Muon g-2: BMW calculation of the hadronic vacuum polarization contribution

Bálint C. Tóth

Budapest–Marseille–Wuppertal-collaboration

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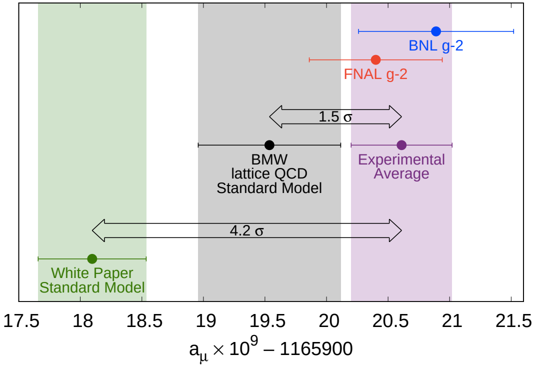
Sz. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato, K. K. Szabo, F. Stokes, B. C. Toth, Cs. Torok, L. Varnhorst

Related parallel talks:

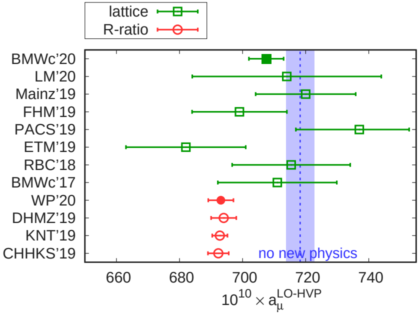
[F. Stokes, Mon 1:00pm EDT]
[K. Szabo, Mon 1:15pm EDT]
[L. Varnhorst, Mon 2:00pm EDT]
[L. Parato, Tue 6:15am EDT]

Overview



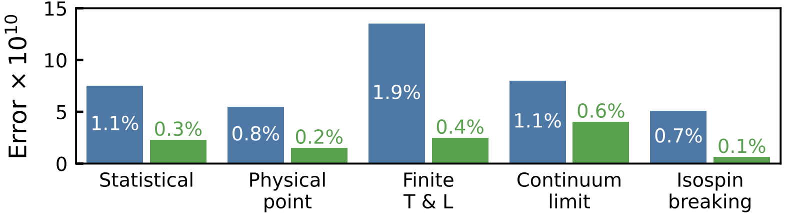
- 2.1 σ higher than R-ratio value [WP'20]
- Consistent with experiment within 1.5 σ

Comparison with other determinations of HVP



- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- Compatible with other lattice calculations
- First lattice calculation with errors comparable to R-ratio results

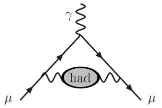
Key improvements

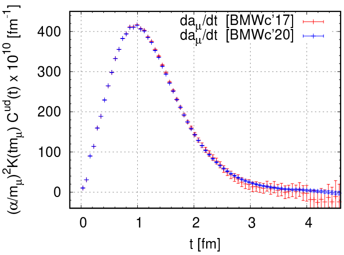


- Error reduction is essential to confirm or refute the existence of new physics
- Incorporated many improvements and recent developments in lattice techniques
- Reduced uncertainty by factor 3.4 compared to [BMWc '17]

$a_\mu^{\text{LO-HVP}}$ from lattice QCD

- $a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$





- $C(t)$: Current-current correlator

$$C(t) = \frac{1}{3}\sum_{i=1}^{3}\langle J_i(t)J_i(0)\rangle$$

- $K(t)$ describes the leptonic part of diagram [Bernecker,Meyer '11], [HPQCD'14], ...

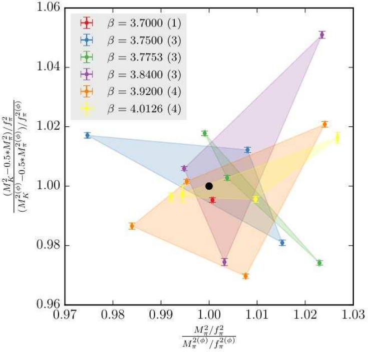
$$K(t) = \int_0^{Q^2_{\text{max}}} \frac{dQ^2}{m_\mu^2}\,\omega\!\left(\frac{Q^2}{m_\mu^2}\right)\left[t^2 - \frac{4}{Q^2}\sin^2\!\left(\frac{Qt}{2}\right)\right]$$

$$\omega(r) = \left[r + 2 - \sqrt{r(r+4)}\right]^2 / \sqrt{r(r+4)}$$

- only integrate up to $Q^2_{\text{max}} = 3\,\text{GeV}^2$
- $Q^2 > Q^2_{\text{max}}$: perturbation theory

Simulations

- Tree-level Symanzyk gauge action
- $N_f = 2 + 1 + 1$ staggered fermions
- stout smearing 4 steps, $\varrho = 0.125$
- $L \sim 6\,\text{fm}$, $T \sim 9\,\text{fm}$
- M_π and M_K are around physical point



β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980

- Ensembles for dynamical QED

β	a[fm]	$L \times T$	#conf
3.7000	0.1315	24×48	716
		48×64	300
3.7753	0.1116	28×56	887
3.8400	0.0952	32×64	4253

Challenges & Improvements

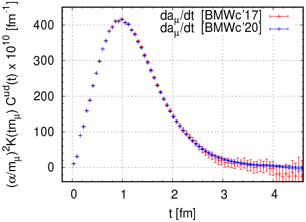
Noise reduction

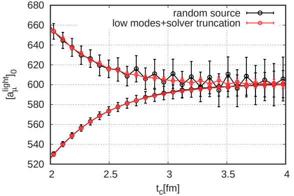
- Treat lowest eigenmodes of Dirac operator exactly (LMA)

[Neff *et.al.* 2001] [Giusti et.al. 2004] [Li *et.al.* 2010] ...

  - $L = 6\,\mathrm{fm}$ ≈ 1000 eigenvectors up to $\approx m_s/2$
  - $L = 11\,\mathrm{fm}$ ≈ 6000 eigenvectors

- Truncated solver method (AMA) [Bali *et.al.* 2010][Blum *et.al.* 2013]





- Replace $C(t)$ by upper/lower bounds above t_c [Lehner 2016] [Borsanyi *et.al.* 2017]

$$0 \leq C(t) \leq C(t_c)e^{-E_{2\pi}(t-t_c)}$$

$\longrightarrow$ factor 5 gain in precision

$\longrightarrow$ bounding t_c: $3\,\mathrm{fm} \to 4\,\mathrm{fm}$

$\longrightarrow$ few permil accuracy on each ensemble

Scale determination

Lattice spacing a enters into a_μ determination:

- physical values of m_μ, m_π, m_K

$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 1.8 \cdot \Delta(\text{scale})$ [Della Morte *et.al.* '17]

1. For final results: M_{Ω^-} scale setting $\longrightarrow$ $a = (aM_{\Omega^-})^{\text{lat}} / M_{\Omega^-}^{\text{exp}}$
   Experimentally well known: 1672.45(29) MeV [PDG 2018]
   - 4-state fits + GEVP [Aubin & Orginos 2011] [DeTar & Lee 2015]
   - include all $\mathcal{O}(e^2)$ QED effects
   - $\approx 0.1\%$ precision on each ensemble

2. For separation of isospin breaking effects: w_0 scale setting
   No experimental value [Lüscher 2010] [BMWc 2012]
   $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

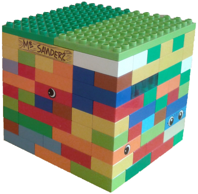
$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

More details: [L. Varnhorst, Mon 2:00pm EDT]

Finite-size effects

- Typical lattice runs use $L \lesssim 6\,\text{fm}$, earlier model estimates gave $O(2)\%$ FV effect

[Aubin *et.al.* '16]

$L_{\text{ref}} = 6.272\,\text{fm}$   $L_{\text{big}} = 10.752\,\text{fm}$

1. $a_\mu(\text{big}) - a_\mu(\text{ref})$
   - perform numerical simulations in $L_{\text{big}} = 10.752\,\text{fm}$
   - perform analytical computations to check models

lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\text{stat}}(1.4)_{\text{cont}}$	11.6	15.7	17.8	16.7	15.2

[Gounaris & Sakurai '68][Lellouch & Lüscher '01][Bernecker & Meyer '11]
[Hansen & Patella '19, '20] [Chakraborty *et.al.* '17]

2. $a_\mu(\infty) - a_\mu(\text{big})$
   - NNLO XPT: $0.6(0.3)$ [Aubin *et.al.* '20]

$$a_\mu(\infty) - a_\mu(\text{ref}) = 18.7(2.0)_{\text{stat}}(1.4)_{\text{cont}}(0.3)_{\text{big}}(0.6)_{I=0}(0.1)_{\text{qed}}[2.5]$$

More details: [F. Stokes, Mon 1:00pm EDT]

QCD+QED

- Reach sub-percent level: include isospin breaking effects for
  - $\langle jj \rangle$
  - masses
  - scale
- Rewrite dynamical QED as quenched QED expectation values

$$\langle \mathcal{O} \rangle_{\text{QCD+unquenched QED}} = \frac{\left\langle \left\langle \mathcal{O}(U,A) \frac{\det M(U,A)}{\det M(U,0)} \right\rangle_{\text{quenched QED}} \right\rangle_{\text{QCD}}}{\left\langle \left\langle \frac{\det M(U,A)}{\det M(U,0)} \right\rangle_{\text{quenched QED}} \right\rangle_{\text{QCD}}}$$

- Take isospin symmetric gluon configurations: U
- Compute derivatives

$$m_l \frac{\partial X}{\partial \delta m} \qquad \frac{\partial X}{\partial e} \qquad \frac{1}{2} \frac{\partial^2 X}{\partial e^2}$$

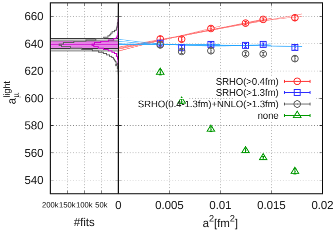
- Hybrid approach:
  - sea effects: derivatives [De Divitiis *et.al.* 2013]
  - valence effects: finite differences [Eichten *et.al.* 1997]

More details: [L. Parato, Tue 6:15am EDT]

Continuum limit – Taste improvement

Controlled $a \to 0$ extrapolation

- 6 lattice spacings: 0.132 fm $\longrightarrow$ 0.064 fm
- Leading cutoff effects at large t are taste breaking effects $\longrightarrow$ mass effects
- Distortion in spectrum: cured by taste improvement rho-pion-gamma model (SRHO)
  [Sakurai '60][Bijnens *et.al.* '99][Jegerlehner *et.al.* '11][Chakraborty *et.al.* '17]
- Our data confirms: Taste violation according to SRHO describes most of the lattice artefacts in a_μ^{light}



- Central value obtained using SRHO improvement
- At $t > 1.3$ fm add and subtract (NNLO – SRHO) [Aubin *et.al.* '20]
- Error corresponding to this variation $\longrightarrow$ Add to systematic error in quadrature

More details: [K. Szabo, Mon 1:15pm EDT]

Continuum limit – Global fit procedure

- For full result: physical point is set via

$$M_{\Omega_-},\quad M^2_{K_\chi}=\frac{1}{2}\left(M^2_{K_0}+M^2_{K_+}-M^2_{\pi_+}\right),\quad \Delta M^2_K,\quad M^2_{\pi_0} \qquad \longleftarrow \text{Type-I}$$

- For IB-decomposition: match QCD+QED and $\mathrm{QCD}_{\mathrm{iso}}$ via

$$w_0,\quad M^2_{ss},\quad \Delta M^2=M^2_{dd}-M^2_{uu},\quad M^2_{\pi_\chi}=\frac{1}{2}\left(M^2_{uu}+M^2_{dd}\right) \qquad \longleftarrow \text{Type-II}$$

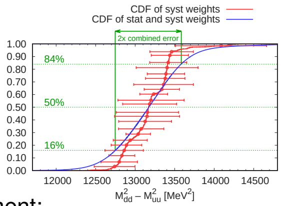
- Expand observable around physical point

$$Y=A+BX_l+CX_s+DX_{\delta m}+Ee_v^2+Fe_ve_s+Ge_s^2$$

- Combined χ^2 fit for all components

- Several hundreds of thousands of analyses, combined using histogram method

  linear vs. quadratic, a^2 vs $a^2\alpha_s(1/a)^3$ [Husung *et.al* 2020]
  cuts in lattice spacing, hadron mass fit ranges, . . .



- Uncertainty arising from choice of taste improvement: Added to systematic error in quadrature

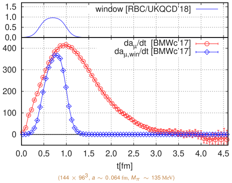
More details: [L. Varnhorst, Mon 2:00pm EDT]

Window observable

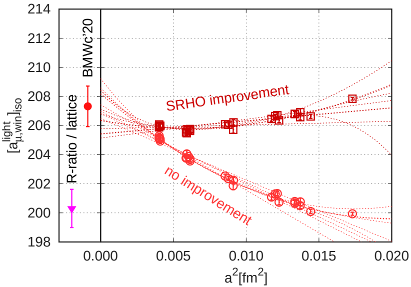
Window observable

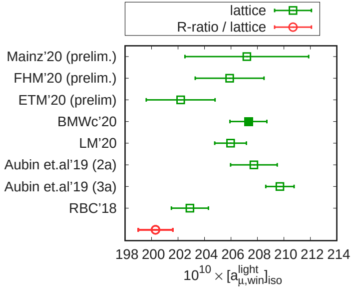
- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

[RBC/UKQCD'18]



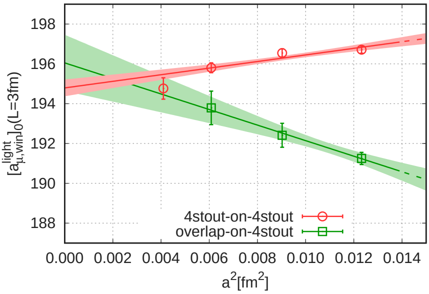
- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)





Overlap crosscheck

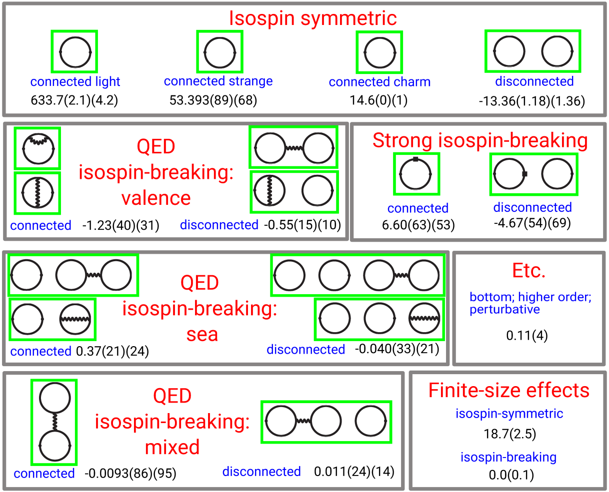
- $L = 3\,\text{fm}$
- Valence: overlap fermions, local current
- Sea: `4stout` staggered



- Continuum limit is consistent with staggered valence

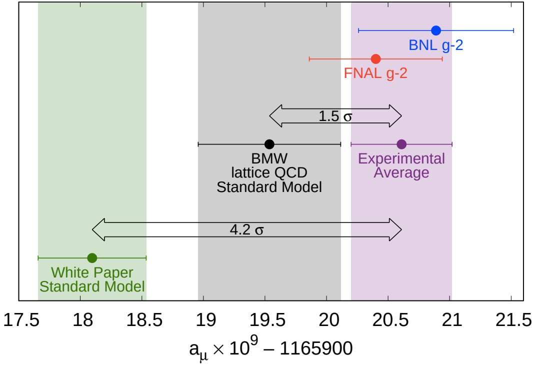
Conclusions

Summary of contributions to $a_\mu^{\text{LO-HVP}}$



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Conclusions



- Consistent with experiment within $1.5\,\sigma$
- $2.1\,\sigma$ higher than R-ratio value [WP'20]
- Important to have crosschecks from other lattice groups
- Important to understand disagreement with R-ratio, in particular in the window