An overview of the QCD phase diagram at finite T and μ

Jana N. Guenther
Wuppertal-Budapest collaboration

July 26th 2021

Aix*Marseille université

Many thanks to everyone who sent me material beforehand

- Felipe Attanasio
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- Maxim Chernodub
- Francesca Cuteri
- Tom DeGrand
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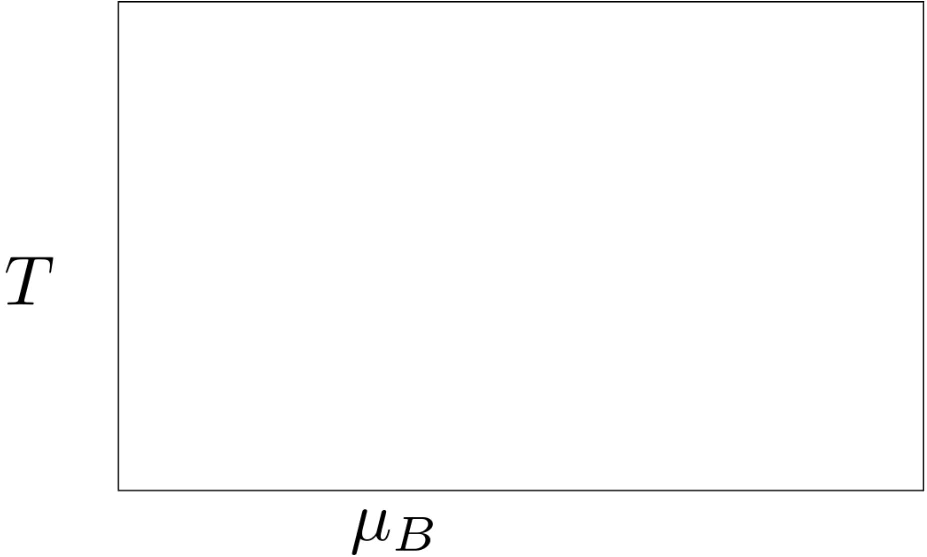
- Rajiv V. Gavai
- Jishnu Goswami
- Prasad Hegde
- Benjamin Jäger
- Kazuyuki Kanaya
- Frithjof Karsch
- Andrey Kotov
- Aleksi Kurkela
- Sourav Mondal
- Swagato Mukherjee
- Yoshifumi Nakamura
- Jun Nishimura
- Mike Ogilvie
- Marco Panero
- Owe Philipsen
- Christian Schmidt
- Alessandro Sciarra
- Jon-Ivar Skullerud
- Marianna Sorba
- Judah Unmuth-Yockey
- Aleksi Vuorinen
- Felix Ziegler

Many apologies to all the fascinating results (and their authors) that I cannot cover due to time constrains!

The (T, μ_B)-phase diagram of QCD



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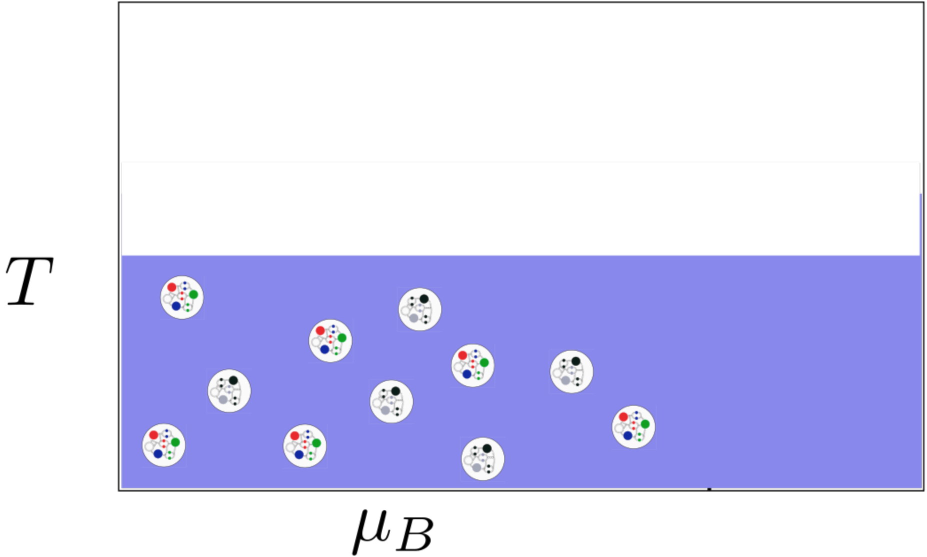

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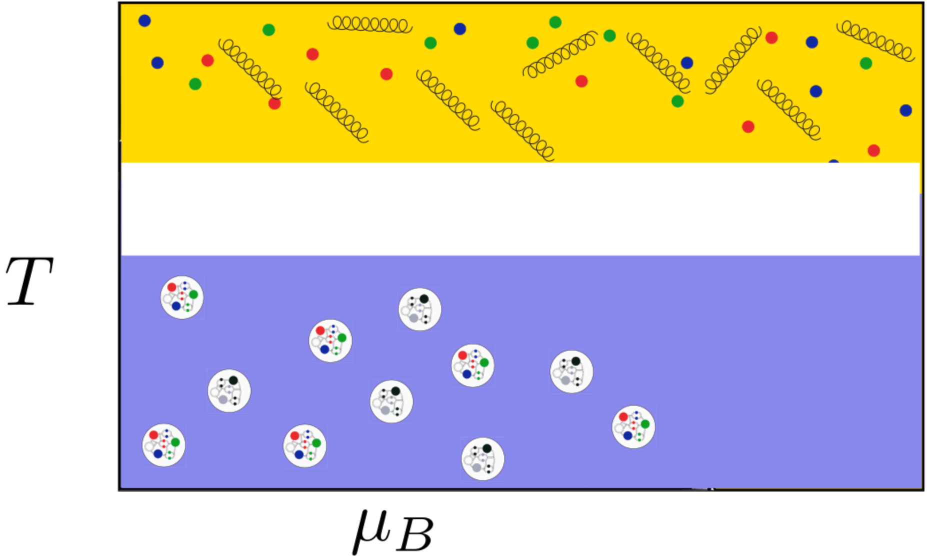

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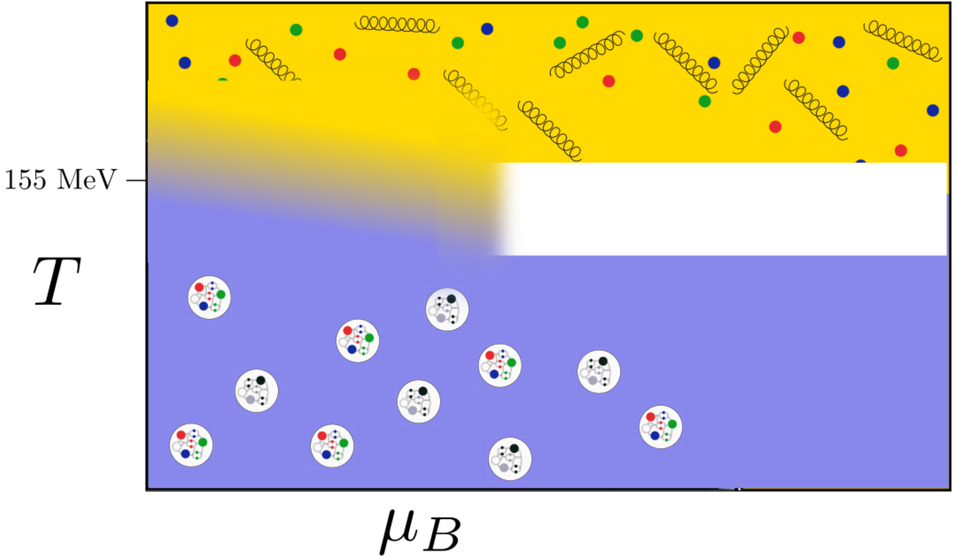

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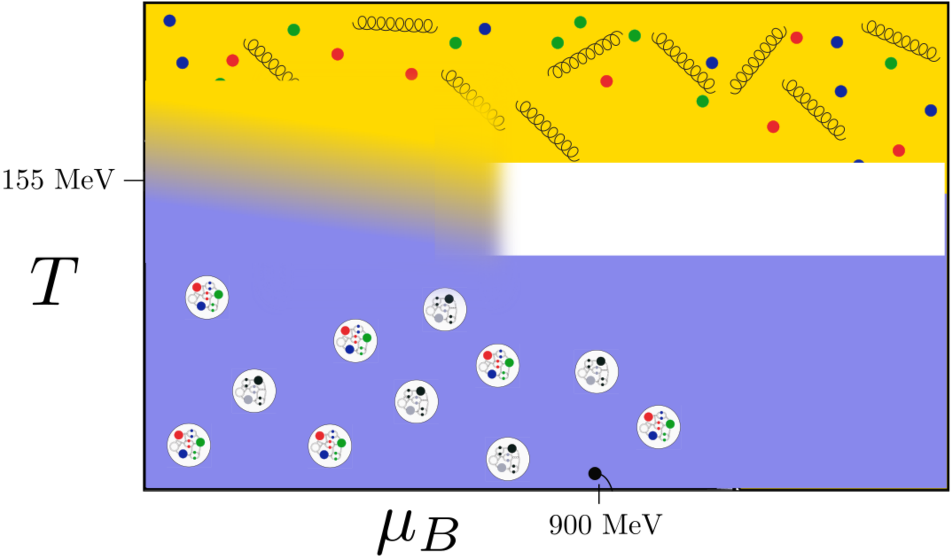

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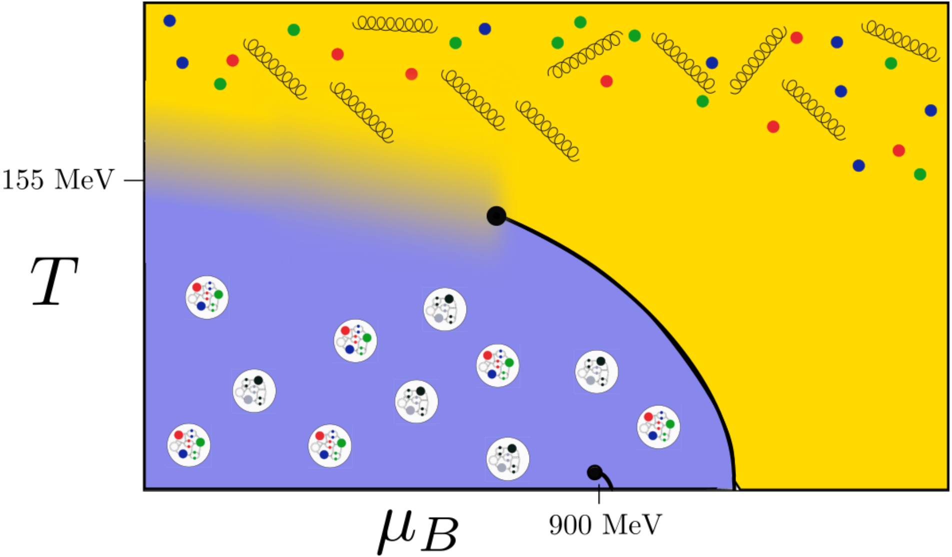

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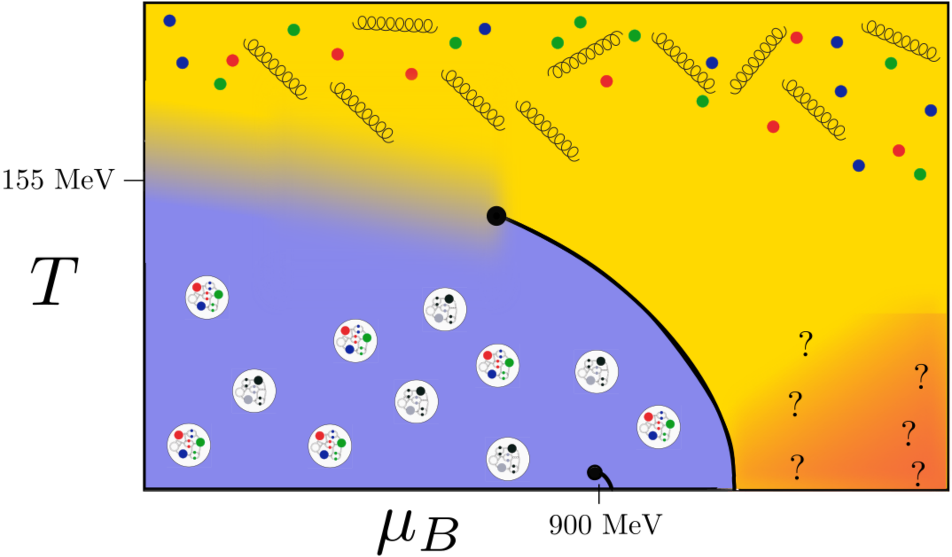

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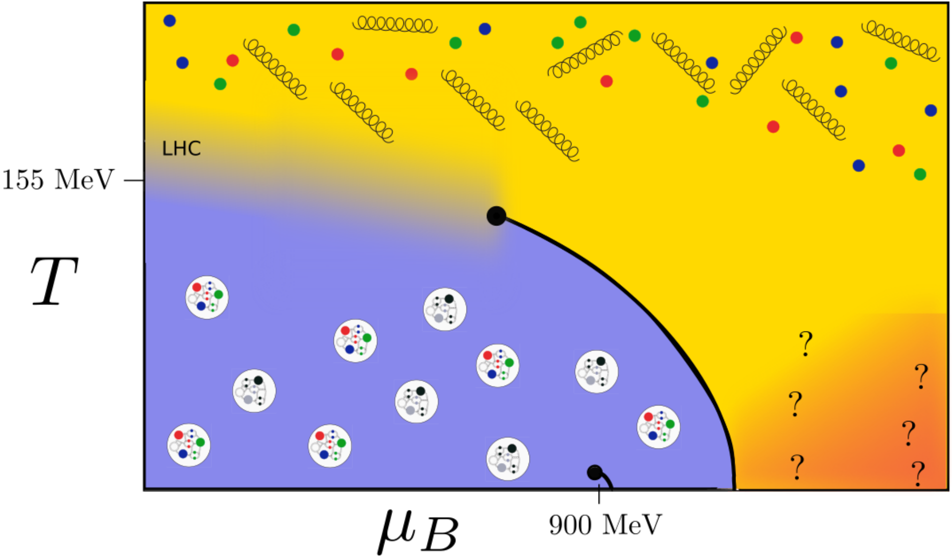

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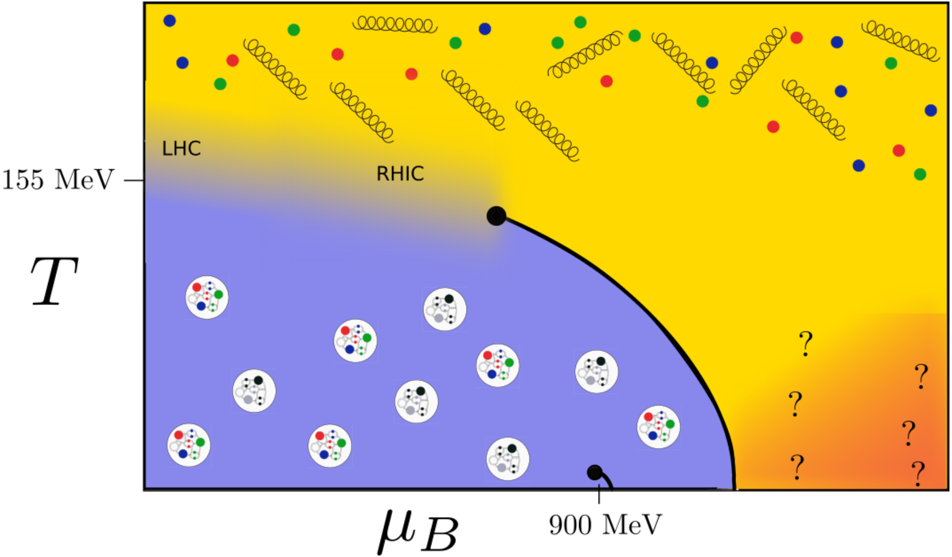

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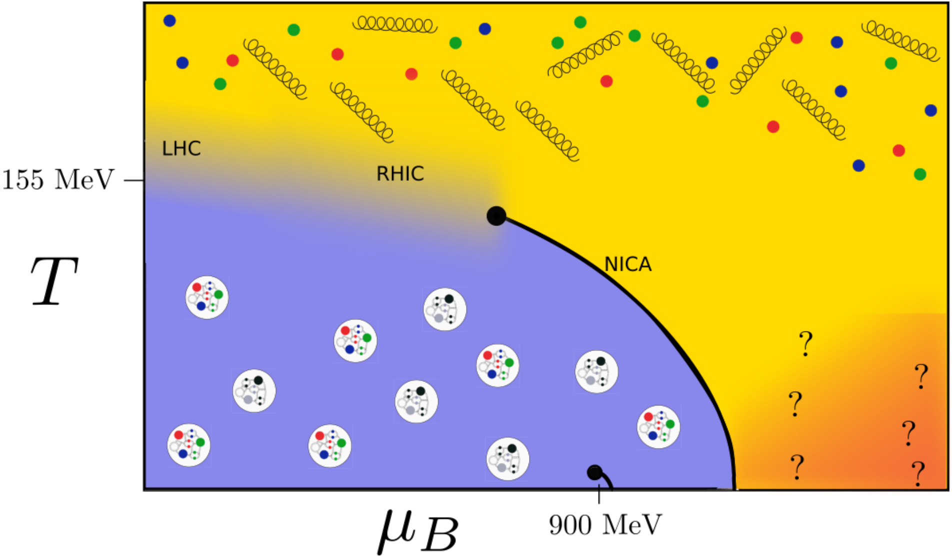

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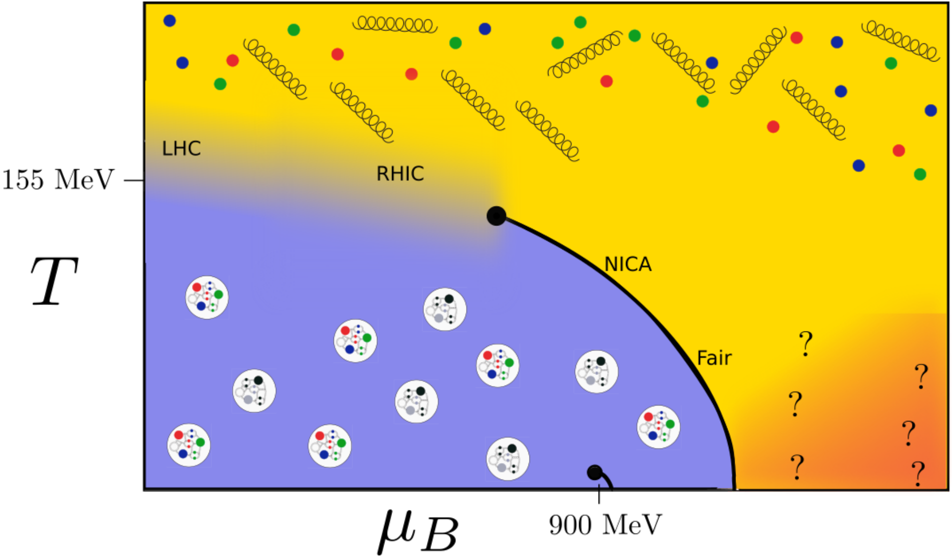

Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Complex Langevin
- ...

- Attila Pasztor, Tue 5:30.
- Kornél Kapás, Tue 6:00
- Prasad Hegde, Tue 7:15
- Francesco di Renzo, Mon 14:15
- Kevin Zambello, Tue 6:15
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- Volodymyr Chelnokov, Mon 13:15
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- Yusuke Namekawa, Wen 21:15
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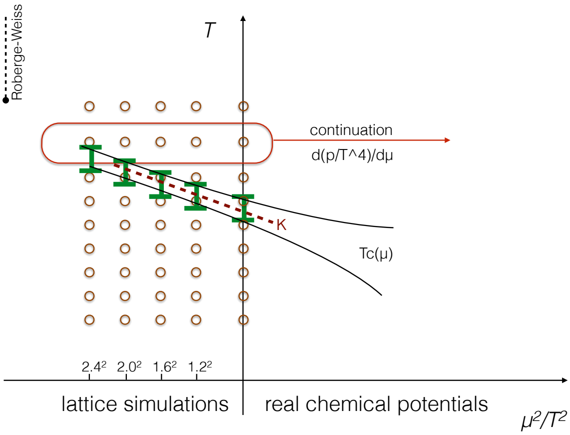
- (Taylor) expansion
- Imaginary μ

- Paolo Parotto, 7:00
- David Pesznyak, 7:30

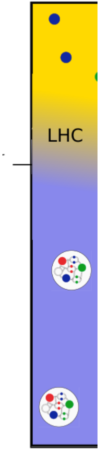
Analytic continuation from imaginary chemical potential



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- [Borsanyi:2018grb]
- [Borsanyi:2020fev]
- [Bellwied:2021nrt]
- ...

Expansion from $\mu = 0$



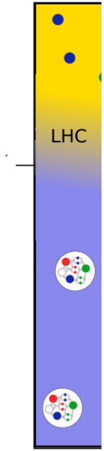
Taylor expansion

$$\frac{p}{T^4} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!k!} \chi_{jk}^{BS} \hat{\mu}_B^j \hat{\mu}_S^k$$

with $\hat{\mu} = \frac{\mu}{T}$

- rapid convergence in Stephan-Boltzmann ($T = \infty$) limit
- expansion coefficients are lattice observables

Expansion from $\mu = 0$



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with $\hat{\mu} = \frac{\mu}{T}$

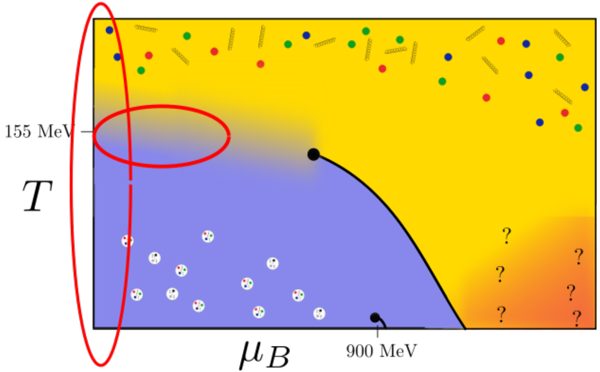
- rapid convergence in Stephan-Boltzmann ($T = \infty$) limit
- expansion coefficients are lattice observables

Fugacity expansion/sector method

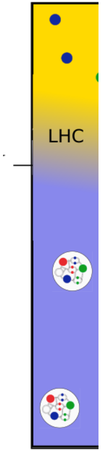
$$\frac{p}{T^4} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} P_{jk}^{BS} \cosh(j\hat{\mu}_B - k\hat{\mu}_S)$$

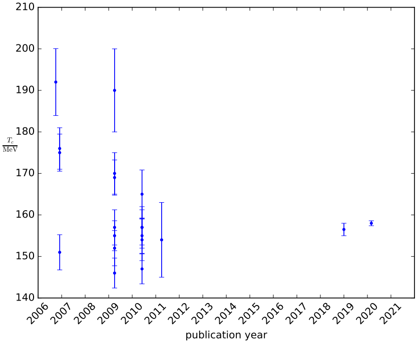
with $\hat{\mu} = \frac{\mu}{T}$

- rapid convergence in hadronic phase
- information about particle content

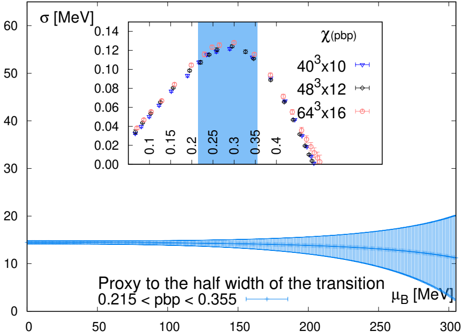

155 MeV
T
μ_B
900 MeV
?
?
?
?
?
?

The transition temperature





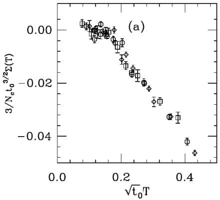
[Cheng:2006qk], [Aoki:2006br], [Aoki:2009sc], [Bazavov:2009zn], [Borsanyi:2010bp], [Bazavov:2011nk], [Bazavov:2018mes], [Borsanyi:2020fev]



[Borsanyi:2020fev]

The transition temperature under different conditions

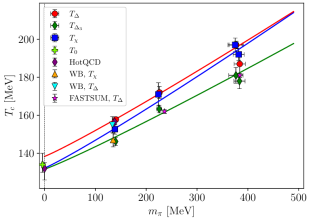
Many color influences



- [DeGrand:2021zjw]
- rescaled chiral condensate
- $N_c = 3, 4, 5$, $N_f = 2$, Wilson-Clover
- $(m_{PS}/m_V)^2 \sim 0.63$

- The rescaled condensate temperature dependence is independent of N_c
- Thomas DeGrand, Tue 21:00

Various quark masses



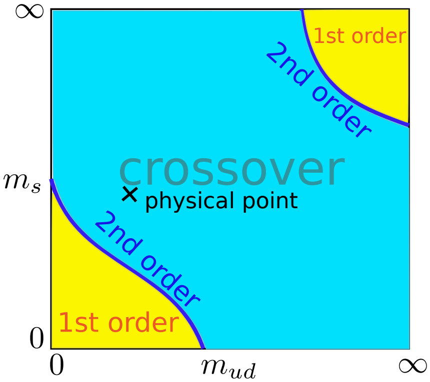
- [Kotov:2021rah]
- $a = 0.06 - 0.08$ fm
- $N_f = 2 + 1 + 1$, Wilson-Clover twisted mass

[HotQCD:2018pds], [Ding:2019fzc], [Borsanyi:2020fev], [Aarts:2019hrg], [Aarts:2020vyb]

- Andrey Kotov, Tue 13:15

The chiral limit will be covered in plenary talk by Anirban Lahiri later in this session.
On Dirac eigenvalues around T_c: Wei-Ping Huang, Wed 6:00

Columbia plot



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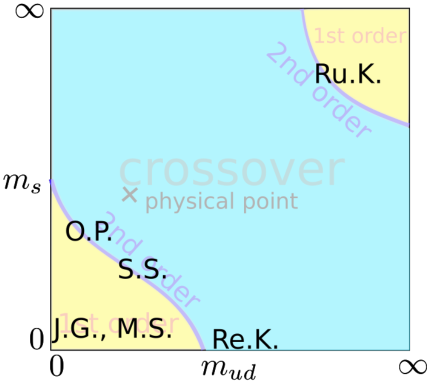

- Reinhold Kaiser, Tue 13:30
- Jishnu Goswami, Tue 13:45
- Mugdha Sarkar, Thu 6:00
- Ruben Kara, Thu 6:30.
- Sipaz Sharma, Thu 7:00
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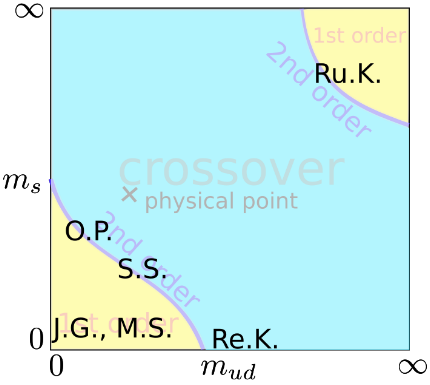
Columbia plot



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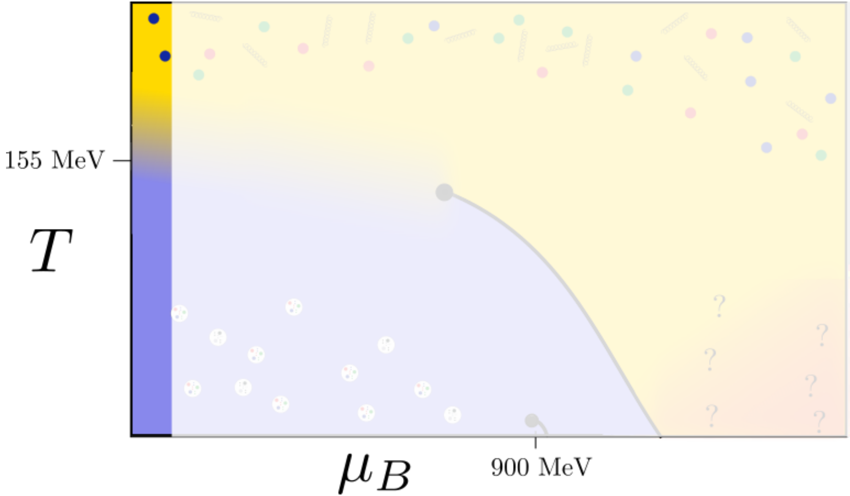
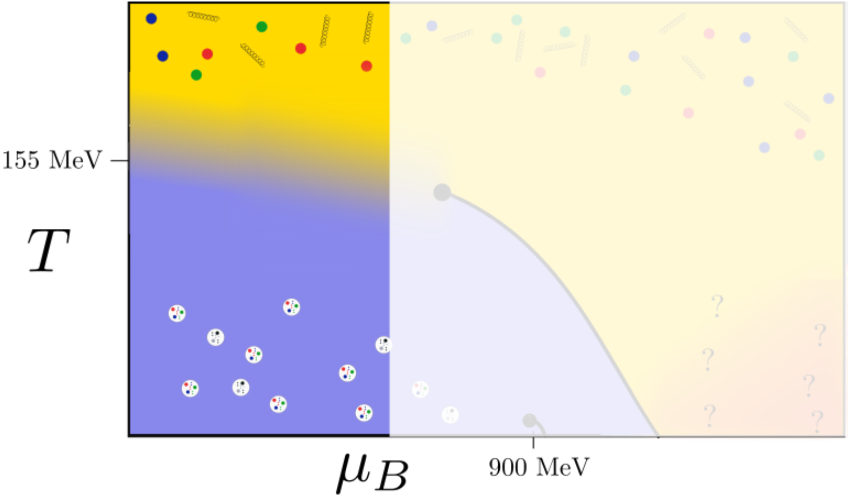
Discussed in the plenary talk by Anirban Lahiri later in this session.

Also in this session: Plenary talk by Johannes Heinrich Weber on Heavy quarks at finite temperature.

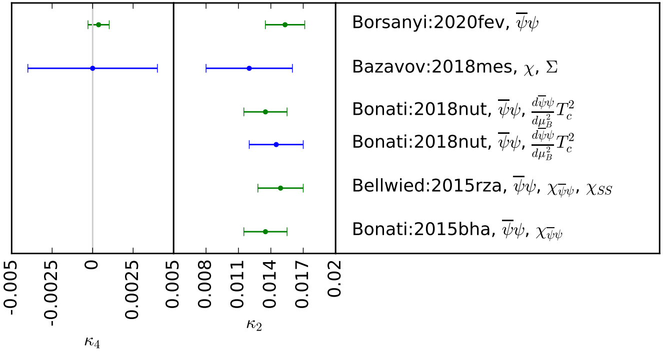
Lifting the curtain - Looking at finite μ

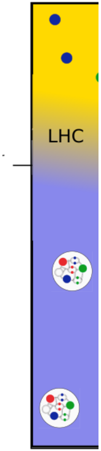


Lifting the curtain - Looking at finite μ



Curvature of the transition temperature



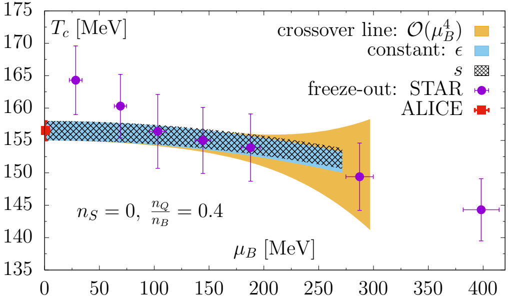


$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

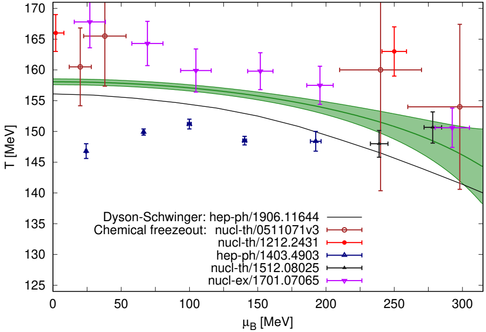
green: from imaginary μ_B, blue: from Taylor expansion method

Extrapolation of the transition temperature

[Bazavov:2018mes]
Results from the Taylor expansion method
HISQ quarks
Continuum limit from $N_t = 6, 8, 12$



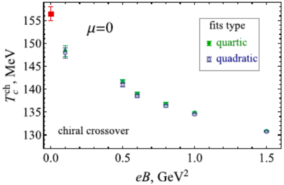
[Borsanyi:2020fev]
Results from the imaginary potential method
staggered quarks
Continuum limit from $N_t = 10, 12, 16$



chemical freezeout: abundancies of hadrons are fixed (frozen-in)
kinetic freezeout: momentum distributions are fixed

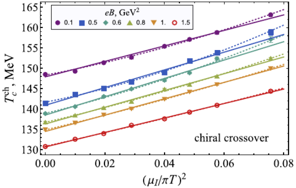
The influence of a magnetic field

- [Braguta:2019yci]



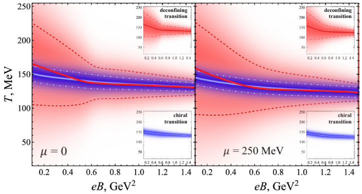
[Bali:2011qj], [Bali:2012zg], [Ilgenfritz:2013ara], [Bornyakov:2013eya], [Bali:2014kia], [Endrodi:2019zrl], [Tomiya:2019nym]...
Kenneth G. Wilson award 2014: Gergely Endrödi

- $N_t = 6$



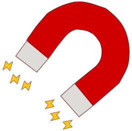
μ_I: imaginary μ_B

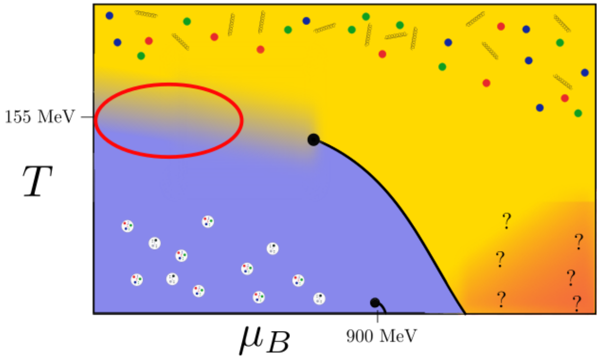
- $N_f = 2 + 1$ staggered



Talks on influences of the magnetic field:

- Artem Roenko, Wed 13:00
- Natalia Kolomoyets, Wed 13:15
- Lorenzo Maio, Wed 13:30
- Adeilton Dean Marques Valois, Wed 13:45
- Xiaodan Wang, Wed 22:00
- Shengtai Li, Wed 22:15



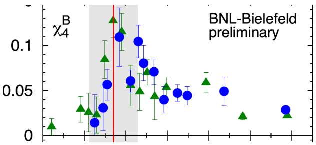


155 MeV
T
μ_B
900 MeV
?

χ_4^B, χ_6^B and χ_8^B on finite lattices

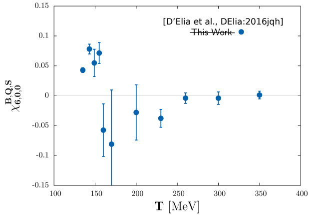
$$\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,,\quad \hat{\mu}_i = \frac{\mu}{T}$$

2012 up to χ_4^B
$N_t = 6, 8$



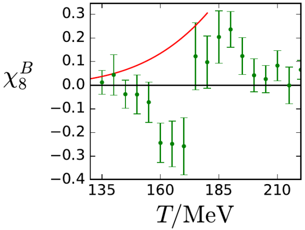
[Schmidt:2012ka]

2016 up to χ_6^B
$N_t = 8$



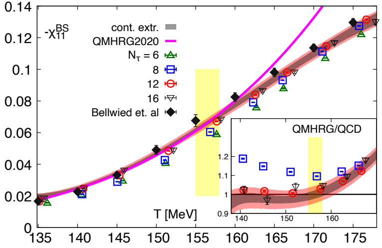
[DElia:2016jqh], see also [Bazavov:2017dus]

2018 up to χ_8^B
$N_t = 12$



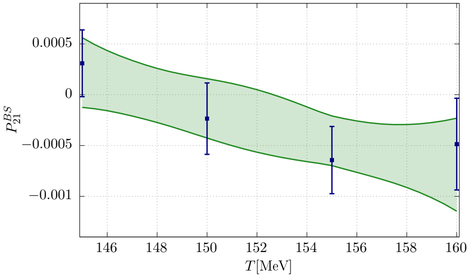
[Borsanyi:2018grb], see also [Bazavov:2020bjn]

Low order fluctuations with high precision



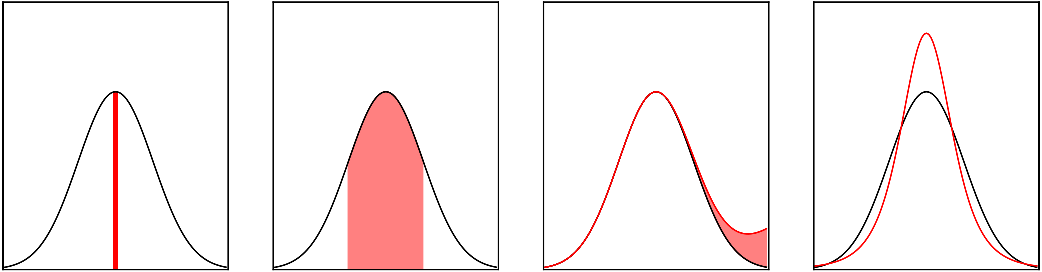
- [Bollweg:2021vqf]
- HISQ
- New continuum extrapolated results ($N_t = 6, 8, 12, 16$) allow for detailed comparisons with various models
- Quark model states are needed for HRG

- [Bellwied:2021nrt]
- continuum estimate from $N_t = 8, 10, 12$
- stout smeared staggered
- contributions vom $N - \Lambda$, $N - \Sigma$ scattering
- negative contribution in the Fugacity expansion indicate repulsive interaction that cannot be described with more resonances



Observables

Cumulants of the net baryon number distributions:



mean
$M_B = \chi_1^B$

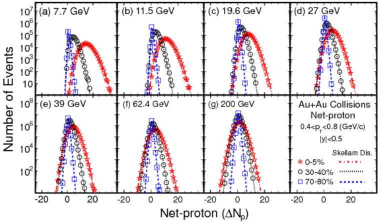
variance
$\sigma_B^2 = \chi_2^B$

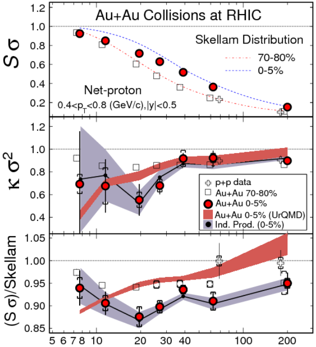
skewness
$S_B = \frac{\chi_3^B}{(\chi_2^B)^{3/2}}$
asymmetry of the distribution

kurtosis
$\kappa_B = \frac{\chi_4^B}{(\chi_2^B)^2}$
"tailedness" of the distribution

Calculating observables

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,,\quad \hat{\mu}_i = \frac{\mu}{T}$$





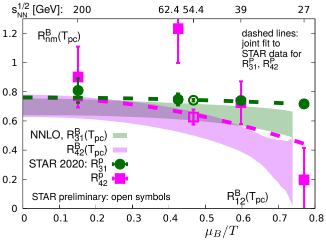
$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} \qquad \frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)}$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)}$$

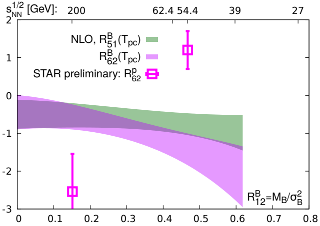
[Bazavov:2017dus], [Karsch:2017zzw],
figs: [STAR, Adamczyk:2013dal]

Comparison with heavy ion collision experiments

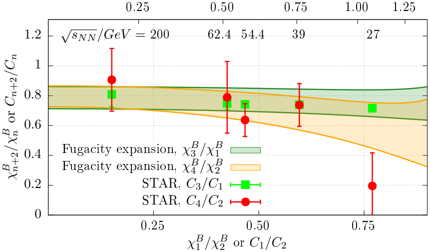
Continuum estimate from $N_t = 8, 12$



$N_t = 8$



- [Bazavov:2020bjn]
- Taylor method
- HISQ

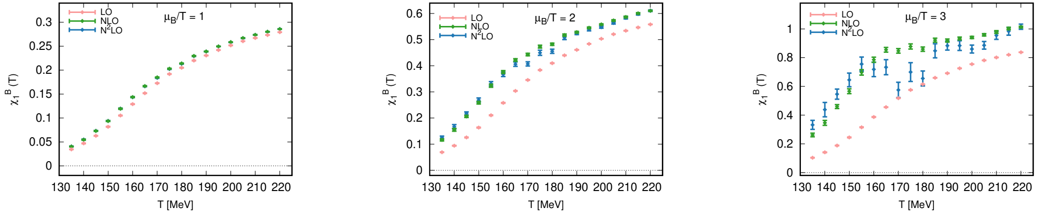


- [Bellwied:2021nrt]
- continuum estimate from $N_t = 8, 10, 12$
- stout smeared staggered

- 2d-extrapolation in μ_B and μ_S
- Fugacity expansion and imaginary chemical potential
- David Pesznyak, Tue 7:30

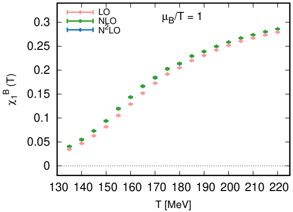
Extrapolations are done along the transition line.

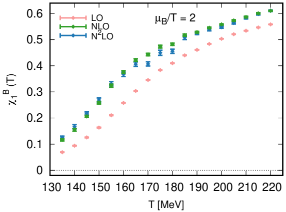
Trouble with the equation of state

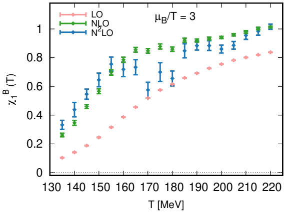


[Borsanyi:2021sxv], [Borsanyi:2018grb], $N_t = 12$

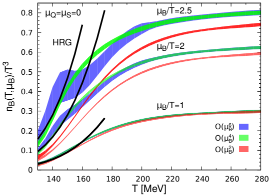
Trouble with the equation of state





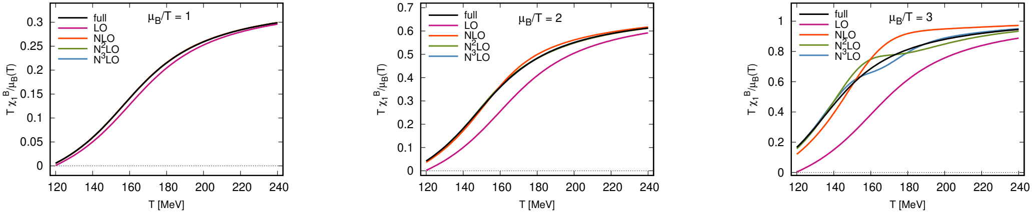


[Borsanyi:2021sxv], [Borsanyi:2018grb], $N_t = 12$

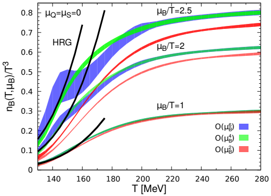


[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

Trouble with the equation of state



[Borsanyi:2021sxv]



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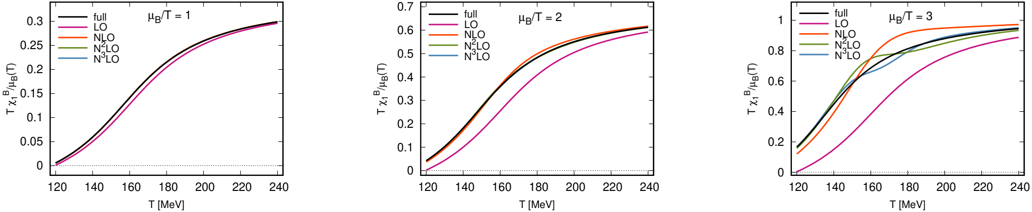
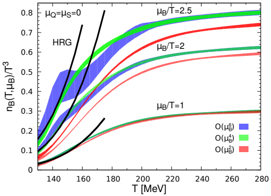


[Borsanyi:2021sxv]

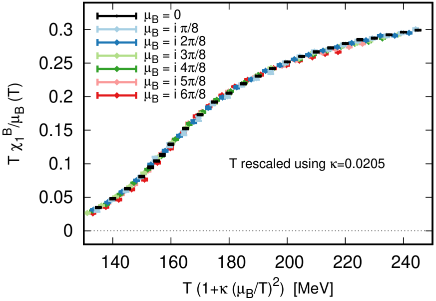


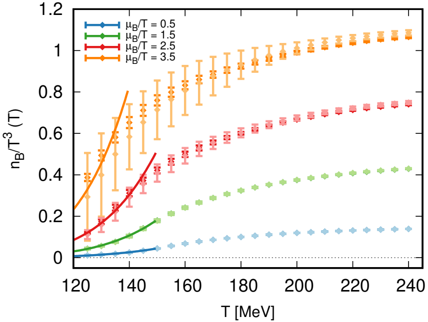
[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

- extrapolation at fixed T cross the transition line
- bad convergence with low order Taylor coefficients

Equation of state

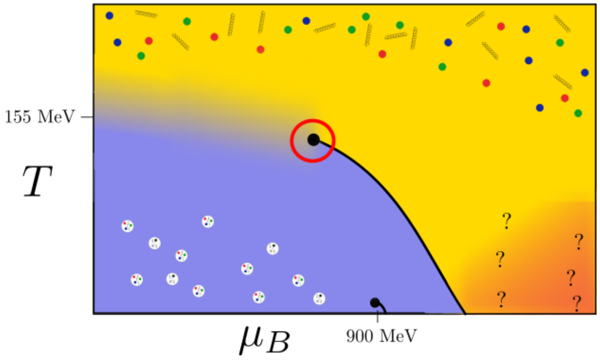
Find a different extrapolation scheme for extrapolating to higher μ_B.





- [Borsanyi:2021sxv]
- $N_t = 10, 12, 16$
- Paolo Parotto, Tue 7:00

On a resummation of the Taylor expansion: Prasad Hegde, Tue 7:15



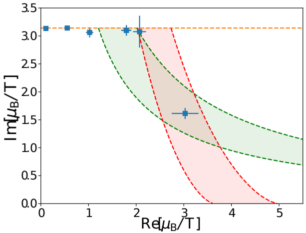
155 MeV
T
μ_B
900 MeV
?

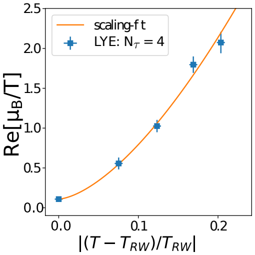
Complex singularities

For the QCD phase diagram:

Roberge-Weiss (Z(2)), Chiral ((O(4)/O(2)), Critical endpoint (Z(2))

Direct detection with Lee-Yang edge singularities by Bielefeld-Parma Collaboration:



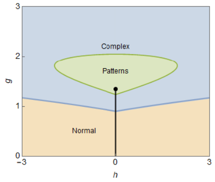


- $N_t = 4, 6$
- Kevin Zambello, Tue 6:15
- Simran Singh, Tue 6:30
- Guido Nicotra, Tue 6:45

Lee-Yang edge and Taylor: Gokce Basar, Mon 21:00
Critical endpoint with Wilson-Clover fermions: Hiroshi Ohno, Mon 22:00
Reweighting: Attila Pasztor, Tue 5:30, Sandor Katz, Tue 5:45

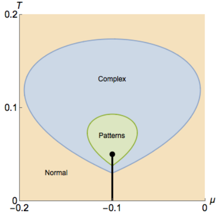
The critical endpoint in models

On the scaling region of the Ising universaltiy class: Marianna Sorba, Thu 5:30



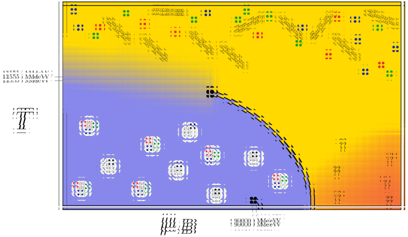
- [Schindler:2021otf]
- $\mathcal{PT}$ symmetric model with $Z(2)$ symmetry
- in QCD-inspired heavy quark model patterns around critical endpoint

- Similar patterns as in nuclear pasta but with confined and deconfined quarks instead of protons and neutrons
- Michael Oglivie, Tue 21:30
- Stella Schindler, Tue 21:45



Outlook on higher μ_B with Complex Langevin

Evolution in a fictitious Langevin time generates configurations with a complex measure.



Outlook on higher μ_B with Complex Langevin

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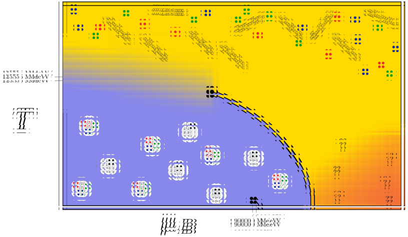
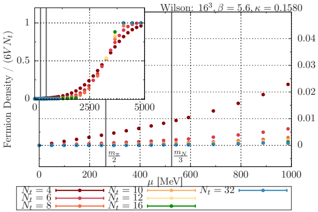
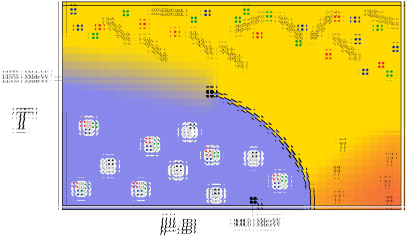


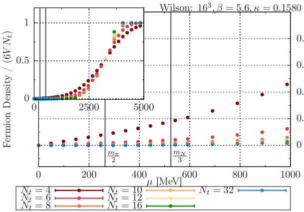


- $N_f = 2$, $m_\pi = 550$ MeV, Wilson fermions
- Benjamin Jaeger, Tue 5:15

Outlook on higher μ_B with Complex Langevin

Evolution in a fictitious Langevin time generates configurations with a complex measure.





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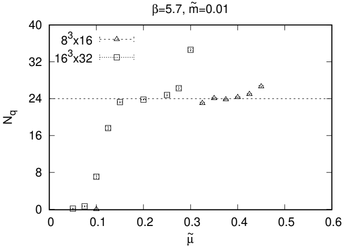
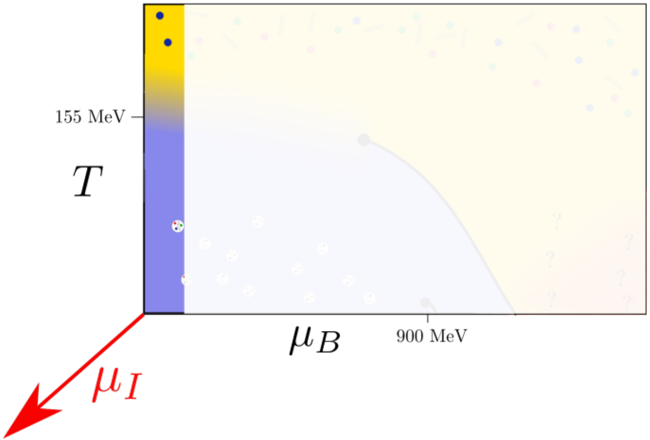
- Benjamin Jaeger, Tue 5:15



- [Ito:2020mys]
- $N_f = 4$, $\tilde{\mu} = \mu a$, $a^{-1} \approx 4.7$ GeV
- plateau might be connected to a Fermi surface and color superconductivity
- Shoichiro Tsutsui, Wed 21:45

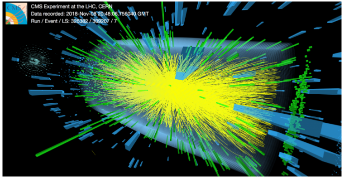


155 MeV
T
μ_B
900 MeV
μ_I

Isospin chemical potential μ_I

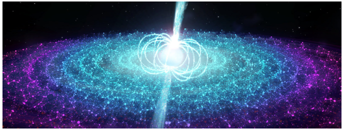
Popular systems with more neutrons than protons:

Heavy ion collisions



[https://cms.cern]

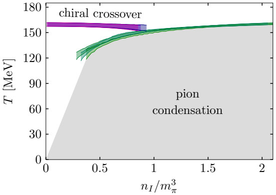
Neutron stars:



[www.ectstar.eu]

For $\mu_I \neq 0$ and $\mu_B = 0$ there is no sign problem

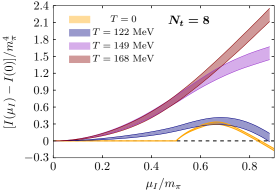
$\mu_B = \mu_S = 0$:



$N_t = 8, 10, 12$

Equation of state on $N_t = 8$

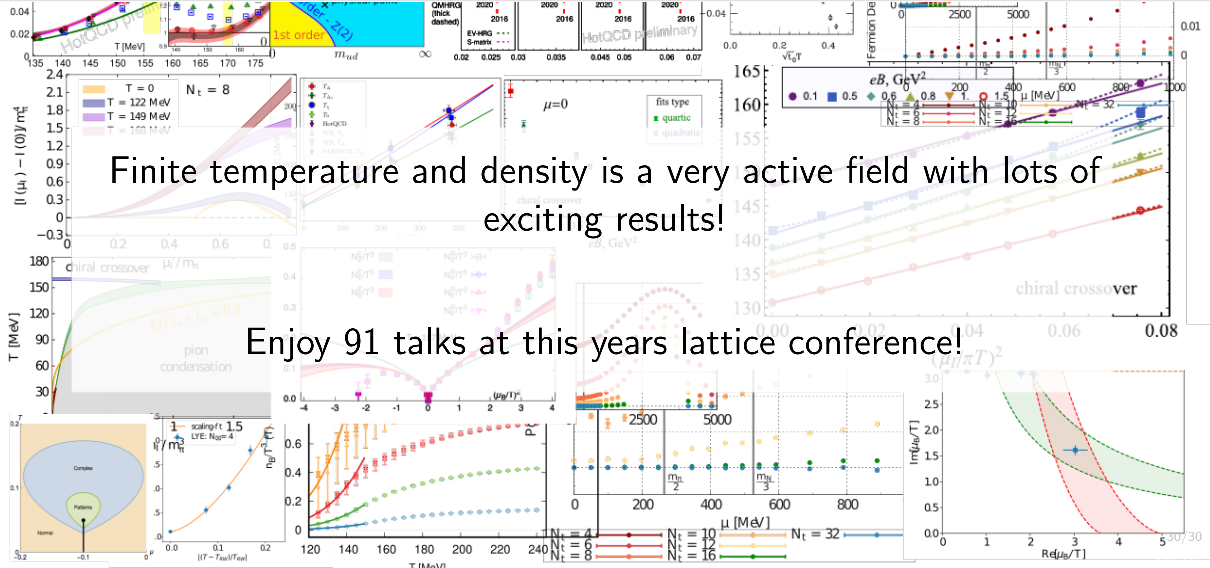
Bastian Brandt, Tue 14:15



Finding phase boundary at large μ_I: Francesca Cuteri, Tue 14:30

Imaginary μ_I (and μ_B): Christopher Winterowed, Tue 14:00, Amine Chabane, Thu 6:15

Conclusion



Finite temperature and density is a very active field with lots of exciting results!

Enjoy 91 talks at this years lattice conference!