QFT Dynamics from CFT Data with Lightcone Conformal Truncation

Liam Fitzpatrick

Lattice 2021

Motivation

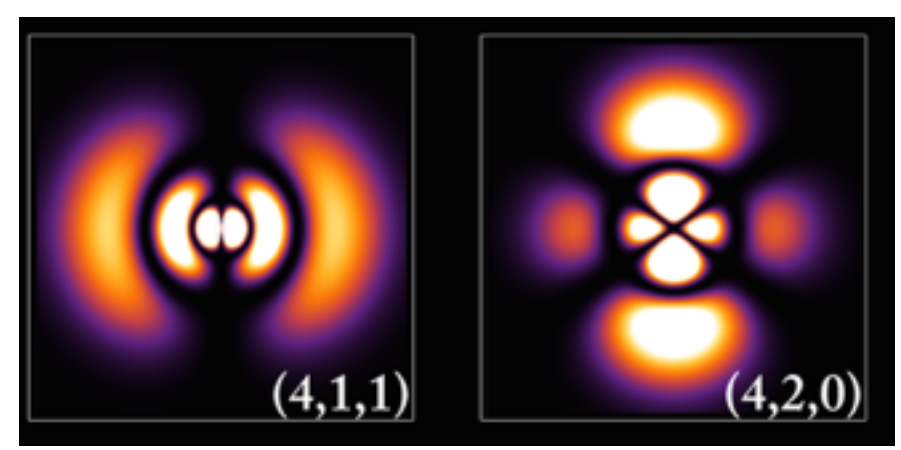
Two Goals:

1) Explore the space of QFTs at strong coupling

coupling

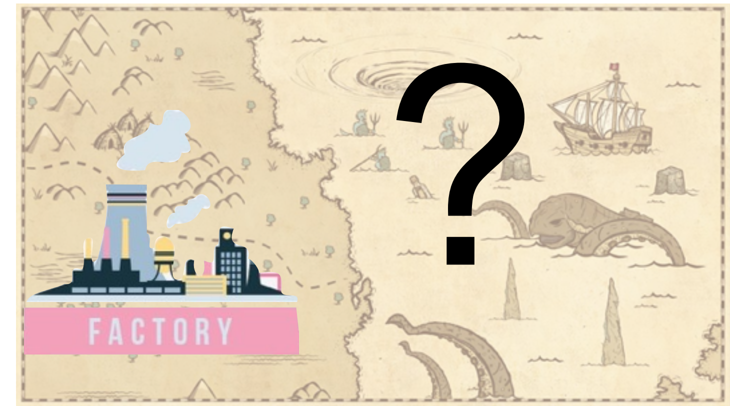
2) Develop variational methods for QFT (in infinite volume, continuum limit)

Get spectrum ***and wavefunctions***



QFT at Strong Coupling

1) Explore the space of QFTs at strong coupling



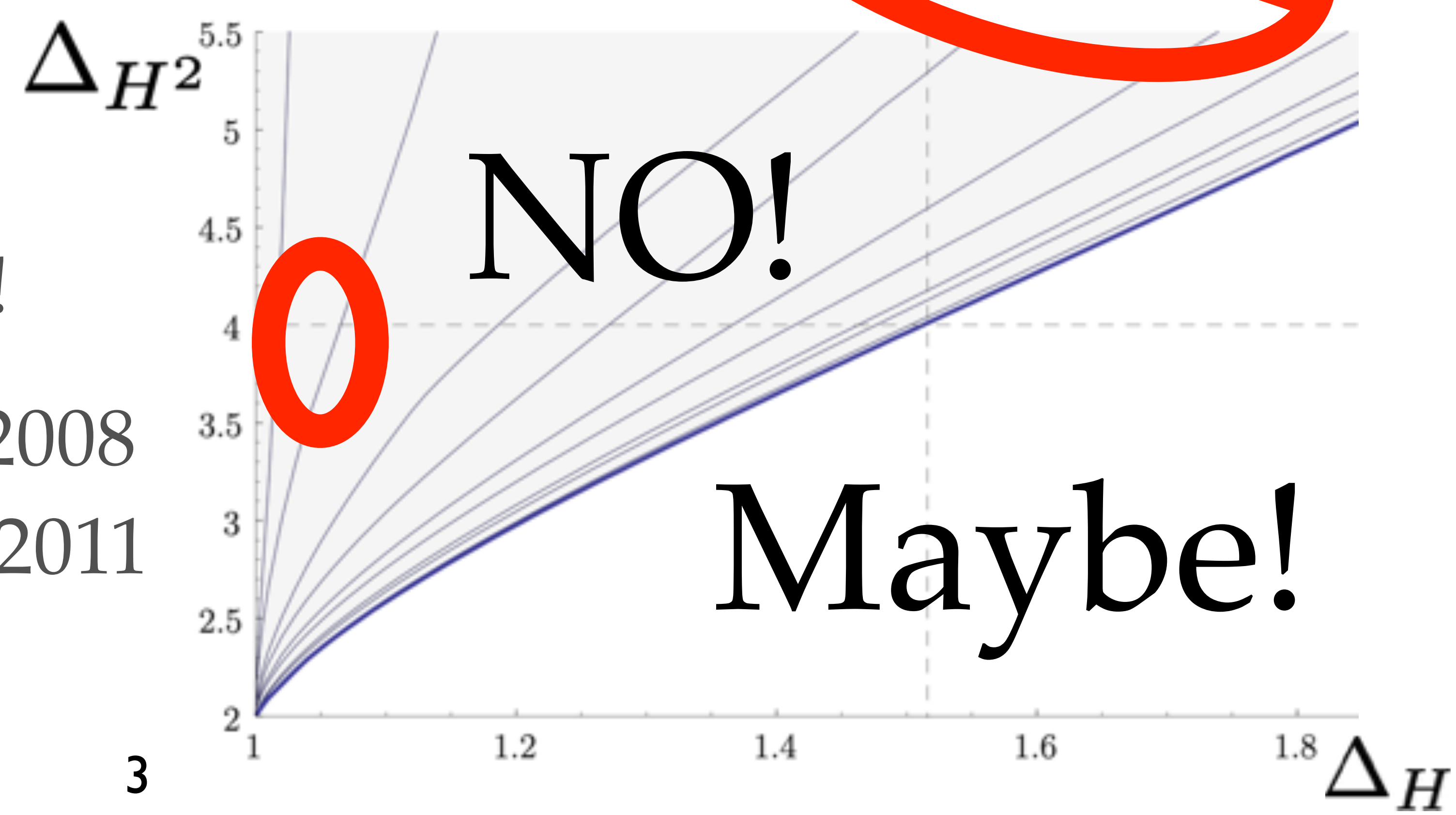
BSM Model-building - hard to do more than speculate at strong coupling

E.g. "Conformal Technicolor" Luty & Okui 2004, Proposal $\Delta_H \approx 1$
to solve hierarchy problem with novel scaling dimensions: $\Delta_{H^2} \sim 4$

Why not? Who knows?

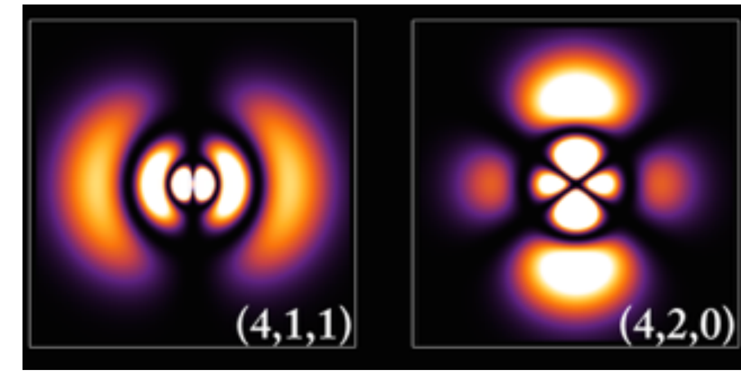
Conformal Bootstrap knows!

Rattazzi, Rychkov, Tonni, Vichi 2008
Poland, Simmons-Duffin, Vichi 2011



QFT at Strong Coupling

2) Develop variational methods for QFT
in infinite volume, in continuum limit



Variational methods give you wavefunctions! Useful for computing correlators

E.g. $$\langle \mathrm{vac} | \Phi(x) \Phi(y) | \mathrm{vac} \rangle$$

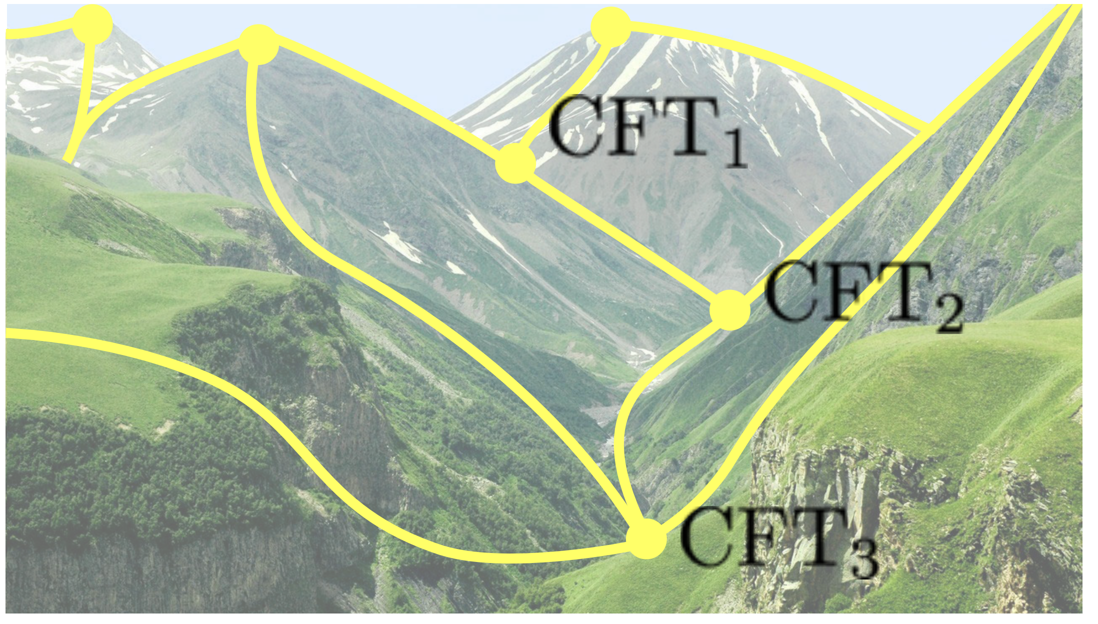
Especially useful if you can get wavefunctions for excited states (e.g. particles), not just for vacuum

E.g. form factors $$\langle p | \Phi(x) | p' \rangle$$

particle at momentum p, p′

Variational Methods and the Space of QFTs

Some philosophy: QFTs are RG flows between UV and IR CFT fixed points



Practical point: if your QFT is conformal at high energies, this can actually be useful for your variational method!

Hamiltonian Truncation

Basic idea of Hamiltonian truncation:

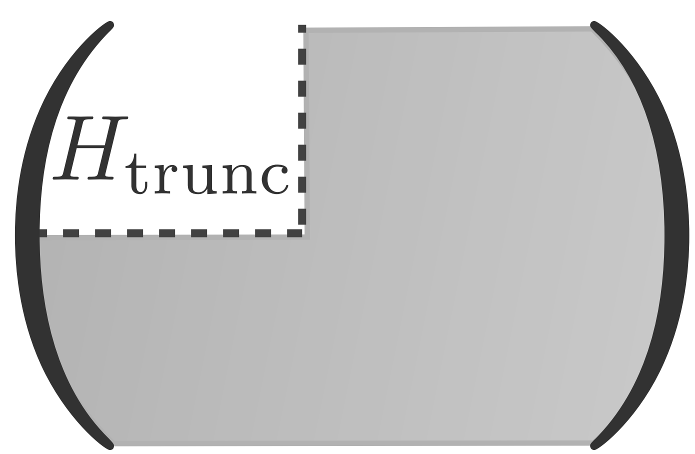
$$H = \overset{\text{solvable}}{H_0} + \overset{\text{“interaction”}}{V}$$

Do exact (numerical) diagonalization of H

Can’t numerically dragonalize an ∞-by-∞ matrix!

Restrict to a finite basis of states:

eigenstates of H_0 with $E<E_{max}$



Hamiltonian Truncation Warm-up: Anharmonic Oscillator in QM

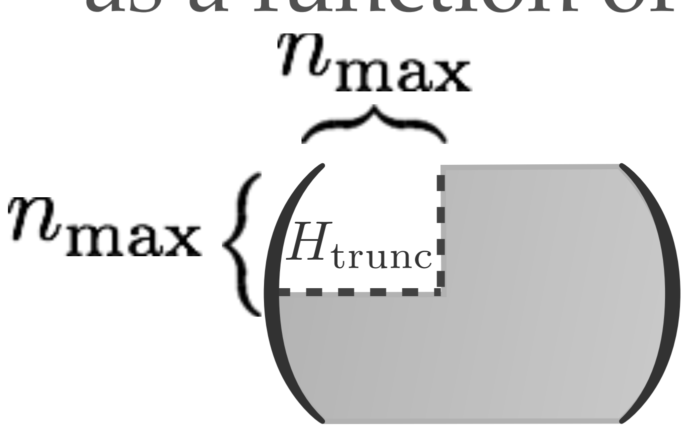
$$\mathcal{L} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}m^2x^2 - \overbrace{gx^4}^{V} \longrightarrow H = H_{\text{SHO}} + V$$

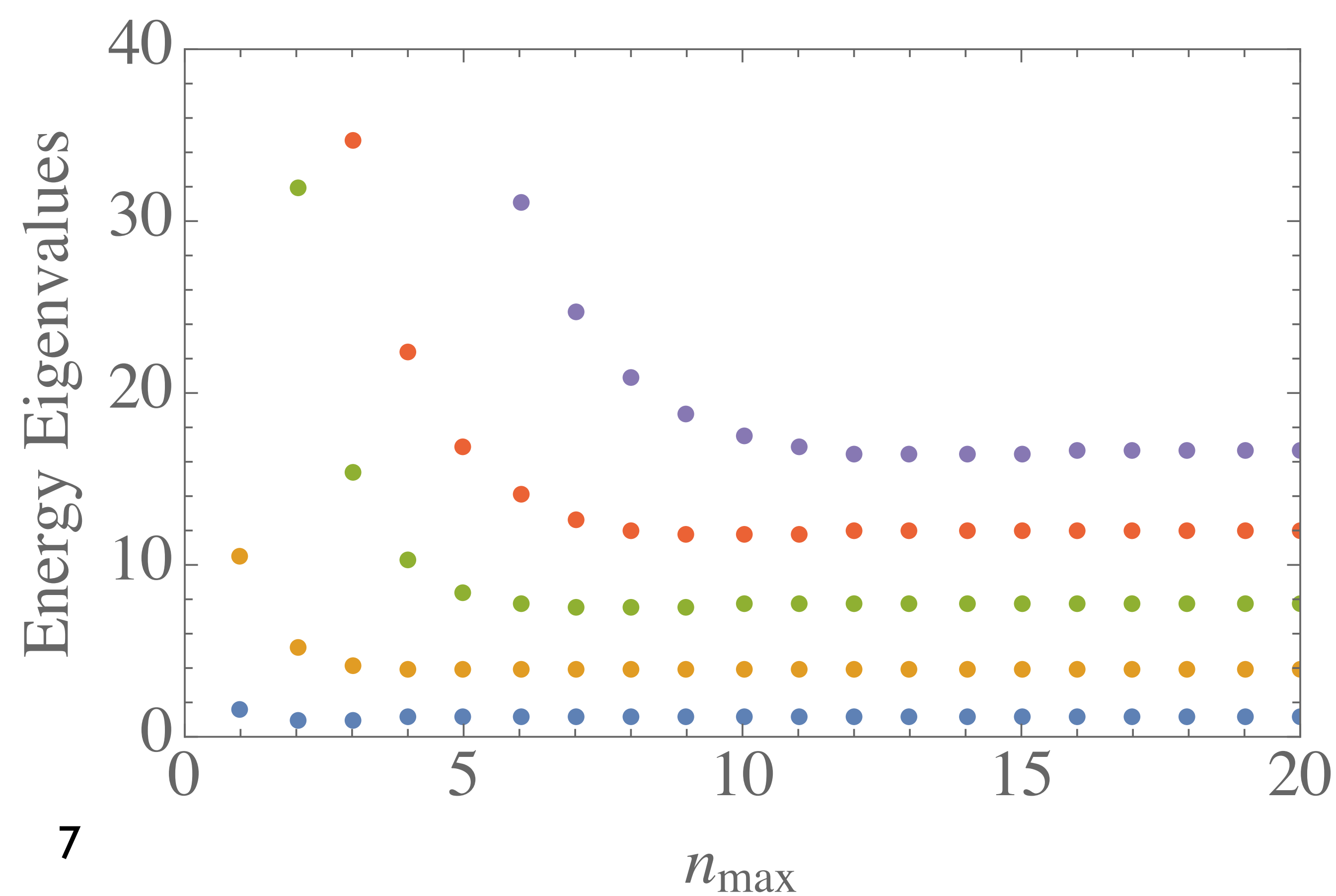
$$V \sim g(a+a^\dagger)^4$$

Basis: $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$ $\qquad \langle n|H|m\rangle = \delta_{nm}E_n^{(\text{SHO})} + \langle n|V|m\rangle$

Take finite n_{max} and diagonalize n_{max} x n_{max} matrix

Eigenvalues converge quickly as a function of n_{max}





Truncation for QFT

Same idea, applied to QFT Hamiltonian

Our QFT is a CFT plus relevant deformation:

$$H = H_{\text{CFT}} + V_{\text{relevant}}$$

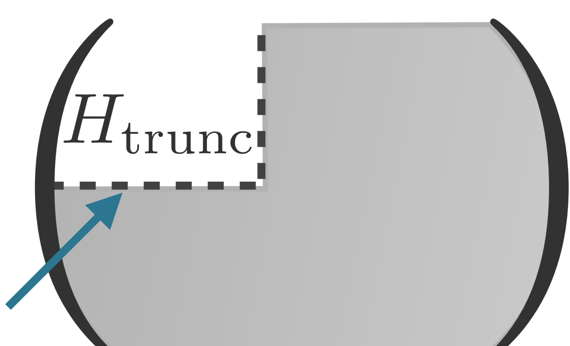
For example:

Free massless scalar

(d<4)

$$\int d^{d-1}x \left(\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right)$$

Truncated basis: states created by CFT operators with dimension < Δ_{max}

$H = H_{\text{CFT}} + V_{\text{relevant}} =$ 

Δ_{max}

User’s Guide(s)

I will skip all the technical details of how the basis states are constructed and how the Hamiltonian matrix elements are calculated

Recent pedagogical introduction to Lightcone Conformal Truncation (LCT) in d=2 with Mathematica code: 2005.13544

		Pedagogical			Radial Quantization		
$\Delta_{\max}$	num of states	basis	mass	quartic	basis	mass	quartic
10	42	0.19	0.26	2.36	0.02	0.06	0.07
20	627	3061	170.1	5183	0.46	1.09	3.96
30	5604	weeks?	hours?	weeks?	7.88	17.93	111.9
40	37338	Good luck			231	410	3579

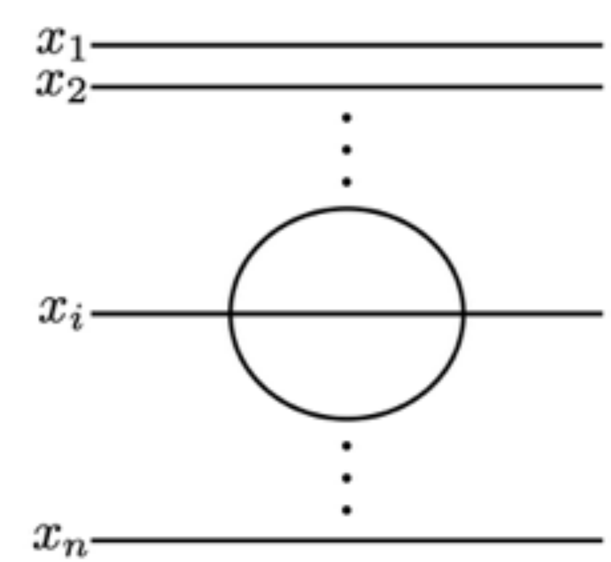
Anand, ALF, Katz, Khandker, Walters, Xin

Can do d=2 with scalars, fermions, gauge fields

Recent LCT study of ϕ^4 in d=3:
2010.09730 Anand, Katz, Khandker, Walters

Also equal-time 2003.08405 Elias-Miró, Hardy

Treatment of UV divergences



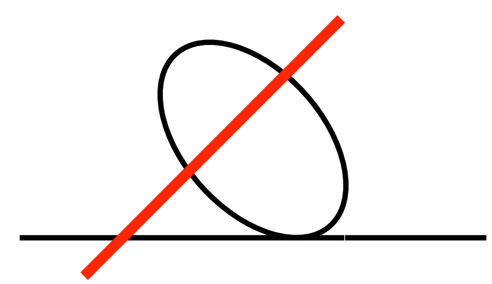
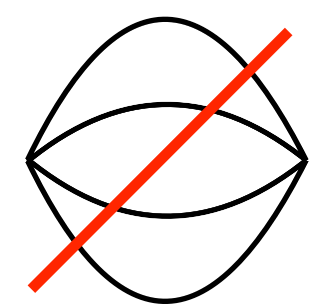
Lightcone Quantization

Main advantages:

1) Preserves boosts (unlike "equal-time" Hamiltonian)

$\Rightarrow$ Don't need to diagonalize a separate Hamiltonian for each momentum

2) Eliminates "particle production" diagrams, including all vacuum bubbles

e.g.  and 

$\Rightarrow$ a) Fewer UV divergences to handle

b) Vacuum doesn't mix with other states, ***can take infinite volume limit from the very beginning***

Example: 2d ϕ^4

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

dimensionless $\bar{\lambda} = \frac{\lambda}{m^2}$

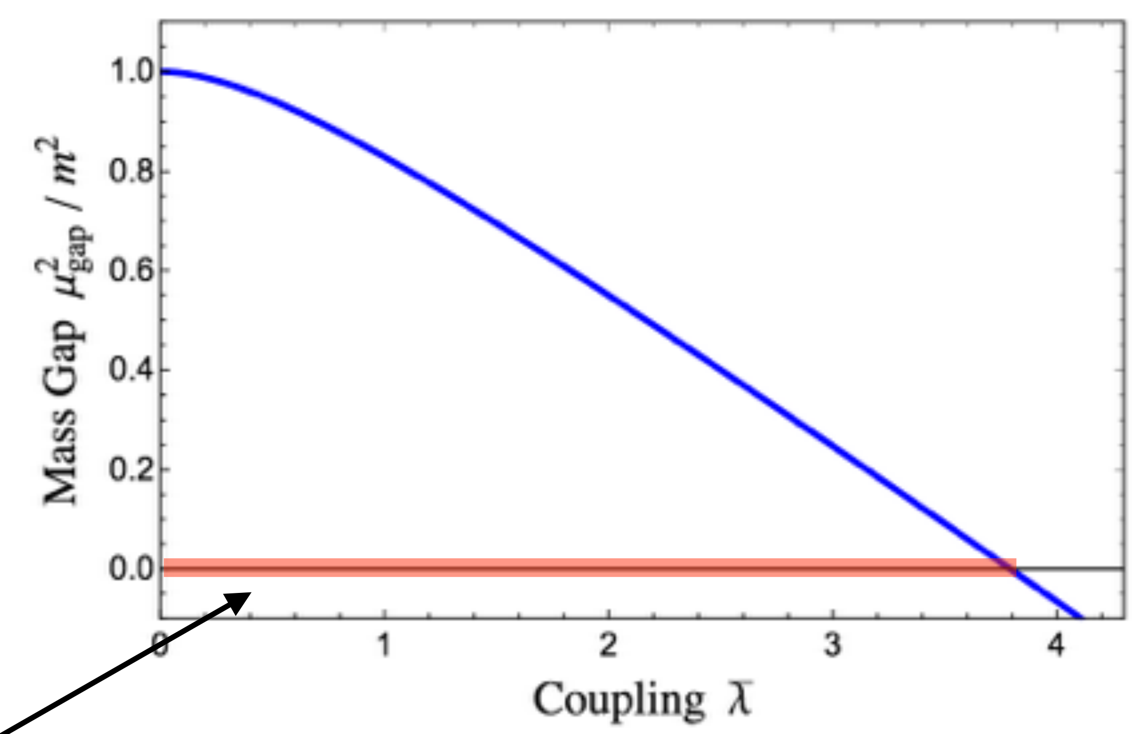
Warm-up with two basis states ($\Delta_{\text{max}} = 3$):

$$\mathcal{O}_1 = \partial_-\phi \qquad \mathcal{O}_2 = (\partial_-\phi)^3$$

Hamiltonian is 2x2:

$$M^2 \equiv P^2 = 2P_-(P_+^{(\text{CFT})} + \delta P_+)$$

$$= m^2 \begin{pmatrix} 1 & 0 \\ 0 & 15 \end{pmatrix} + \bar{\lambda} m^2 \begin{pmatrix} 0 & \sqrt{5} \\ \sqrt{5} & 15 \end{pmatrix}$$



Variational method, so critical point must be somewhere in here

Example: 2d ϕ^4

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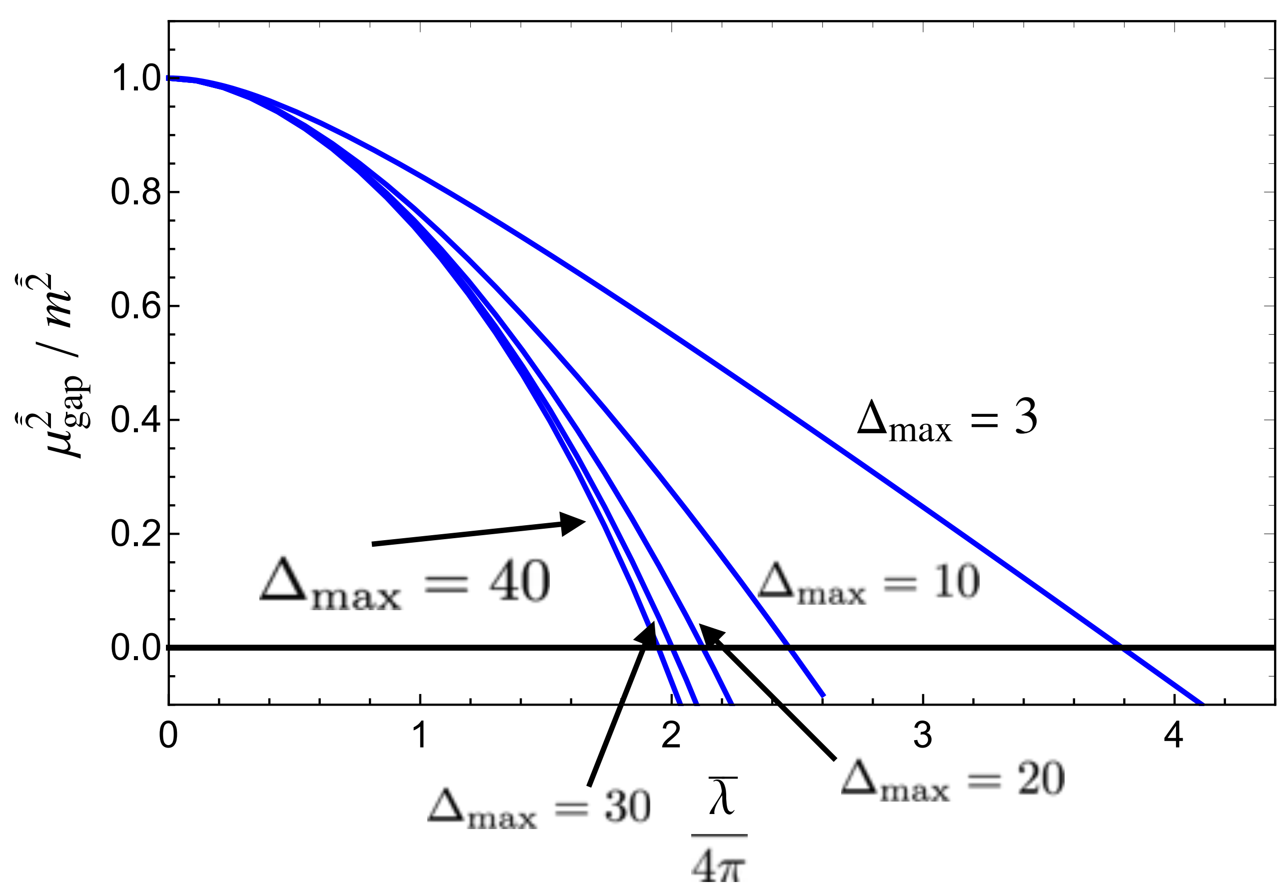
Increasing Δ_{max}



Δ_{max}=3: 3 states

Δ_{max}=10: 42 states

Δ_{max}=20: 627 states

Δ_{max}=30: 5604 states

Δ_{max}=40: 37338 states

Dynamical Quantities: Spectral Densities

Once one has the eigenvectors of the Hamiltonian in a fixed momentum frame, one can compute spectral densities

$$\rho_{\mathcal{O}}(s) = \sum_j |\langle \mathcal{O}(0)|j\rangle|^2 \delta(s - p_j^2)$$

eigenvectors

eigenvalues

Sum over eigenstates $P^2|j\rangle = p_j^2|j\rangle$

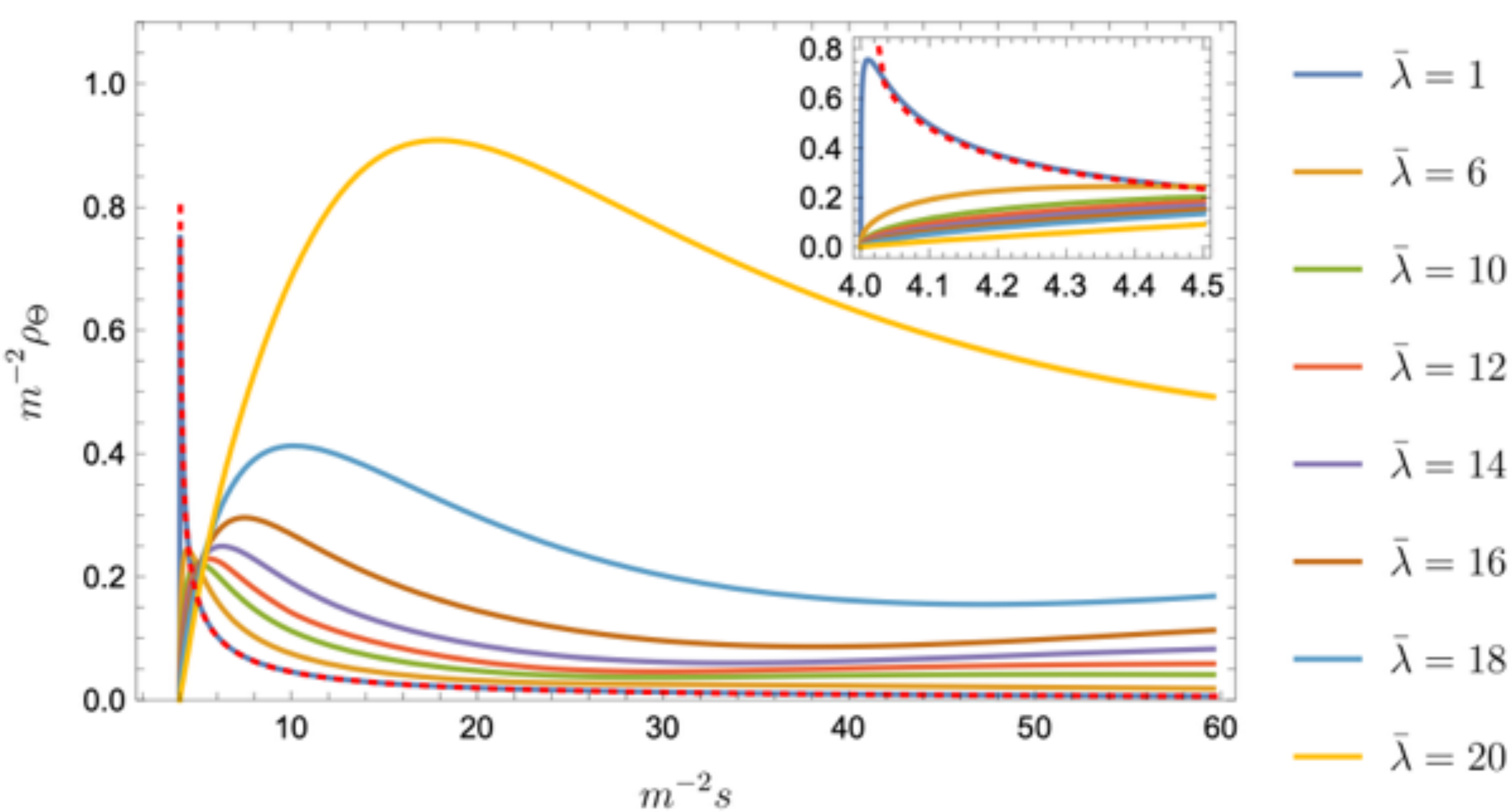
Then can get time-ordered correlators from spectral densities

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = \int d^2x e^{ipx} \langle \mathcal{T}\{\mathcal{O}(x)\mathcal{O}(0)\}\rangle = \int ds \frac{\rho_{\mathcal{O}}(s)}{p^2 - s + i\epsilon}$$

Example: 2d ϕ^4

Stress Tensor Spectral Density

$$\rho_{\mathcal{O}}(s) = \sum_j |\langle \mathcal{O}(0)|j\rangle|^2 \delta(s - p_j^2)$$

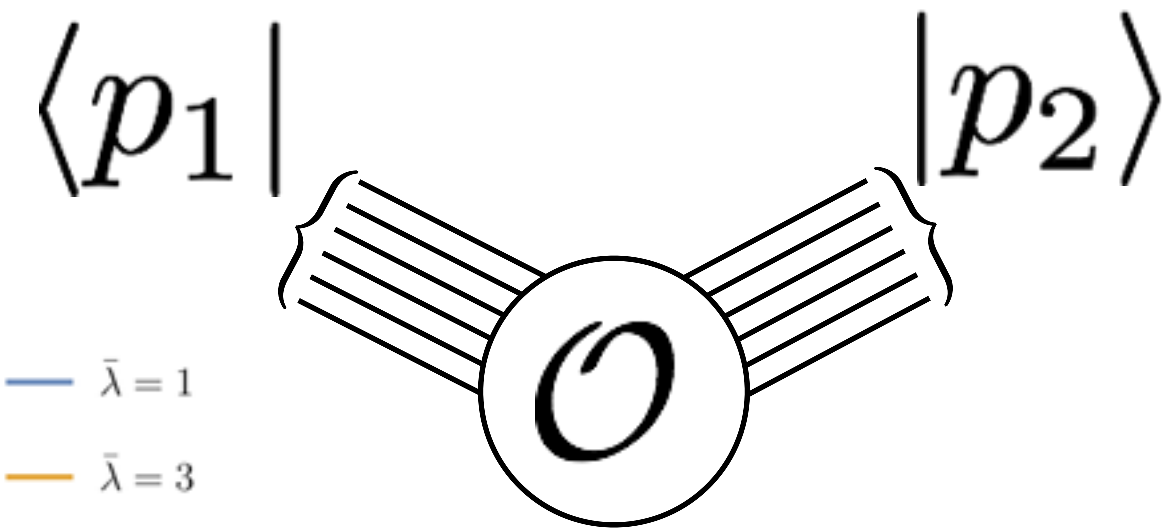


$$\bar\lambda_{\text{crit}} \approx 23.1$$

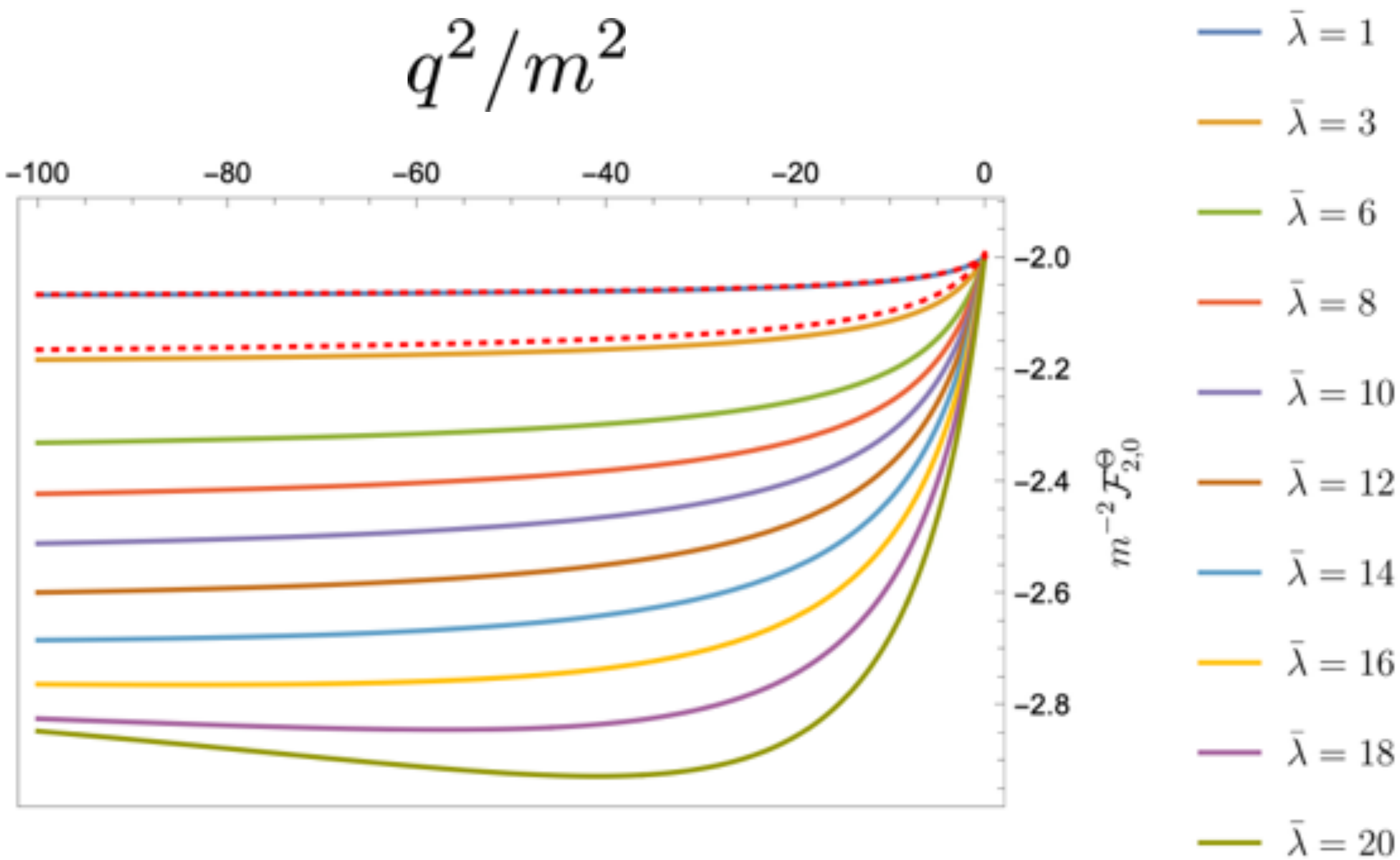
Form Factors

One can obtain form factors of operators using the one-particle eigenstates

$$\langle p_1|\mathcal{O}(0)|p_2\rangle = F_{\mathcal{O}}(q^2)$$

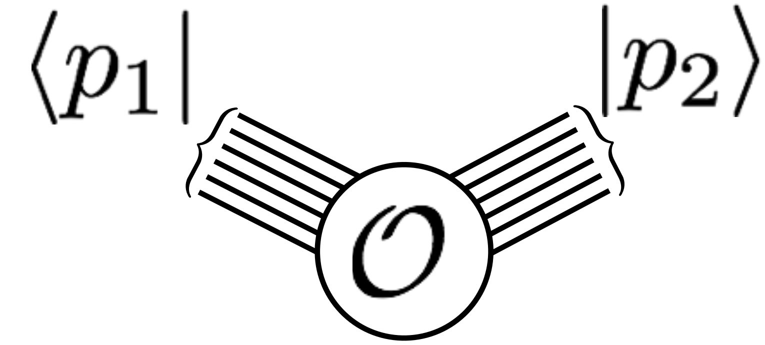


$$q = p_1 - p_2$$



$\bar{\lambda}_{\text{crit}} \approx 23.1$

Form Factors



A worked example: Δ_{max}=5 (4 states)

$$\mathcal{O}_1 \propto \partial\phi, \quad \mathcal{O}_2 \propto (\partial\phi)^3, \quad \mathcal{O}_3 \propto (6\partial^3\phi(\partial\phi)^2 - 9(\partial^2\phi)^2\partial\phi), \quad \mathcal{O}_4 \propto (\partial\phi)^5$$

$$P^2 = m_0^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 15 & 4\sqrt{3} & 0 \\ 0 & 4\sqrt{3} & 27 & 0 \\ 0 & 0 & 0 & 45 \end{pmatrix} + \lambda \begin{pmatrix} 0 & \frac{\sqrt{5}}{4\pi} & \frac{\sqrt{15}}{8\pi} & 0 \\ \frac{\sqrt{5}}{4\pi} & \frac{15}{4\pi} & \frac{\sqrt{3}}{\pi} & \frac{\sqrt{105}}{2\pi} \\ \frac{\sqrt{15}}{8\pi} & \frac{\sqrt{3}}{\pi} & \frac{33}{8\pi} & 0 \\ 0 & \frac{\sqrt{105}}{2\pi} & 0 & \frac{45}{2\pi} \end{pmatrix}$$

$$\bar{\lambda} = 1: \; |p\rangle = 0.99994|\mathcal{O}_1, p\rangle - 0.01036|\mathcal{O}_2, p\rangle - 0.00280|\mathcal{O}_3, p\rangle + 0.00033|\mathcal{O}_4, p\rangle$$

Fourier transforms of CFT three-point functions - fixed by scaling dimensions and OPE coefficients!

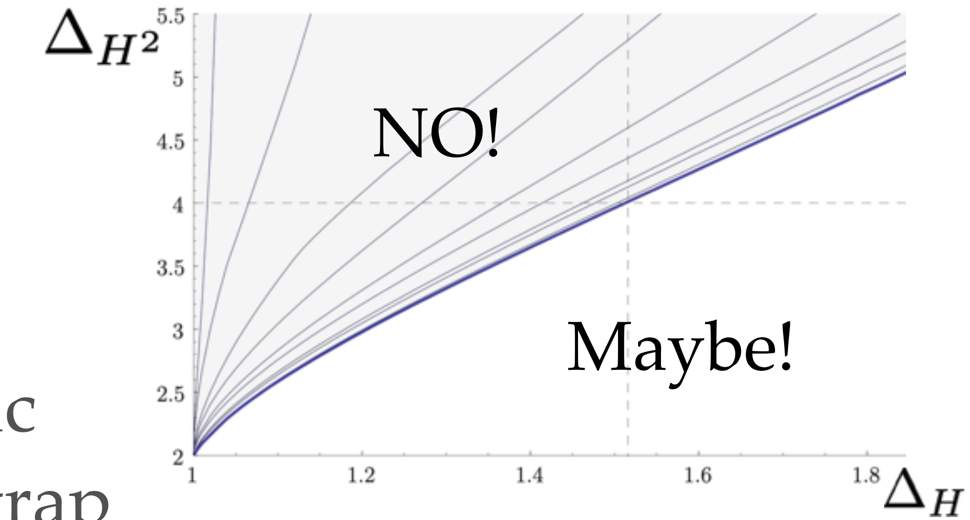
$$\langle p|T_{--}(0)|p'\rangle = \sum_{j,j'} c_j^* c_{j'} \overbrace{\langle \mathcal{O}_j, p|T_{--}(0)|\mathcal{O}_{j'}, p'\rangle}$$

$$= -0.072226 + 0.399852\frac{p_-}{p'_-} - 1.01012\left(\frac{p_-}{p'_-}\right)^2 + 1.3543\left(\frac{p_-}{p'_-}\right)^3 - 0.91653\left(\frac{p_-}{p'_-}\right)^4 + 0.24472\left(\frac{p_-}{p'_-}\right)^5$$

Closing Thoughts

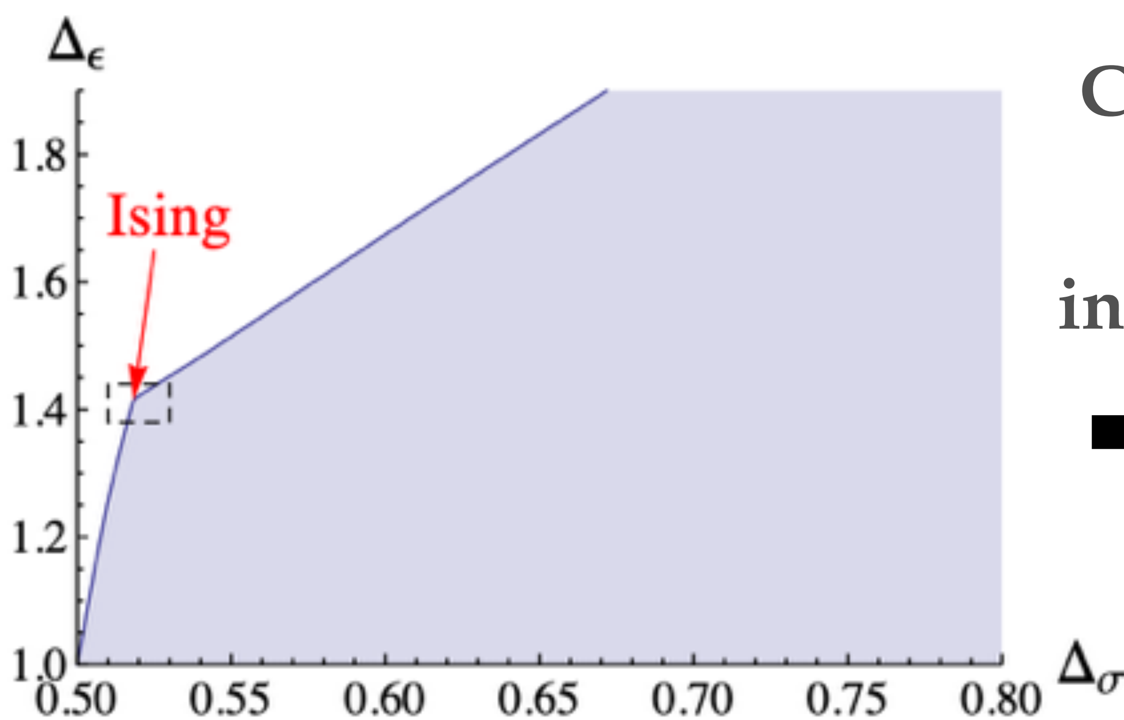
Better Together

But what about when answer is "maybe"?



In fact, if you have results for your specific model, you can often inject them into bootstrap

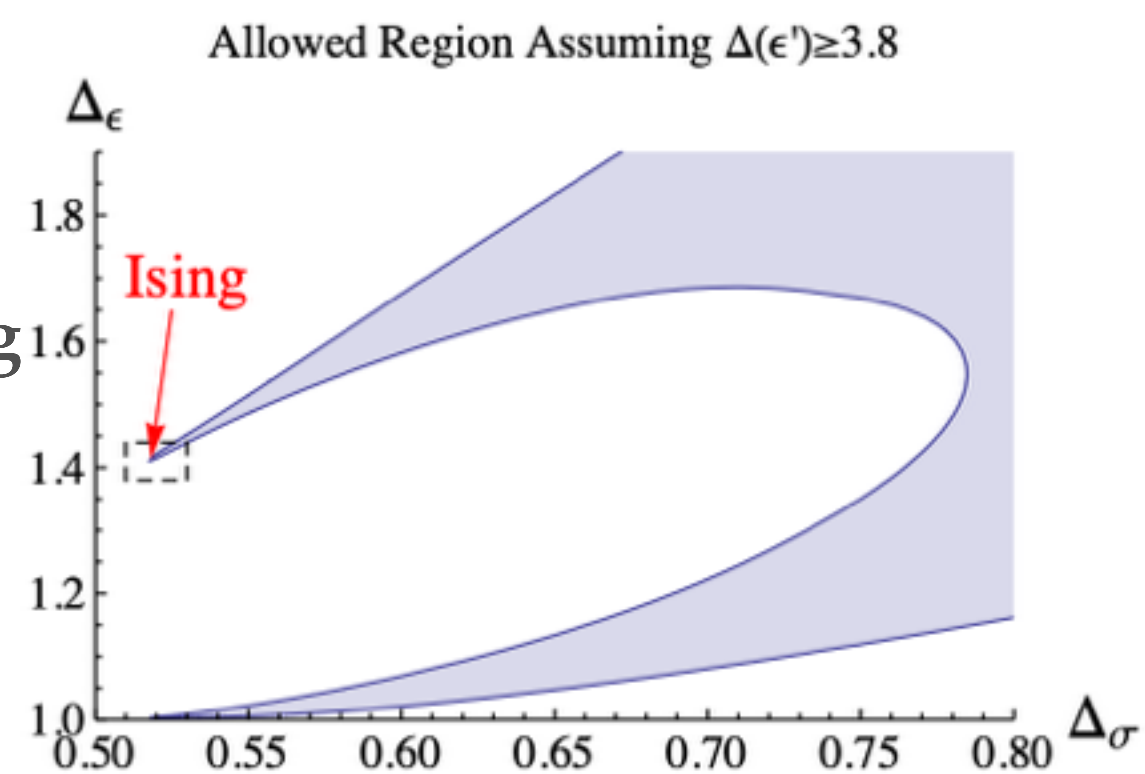
E.g. Critical 3d Ising model: knowing certain scaling dimensions is a big help!



Calculate

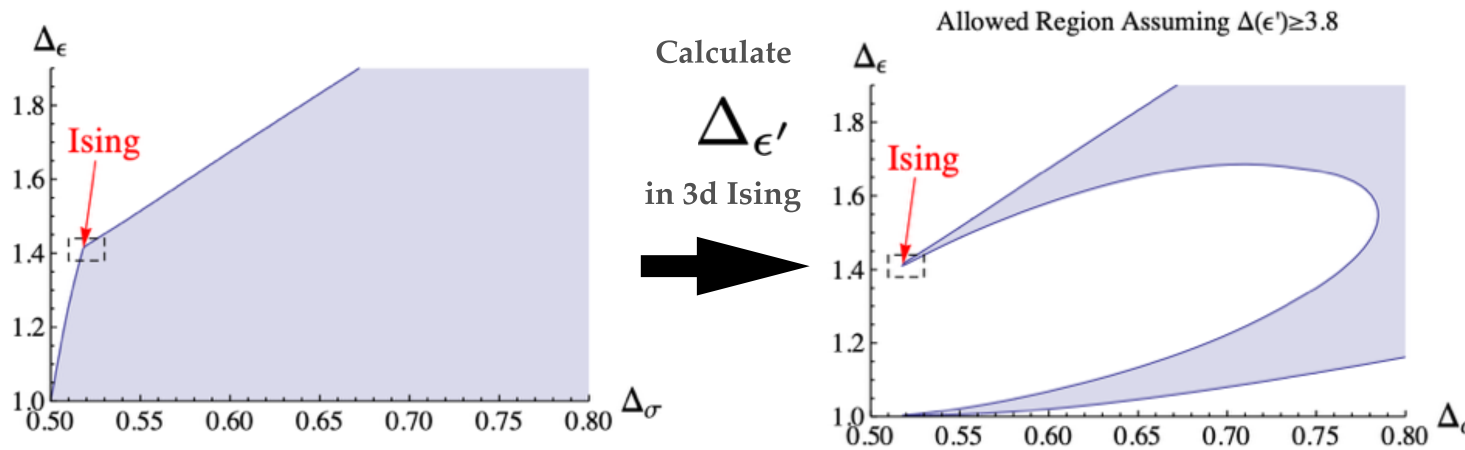
$\Delta_{\epsilon'}$

in 3d Ising



El-Showk et al. 2012

Combine Conformal Bootstrap & Lattice to explore space of CFTs??

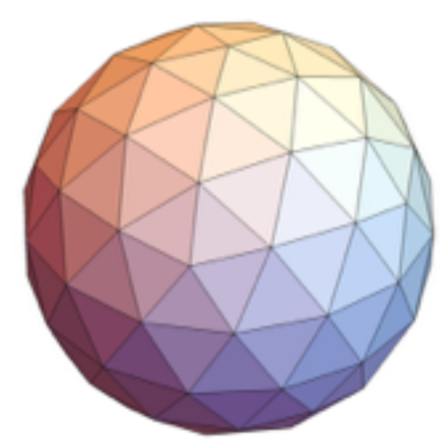


Could Lattice Monte Carlo provide the scaling dimensions of lowest ***several*** scaling dimensions in theory of interest (not just the standard α,β,γ,δ etc, which are given by lowest dimension op in each sector, e.g. Δ_σ & Δ_ε), and conformal bootstrap does the rest? (E.g. QED_3 in conf. window?)

Radial quantization: Dimensions of operators are scaling dimensions of theory quantized on $\mathbb{R} \times S^{d-1}$

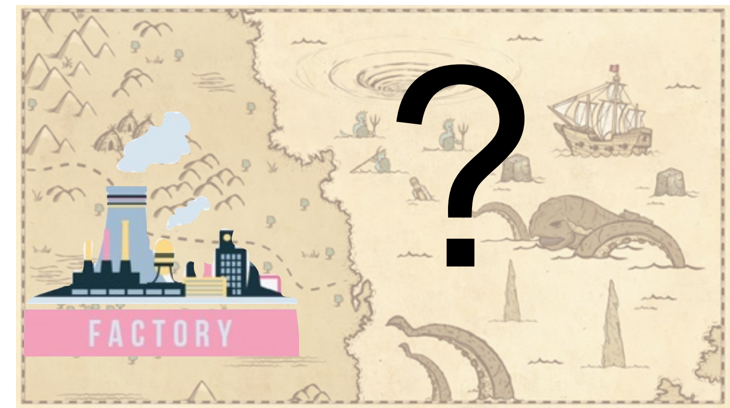
E.g. Brower et al. 2006.15636

Studied 3d ϕ^4 at critical coupling on sphere



QFT at Strong Coupling

1) Explore the space of QFTs at strong coupling



More philosophically:
Everything we have ever observed can be described as arising from a QFT
QFT is the language of nature, we should explore its full range for its own sake

"The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy" - Steven Weinberg

The End