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Recent Progress on Nucleon Form Factors

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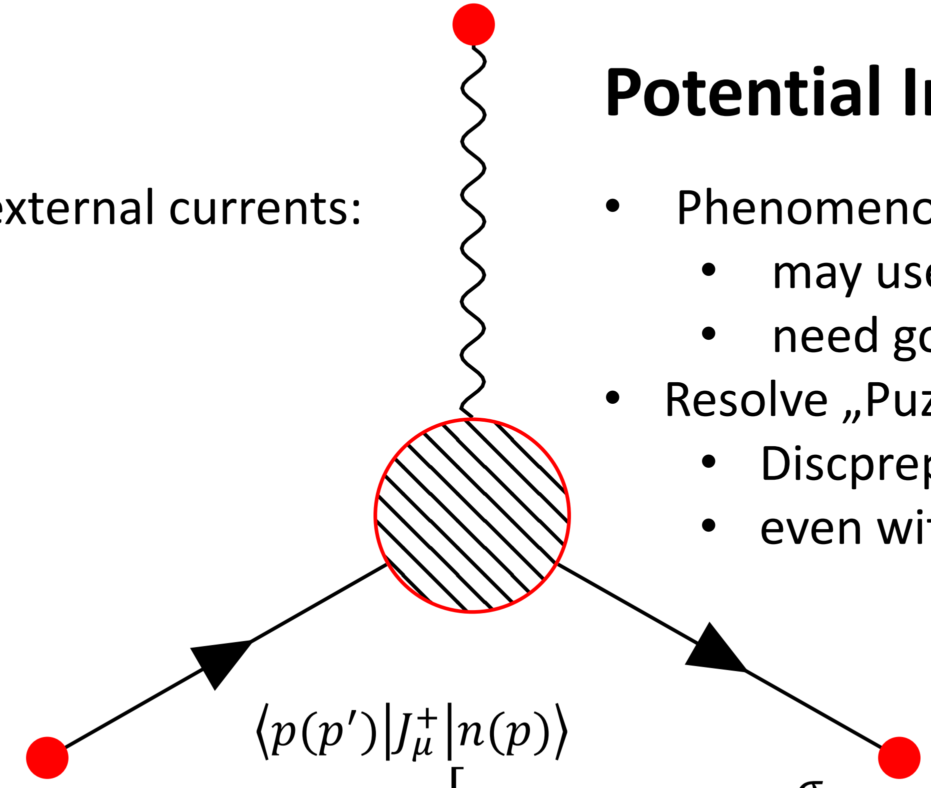
Form Factors

What are they?

- Parameterize response to external currents: Form Factors
- Form Factor universal: Enter many processes
- In principle measurable

Potential Impact of Lattice

- Phenomenology :
  - may use Lattice results as input
  - need good accuracy
- Resolve „Puzzles“:
  - Discprepancies between experiments
  - even with less accuracy

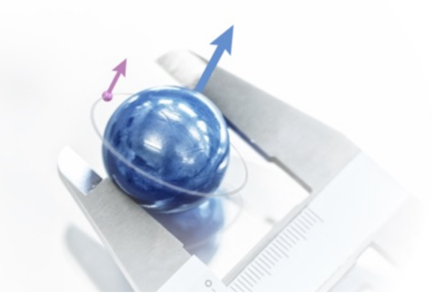


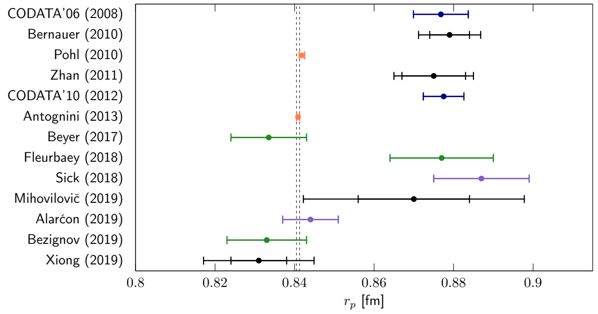
Example V-A current:

$$\langle p(p')|J_\mu^+|n(p)\rangle$$
$$= \overline{u^p}(p')\Big[\gamma_\mu F_1^{CC}(q^2) + i\,\frac{\sigma_{\mu\nu}}{2m_N}\,q^\nu F_2^{CC}(q^2) + \frac{q_\mu}{m_N}F_S^{CC}(q^2)$$
$$+ \gamma_\mu\gamma_5 F_A^{CC}(q^2) + \frac{\gamma_5 q_\mu}{m_N}F_P^{CC}(q^2)$$
$$+ \frac{\gamma_5(p'+p)_\mu}{m_N}F_T^{CC}(q^2)\Big]u^n(p)$$

Impact of Form Factors

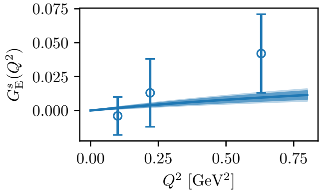
- Proton Radius Puzzle
  - » Provide ab-initio calculation





Taken from Bernauer, EPJ Web Conf. 234 (2020)

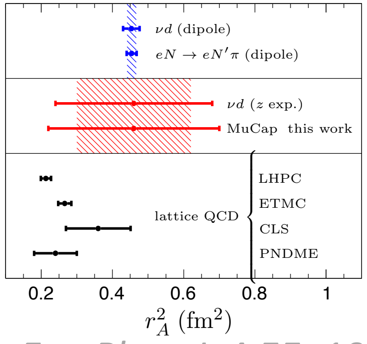
- Precision Tests of SM
  - » Via strangeness FF → Parity Violation Experiments
    - Lattice determinations of strange FF very precise



Taken from D.D. et al. *Phys.Rev.Lett.* 123 (2019) 21, 212001

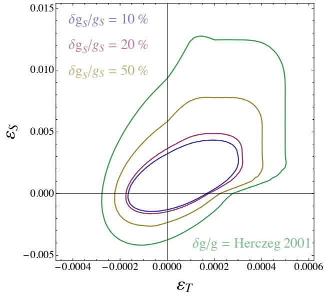
  - » Via axial FF → Vital input to neutrino-nucleus scattering
    - Lattice competitive to z-exp extractions of experiments

A. Meyer, Tue ID: 291



Taken from A. Kronfeld, et al. *Eur. Phys. J. A* **55,** 196 (2019)

  - » Via Charges → Constraining BSM EFT couplings



Taken from T. Bhattacharya et al., Phys. Rev. D85, 054512 (2012)

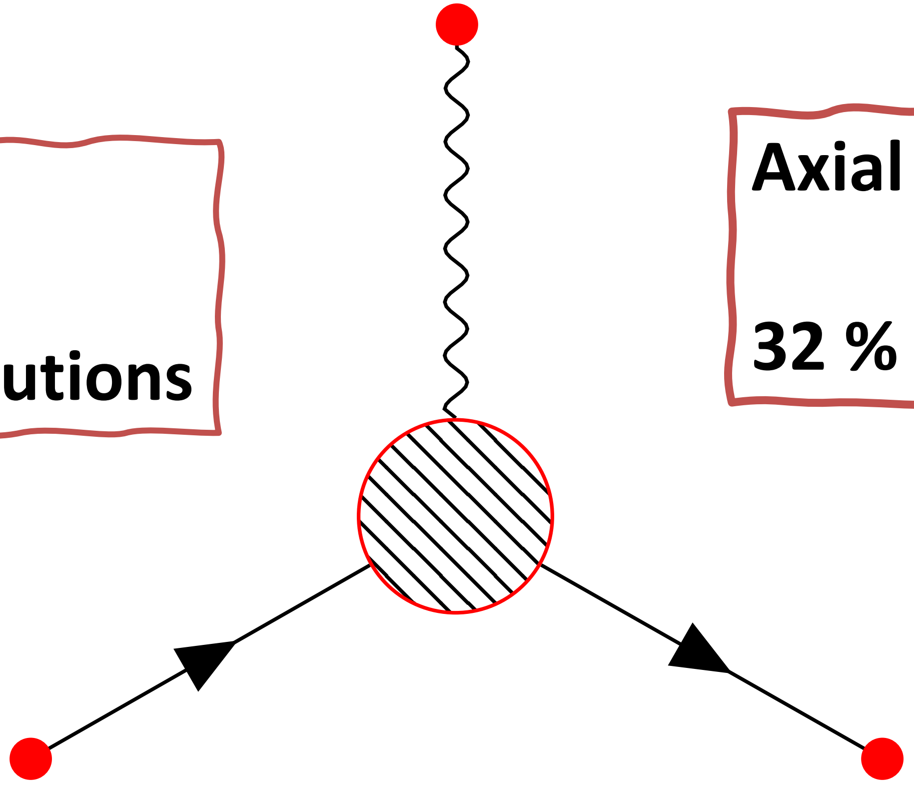
My apologies for omissions …

Vector FF:

37 % Contributions

Axial FF:

32 % Contributions



Charges and Sigma-Term:

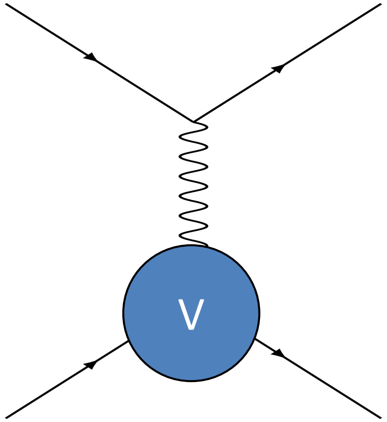
23 % Contributions

Gravitational FF:

8 % Contributions

Hacket, Wed ID: 605
Pefkou, Wed ID: 434

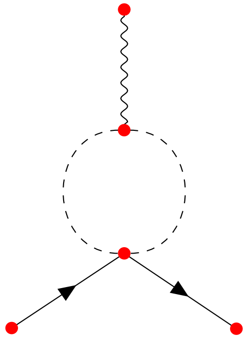
Nucleon Form Factors



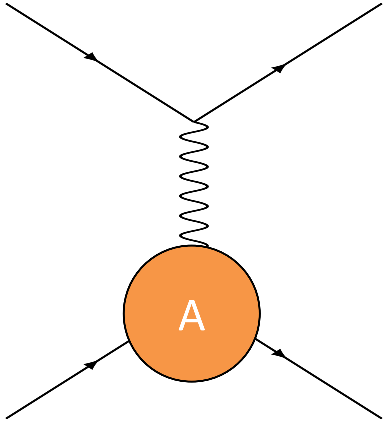
$$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle$$
$$= \bar{u}(p')\left[\gamma_\mu\, F_1^q(Q^2=-q^2) + i\,\frac{\sigma_{\mu\nu}}{2m_N}\, q^\nu\, F_2^q(Q^2)\right]u(p)$$
$$G_E(Q^2) = F_1^q(Q^2) - \frac{Q^2}{2m_N} F_2^q(Q^2) = \left(1 - \frac{1}{6}\langle r_E^2\rangle\, Q^2 + \cdots\right)$$
$$G_M(Q^2) = F_1^q(Q^2) + F_2^q(Q^2) = \mu\left(1 - \frac{1}{6}\langle r_M^2\rangle\, Q^2 + \cdots\right)$$

Dirac Pauli

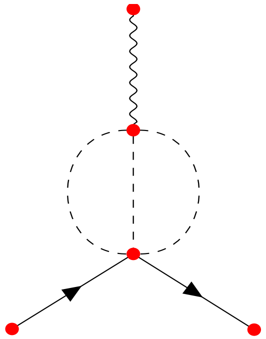
$$\langle r_{E/M}^2\rangle \xrightarrow{M_\pi\to 0} \{\ln M_\pi, \frac{1}{M_\pi}\}$$



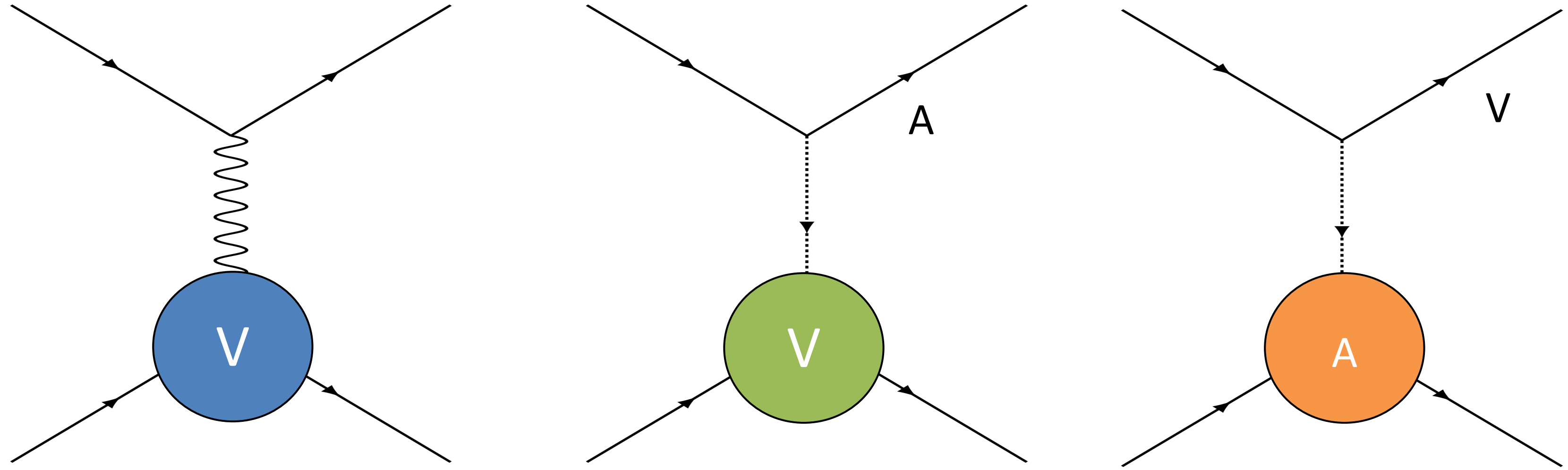
$$\langle r_A^2\rangle \xrightarrow{M_\pi\to 0} const.$$



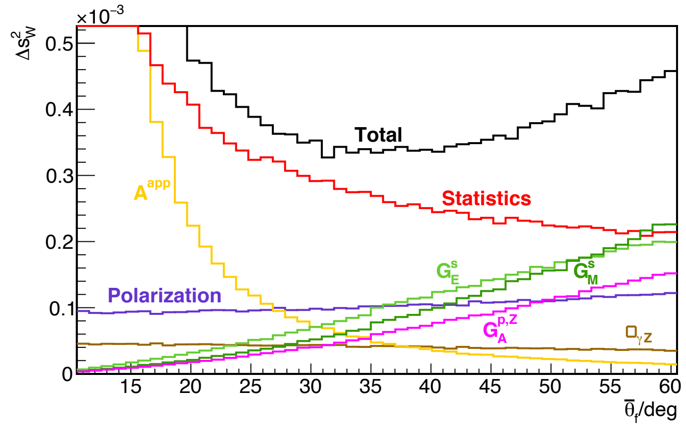
$$\langle N(p')|\bar{q}\gamma_\mu\gamma_5 q|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu\, G_A(Q^2) + i\,\frac{q^\mu}{2m_N}\, G_P(Q^2)\right]\gamma_5\,\frac{\tau^3}{2}u(p)$$
$$G_A(Q^2) = g_A\left(1 - \tfrac{1}{6}\langle r_A^2\rangle\, Q^2 + \cdots\right)$$
$$2m_N G_A(Q^2) - \frac{Q^2}{2\,m_N} G_P(Q^2) = \frac{2\,M_\pi^2 F_\pi}{M_\pi^2+Q^2} G_{\pi N}(Q^2)$$ (PCAC)



Parity Violation



Projected Error Budget for P2:



Taken from arxiv:1802.04759

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{\epsilon\, G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\sin^2\theta_W)\,\epsilon' G_M^\gamma\, G_A}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

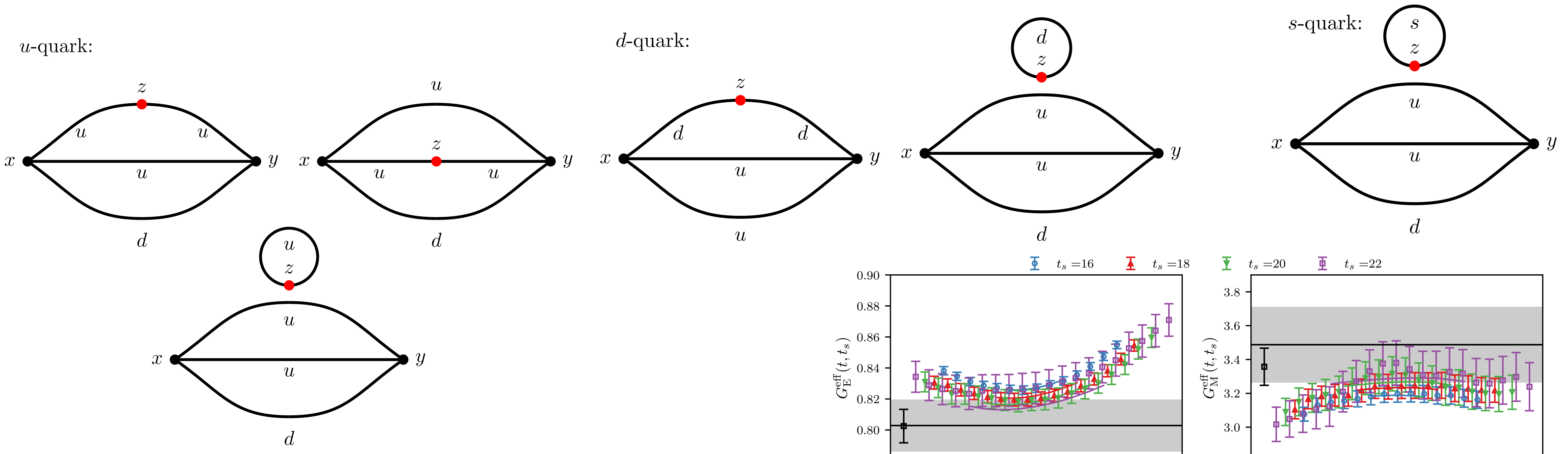
$$\tau = \frac{Q^2}{4\, m_N^2}, \epsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1}, \epsilon' = \sqrt{\tau(1+\tau)(1-\epsilon^2)}$$

$$G_{E/M}^{Z,p} = (1 - 4\sin^2\theta_W)\, G_{E/M}^{\gamma,p} - G_{E/M}^{\gamma,n} - G_{E/M}^{s}$$

- Sensitive to the Weak Charge
  Test of SM at low energies
- Need e/m FF (strange)
- Need axial FF (strange)

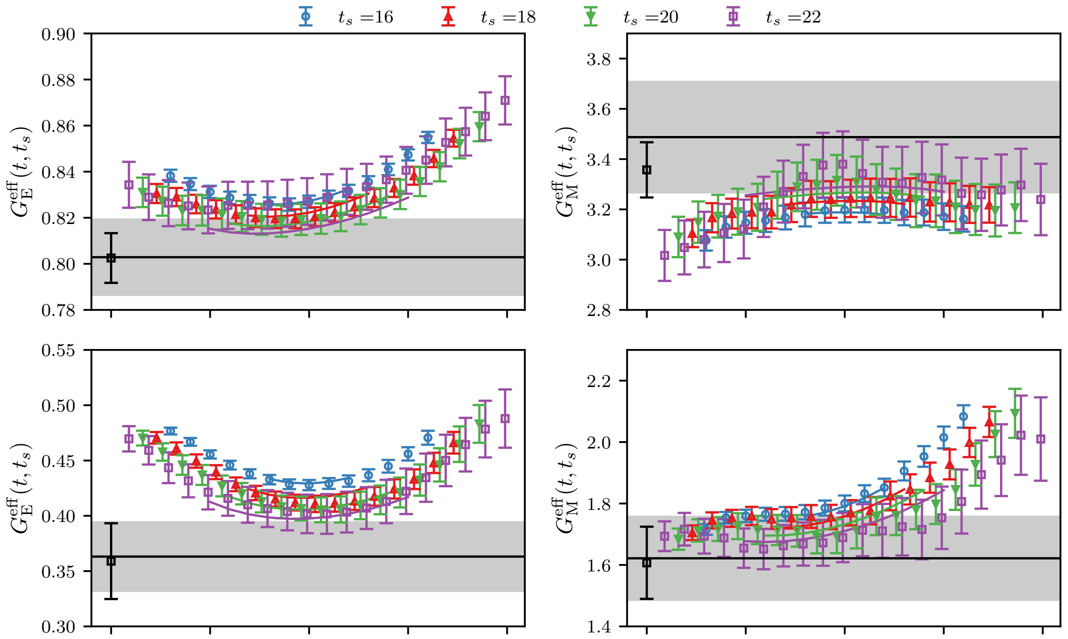
(Decomposition assuming Isospin Symmetry)

Calculational Setup Lattice



Ratio gives access to Form Factors:

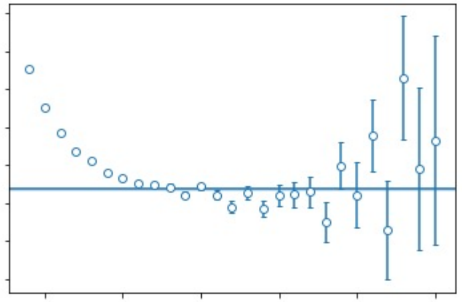
$$R^s_{V_\mu}(z_0,\boldsymbol{q};y_0,\boldsymbol{p}';\Gamma_\nu)=\frac{C^s_{3,V_\mu}(\boldsymbol{q},z_0;\boldsymbol{p}',y_0;\Gamma_\nu)}{C_2(\boldsymbol{p}',y_0)}\times\sqrt{\frac{C_2(\boldsymbol{p}',y_0)C_2(\boldsymbol{p}',z_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0\text{-}z_0)}{C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},y_0)C_2(\boldsymbol{p}'\text{-}\boldsymbol{q},z_0)C_2(\boldsymbol{p}',y_0\text{-}z_0)}}.$$

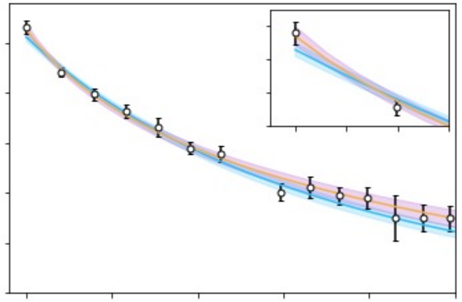


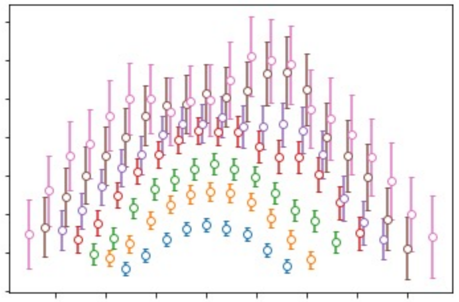
Example of these Ratios

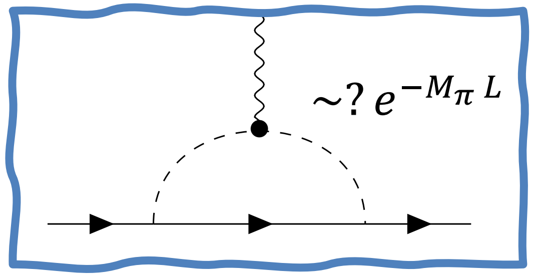
Sources of Uncertainty

- Statistical Accuracy
- Excited State Contamination
- Model Dependence
- Extrapolations
  - Chiral
  - Continuum
  - Finite Size



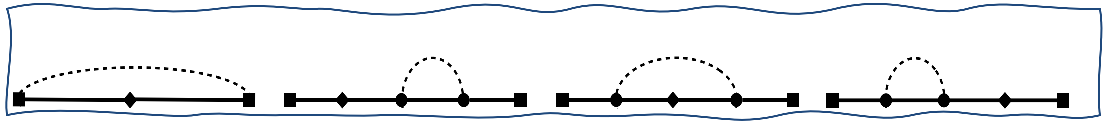






Aside: Can calculate Excited States directly in ChPT even for FF (reliable for large t_{sep})

O. Bär, Tue ID: 70



Taken from O. Bär, H.Colic, *Phys.Rev.D* 103 (2021) 11, 114514

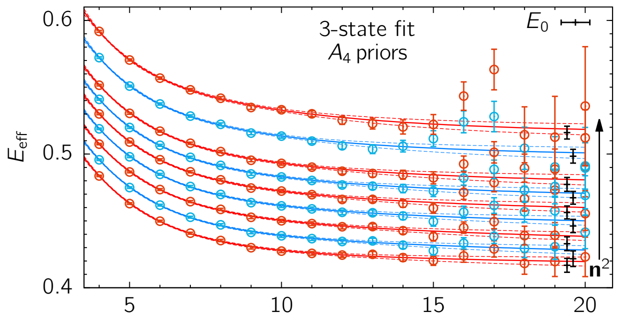
Excited States

- Write any correlation function generically as

$$C(t, t_{sep}) = a + b\, e^{-t\, \Delta E_1} + c\, e^{-(t_{sep}-t)\Delta E_1} + \cdots$$

- „Just“ do the fit (Multistate method)
- Summation method

$$\sum_{t=1}^{t_{sep}-1} C(t, t_{sep}) = a(t_{sep}-1) + \tilde{b} e^{-t_{sep}\Delta E_1} + \tilde{c} e^{-t_{sep}\Delta E_1} + \cdots$$

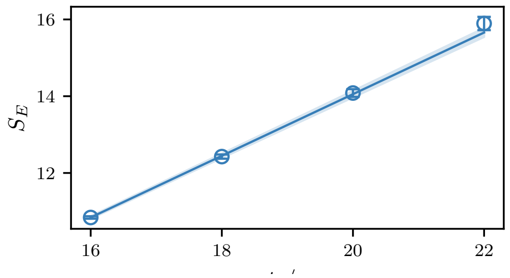


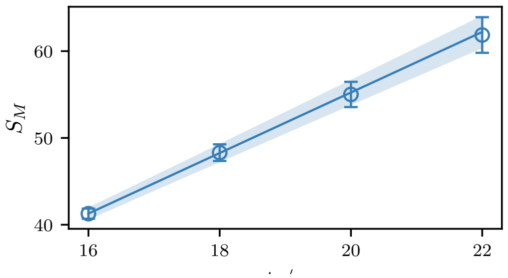
Taken from Y-C. Jang Talk

– Note that: $t_{sep} \gg t, (t_{sep} - t)$

– a = ground state ME

– b, $\tilde{b}$- and c, $\tilde{c}$ -terms ES

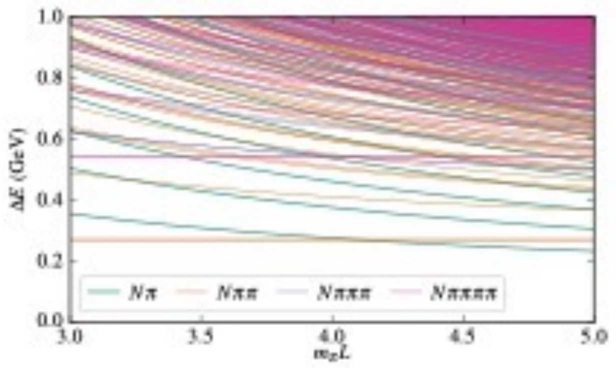




Multistate

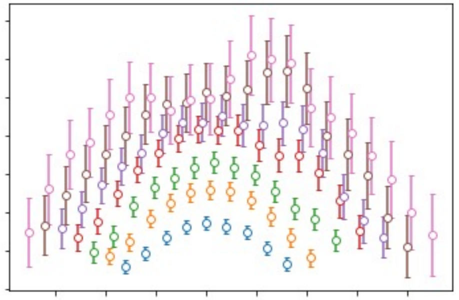
- Multistate Fits
  - How many states to include? (states become dense)
  - Use results from Spectrum?
  - Gaps universal (?)

    $\Longrightarrow$ Do Simultaneous Fits



Taken from J. Green arxiv:1812.10574

» Possibly (very) large covariance matrices
- Make this less demanding by taking 2pt-functions spectrum
  - as is or
  - as priors for 3pt-corr.



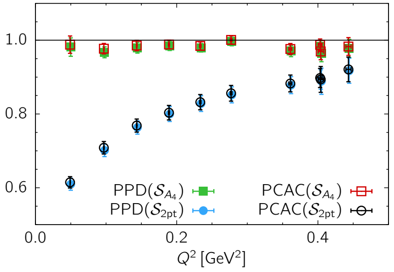
» Data might NOT constrain all parameters
$\Longrightarrow$ Stabilize via priors

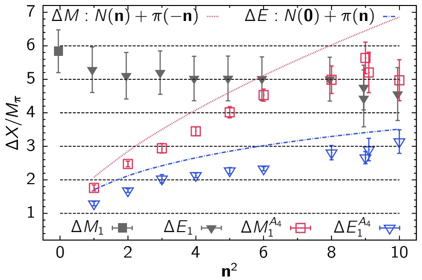
» Assumptions about universality between 2pt- and 3-pt might not be justified

See e.g. R. Gupta, Mon ID: 527

Multistate

- How do we know if asymptotics is reached?
  In general not easy to see
  - Axial Form Factors use PCAC as consistency check



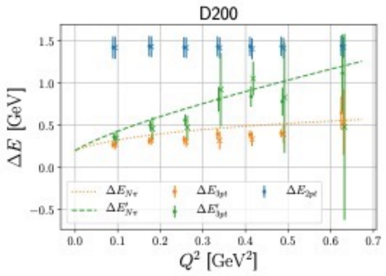


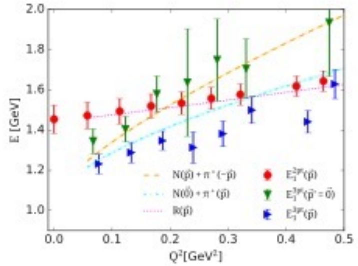
$$2\widehat{m}G_P(Q^2) = 2MG_A(Q^2) - \frac{Q^2}{2M}\tilde{G}_P(Q^2)$$

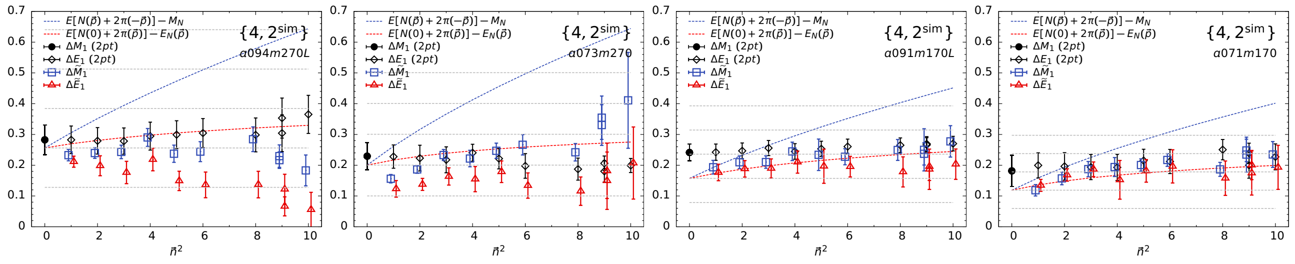
Y-C. Jang, R. Gupta, B. Yoon, T. Bhattacharya, Phys. Rev. Lett. 124, 072002 (2020)

Y-C. Jang, Tue ID:519

- Gaps might not be universal between 2-pt and 3-pt (Less severe for vector?)







Bali et. al (RQCD) JHEP05 (2020) 126

Alexandrou, et al. (ETMC) Phys.Rev.D 103 (2021) 3

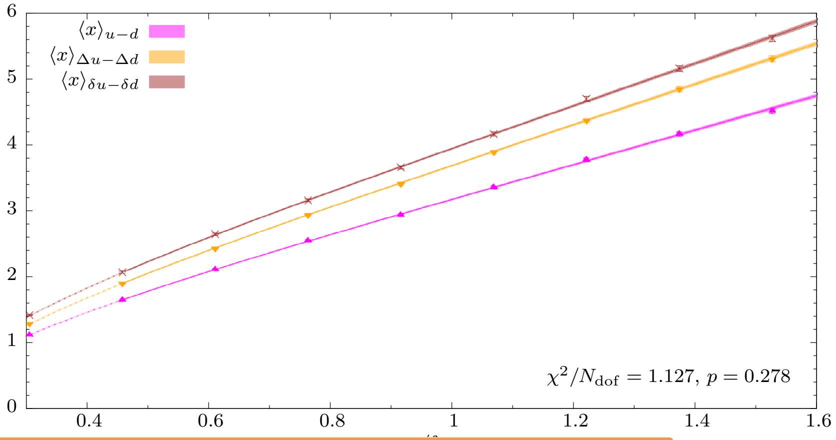
S. Park, et. al (NME) arXiv:2103.05599

G. Koutsou, Tue ID: 401

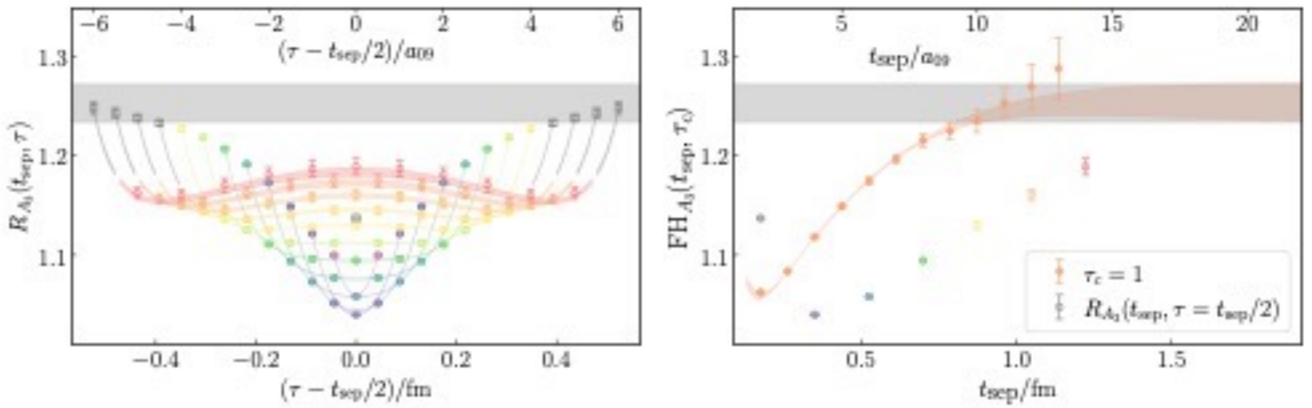
Summation

- Easy to apply but errors usually larger
- Supression of ES paramterically larger
  - » Use more t_{seps} in Fits (Deviation from linearity)

$$\sum_{t=1}^{t_f-1} C(t,t_f) = a(t_{sep}-1) - \tilde{b}e^{-t_{sep}\Delta E_1} - \tilde{c}e^{-t_{sep}\Delta E_1} + \cdots$$



Taken from K. Ottnad, Mon ID: 229



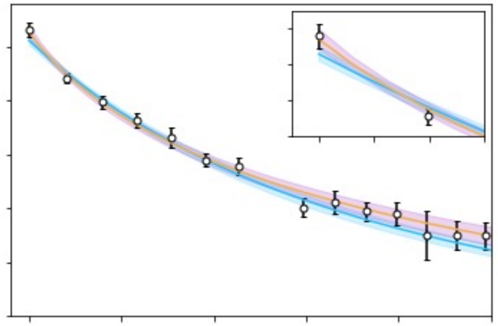
Taken from A. Walker-Loud, Mon ID: 612

CalLat: arXiv:2104.05226

Model Dependence

– Derived Quantities depend on model for Q^2 dependence

$$c^q\left(1-\frac{1}{6}\langle r_E^2\rangle\, Q^2+\cdots\right)$$



» Dipole is not very flexible!

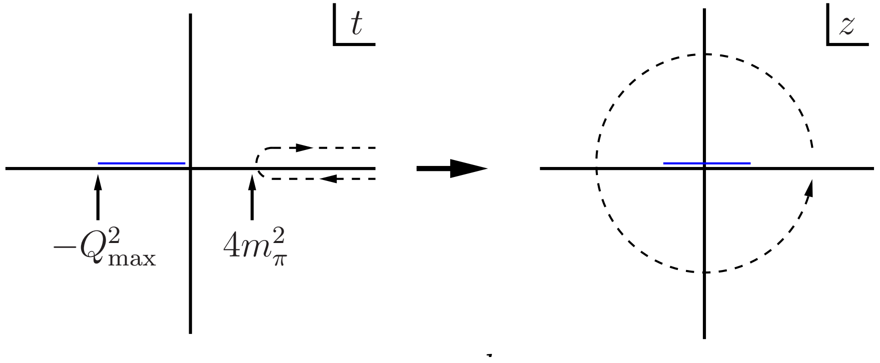
J. Bernauer et al., *Phys.Rev.C* 90 (2014) 1, 015206

….

» Use z-expansion, e.g.

$$G_{E/M}(Q^2)=\sum_{k=1/0}^{5} a_k^{E/M} z(Q^2)^k,$$

$$z(Q^2)=\frac{\sqrt{t_{\rm cut}+Q^2}-\sqrt{t_{\rm cut}}}{\sqrt{t_{\rm cut}+Q^2}+\sqrt{t_{\rm cut}}}.$$

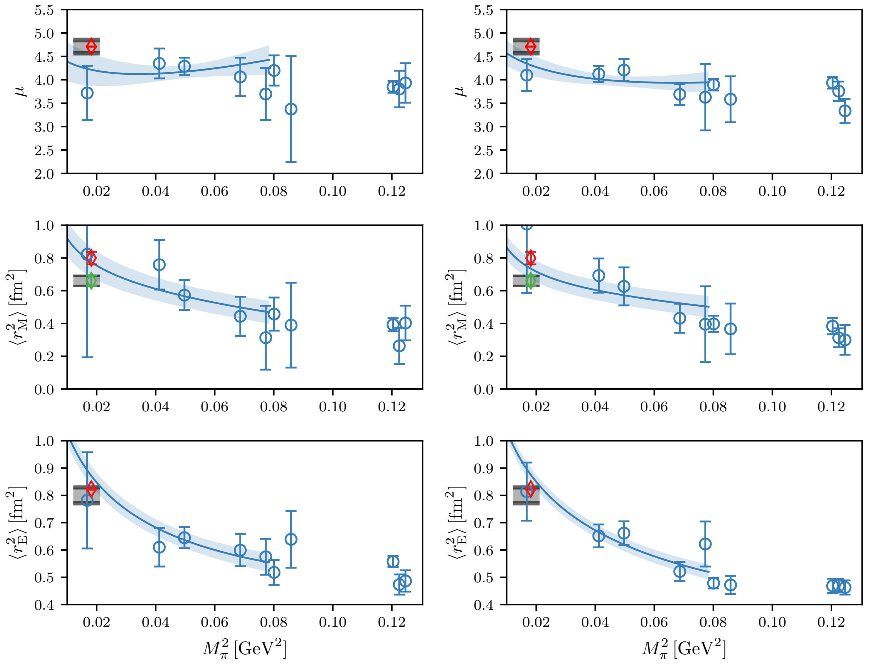


R. Hill, G. Paz, Phys. Rev. D 82 (2010), 113005

» Direct methods avoid this altogether!

- Most recently K-I. Ishikawa (PACS) arxiv:2107.07085

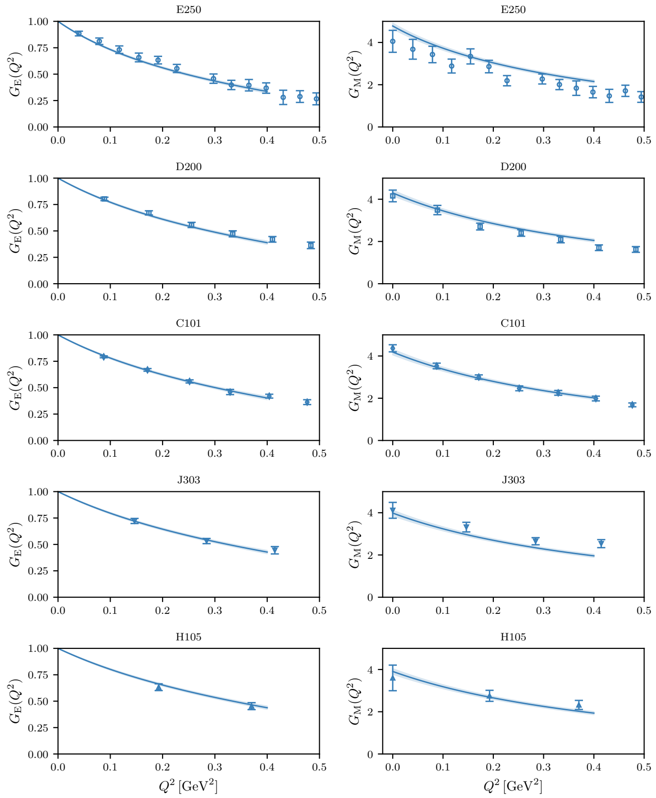
Model Dependence & CCF - Combine



D.D. et. al, Phys. Rev. D 103, 094522 (2021)

Direct Fits Using Chiral EFT

- Use Chiral EFT T. Bauer, et al, Phys. Rev. C86, 065206 (2012).
  Less freedom at small Q^2 vs z-exp (smaller errors)
- Can be more agressive with cuts in Q^2
- Results from usual z-expansion consistent



Z-Expansion Fits

- Do the z-exp for each ensemble
- Perform CCF Fits
- Number of Q^2 points lost at this stage
- Combine z–exp and CCF (larger errors)

Y-C. Yang (PNDME) Phys. Rev. D 101, 014507 (2020)

Lots of Variations

- In the end still have lots of variations $\left(Q^2, M_\pi^2, \mathcal{O}(a^2), \mathcal{O}(e^{-M_\pi L})\right)$
- No clear winner
- Perform averages based on AIC weights

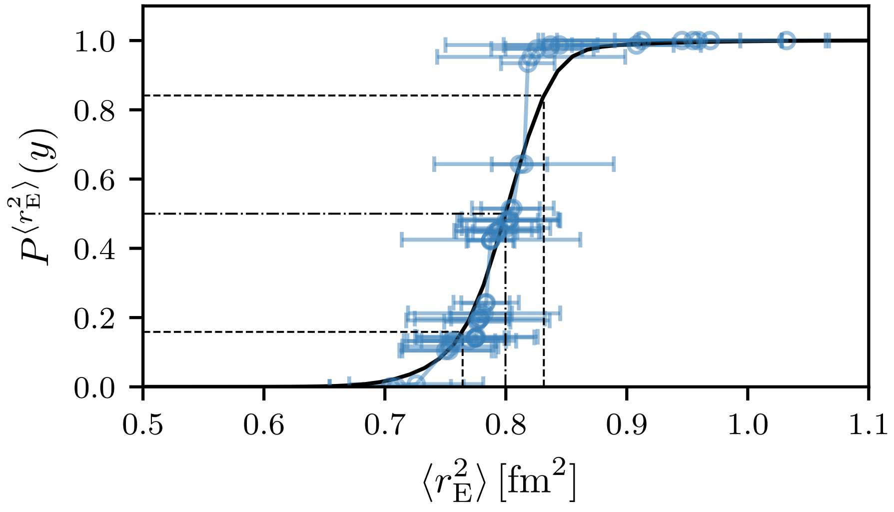
E. Neil, Tue ID: 316

$$w_i^{\mathrm{AIC}} = \frac{e^{-\frac{1}{2}\mathrm{AIC}_i}}{\sum_j e^{-\frac{1}{2}\mathrm{AIC}_j}},$$

- Treat estimates as random variable

$$P^x(y) = \int_{-\infty}^{y} \sum_i^n w_i \mathcal{N}(y'; x_i, \sigma_i^2) dy'$$

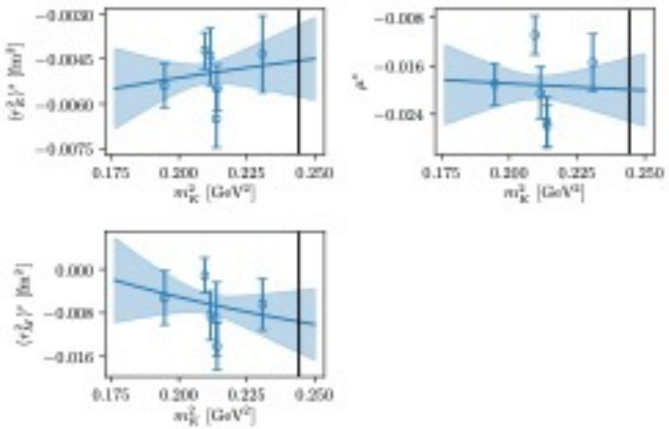
Strategy from S. Borsanyi et al. (2020), *Nature* **593,** 51–55 (2021)



Weighting applied to isovector electric radius.

Results for Isovector Strange FF

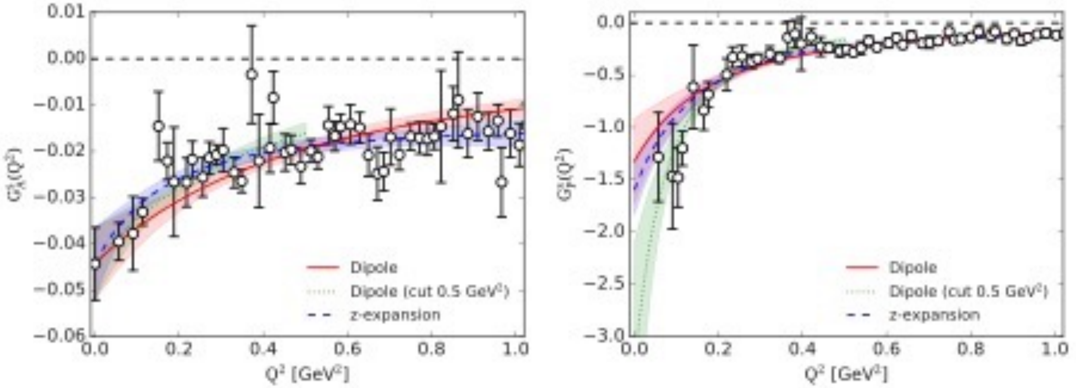
- Disconnected Diagrams only
- Very precise estimates
- CCF mild



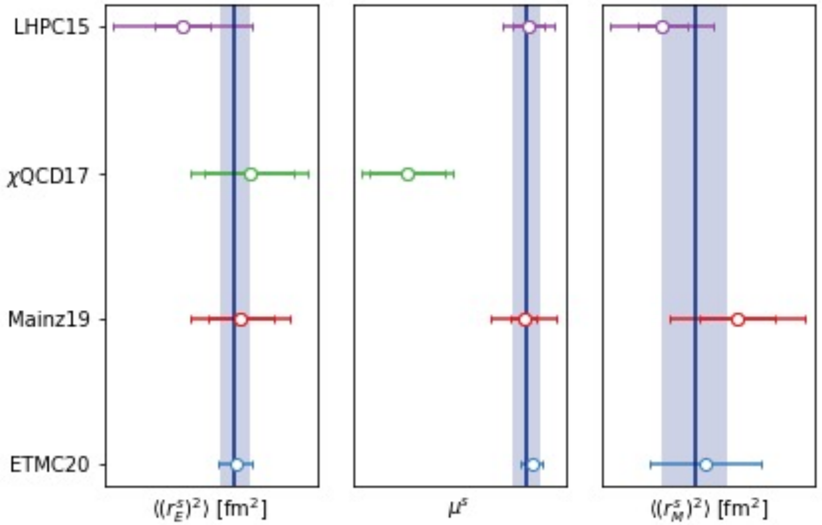
Taken from *PRL* 123 (2019) 21, 212001

- Consistent picture from Lattice non-zero radii

(Blue Band PDG-style average)

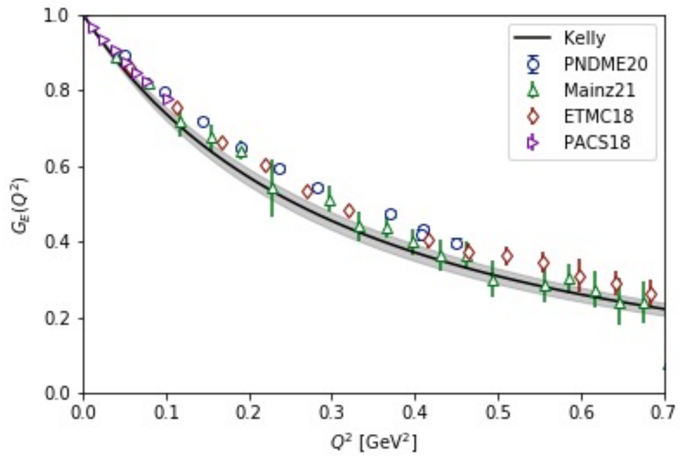


Taken from C. Alexandrou, et al. arxiv:2106.13468

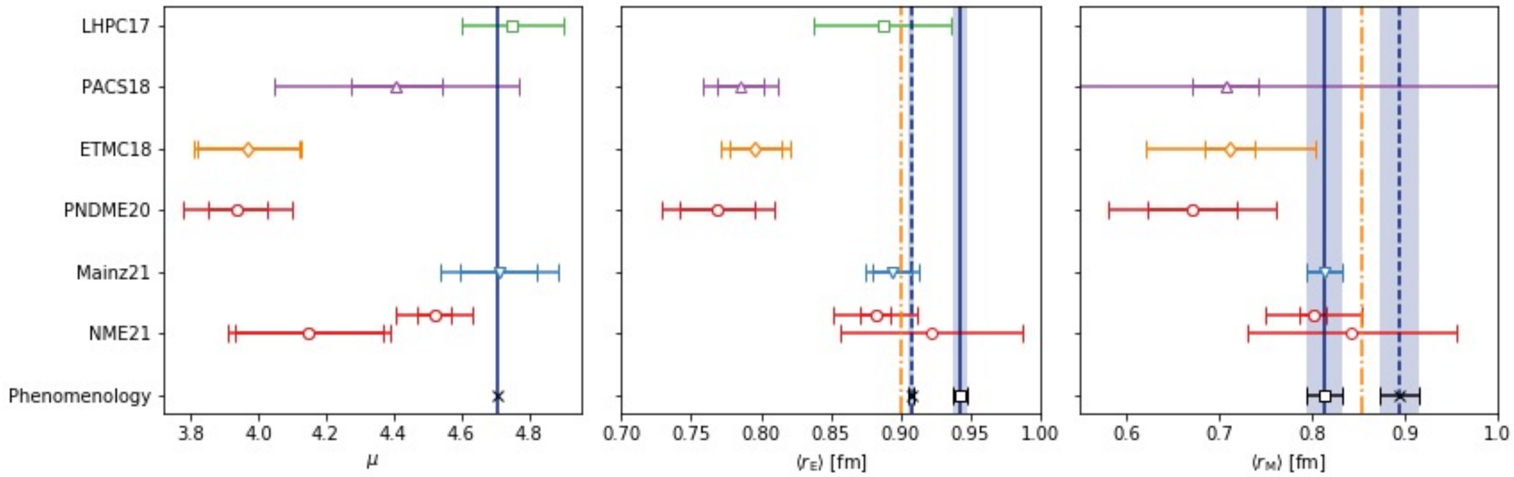


Results Isovector Vector FF

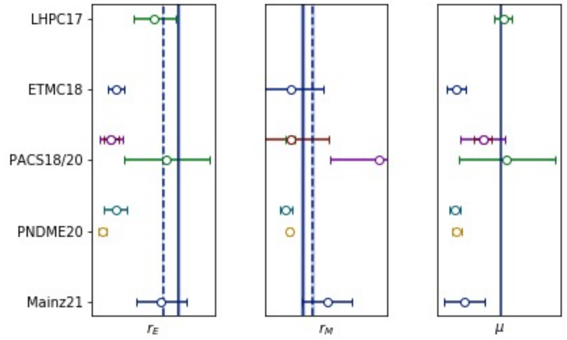
- Excited State Contamination: Summation or Multistate
- Q^2 -dependence: via z-Expansion/Dipole/EFT/Pade
- CCF extrapolations performed



Recent ensembles @ Physical pion mass



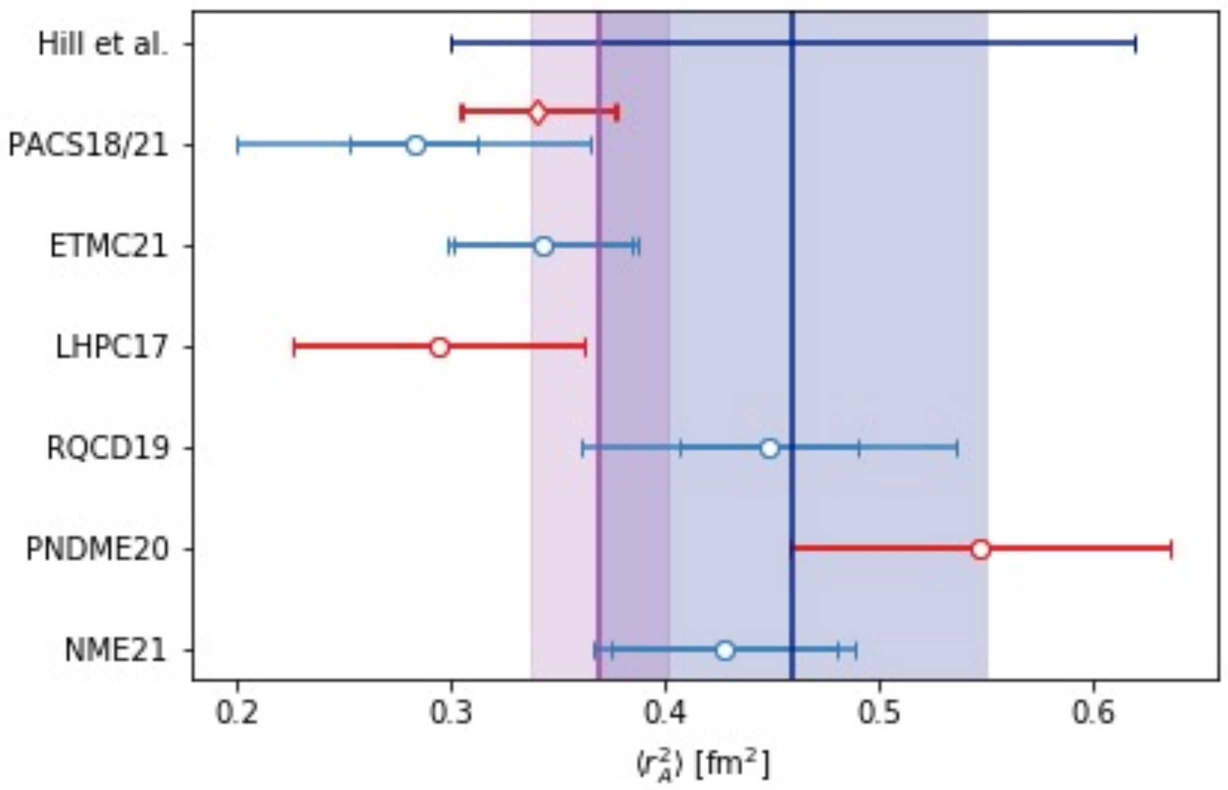
Current status after CCF



Isoscalar FF e.g. M. Salg, Tue ID: 406

Results Isovector Axial FF

- Excited State Contamination: Summation/Multistate (πN)
- Q^2 -dependence param. via z-Expansion
- CCF extrapolations are performed
- Lattice: $\sim$10 % statistical error



(Determination with 20 % acc blue shaded area)

(Purple Band PDG-Style Average)

New analyses e.g. T. Schulz, Tue ID: 86

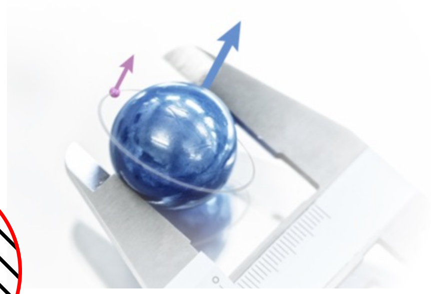
Lots of experiments

A SEARCH FOR NEW PHYSICS
Qweak

A1

$P2$ $\sin^2\vartheta_W$

COMPASS



MAGIX

PRoton Radius

SUPER K

Hyper-Kamiokande

DUNE