NLP resummation and the endpoint divergent contribution in DIS

Sebastian Jaskiewicz

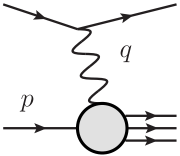
World SCET

April 22nd, 2021

[JHEP 10 (2020) 196] with Martin Beneke, Mathias Garny, Robert Szafron, Leonardo Vernazza and Jian Wang

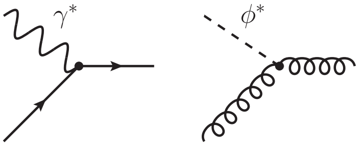
Durham University

Motivations and focus



Deep inelastic scattering (DIS) at threshold contains a hierarchy of scales: $Q^2 \gg P_X^2 \sim Q^2(1-x)$

$$x = \frac{Q^2}{2p \cdot q} \to 1 \quad \text{(large } N \text{ in Mellin space)}$$



DIS is well understood at leading power. The coefficient function known to N^{3}LL

$$C(Q^2) \sim \exp\left[g_1 \ln(N) + ...\right] + \mathcal{O}(N^{-1} \ln^n(N))$$

via traditional resummation techniques

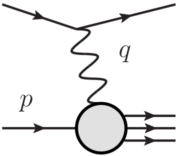
[S. Moch, J.A.M. Vermaseren, A. Vogt, hep-ph/0506288]

and equivalent results obtained in SCET using RG equations directly in momentum space

[T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]

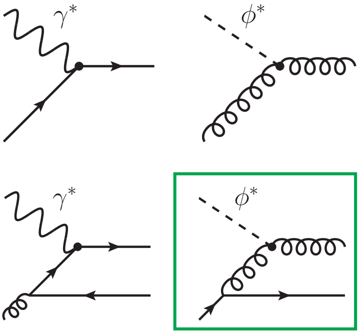
$$P_{qq/gg}^{(n-1)} \sim \frac{A^{(n)}}{(1-x)_+} + B^{(n)} \ln(1-x) + ...$$

Motivations and focus



Deep inelastic scattering (DIS) at threshold contains a hierarchy of scales: $Q^2 \gg P_X^2 \sim Q^2(1-x)$

$$x = \frac{Q^2}{2p\cdot q} \to 1 \quad (\text{large } N \text{ in Mellin space})$$



DIS is well understood at leading power. The coefficient function known to $\mathrm{N^3LL}$

$$C(Q^2) \sim \exp\left[g_1 \ln(N) + ...\right] + \mathcal{O}(N^{-1}\ln^n(N))$$

via traditional resummation techniques

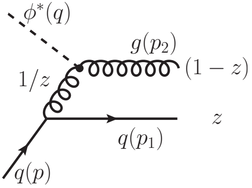
[S. Moch, J.A.M. Vermaseren, A. Vogt, hep-ph/0506288]

and equivalent results obtained in SCET using RG equations directly in momentum space

[T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]

$$P_{i\neq j}^{(n)}(x) = \sum_{\ell=0}^{2n-1} D_{ij}^{(n,\ell)} \ln^{2n-\ell}(1-x)$$

Off-diagonal Deep Inelastic Scattering



Off-diagonal DIS at threshold: $x = Q^2/2p\cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

gives access to

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty} \frac{B_n}{(n!)^2} x^n$$

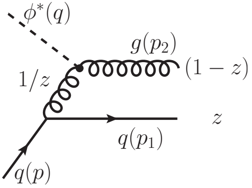
with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...
Conjectured in

[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352]
[A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

The resummed coefficient function is

$$C_{\phi,q}^{\mathrm{LL}}(N,\alpha_s) = \frac{1}{2N\ln N}\,\frac{C_F}{C_F - C_A}\Big\{\exp\left[2C_A\alpha_s\ln^2 N\right]\mathcal{B}_0(a) - \exp\left[2C_F\alpha_s\ln^2 N\right]\Big\}$$

Off-diagonal Deep Inelastic Scattering



Off-diagonal DIS at threshold: $x = Q^2/2p\cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

gives access to

$$P_{gq}^{\rm LL}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...
Conjectured in

[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352]
[A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

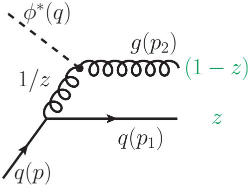
Partonic structure function:

$$W_{\phi,i=q} = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\,\langle i(p)|\big[G^A_{\mu\nu}G^{\mu\nu A}\big](x)\big[G^B_{\rho\sigma}G^{\rho\sigma B}\big](0)|i(p)\rangle$$

At lowest order

$$q(p) + \phi^*(q) \to q(p_1) + g(p_2)$$

Off-diagonal Deep Inelastic Scattering



Off-diagonal DIS at threshold: $x = Q^2/2p \cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

gives access to

$$P_{gq}^{\rm LL}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...
Conjectured in

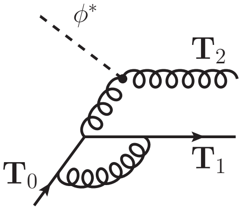
[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352]
[A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

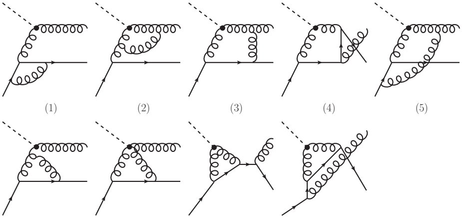
Parametrise with momentum fraction z:

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz\,\left(\frac{\mu^2}{s_{qg}z\bar{z}}\right)^{\epsilon}\mathcal{P}_{qg}(s_{qg},z) \qquad z \equiv \frac{n_-p_1}{n_-p_1 + n_-p_2}$$

$$\mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}\,Q^2}{16\pi^2\Gamma(1-\epsilon)}\,\frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2} \qquad \mathcal{P}_{qg}(s_{qg},z)\big|_{\rm tree} = \frac{\alpha_s C_F}{2\pi}\,\frac{\bar{z}^2}{z}$$

Momentum distribution function



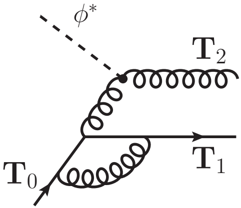


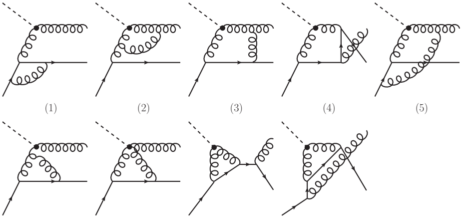
$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left\{ \mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right\}$$

Colour operator notation [S. Catani, hep-ph/9802439]

$$\mathbf{T}_1\cdot\mathbf{T}_0 = C_A/2 - C_F, \quad \mathbf{T}_2\cdot\mathbf{T}_0 = \mathbf{T}_1\cdot\mathbf{T}_2 = -C_A/2$$

Momentum distribution function





[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left\{ \mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right\}$$

Colour operator notation [S. Catani, hep-ph/9802439]

$$\mathbf{T}_1\cdot\mathbf{T}_0 = C_A/2 - C_F, \quad \mathbf{T}_2\cdot\mathbf{T}_0 = \mathbf{T}_1\cdot\mathbf{T}_2 = -C_A/2$$

Momentum distribution function

Divergences in the final integral!

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3}$$

It is important to keep full ϵ information.

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right)$$
$$= \quad -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots$$

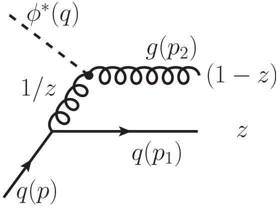
$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} \quad = \quad \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left\{\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{z s_{qg}}\right)^{\epsilon}\right]\right\}$$

Colour operator notation [S. Catani, hep-ph/9802439]

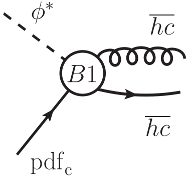
$$\mathbf{T}_1\cdot\mathbf{T}_0 = C_A/2 - C_F, \quad \mathbf{T}_2\cdot\mathbf{T}_0 = \mathbf{T}_1\cdot\mathbf{T}_2 = -C_A/2$$

The EFT perspective - beyond LP

Is the SCET_I set-up sufficient beyond LP?

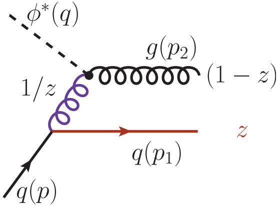


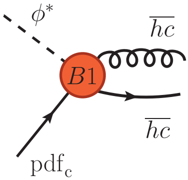
Operator matching $\rightarrow$



The EFT perspective - beyond LP

Is the SCET_{I} set-up sufficient beyond LP?





If the quark is soft $z \to 0$, the matching coefficient contains a $1/z$ divergence.

$$C^{B1} = \frac{\kappa}{2}\frac{\bar{z}}{z} + \mathcal{O}(\alpha_s)$$

Refactorization

Endpoint divergence points to a new scale in the problem.
$\rightarrow$ Refactorization required

New power counting parameter z: $\quad 1 \gg z \gg \lambda$

Name	$(n_+ l\,,\, l_\perp\,,\, n_- l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

Refactorization

Endpoint divergence points to a new scale in the problem.
→ Refactorization required

New power counting parameter z: $1 \gg z \gg \lambda$

Name	$(n_+ l\,,\, l_\perp\,,\, n_- l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

Expansion-by-regions method → Large $\ln(z)$ from hard and z-hardcollinear regions.

$$\int d^dx\, T\left\{J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z-\overline{sc}}}(x)\right\} = D^{B1}(zQ^2, \mu^2)\, J^{B1}$$

Similar refactorization observed in

[Z. L. Liu, B. Mecaj, M.Neubert, X.Wang, 2009.06779]

Refactorization

→ Then solve RGEs in d-dimensions

First step matching

$$\left[C^{A0}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s\,C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right]$$

Second step matching

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]$$

Final step in the Soft Sudakov derivation: combination of these terms gives the exponentiated $\mathcal{P}_{qg}$ conjectured by [I. Moult, I.W. Stewart, G. Vita, H.X. Zhu, 1910.14038]

$$\mathcal{P}_{qg}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}+\left(C_A-C_F\right)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right]$$

[M. Beneke, M. Garny, **SJ**, R. Szafron, L. Vernazza, J. Wang, 2008.04943]

Consistency relations

- Working in d-dimensions, we can still obtain all-order results.
- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

The general expansion for the cross section is

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon}$$

The scaling of the regions: hard (Q^2), anti-hardcollinear (Q^2/N), collinear (Λ^2), softcollinear (Λ^2/N). Where $f_q(\Lambda)$ is a partonic PDF.

Consistency relations

- Working in d-dimensions, we can still obtain all-order results.
- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Invoking pole cancellation

$$\sum_{k=0}^{n}\sum_{j=0}^{n} j^r k^s c_{kj}^{(n)} = 0 \qquad \text{for } s+r < 2n-1,\ r,s \geq 0$$

allows $(n+1)^2$ coefficients $c_{kj}^{(n)}$ to be determined from $2n^2 - n$ equations up to three unknowns.
Use boundary conditions:

$$c_{n0}^{(n)} = 0, \qquad c_{00}^{(n)} = 0 \qquad \text{for all } n\,.$$

The third initial condition, $c_{n1}^{(n)}$, is the discussed exponentiated $\mathcal{P}_{qg}$.

Resummed result

A closed form solution from all-order algebraic relations for $\tilde{C}_{\phi,q}^{\mathrm{NLP,LL}}$ in agreement with [A. Vogt, 1005.1606]. We arrive at identical splitting kernels:

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...

Summary

- Divergence in the convolution integral takes the considerations outside the standard SCET paradigm.
- In SCET_{I} problems new modes appear due to endpoint divergences.
- We require a consistent refactorization of the operator to truly separate the scales.
- In SCET_{I}, endpoint divergences are regularized by ϵ and resummation can be performed in d-dimensions.

Thank you