-1}(z) - [\![H_2^{(0)}(z)]\!][\![Z_{21}^{-1}(z)]\!] - [\![H_2^{(0)}(\bar{z})]\!][\![Z_{21}^{-1}(\bar{z})]\!]\Big]$$

Some Results: Logarithms at 3-loop and "NLL"

$$\mathcal{M}_b^{\gamma\gamma} \propto \frac{L^2}{2} - 2 + \frac{C_F \alpha_s(\hat{\mu}_h)}{4\pi}\left[-\frac{L^4}{12} - L^3 - \frac{2\pi^2}{3}L^2 + \left(12 + \frac{2\pi^2}{3} + 16\zeta_3\right)L - 20 + 4\zeta_3 - \frac{\pi^4}{5}\right]$$
$$+ C_F\left(\frac{\alpha_s(\hat{\mu}_h)}{4\pi}\right)^2 \left[\frac{C_F}{90}L^6 + \left(\frac{C_F}{10} - \frac{\beta_0}{30}\right)L^5 + d_4^{\rm OS}L^4 + d_3^{\rm OS}L^3 + \ldots\right]$$

$0.01975L^6 - 0.31111L^5 - 8.74342L^4 - 68.6182L^3$

- It is in perfect agreement with fixed order calculation in [Czakon, Niggetiedt, '20].
- The subleading logs are not smaller at all than the leading ones, due to the larger coeffs. So it only makes sense to consider it in RG-improved perturbation theory. But we present below resummation at "NLL" for just academic purpose:

$$\mathcal{M}_{\gamma\gamma}^{\rm NLL} \propto \frac{L^2}{2}\sum_{n=0}^{\infty}(-\rho_\gamma)^n \frac{2\Gamma(n+1)}{\Gamma(2n+3)}\left[1 + \frac{3\rho_\gamma}{2L}\frac{2n+1}{2n+3} - \frac{\beta_0}{C_F}\frac{\rho_\gamma^2}{4L}\frac{(n+1)^2}{(2n+3)(2n+5)}\right] \qquad \rho_\gamma = \frac{C_F\alpha_s(\mu_h)L^2}{2\pi}$$

$$\mathcal{M}_{gg}^{\rm NLL}(\hat{\mu}_h) \propto \frac{L^2}{2}\sum_{n=0}^{\infty}(-\rho_g)^n \frac{2\Gamma(n+1)}{\Gamma(2n+3)}\left[1 + \frac{C_F}{C_F - C_A}\frac{3\rho_g}{2L}\frac{2n+1}{2n+3} - \frac{\beta_0}{C_F - C_A}\frac{\rho_g^2}{4L}\frac{(n+1)^2}{(2n+3)(2n+5)}\right] \qquad \rho_g = \frac{(C_F - C_A)\alpha_s(\mu_h)L^2}{2\pi}$$

- At NLL, non-abelian case is the same as its abelian cousin by $C_F \to C_F - C_A$ (**not true beyond cusp**).
- For details and how to obtain predictions in RG-improved perturbation theory, see Bianka's talk.

Conclusion and take-home message

- ☑ We derived the renormalized factorization formula in the "plus-type subtraction" scheme to get rid of endpoint divergences;
- ☑ Its prediction is in perfect agreement with QCD three-loop calculations;
- ☑ As far as "cusp" terms are concerned, abelian and non-abelian seem the same under the replacement $C_F \to C_F - C_A$;
- ☑ See Bianka's talk about RGEs and resummation beyond "cusp";
- ☑ Re-factorization conditions play a key role in our case, and we believe they are generic to get rid of endpoint divergences $\longrightarrow$ hand it over to Philipp's talk.

Thank you! See you in the discussion session.

Backup: Renormalization ($h \to \gamma\gamma$)

$$\{O_1,\ O_2(z),\ [\![O_2(z)]\!]\} \qquad O_i(\mu) = Z_{ij} \otimes O_j^{(0)} \qquad \boldsymbol{Z} = \begin{pmatrix} Z_{11} & 0 & 0 \\ Z_{21} & Z_{22} & 0 \\ [\![Z_{21}]\!] & 0 & [\![Z_{22}]\!] \end{pmatrix}.$$

- Renormalization of O_1 is trivial, which is just the quark mass renormalization
- The diagonal Z_{22} can be understood by noticing that the coloured fields in O_2 have the same structure as in leading-twist LCDA of a transversely polarized vector meson: Brodsky-Lepage kernel
- Z_{22} is not enough to absorb all the UV divergence in O_2. The remaining can be absorbed by the mixing with O_1, which is just Z_{21}. Since the final states are photons, the mixing is natural
- The renormalization of $[\![O_2(z)]\!]$ can be obtained by the limiting behaviour of that of O_2

$$J^{(0)} \otimes J^{(0)} \otimes S^{(0)} = O_3^{(0)} = \mathrm{T}\left\{h\bar{\xi}_{n_1}\xi_{n_2}, i\int d^D x \mathcal{L}^{(1/2)}_{q\xi_{n_1}}(x), i\int d^D y \mathcal{L}^{(1/2)}_{\xi_{n_2}q}(y)\right\} + \text{h.c.}$$

- NLP SCET Lagrangian doesn't need renormalization, so the renormalization of $O_3^{(0)}$ comes from that of the scalar current $J_S = h\bar{\xi}_{n_1}\xi_{n_2}$, which is known to three loops:

$$\int_0^\infty d\ell_- \int_0^\infty d\ell_+ Z_J(\ell'_-, \ell_-) Z_J(\ell'_+, \ell_+) Z_S(\ell_-\ell_+, \omega) = Z_{33}\delta(\omega - \ell'_-\ell'_+)$$

- Z_J is related to $[\![Z_{22}]\!]$ by re-factorization formula and we prove that it can also be obtained from first principle
- Z_S can be obtained from the above relation and recently confirmed by Bodwin et al. first principle calculation at NLO