

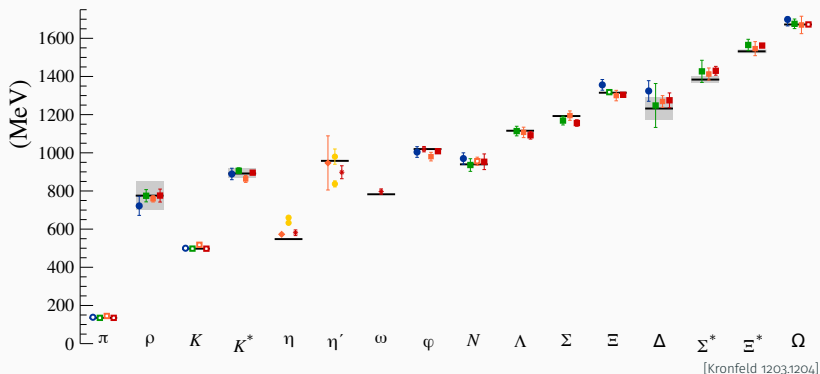
# SPECTROSCOPY AND HADRON INTERACTIONS

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Ben Hörz (Nuclear Science Division, Lawrence Berkeley National Lab)

*The 38th International Symposium on Lattice Field Theory*  
July 30, 2021

# FLAVORS OF SPECTROSCOPY



- general objective: obtain 'hadron masses' from QCD

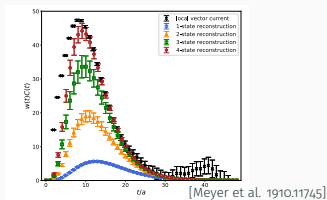
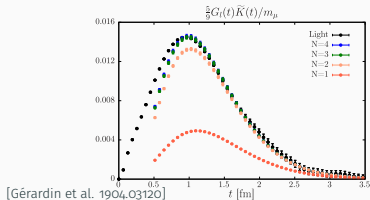
88 talks at this conference

- (QCD-)stable hadrons are precision observables

inclusion of strong isospin breaking + QED

- ...but 'not all' hadrons are stable

# SPECTROSCOPY AS A STEPPING STONE



Multi-hadron methods relevant beyond unearthing resonances, e.g.

- $\gamma^* \rightarrow \pi\pi$  – big contribution to hadronic vacuum polarization

[ $g - 2$  plenary talks later today by El-Khadra & Toth]

- $N \rightarrow \Delta$  – input for realistic nuclear response functions
- $NN$  (and  $3N$ ?) interactions – rooting nuclear physics in QCD
- controlling excited-state effects in nuclear structure

[talk by Barca Mon 13:00]

# SCATTERING AMPLITUDES FROM LATTICE QCD

## Two particle states on a torus and their relation to the scattering matrix

Martin Lüscher (DESY)

Oct, 1990

48 pages

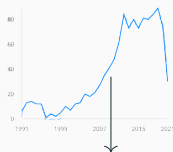
Published in: Nucl.Phys.B 354 (1991) 531-578

DOI: 10.1016/0550-3213(91)90366-6

Report number: DESY 90-131

View in: AMS MathSciNet

Citations per year



*distillation*

[Peardon et al. 0905.2160]

## The Nuclear Force from Lattice QCD

N. Ishii, S. Aoki, T. Hatsuda

Nov, 2006

4 pages

Published in: Phys.Rev.Lett. 99 (2007) 022001

e-Print: nucl-th/0611096 [nucl-th]

DOI: 10.1103/PhysRevLett.99.022001

Report number: UTCCS-P-30, TKYNT-06-19, UTHEP-534

View in: ADS Abstract Service

Citations per year



2-particle channel  
partial wave  
(total angular mom.)

$$\det [\mathcal{M}^{-1}(E_L) + F(E_L, L)] = 0$$

$E_L$  – FV spectrum  
 $\mathcal{M}$  – 2-to-2 scatt. ampl.  
 $F$  – known functions

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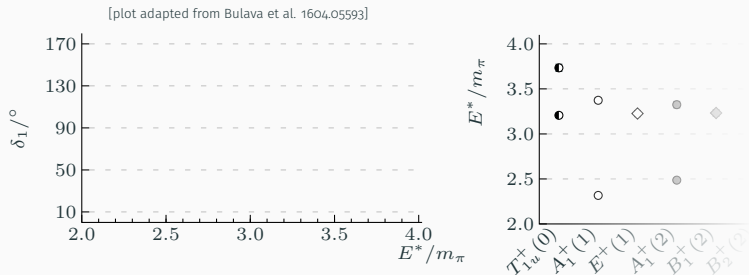
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 $F$  – known functions



# A SIMPLE RESONANCE: $\rho(770)$

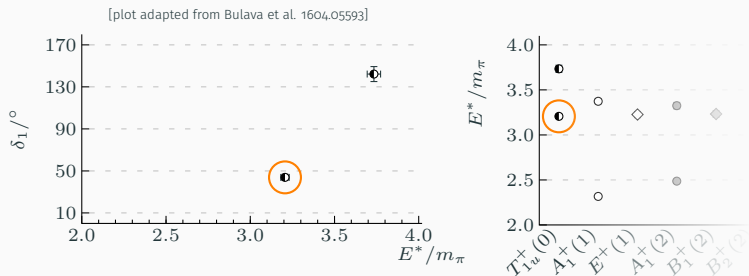


- elastic  $\pi\pi$  scattering neglecting  $\ell \geq 3$  partial wave spectrum  $\Leftrightarrow$  scattering amplitude
- benchmark system for the lattice

e.g. Lang et al. 1105.5636, Aoki et al. 1106.5365, ..., Dudek et al. 1212.0830, ...

- important contribution to hadronic vacuum polarization

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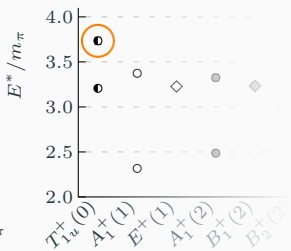
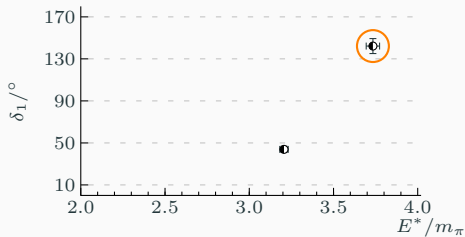
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[plot adapted from Bulava et al. 1604.05593]

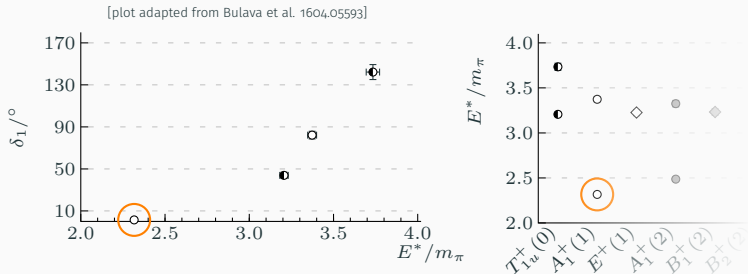


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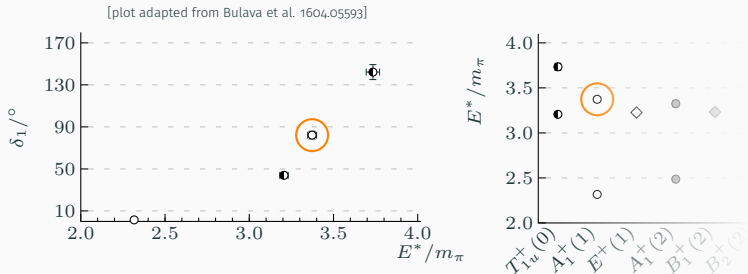


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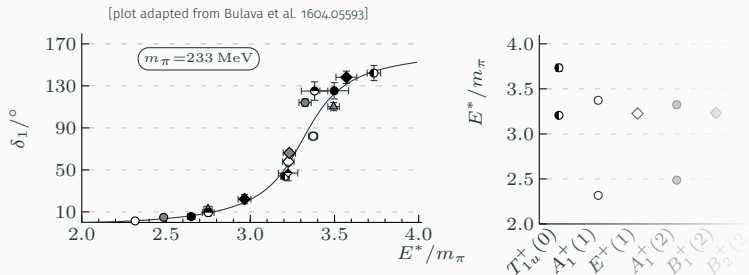


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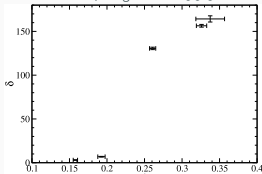
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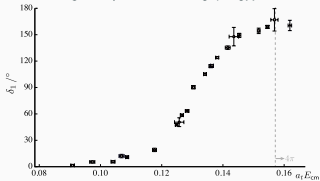
- important contribution to hadronic vacuum polarization

# INCOMPLETE SELECTION OF LATTICE DETERMINATIONS: $\rho(770)$

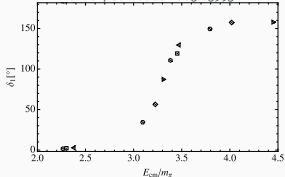
266 MeV, Lang et al. 1105.5636



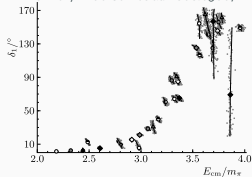
236 MeV, Wilson et al. 1507.02599



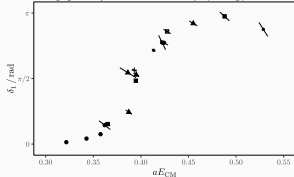
226 MeV, Guo et al. 1605.03993



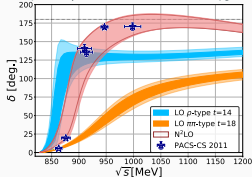
220 MeV, Andersen et al. 1808.05007



315 MeV, Werner et al. 1907.01237



411 MeV, Akahoshi et al. 2106.08175



HALQCD

[talk by Akahoshi Thu 22:15pm]

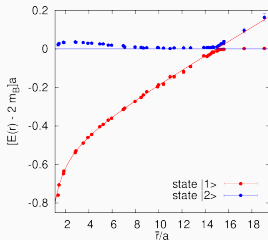
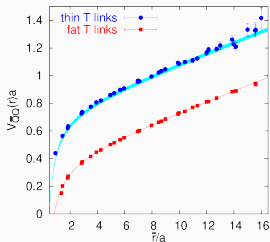
# ASIDE: FINITE-VOLUME SPECTROSCOPY

$$\langle 0|O_i(t)O_j^\dagger(0)|0\rangle = \sum_n \langle 0|O_i|n\rangle \langle n|O_j^\dagger|0\rangle e^{-E_n t}$$

Finite-volume spectrum from two-point correlation functions

Garbage In, Garbage Out  $\leftrightarrow$  'first principles'?

## String Breaking



[Bali et al. hep-lat/0505012]

only  $Q\bar{Q}$  interpolator vs.  $Q\bar{Q}$  & 'meson-meson-like'

[talks by Mueller Wed 6am, Knechtli Wed 2:15pm]

## ASIDE: FINITE-VOLUME SPECTROSCOPY

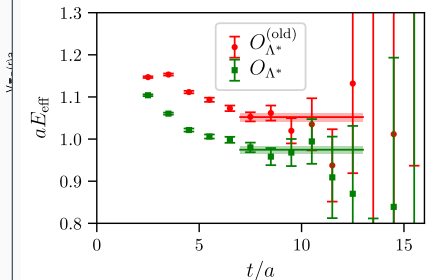
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String Breaking

$\Lambda(1520)$



interpolator with spatial structure required to obtain good overlap with state resembling  $\Lambda(1520)$

[Meinel, Rendon 2009.09313  
Engel et al. 1212.2032  
Edwards et al. 1212.5236]

## ASIDE: FINITE-VOLUME SPECTROSCOPY

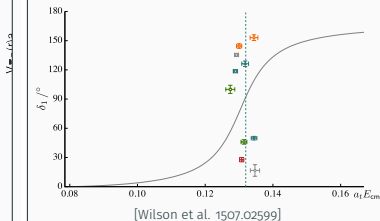
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Finite-volume spectrum from two-point correlation functions

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String Breaking

$\Lambda(1520)$   
 $\rho-\pi\pi$



with only  $\rho$ -like interpolators:  
energy levels around the  
resonance region, but not the  
correct spectrum

## ASIDE: FINITE-VOLUME SPECTROSCOPY

$$\langle 0|O_i(t)O_j^\dagger(0)|0\rangle = \sum_n \langle 0|O_i|n\rangle \langle n|O_j^\dagger|0\rangle e^{-E_n t}$$

Finite-volume spectrum from two-point correlation functions

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String Breaking

$\Lambda(1520)$

$\rho-\pi\pi$

...and yet to be settled: tetraquarks, Roper, **deuteron**?

- with insufficient set of operators
  - best-case scenario: cleanly miss an eigenstate
  - worst-case scenario: exponentials conspire to yield 'in-between' levels
- best-effort basis
  - variational setup with diverse set of operators
  - aided by flexible methods for treating quark propagation
    - new entry: sparsened all-to-all quark propagators

[talk by Wagman Tue 2pm]



meson-meson precision calculations

achieving FLAG-level rigor

more complicated meson-meson systems

chasing coupled-channel resonances

meson-baryon calculations

finding the  $\rho$  of baryons

baryon-baryon calculations

does the deuteron bind at heavier pion masses?

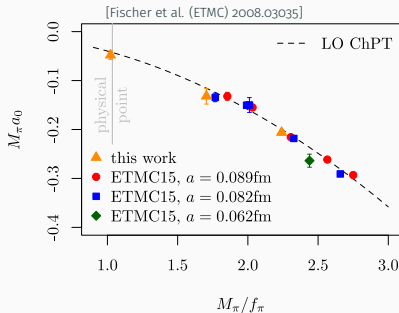
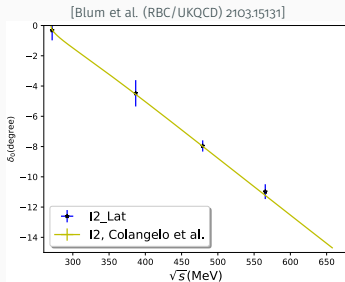
cutoff effects

our favorite nuisance

three-hadron calculations

new frontiers

$$I = 2 \pi \pi$$



- (among) first applications of finite-volume formalism  
but little attention aside from validation

- now more data as a ‘byproduct’

$$K \rightarrow \pi \pi$$

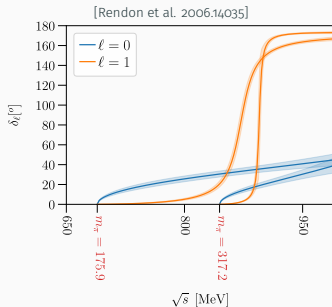
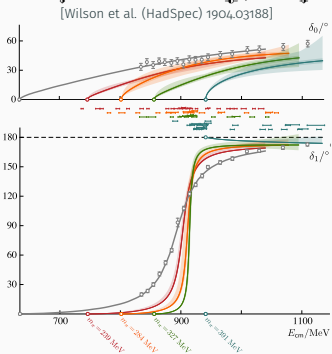
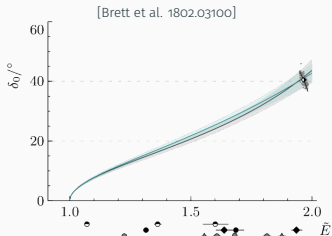
$$3\pi \quad (\text{increased sensitivity})$$

[talk by Wang Tue 21:15]

[talks by Hanlon Thu 13:45, Brett Thu 14:15]

- compilation might soon become feasible

# $I = 1/2$ $K\pi$ SCATTERING



- $K^*(892)$  and  $\kappa$  resonances  
(flavor physics with multi-hadron final states)
- precision test at DIRAC experiment  
(with  $I = 3/2$  scattering length) [1605.06103]
- more data incoming  
(talks by Lachini Tue 5:30, Asmussen Tue 5:45)

# SCATTERING WITH COUPLED CHANNELS

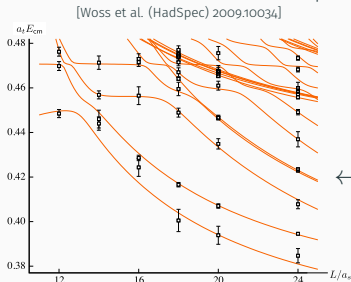
- most resonances have more than one decay mode
- no simple one-to-one between data and amplitude

parametrize scattering amplitude and fit spectrum

obtain amplitude

$$\det [\mathcal{M}^{-1}(E_L) + F(E_L, L)] = 0$$

predict spectrum



- very challenging
  - phenomenology  $\leftrightarrow$  parametrizations
  - implementation of fits

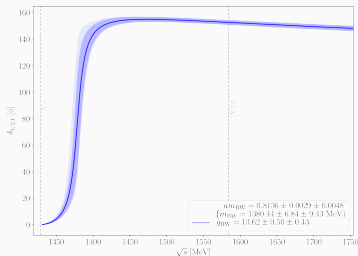
← eight channels [SU(3)-symmetric]  
*Decays of an exotic  $1^{-+}$  hybrid meson resonance*

[talk by Wilson Fri 7:30]

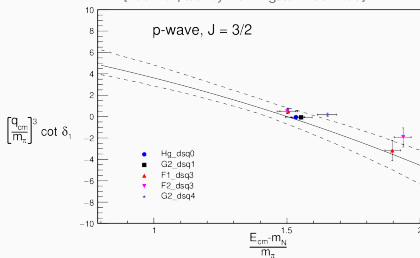
[other coupled-channel talks by Johnson Wed 22:00, Prelovsek Wed 7:30]

# MESON-BARYON SCATTERING

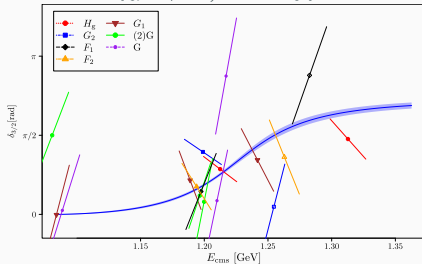
[255 MeV, Silvi et al. 2101.00689]



[200 MeV, talk by Morningstar Wed 21:00]



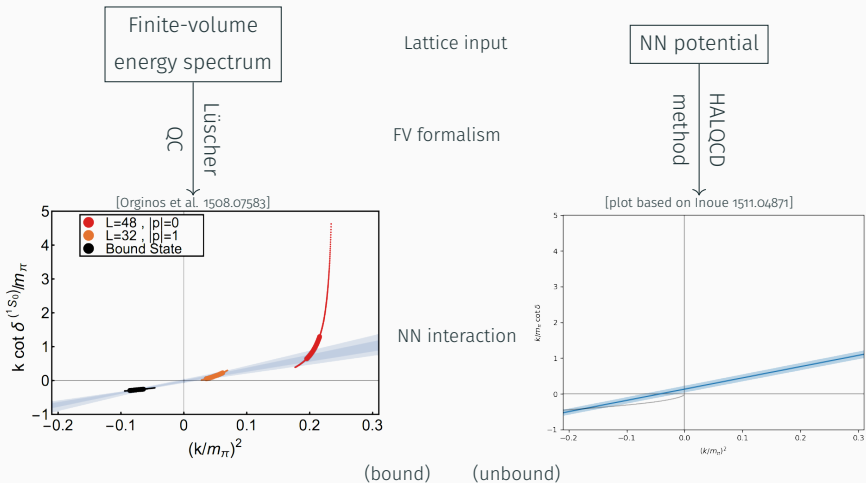
[139 MeV, talk by Pittler Tue 6:30]



- meson-baryon systems less studied  
proliferation of contractions  
(typically) worse signal-to-noise
- benchmark calculation:  $\Delta(1232)$   
virtually elastic in  $I=3/2 N\pi$
- ← same talk: first look at  $\Lambda(1405)$
- HALQCD [talk by Murakami Thu 21:00]

# BARYON-BARYON INTERACTIONS

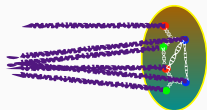
Are there two-nucleon bound states at  $m_\pi \approx 450$  MeV?



(similar disagreement for deuteron and  $m_\pi \gtrsim 700$  MeV)

# WHAT'S THE RIGHT WAY TO CREATE A DEUTERON?

[illustrations courtesy of M. Wagman]

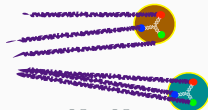


$N_x N_x$

point sources

NPLQCD

[Beane et al. 1012.3812]

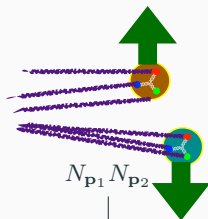


$N_{x_1} N_{x_2}$

displaced point sources

CalLat

[Berkowitz et al. 1508.00886]



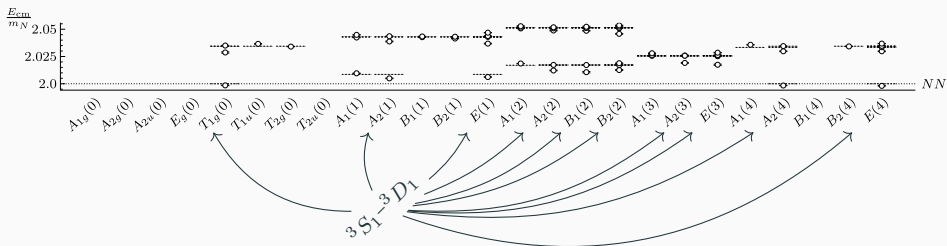
$N_{p_1} N_{p_2}$

'all-to-all'

(Mainz)

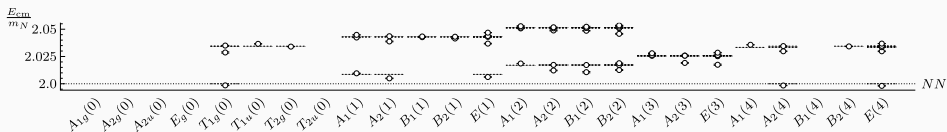
[Francis et al. 1805.03966]

# $I = 0$ NN AT $m_\pi \approx 710$ MeV WITH STOCHASTIC LAPH

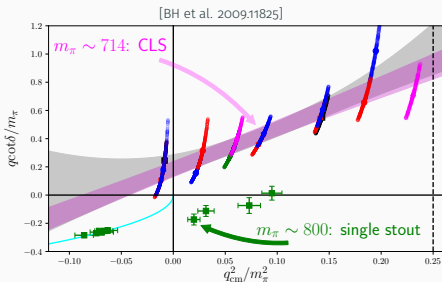


variational analysis using  
momentum-projected two-nucleon operators

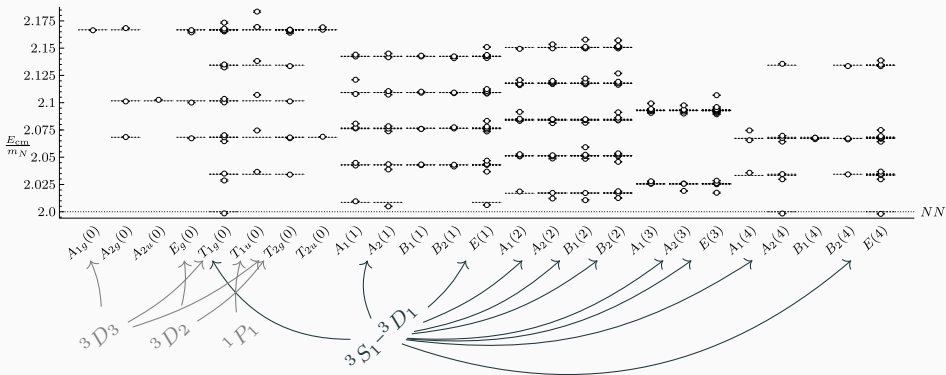
# $I = 0$ NN AT $m_\pi \approx 710$ MeV WITH STOCHASTIC LAPH



(s-wave only)  $\rightarrow$   
disfavors bound state



# $I = 0$ NN AT $m_\pi \approx 710$ MeV WITH STOCHASTIC LAPH



(only shown for total zero momentum)

ongoing: amplitude analysis including higher partial waves

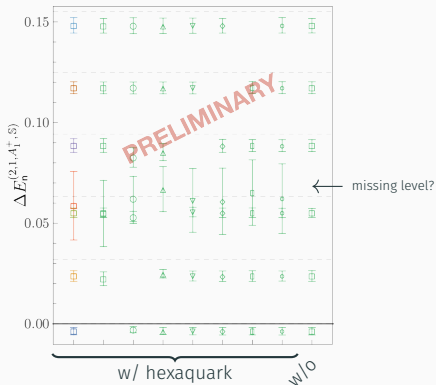
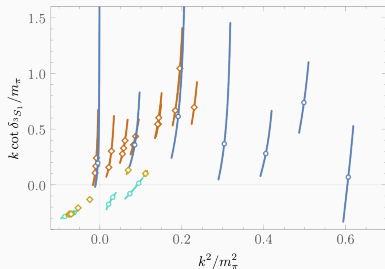
other spectroscopy methods on same ensemble

[talk by Nicholson Tue 14:15]

# NPLQCD UPDATE AT THIS CONFERENCE

[talk by Wagman Tue 14:00 (plots adapted)]

○ This work   ◇ Hörz *et al.* 21   ○ NPLQCD 17   ◇ CalLat 17



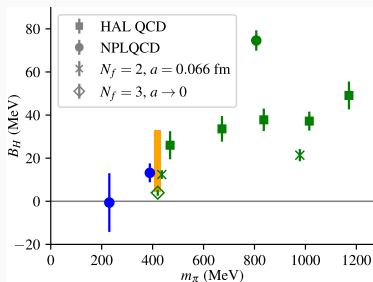
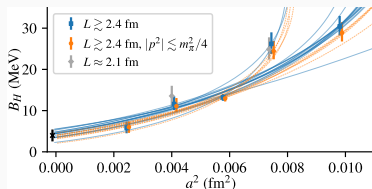
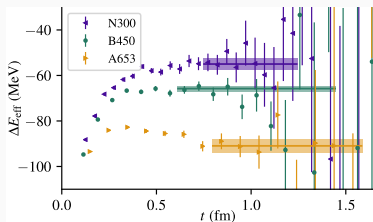
- sparsened all-to-all propagators [Detmold et al. 1908.07050]
- localized **and** momentum-projected two-nucleon operators
- additional level with hexaquark-like operator  
silver lining: cleanly missed  
volume/momentum dependence could elucidate

# A SURPRISING CASE OF CUTOFF EFFECTS

Weakly bound  $H$  dibaryon from  $SU(3)$ -flavor-symmetric QCD

[Green et al. 2103.01054]

Jeremy R. Green\*   Andrew D. Hanlon†   Parikshit M. Junnarkar‡   Hartmut Wittig§



- binding energy 8x larger at coarsest  $a$
- $a^2$  scaling at finer lattice spacings
  - consistent extrapolations at level of
    - energy levels
    - amplitude parameters
- Is  $\Lambda\Lambda$  special?
  - What to expect for  $NN$ ?

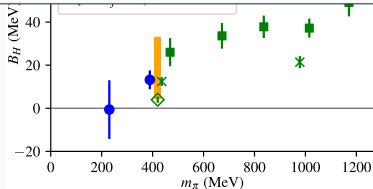
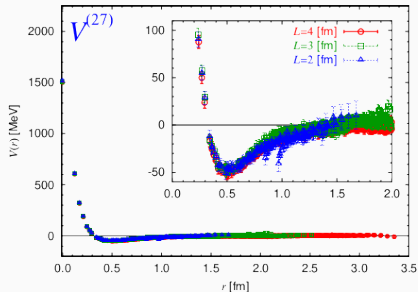
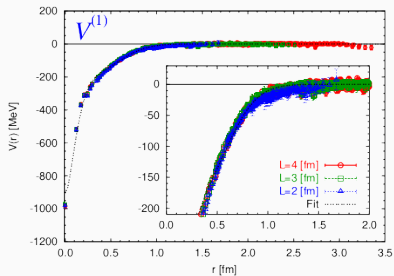
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Weakly bound  $H$  dibaryon from  $SU(3)$ -flavor-symmetric QCD

[Green et al. 1002.0105]

HALQCD potentials at  $m_\pi \approx 1$  GeV

[Inoue et al. 1012.5928]



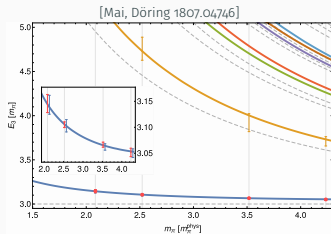
consistent extrapolations at level of

- energy levels
- amplitude parameters

• Is  $\Lambda\Lambda$  special?

What to expect for  $NN$ ?

# A NEW FRONTIER: THREE-HADRON SYSTEMS



- formalism developed over the last decade  
scattering matrices  $\Rightarrow$  three-particle spectrum

[Polejaeva, Rusetsky 1203.1241]

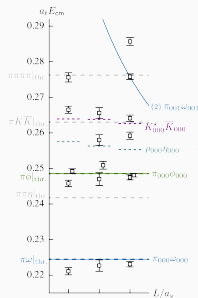
- three approaches [\*a\* review: Hansen, Sharpe 1901.00483]
  - relativistic field theory (RFT)
  - finite-volume unitarity (FVU)
  - nonrelativistic effective field theory (NREFT)

- ← limited lattice data for many years  
three-pion ground states for various pion masses

[Beane et al. 0710.1827; Detmold et al. 0803.2728]

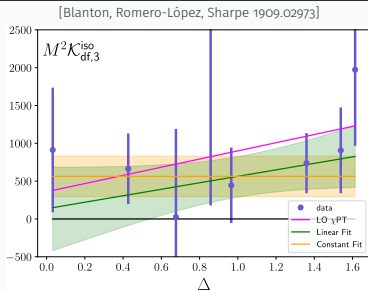
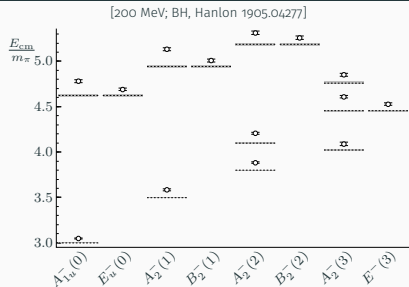
- three-body energies in HadSpec  $b_1$  study  $\rightarrow$   
out of reach for rigorous analysis
- now several studies of three equal hadrons

$$\begin{array}{l}
 I=3 \quad 3\pi \\
 I=3/2 \quad 3K \\
 I=1 \quad 3\pi \quad a_1(1260) \quad [3 \text{ weeks ago}]
 \end{array}
 \left. \vphantom{\begin{array}{l} I=3 \\ I=3/2 \\ I=1 \end{array}} \right\} \text{weakly interacting}$$



[Woss et al. (HadSpec) 1904.04136]

# MAKING SENSE OF THREE-HADRON SPECTRA



- energy shifts dominated by two-hadron dynamics  
need control to tease out three-hadron interaction
- fits to  $2\pi$  and  $3\pi$  spectra  
three-hadron interaction

$$\mathcal{K}_{df,3} = \mathcal{K}^{\text{iso},0} + \mathcal{K}^{\text{iso},1} \Delta, \quad \Delta = (E_{\text{cm}}^2 - 9m_{\pi}^2)/9m_{\pi}^2$$

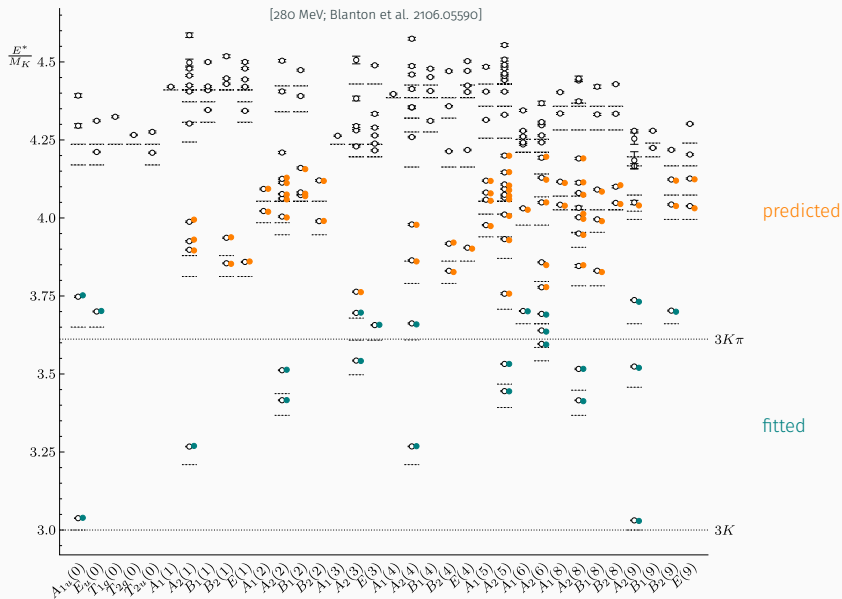
- three different analyses yield good description of spectrum

Blanton 1909.02973

Mai et al. 1909.05749

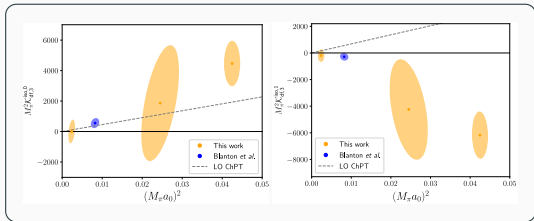
Guo, Long 2002.09266

# THREE-BODY QUANTIZATION CONDITION WORKS!

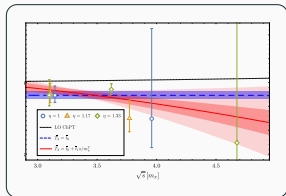


# MORE THREE PIONS

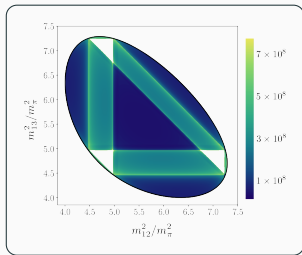
[Fischer et al. 2008.03035]



[220 MeV, Brett et al. 2101.06144]



[391 MeV; Hansen et al. 2009.04931]



chiral dependence of  $\mathcal{K}_{df,3}$   
(ETMC)

FVU three-body interaction  
(GWU)

three-body scattering amplitude  
(HadSpec)

... and a whole session on three hadrons  
including first peek at a resonance

[talk by Mai Thu 14:00]

# CONCLUSION

- spectrum of meson-meson calculations from precision to exploration
- meson-baryon calculations are picking up
- new chapter in baryon-baryon calculations  
mind those cutoff effects
- new frontier: three-hadron calculations
- enabled by the work of many on finite-volume formalism!  
two-body matrix elements, three-body  
([post/para-]Lüscher methods on the horizon?)

[talks by Bruno Tue 6:45, Garofalo Tue 6:15]