

Direct CP violation and the $\Delta I=1/2$ rule in $K \rightarrow \pi\pi$ decay from the Standard Model

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Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \rightarrow \pi\pi$:

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}, \quad \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)}.$$

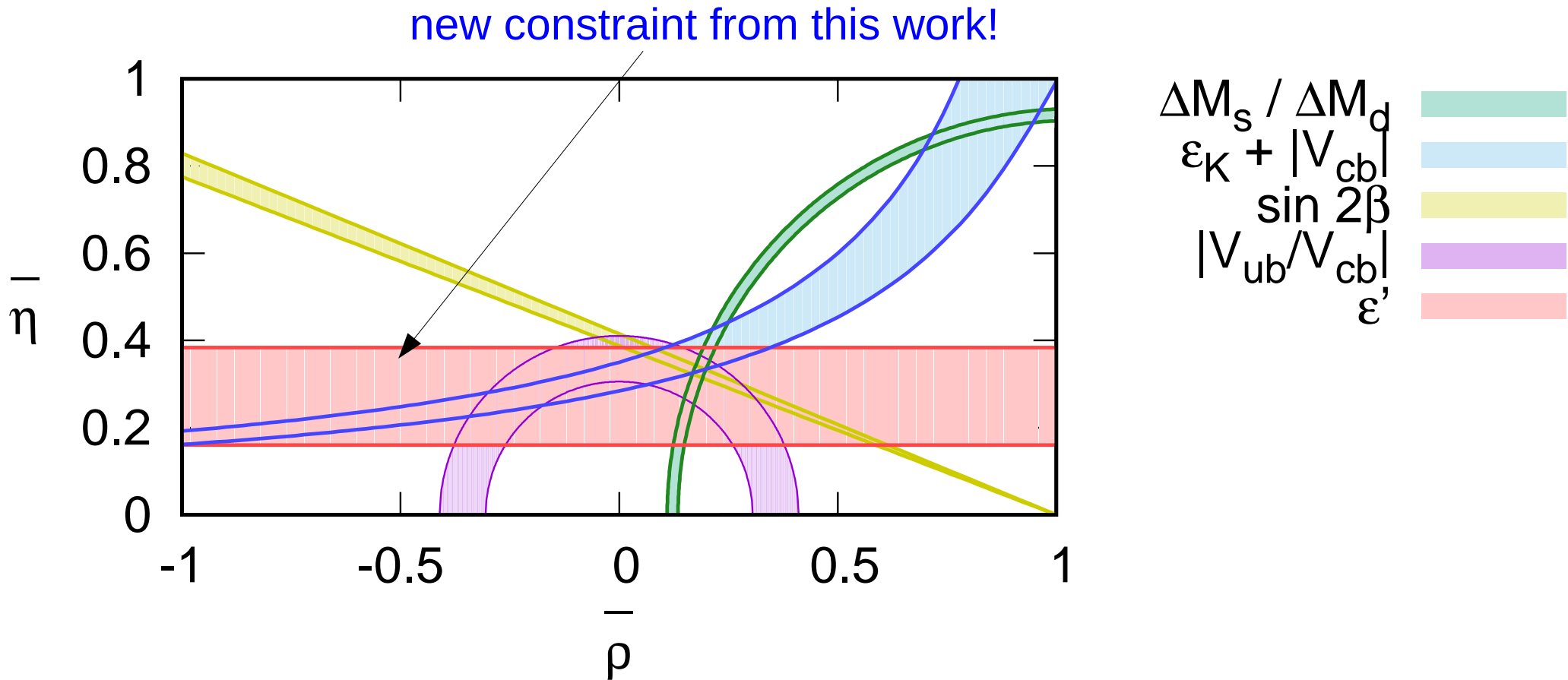
$$\text{Re}(\epsilon'/\epsilon) \approx \frac{1}{6} \left(1 - \left| \frac{\eta_{00}}{\eta_{\pm}} \right|^2 \right) = 16.6(2.3) \times 10^{-4} \quad (\text{experiment})$$

measure of direct CPV

measure of indirect CPV

- **Small size of ϵ' makes it particularly sensitive to new direct-CPV introduced by many BSM models.**
- Looking for deviations from experiment may help shed light on origin of M/AM asymmetry.

- A Standard Model prediction of ε' also provides a new horizontal band constraint on CKM matrix in ρ - η plane:



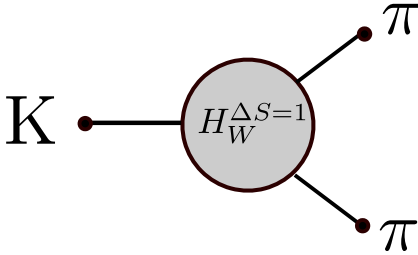
- While underlying weak process occurs at high energies $\sim M_W = 80$ GeV, $K \rightarrow \pi\pi$ decays receive large corrections from low-energy hadronic physics $O(\Lambda_{\text{QCD}}) \sim 250$ MeV.
- Lattice QCD is the only known *ab initio*, **systematically improvable** technique for studying non-perturbative QCD.

Overview of calculation

$$\begin{aligned}
 A(K^0 \rightarrow \pi^+\pi^-) &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2e^{i\delta_2}, \\
 A(K^0 \rightarrow \pi^0\pi^0) &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2e^{i\delta_2}.
 \end{aligned}
 \rightarrow \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right)$$

$\omega = \text{Re}A_2/\text{Re}A_0$
 $\pi\pi$ phase shifts
I=2 decay I=0 decay

- Hadronic energy scale $\ll M_W$ – use weak effective theory (3 flavors)



LL finite-volume correction

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \sum_{j=1}^7 \left[(z_i(\mu) + \tau y_i(\mu)) Z_{ij}^{\text{lat} \rightarrow \overline{\text{MS}}} M_j^{I, \text{lat}} \right]$$

perturbative Wilson coeffs. $M_j^{I, \text{lat}} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

renormalization matrix (mixing)
 Use RI-SMOM
 convert to MSbar
 perturbatively

10 effective four-quark operators

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + \mathbf{0.00060408i}$$

Imaginary part solely responsible for CPV
 (everything else is pure-real)

$I=2$ calculation

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 can be measured very precisely using “standard” lattice techniques.
- Most recent result (2015):
 - Computed with large, $\sim (5.5 \text{ fm})^3$ volumes
 - Physical quark masses
 - Two lattice spacings (2.36 GeV and 1.73 GeV) \rightarrow **Continuum limit taken.**
- $<1\%$ statistical error!
- 10% and 12% total errors on $\text{Re}(A_2)$ and $\text{Im}(A_2)$ resp.
- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

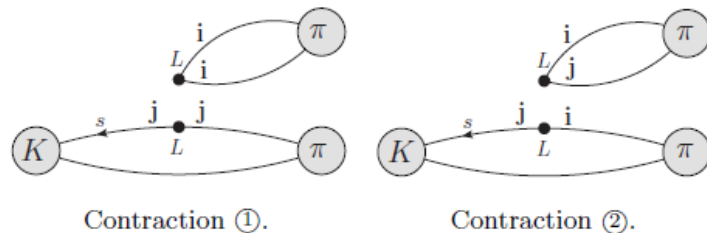
$\Delta I=1/2$ rule

- In experiment kaons $\sim 450x$ (!) more likely to decay into $I=0$ pi-pi states than $I=2$.

$$\frac{\text{Re}A_0}{\text{Re}A_2} = 22.45(6) \quad (\text{the } \Delta I=1/2 \text{ rule})$$

- Perturbative running to charm scale accounts for about a factor of 2. Where does the remaining 10x come from? New Physics?
- The answer is low-energy QCD!** [arXiv:1212.1474, arXiv:1502.00263]

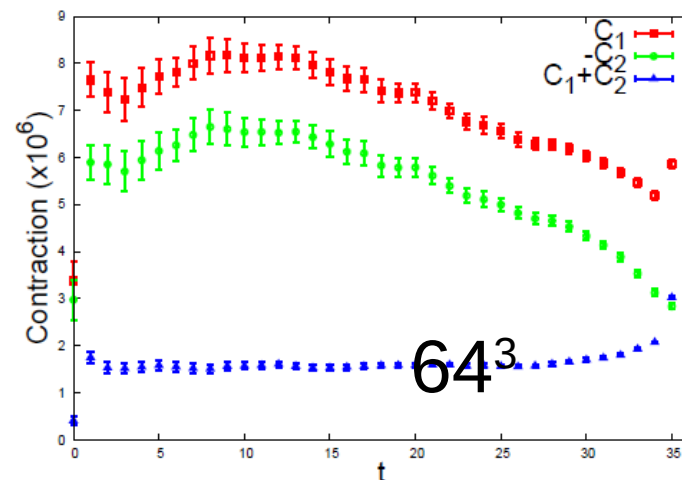
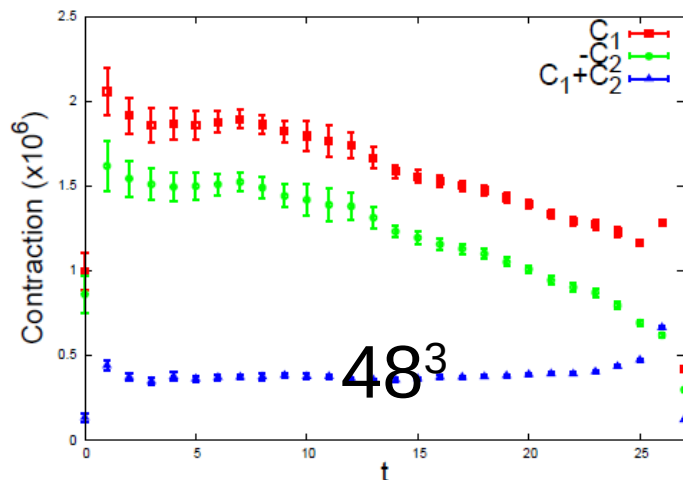
Strong cancellation between the two dominant contractions not predicted by naive factorization:



$$\text{Re}(A_2) \sim \textcircled{1} + \textcircled{2}$$

find $\textcircled{2} \approx -0.7\textcircled{1}$ heavily suppressing $\text{Re}(A_2)$.

[Phys.Rev. D91 (2015) no.7, 074502]



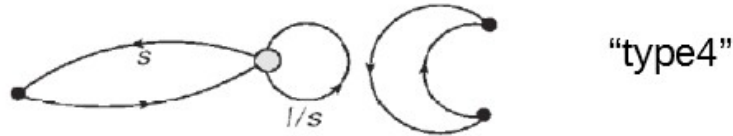
Pure-lattice calculation

$$\frac{\text{Re}(A_0)}{\text{Re}(A_2)} = 19.9(5.0)$$

[$\text{Re}(A_0)$ agrees with expt.]

I=0 Calculation

- A_0 is more difficult than A_2 , primarily because $I=0$ $\pi\pi$ state has *vacuum quantum numbers*.
- “Disconnected diagrams” dominate statistical noise

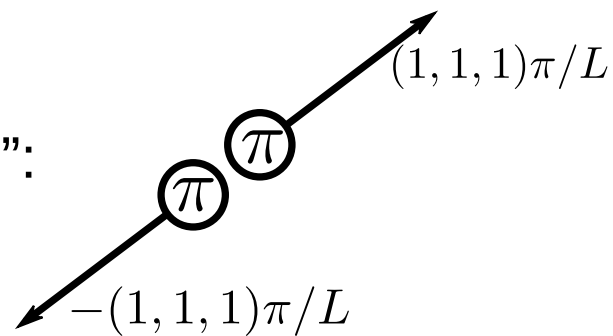


2015 calculation

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- Physical quark masses on single, coarse lattice ($a^{-1} = 1.38$ GeV) but with large $(4.6 \text{ fm})^3$ physical volume to control FV errors.
- G-parity boundary conditions remove dominant unphysical contribution from stationary $\pi\pi$ state.
- Single $\pi\pi$ operator:

“ $\pi\pi(111)$ ”:



- **21% and 65% stat errors** on $\text{Re}(A_0)$ and $\text{Im}(A_0)$ due to disconn. diagrams and, for $\text{Im}(A_0)$ a strong cancellation between Q_4 and Q_6 .
- Dominant, 15% systematic error due again to PT truncation errors.

2015 calculation: ε'

- $\text{Re}(A_0)$ and $\text{Re}(A_2)$ from expt.
- Lattice values for $\text{Im}(A_0)$, $\text{Im}(A_2)$ and the phase shifts,

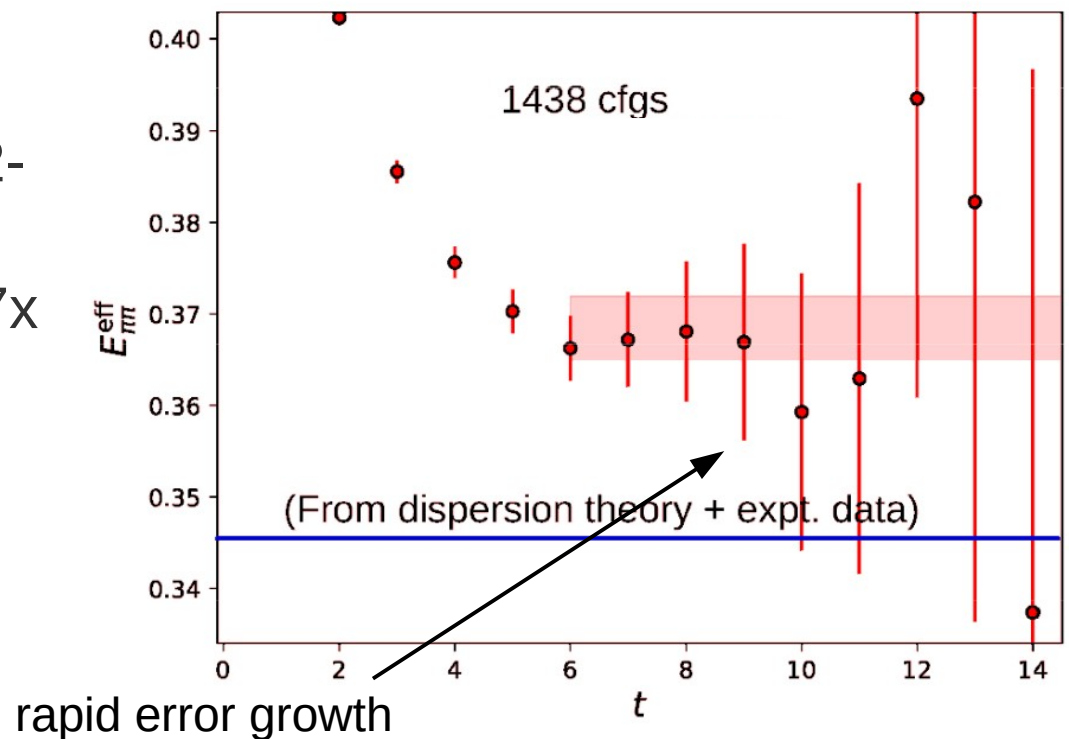
$$\text{Re} \left(\frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\varepsilon} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\}$$

$=$	$1.38(5.15)(4.43) \times 10^{-4}$,	(our result)
	$16.6(2.3) \times 10^{-4}$	(experiment)

- Result is 2.1σ below experimental value.
- Total error on $\text{Re}(\varepsilon'/\varepsilon)$ is $\sim 3x$ the experimental error
- “This is now a quantity accessible to lattice QCD”!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.

The “ $\pi\pi$ puzzle”

- Essential to understand $\pi\pi$ system:
 - Energy needed to extract ground-state matrix element
 - Energy also needed to compute phase-shift (Luscher)
 - Derivative of phase-shift w.r.t. energy is required for Lellouch-Luscher finite-volume correction (F)
- 2015 calculation phase shift $\delta_0(E_{\pi\pi} \approx m_K) = 23.8(5.0)^\circ$ substantially smaller than prediction obtained by combining dispersion theory with experimental input, 36° .
- Result was very stable under varying fit range and also with 2-state fits.
- Increasing statistics by almost 7x did not resolve ($\delta_0 = 19.1(2.5)^\circ$)
- Nevertheless, most likely explanation is excited-state contamination hidden by rapid reduction in signal/noise.

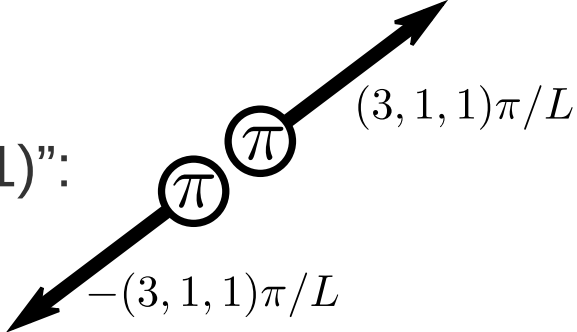


Resolving the $\pi\pi$ puzzle

[arXiv:2103.15131]

- To better resolve the ground-state we have introduced 2 more $\pi\pi$ operators:

“ $\pi\pi(311)$ ”:


$$\sigma = \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d)$$

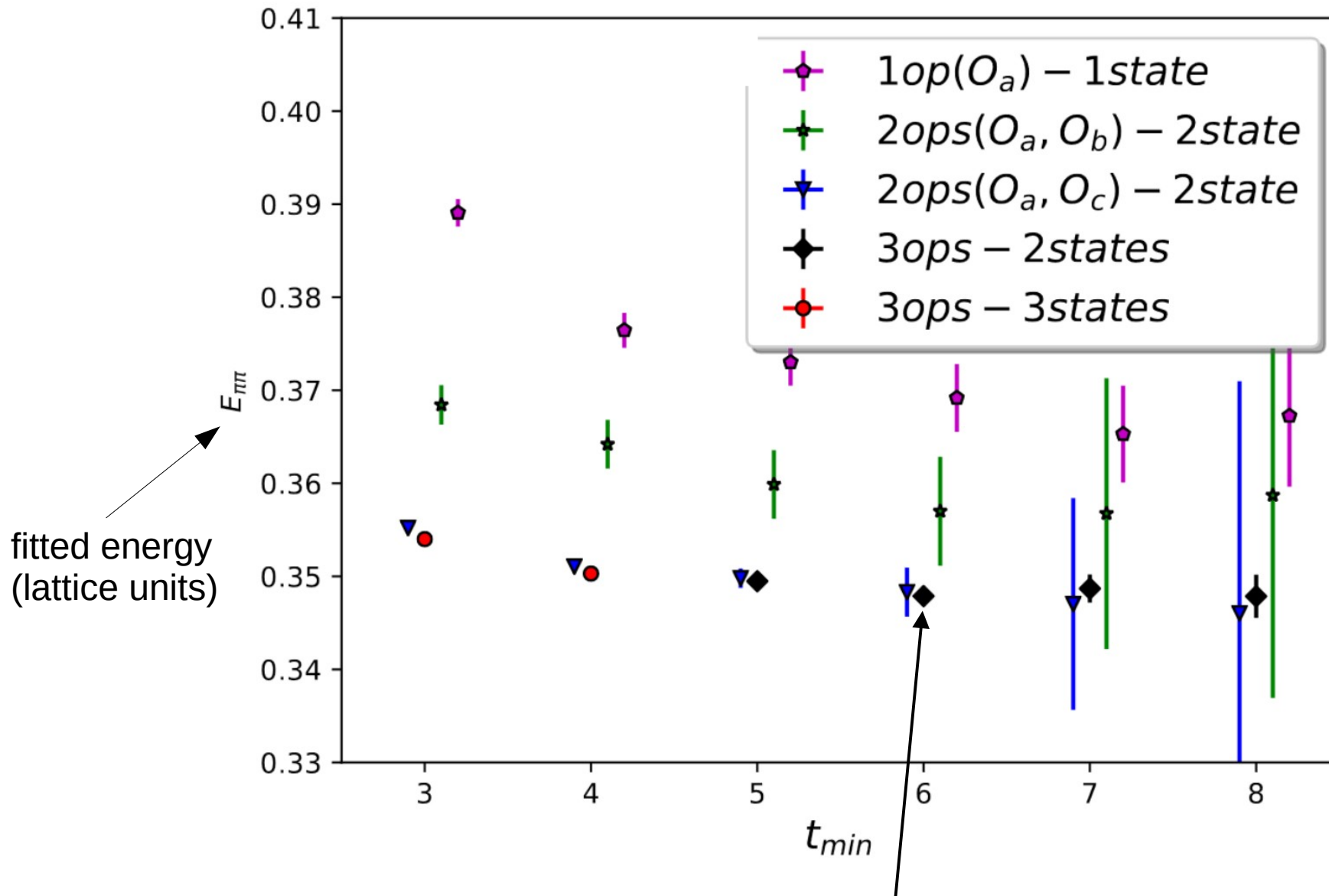
- Obtain parameters by simultaneous fitting to matrix of correlation functions, eg for $\pi\pi$ 2pt Green's function:

$$C_{ij}(t) = \langle 0 | O_i^\dagger(t) O_j(0) | 0 \rangle = C + \sum_{\alpha} A_{i,\alpha} A_{j,\alpha} e^{-E_{\alpha} t}$$

round-the-world single pion propagation
small compared to errors - drop

- A far more powerful technique than just increasing statistics alone.
- 741 configurations measured with 3 operators.

Effect of multiple operators on $\pi\pi$



Result compatible with dispersive value: $\delta_0(479.5 \text{ MeV}) = 32.3(1.0)(1.4)^\circ$

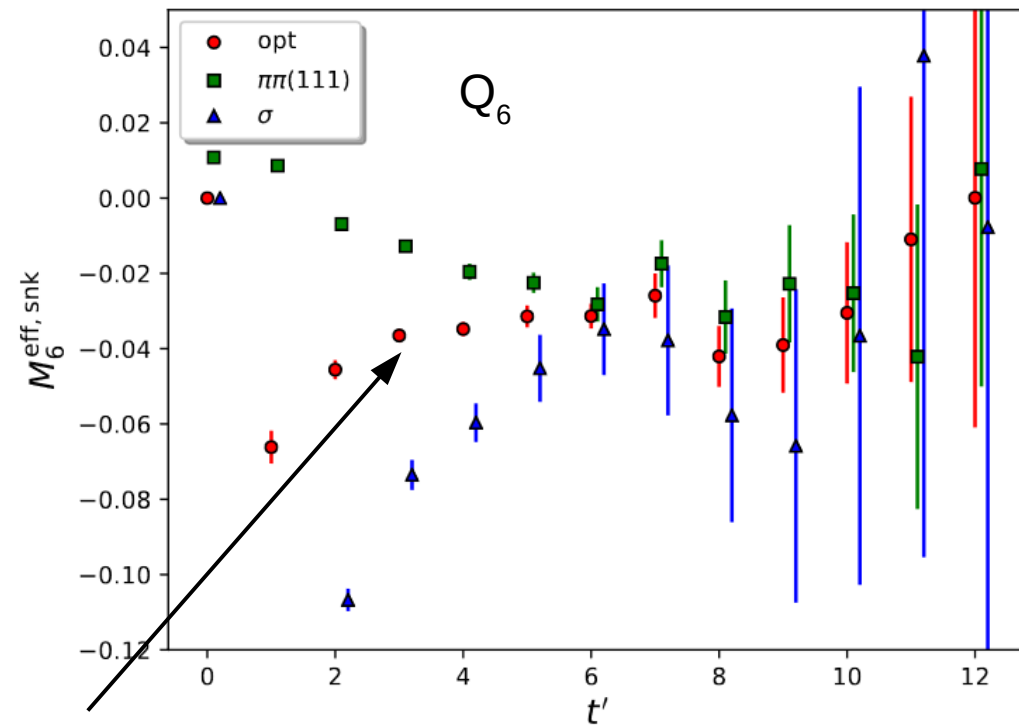
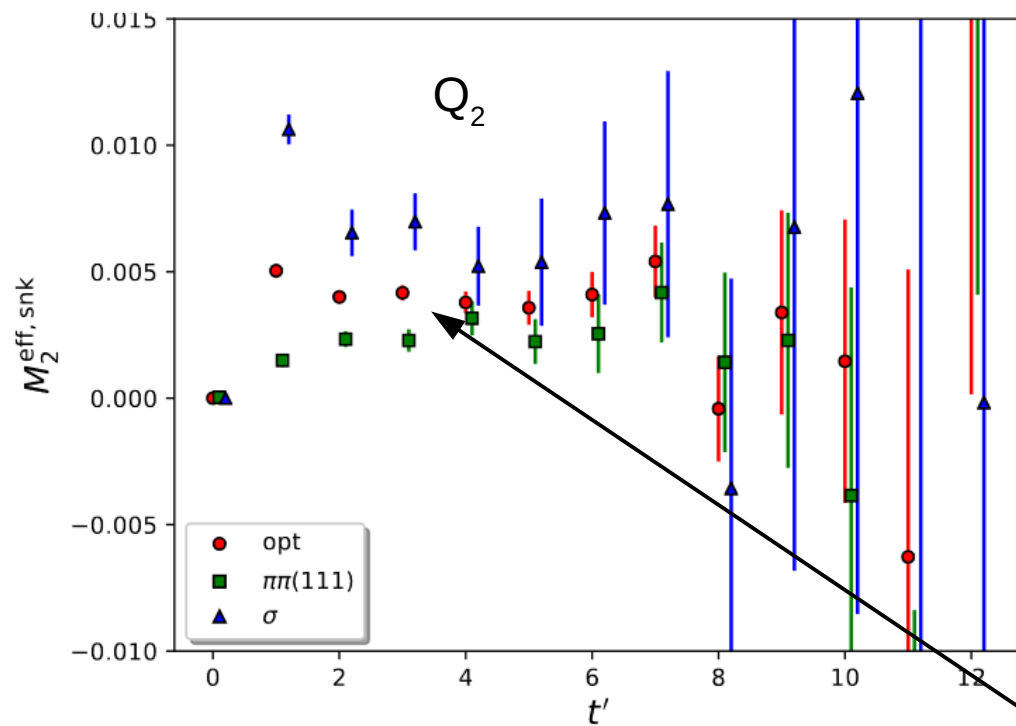
[for more details, cf. talk by T.Wang, today 9.15pm]

Effect of multiple operators on $K \rightarrow \pi\pi$

- Convenient to visualize data by taking “optimal” linear combination of the two most important operators that best projects onto ground-state.

$$\mathcal{O}_{\text{opt}} = r_1 \mathcal{O}_{\pi\pi(111)} + r_2 \mathcal{O}_{\sigma}$$

$$r_1 = 5.24(18) \times 10^{-7} \quad \text{using } \pi\pi \text{ fits}$$
$$r_2 = -2.86(17) \times 10^{-4}$$

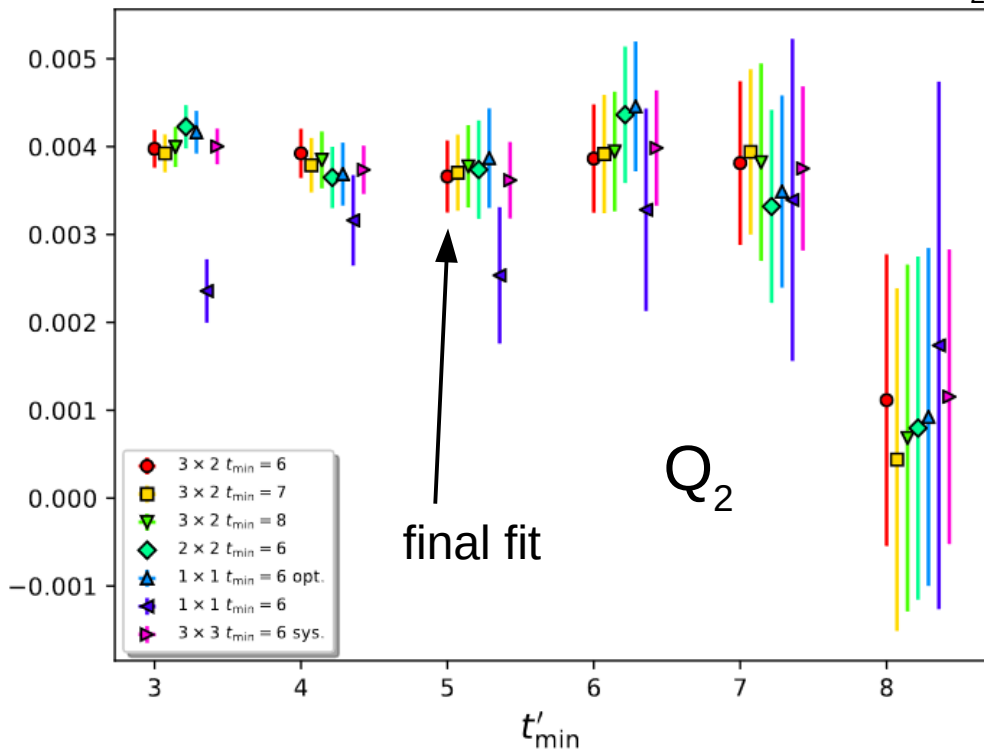


strong, clear plateau + improved precision

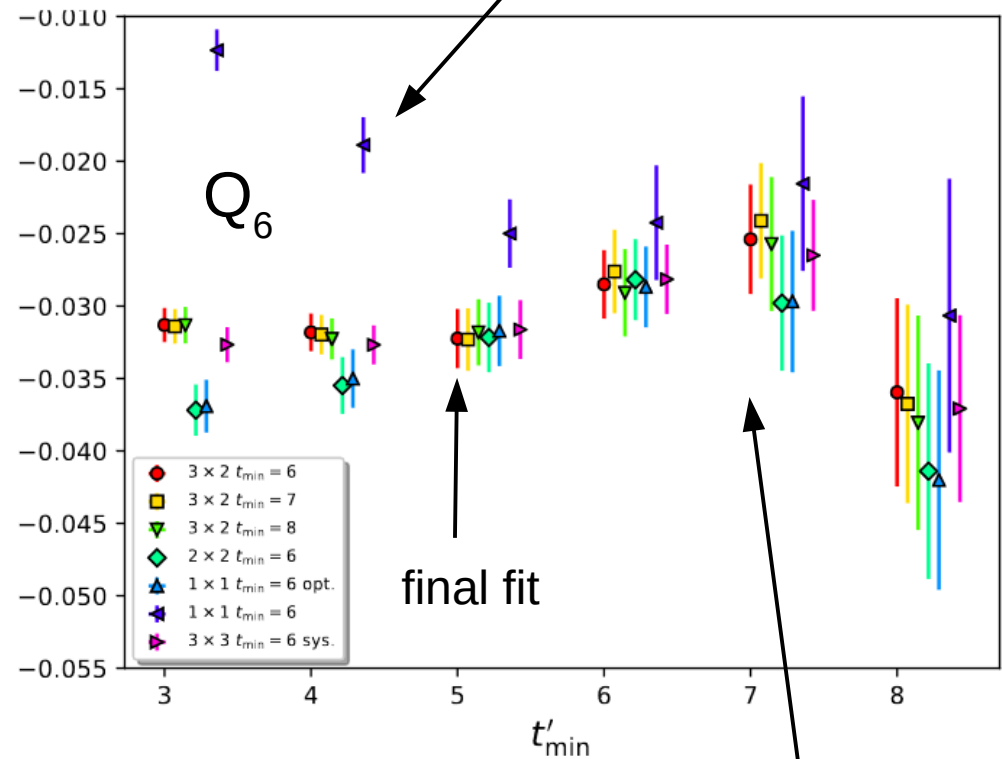
$K \rightarrow \pi\pi$ fit results

- Examine many fit ranges, #states and #operators

little indication of exc. stat. cont. for Q_2



exc. state contam. with 1op
(this was t'_{\min} for 2015 work)



- Adopt uniform fit $t'_{\min}=5$ which is stable for all Q_i

“bump” appears to be statistical

- Evidence that excited state error was significantly underestimated in 2015 work

Systematic error budget

- Primary systematic errors of 2015 work:
 - Finite lattice spacing: 12%
 - Wilson coefficients: 12%
 - Renormalization (mostly PT matching): 15%
 - Excited-state: $\leq 5\%$ but now known to be significantly underestimated
 - Lellouch-Luscher factor (derivative of $\pi\pi$ phase shift wrt. energy): 11%
- In our new work we have used step-scaling to raise the renormalization scale from 1.53 \rightarrow 4.00 GeV: **15% \rightarrow 5%**
- 3 operators have dramatically improved understanding of $\pi\pi$ system: Lellouch-Luscher factor **11% \rightarrow 1.5%**
- Detailed analysis shows no evidence of remaining excited-state contamination: **Excited state error now negligible!**
- Still single lattice spacing: **Discretization error unchanged.**
- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by higher μ :
Wilson coeff error unchanged.

Isospin breaking + EM effects

- Our simulation does not include effects of isospin breaking or EM effects.
- While these effects are typically small $O(1\%)$, heavy suppression of A_2 ($\Delta I=1/2$ rule) means relative effect on A_2 and ε' could be $O(20\%)$.
- Current best determination of effect uses NLO χ PT and $1/N_c$ expansion predicts 23% correction to our result:

Include as separate systematic error.

[Cirigliano *et al*,
JHEP 02 (2020) 032]

Final result for ϵ'

- Combining our new result for $\text{Im}(A_0)$ and our 2015 result for $\text{Im}(A_2)$, and again using expt. for the real parts, we find

$$\begin{aligned} \text{Re} \left(\frac{\epsilon'}{\epsilon} \right) &= \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\epsilon} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\} \\ &= 0.00217(26)(62)(50) \end{aligned}$$

stat sys IB + EM

Consistent with experimental result:

$$\text{Re}(\epsilon'/\epsilon)_{\text{expt}} = 0.00166(23)$$

The road ahead

- Primary pure-lattice systematic is discretization error (12%). Currently estimated using scaling of $I=2$ operators but there may be significant “error on the error”.
- Near-term availability of next-gen supercomputers (Perlmutter, Aurora) opens up opportunity to perform a full continuum extrapolation.
- Current plan is two additional ensembles with following properties:
 - Physical pion and kaon masses.
 - Same gauge action allowing **continuum extrapolation with 3 points**.
 - Same physical volume such that $\pi\pi$ energy remains the same and the interaction remains physical.
- $40^3 \times 64$, $a^{-1}=1.7$ GeV and $48^3 \times 64$, $a^{-1}=2.1$ GeV are computationally feasible while providing a good lever arm (a^2 scaling).
- Already started generating 16^3 test ensembles for tuning.
- Measurement code has been ported to NVidia and Intel GPUs, utilizing Grid’s portable GPU kernel API.

The road ahead pt.2

- Independent calculation of ϵ' using multiple operators to extract on-shell matrix elements as excited-state contributions in a periodic lattice is well under way. [\[cf. talk by M.Tomii, yesterday 10PM\]](#)
 - Avoid complications of using G-parity BCs
 - Uses existing MDWF+I ensembles with physical pion masses
 - 2 lattice spacings allowing continuum limit
- We are developing techniques to perform 3-4f matching in the Wilson coefficients **non-perturbatively** in order to avoid relying on PT at the charm scale. [\[PoS LATTICE2018 \(2019\) 216\]](#)
- Also working on laying the groundwork for the lattice calculation of EM contributions. [\[cf. talk by J.Karpie, today 2.30pm\]](#)

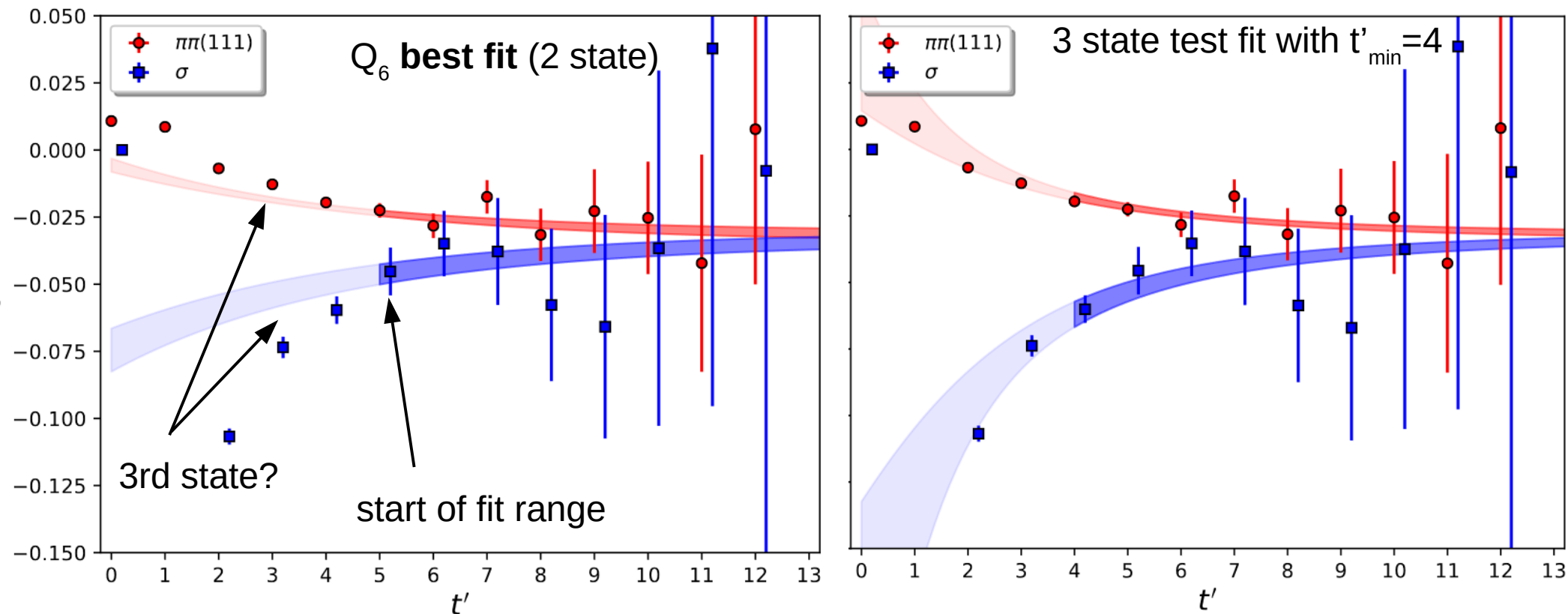
Conclusions

- Completed update on our 2015 lattice determination of A_0 and ϵ'
 - 3.2x increase in statistics.
 - Improved systematic errors, notably use of multi-operator techniques essentially removes excited-state systematic.
- Reproduce experimental value for $\Delta I=1/2$ rule, demonstrating that QCD sufficient to solve this decades-old puzzle.
- Result for ϵ' consistent with experimental value.
- Total error is $\sim 3.6x$ that of experiment.
- ϵ' remains a promising avenue to search for new physics, but greater precision is required.
- The work goes on....



Excited state contamination

- Primary concern is residual excited-state contamination
- See excellent consistency and strong plateaus among fits for 2+ operators with $t'_{\min} \geq 4$
- Also examine 2 and 3-state fits with 3 ops:



- 3-state fit with lower $t'_{\min} = 4$ describes data well outside fit range.
Complete consistency in gnd-state matrix elem of best fit