Progress in x-dependent partonic distributions from lattice QCD

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

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SONATA BIS grant No 2016/22/E/ST2/00013 (2017-2022)

Outline:

Introduction/motivation
Review of results – PDFs/GPDs
Theoretical developments
Review of results – TMDs
Prospects/conclusion

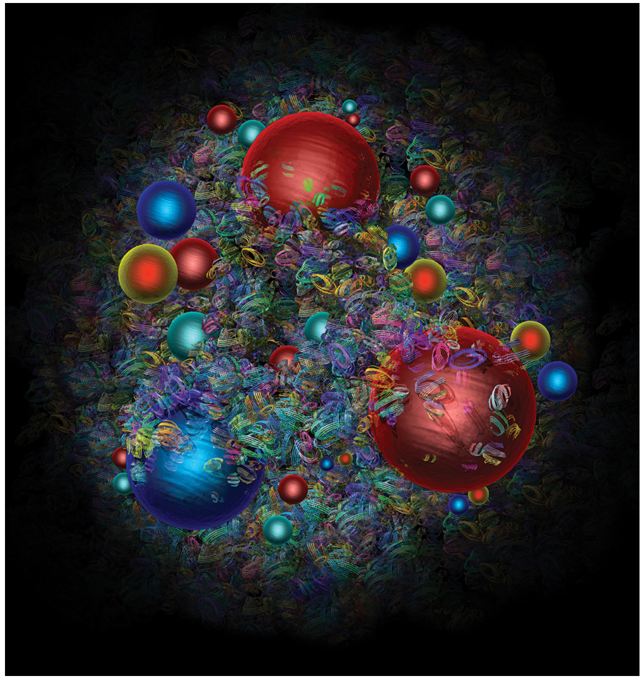
Many thanks to my Collaborators:

C. Alexandrou, M. Bhat, S. Bhattacharya, Y. Chai, M. Constantinou
L. Del Debbio, J. Dodson, X. Feng, T. Giani, J. Green,
K. Hadjiyiannakou, K. Jansen, G. Koutsou, Y. Li, Ch. Liu,
F. Manigrasso, A. Metz, A. Scapellato, F. Steffens, S.-C. Xia

Acknowledgment for discussions/material for this talk:

M. Constantinou, R. Sufian, Y.-B. Yang, S. Zafeiropoulos, Y. Zhao

One of the central aims of hadron physics:
to understand better nucleon structure.



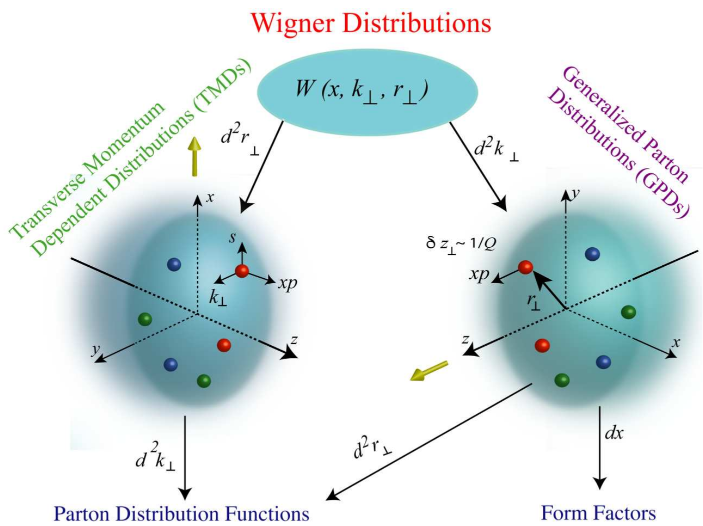
- This is one of the crucial expectations from the approved Electron-Ion Collider (EIC).
- In particular, we want to probe the 3D structure.
- Thus, we need to access new kinds of functions: GPDs, TMDs.
- Also higher-twist is of growing importance for the full picture.
- Both theoretical and experimental input needed.

A. Deshpande Thu 9:00, I. Stewart Thu 10:30

Lattice can provide *qualitative* and eventually *quantitative* knowledge of different functions and their moments:

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function / generalized TMDs

D. Djukanovic Thu 10:10



Approaches to x-dependence

- Recent years (since ≈ 2013): breakthrough in accessing x-dependence.
  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

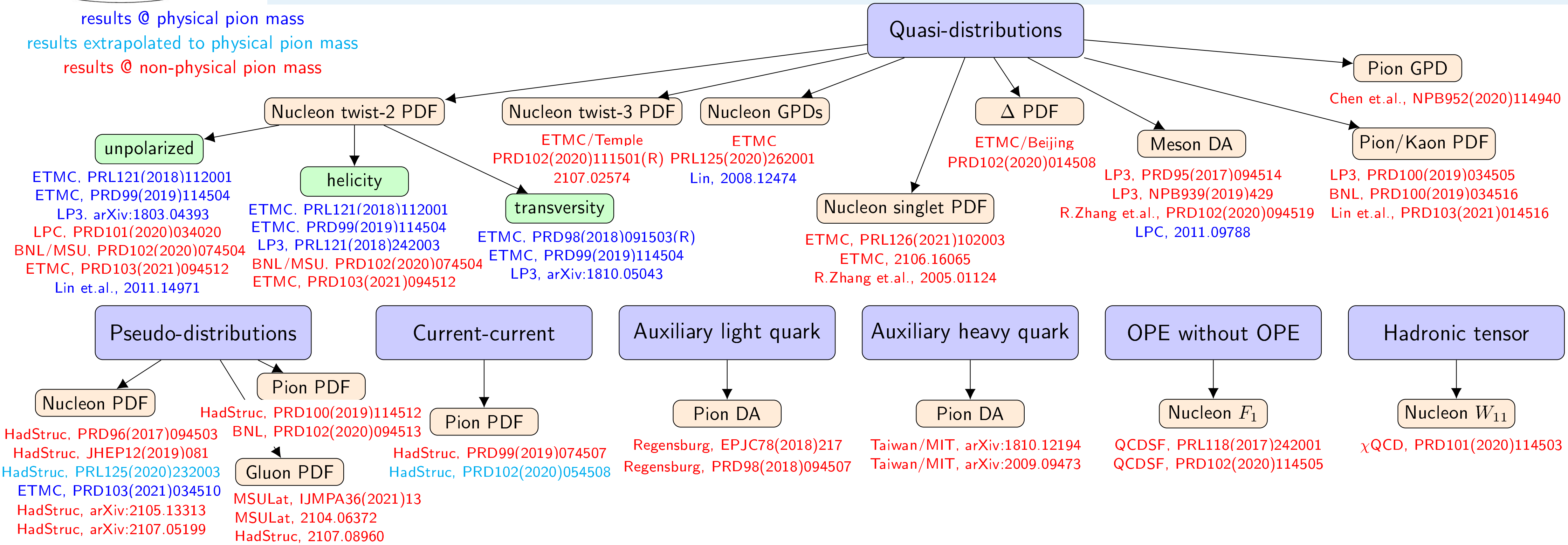
- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y}\, C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$
  with different choices of Γ, Γ' Dirac structures and objects $F(z)$.

  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark (HOPE)** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Lattice PDFs/GPDs: dynamical progress

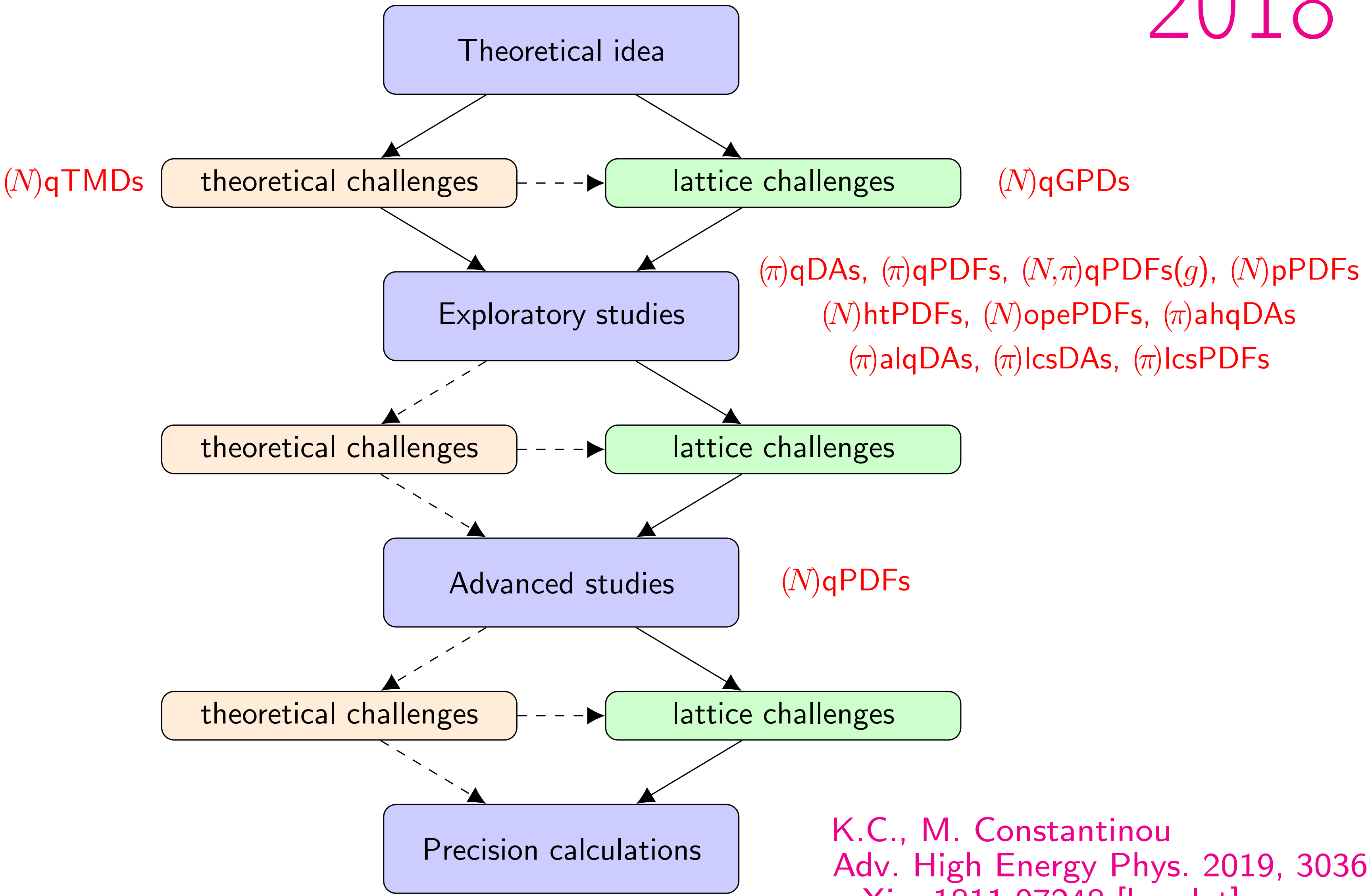


Reviews: K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248
update: M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD*, (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445
X. Ji, Y. Liu, Y.-S. Liu, J.-H. Zhang, Y. Zhao, *Large-Momentum Effective Theory*, 2004.03543
M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, 2006.08636

Some studies already advanced, but still full systematics needs to be investigated
Many exploratory directions: GPDs, twist-3 PDFs/GPDs, singlet PDFs, TMDs

Progress of approaches to x-dependence

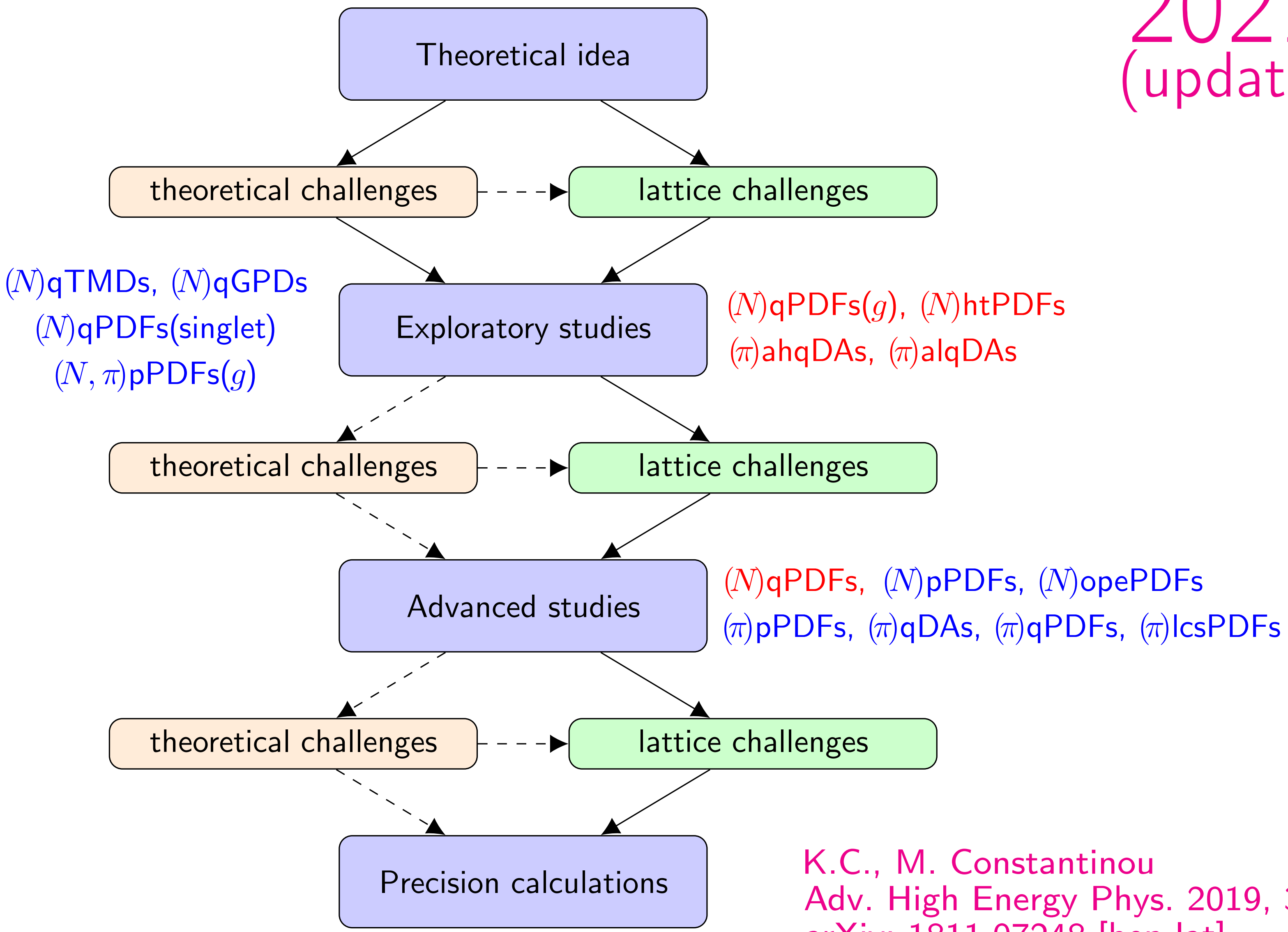
2018



K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Progress of approaches to x-dependence

2021 (update)



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X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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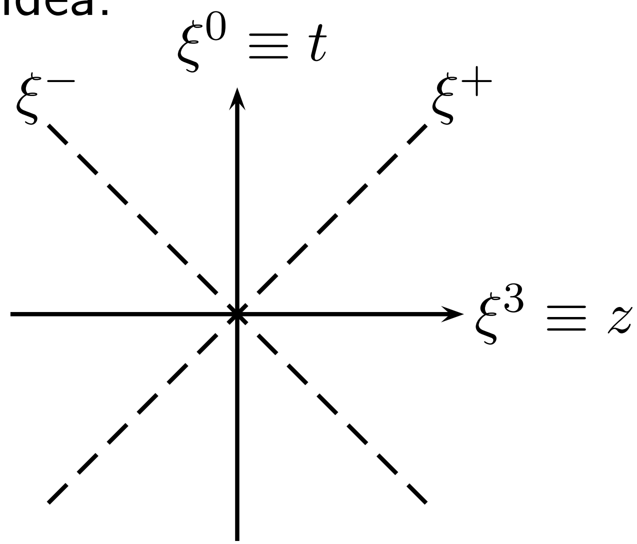

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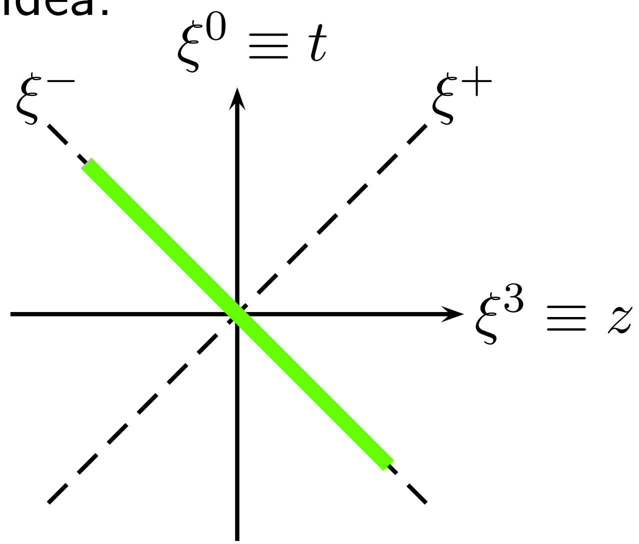

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Quasi-PDFs

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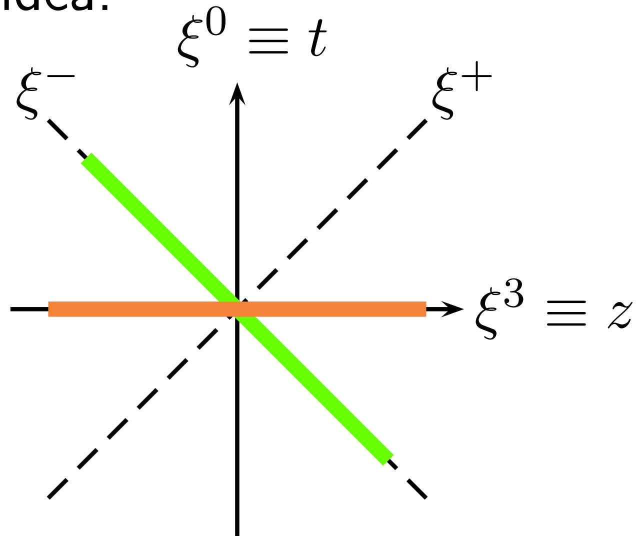

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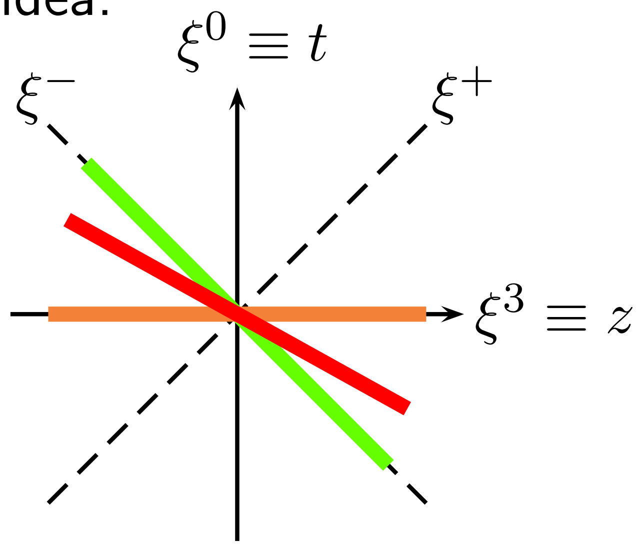

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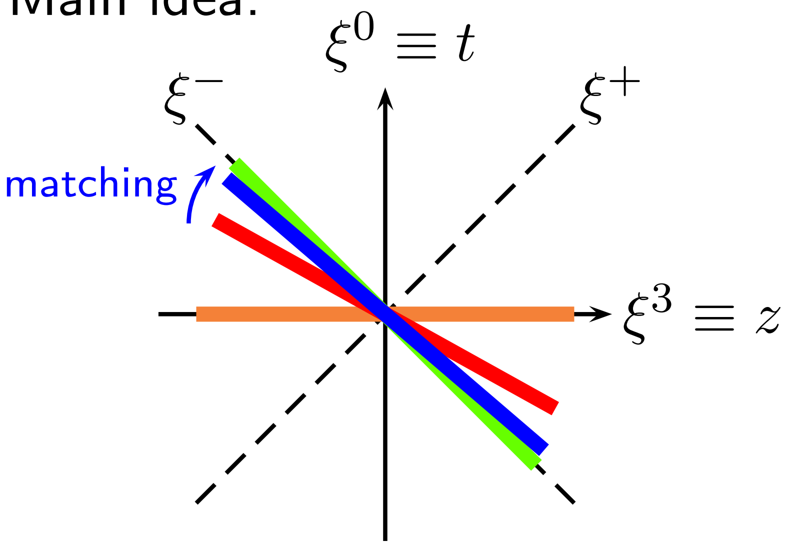

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Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x, \mu, P_3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y, \mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

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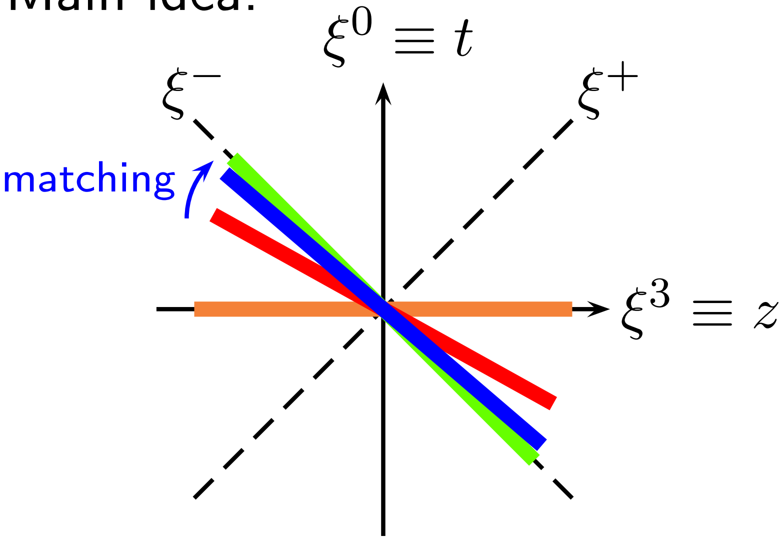

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quasi-PDF

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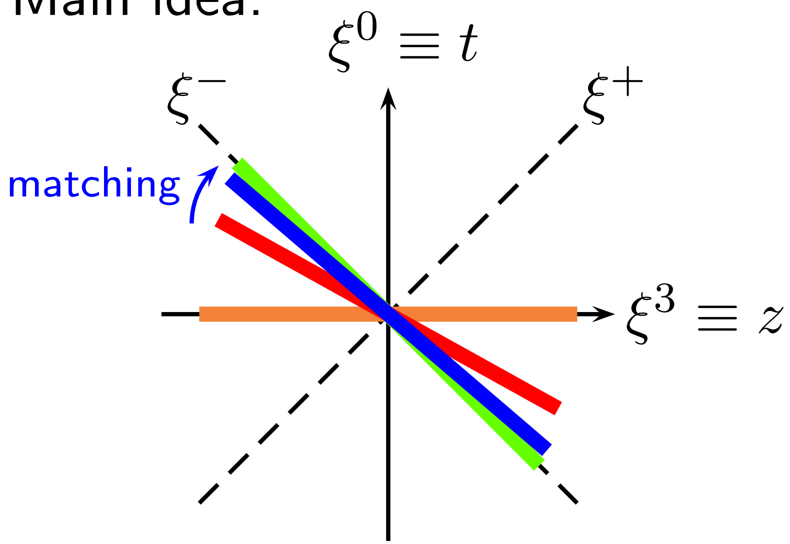

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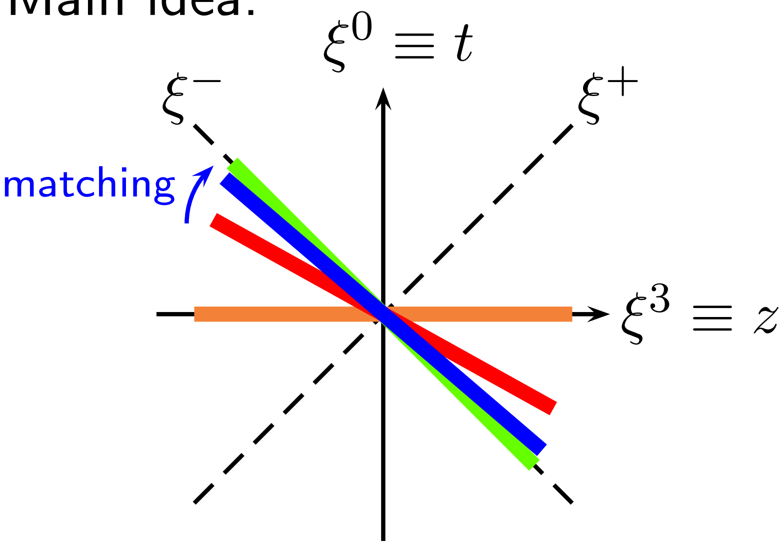

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quasi-PDF   pert.kernel   PDF

Quasi-PDFs

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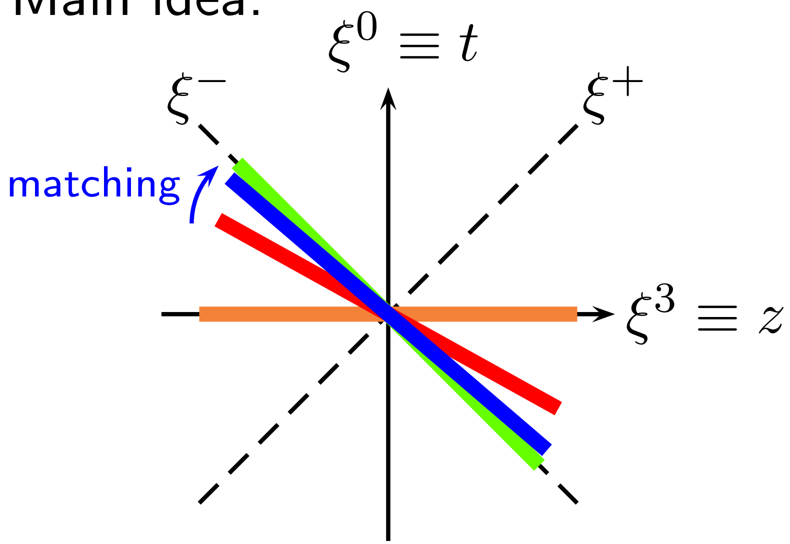

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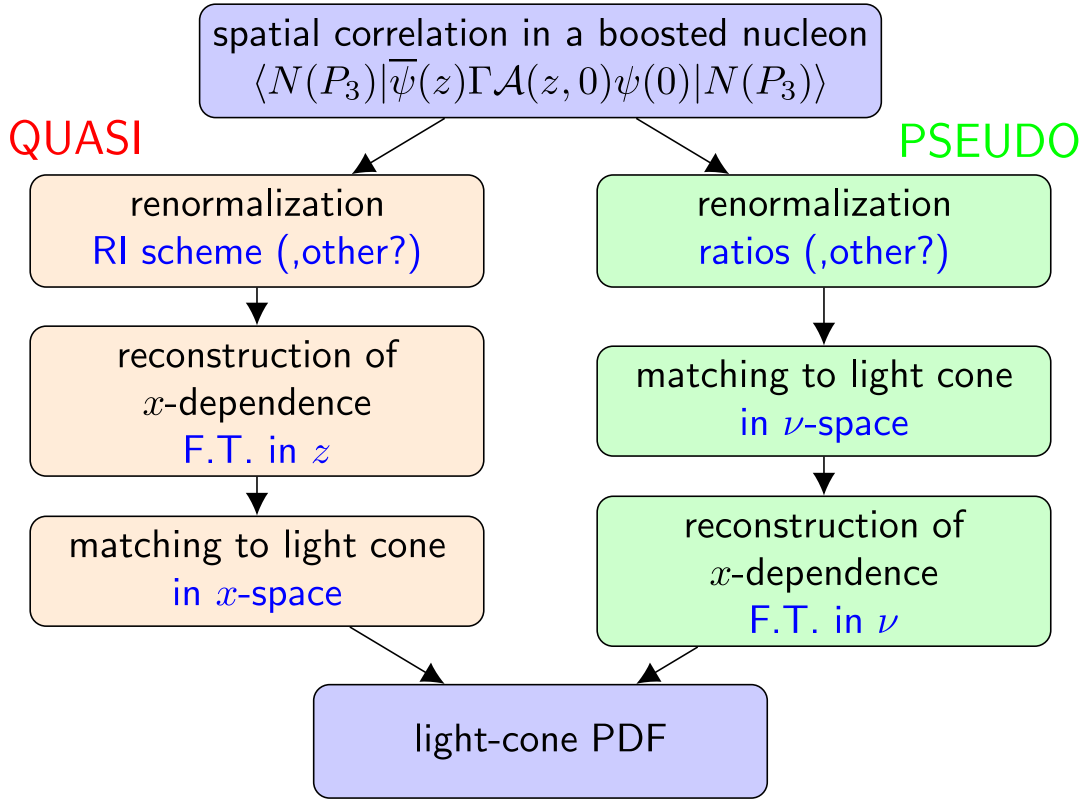
Quasi-PDFs vs. pseudo-PDFs

There is a long-standing debate in the community whether quasi-distributions are "better" than pseudo-distributions or vice versa.

- Large nucleon boost:
  no doubt both need to give the same answer.
- Practicioner's view for realistically achievable momenta:
  certainly different systematics, so worthwhile (almost mandatory?) to use both.
- Quasi-distributions:
  - ⋆ longer on the market and much more explored,
  - ⋆ can utilize all values of z.
- Pseudo-distributions:
  - ⋆ canonical support in x
  - ⋆ fully utilize all nucleon boost data
  - ⋆ allow for easier reconstruction with a fitting ansatz,
  - ⋆ however: z-space factorization requires perturbative z.

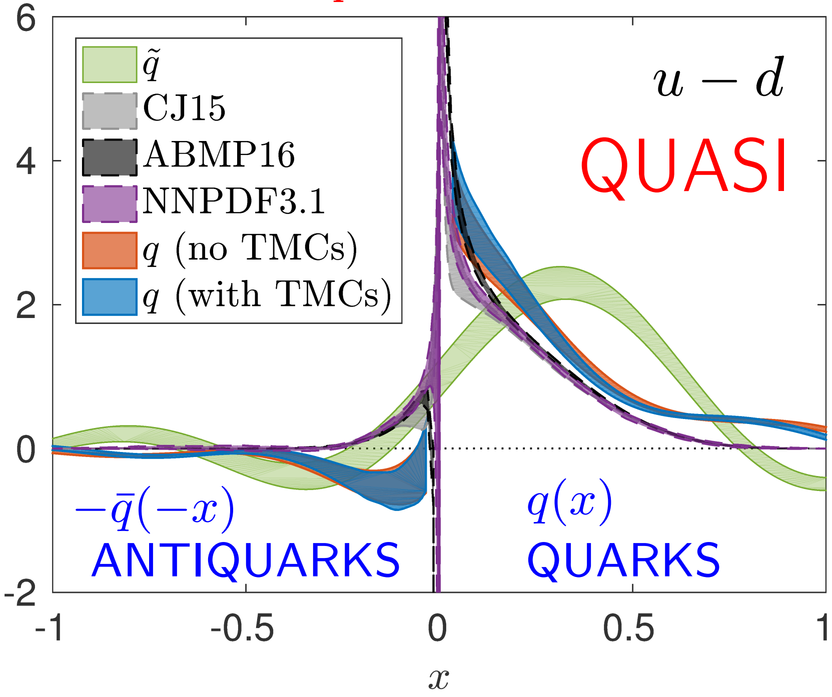


Current state-of-the-art: unpolarized PDFs @ phys.pt.

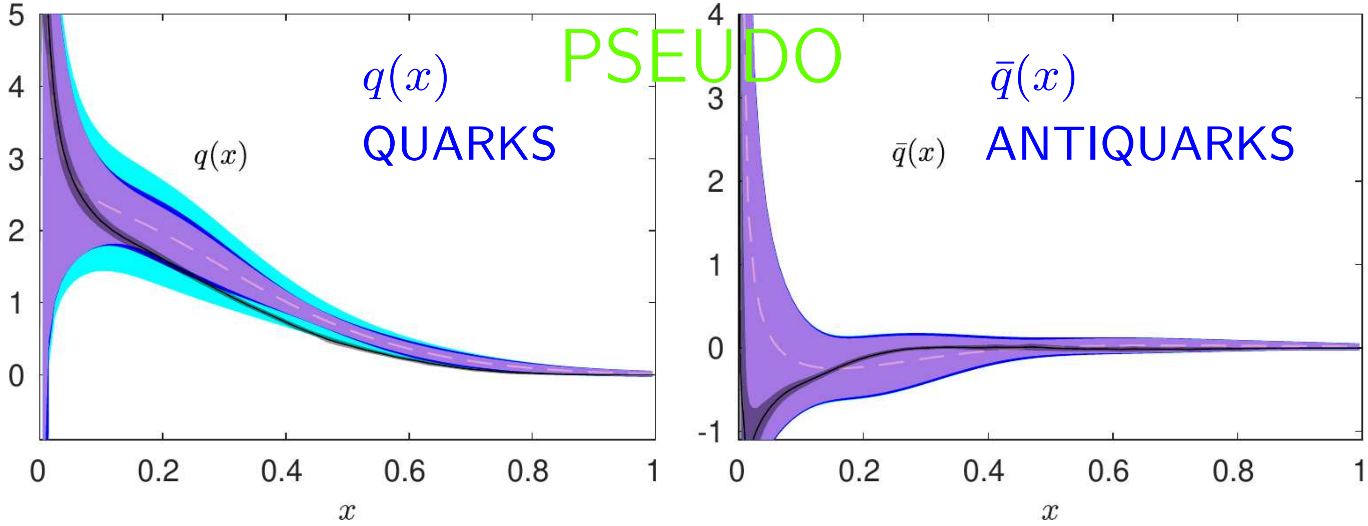
ETMC, Phys. Rev. Lett. 121 (2018) 112001
ETMC, Phys. Rev. D 99 (2019) 114504

QUASI PSEUDO	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm

Unpolarized PDF



$\overline{\text{MS}}$(2GeV) ETMC, Phys. Rev. D103 (2021) 034510



Qualitative agreement with pheno
Systematics to be investigated

cut-off effects
truncation (matching)
higher-twist effects
reconstruction of x-dep.
finite volume effects

Different approach starting from the same MEs
Also: reconstruction using a pheno-inspired ansatz
And: added plausible estimates of systematics

purple – statistical error
blue – quantified systematics
cyan – estimated systematics

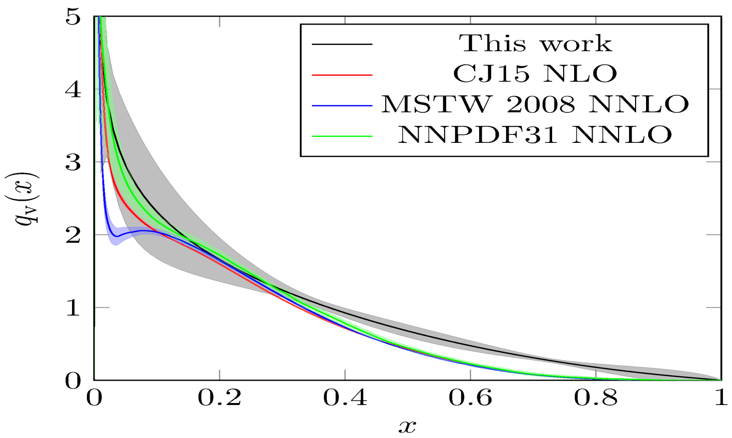
Quantitative agreement with phenomenology within stat. + plausible syst. error!

Pseudo-PDFs: unpolarized @ phys.pt.

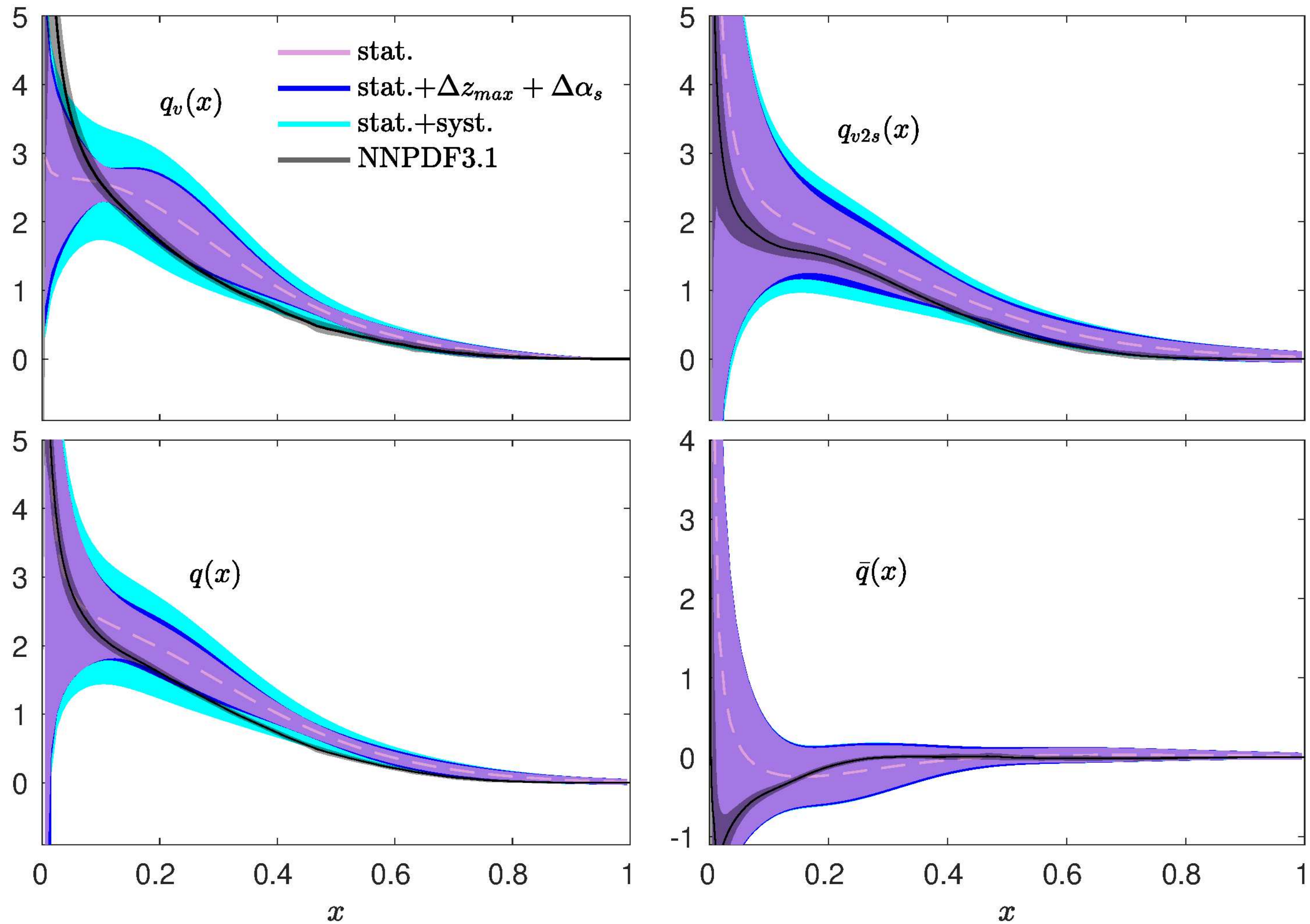
B. Joó et al. (HadStruc)	M. Bhat et al. (ETMC)
Phys. Rev. Lett. 125 (2020) 232003	Phys. Rev. D103 (2021) 034510

PSEUDO	clover	$m_\pi = 358$, 278,172 MeV	$a = 0.094$ fm

PSEUDO	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm

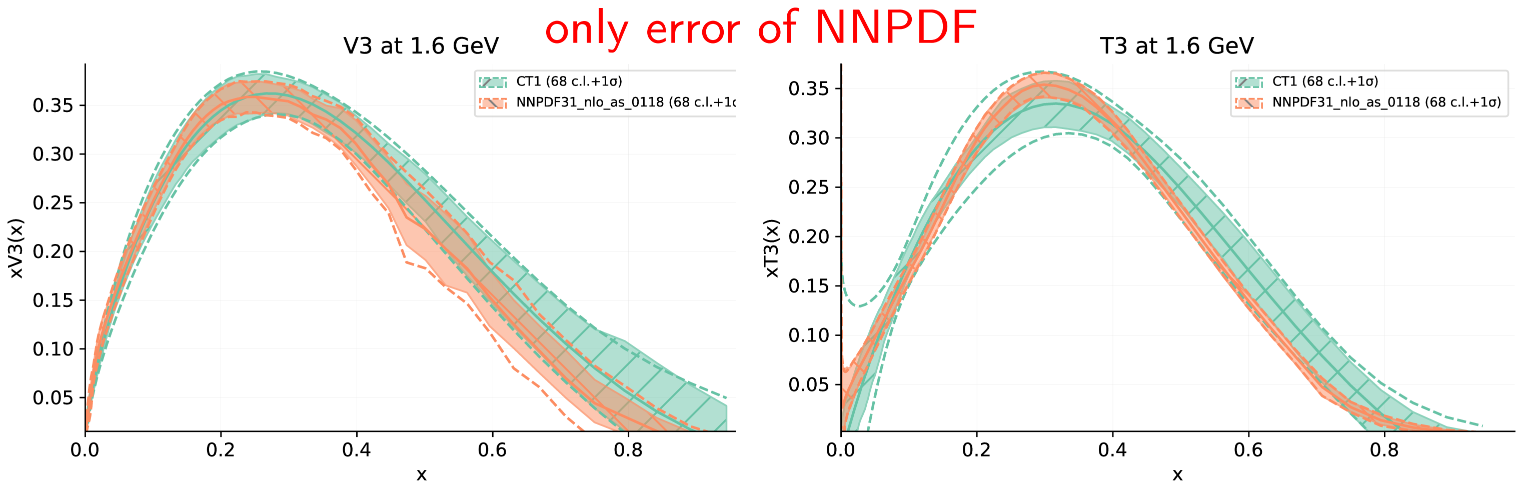


S. Zafeiropoulos Tue 14:00



NNPDF reconstruction from lattice data

- Question: what do we get if we treat lattice observables similarly to cross sections and use the NNPDF framework to reconstruct PDFs?
- Observables: non-singlet distributions V_3 and T_3 (unpolarized):
  $V_3 = u - \bar{u} - (d - \bar{d}) = u_V - d_V\,, \qquad T_3 = u + \bar{u} - (d + \bar{d}) = u_V - d_V + 2(u_S - d_S)$
- Relation between qPDF matrix elements and PDFs: $\mathcal{O}^{\text{Re/Im}}_{\gamma^0}(z,\mu) = \mathcal{C}^{\text{Re/Im}}_3\left(z, \frac{\mu}{P_z}\right) \circledast V_3/T_3\,(\mu)$
  implemented using FastKernel tables (matching+DGLAP evolution) + NN parametrization:
  $V_3/T_3\,(x,\mu) \propto x^{\alpha_{V/T}}\,(1-x)^{\beta_{V/T}}\,\text{NN}_{V/T}\,(x)$.
- Closure tests: generate mock data (e.g. 16 "lattice points" (16 real + 15 imaginary)) from a selected NNPDF and run fitting code over them.



pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

Shows the power of the convolution ⊛ in constraining PDFs! (only 16 lat. points!)

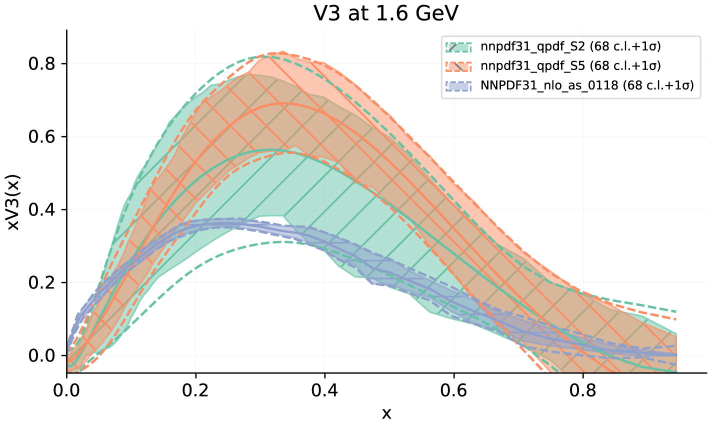
K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137

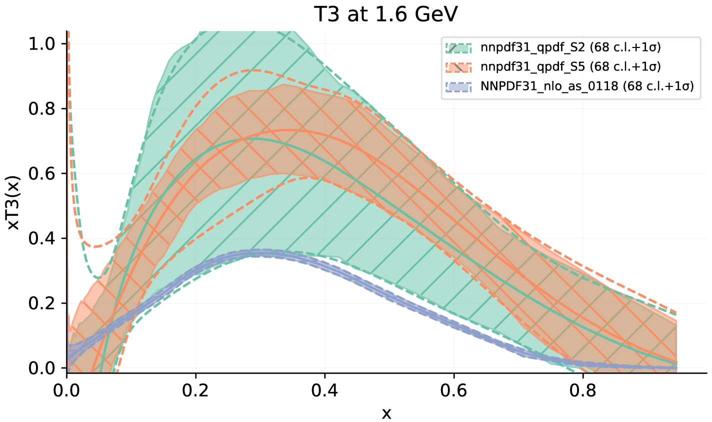
Very robust result!

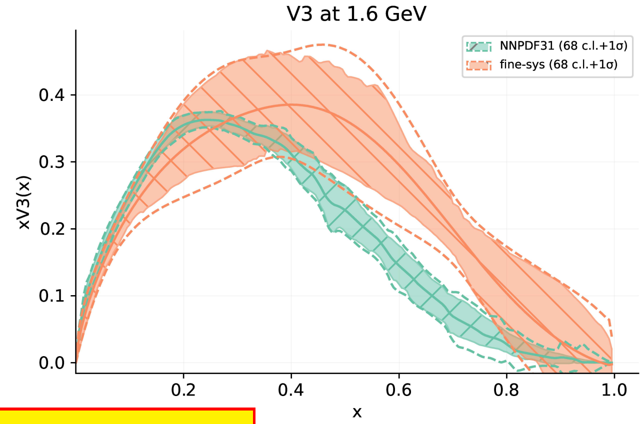
NNPDF reconstruction from actual lattice data

QUASI	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm

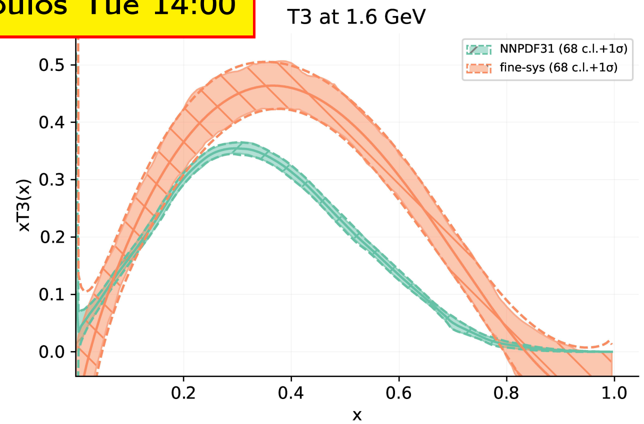
PSEUDO	clover	$m_\pi = 415, 358, 278, 172$ MeV	$a = 0.091, 0.094, 0.127$ fm







S. Zafeiropoulos Tue 14:00



NNPDF (L. Del Debbio, T. Giani) + K.C.
JHEP10(2019)137

Systematics: "plausible estimates"
S2: 20% cutoff, 5% FVE, 10% exc.st., 20% trunc.
S5: 0.2 cutoff, 0.05 FVE, 0.1 exc.st., 0.2 trunc.

NNPDF (L. Del Debbio, T. Giani)
+ JLab (J. Karpie et al.), JHEP02(2021)138

Systematics: additional ensembles
cutoff: 0.094 vs. 0.127 fm, m_π: 172 vs. 278 MeV
FVE: $L \approx 3$ vs. 4 fm @ $m_\pi \approx 415$ MeV

JAM reconstruction from actual lattice data

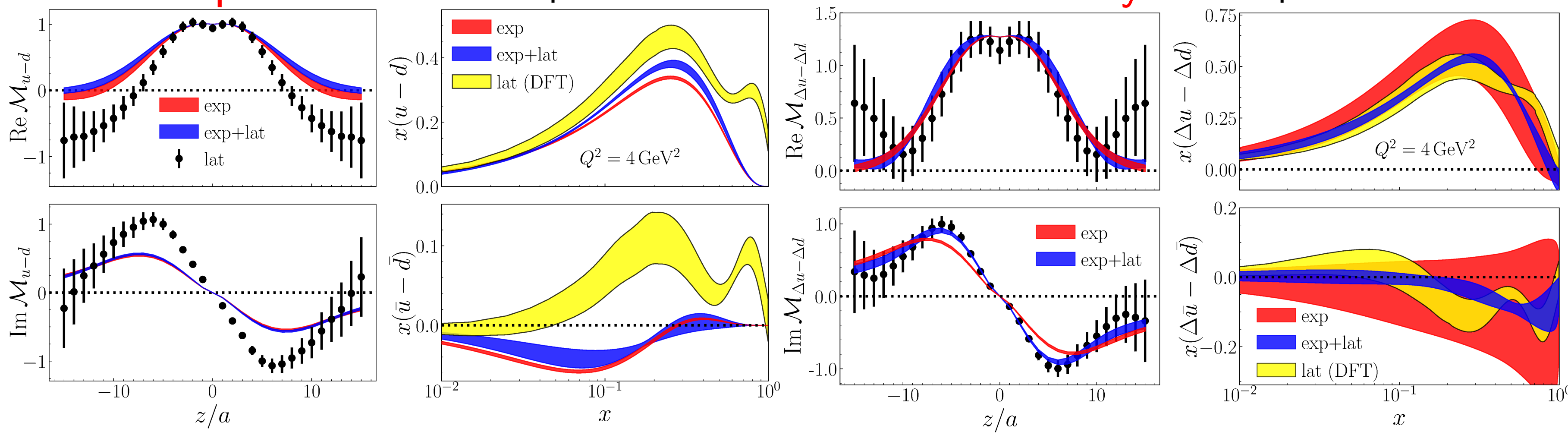
ETMC quasi-PDF lattice data also analyzed within the JAM framework, combining with experimental DIS data:

J. Bringewatt, N. Sato, W. Melnitchouk, J.-W. Qiu, F. Steffens, M. Constantinou, PRD103(2021)016003

QUASI	TMF	$m_\pi = 130$ MeV	$a = 0.094$ fm

unpolarized 2930 datapoints

helicity 650 datapoints



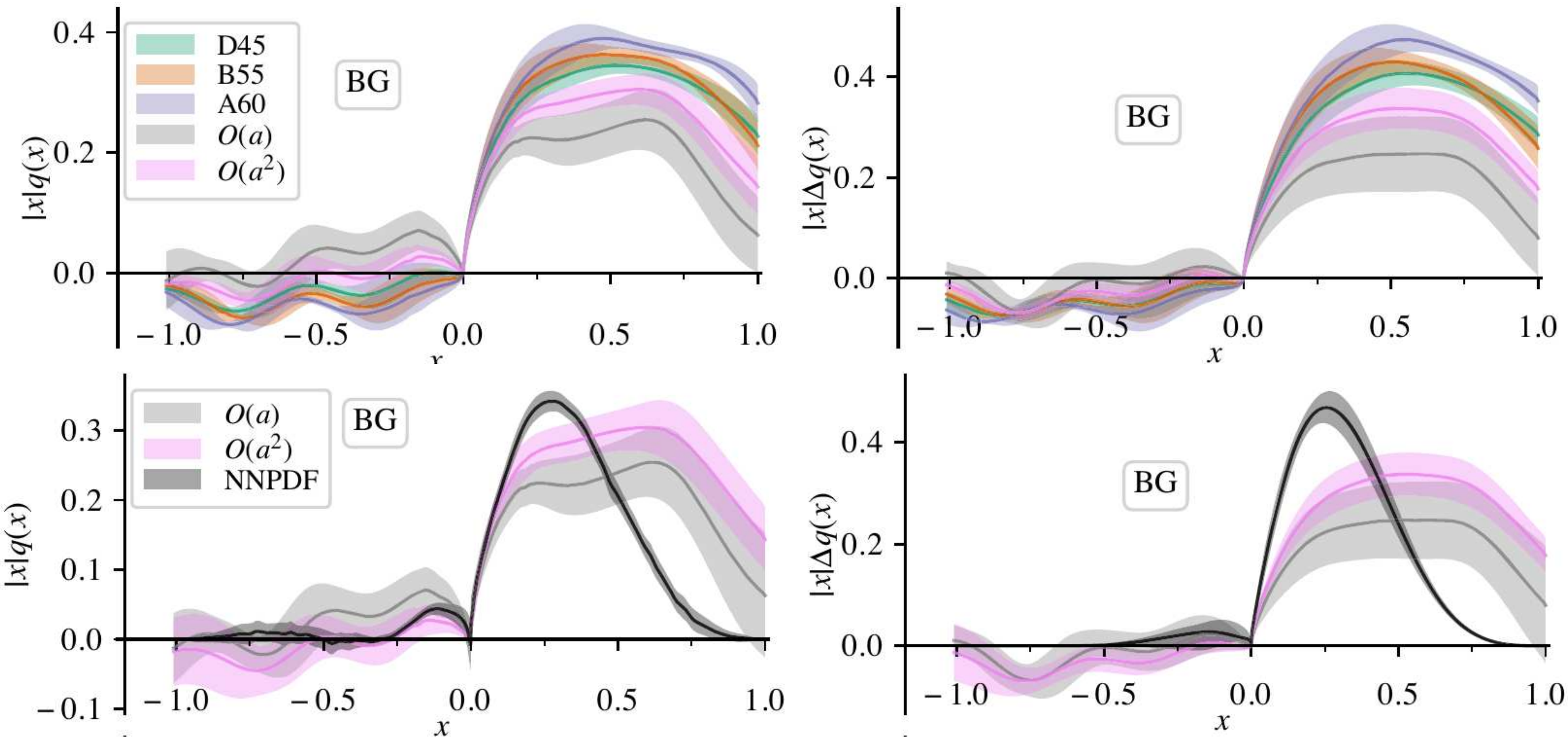
- unpolarized PDFs
  significant tension **lat**↔**exp**, precision of **exp** dominates the PDFs
  much improved precision of **lat** needed for any impact
- helicity PDFs
  promising agreement **lat**↔**exp**
  current precision of **lat** provides significant constraints
  lat does not indicate large $\Delta\bar{u} - \Delta\bar{d}$ asymmetry

Continuum limit

- By now, we basically take $\mathcal{O}(a)$-improvement of lattice observables for granted.
- However, the non-local matrix elements needed for partonic distributions imply the presence of $\mathcal{O}(a)$ effects:
  - ⋆ TM: automatic improvement does not work,
  - ⋆ even chiral symmetry does not prevent these.
    J. Green, K. Jansen, F. Steffens, PRD101(2020)074509
- Nevertheless, there is a framework to calculate improvement coefficients to eliminate terms linear in a.
- Automatic $\mathcal{O}(a)$-improvement of TM can remove some of the contributions and reduce the number of improvement coefficients.
- Some improvement coefficients can be obtained from chiral Ward identities, other from lattice perturbation theory and/or numerically.

Continuum limit – unpolarized + helicity with quasi

QUASI	TMF	$m_\pi = 370$ MeV	$a = 0.064, 0.082, 0.093$ fm



C. Alexandrou et al. (ETMC), Phys. Rev. D103 (2021) 094512

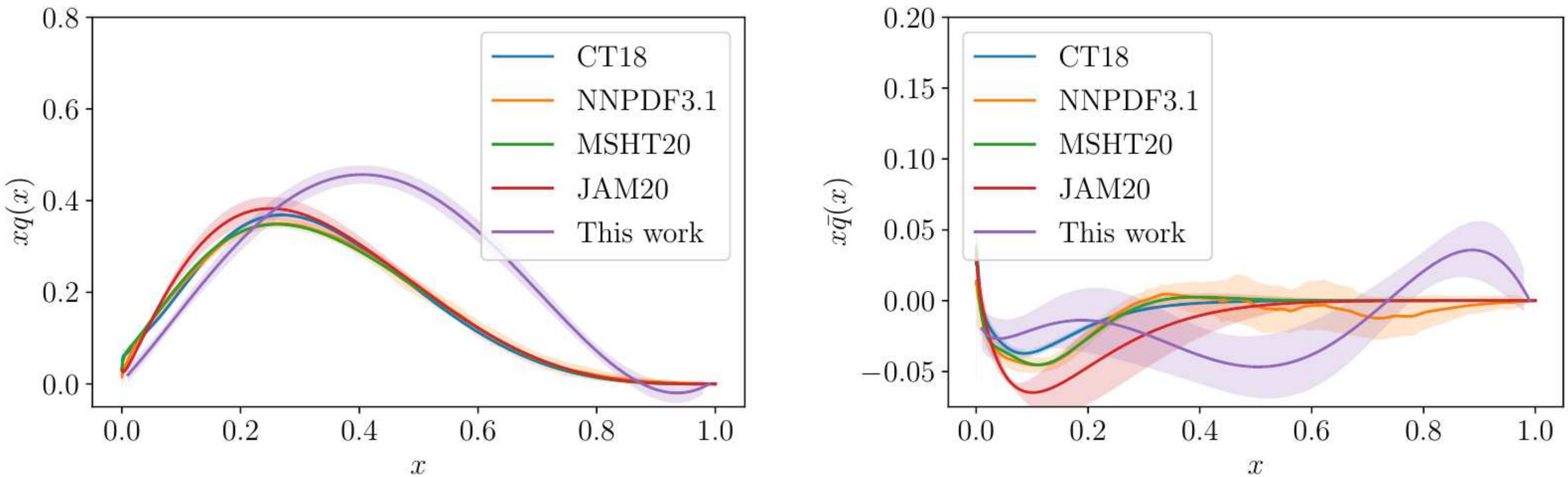
Continuum limit – unpolarized with pseudo

First continuum limit with pseudo-PDFs

J. Karpie, K. Orginos, A. Radyushkin, S. Zafeiropoulos (HadStruc), 2105.13313

- parametrization of systematic uncertainties using Jacobi polynomials to characterize and remove discretization and higher-twist effects
- fits with Bayesian priors

PSEUDO	clover	$m_\pi \approx 440$ MeV	$a = 0.048, 0.065, 0.075$ fm



S. Zafeiropoulos Tue 14:00

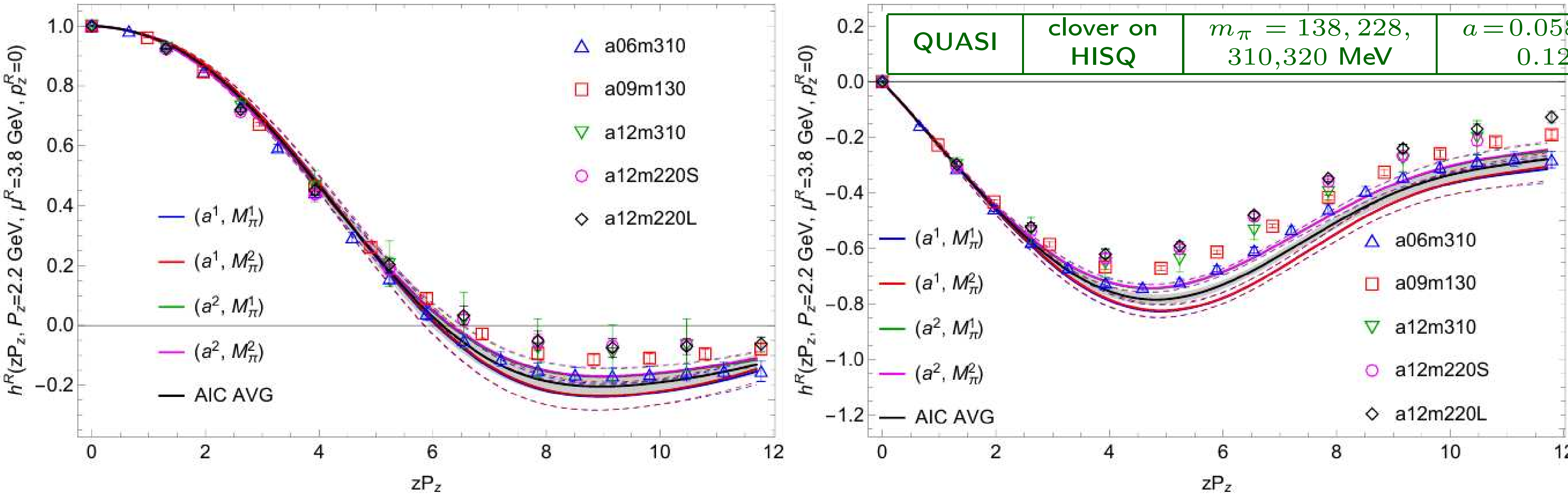
Combined physical pion mass – continuum limit with quasi-PDFs

H.-W. Lin, J.-W. Chen, R. Zhang, 2011.14971

H.-W. Lin Wed 14:45

- nucleon boosts up to 3.1 GeV
- striking feature: tiny statistical errors even at the physical point

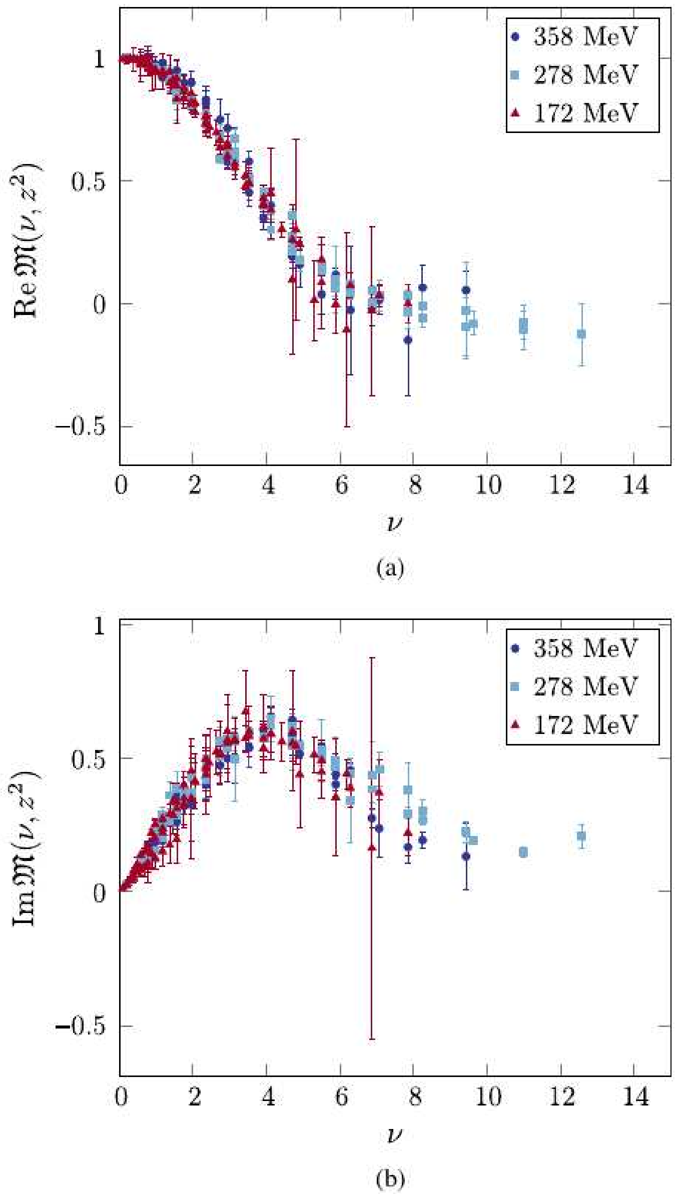
QUASI	clover on HISQ	$m_\pi = 138, 228, 310, 320$ MeV	$a = 0.058, 0.087, 0.12$ fm



How are such small errors possible at the physical point?

Another approach – superfine lattice: unpolarized/helicity PDFs at $a = 0.042$ fm

Z. Fan et al. (BNL+MSULat), Phys. Rev. D102 (2020) 074504



B. Joó et al. (HadStruc), PRL125(2020)232003

Table 1. Parameters of various lattice calculations and comparison of the noise-to-signal ratio.

Ref.	m_π(MeV)	P_3(GeV)	$\frac{n}{s}\Big\vert_{z=0}$
quasi/pseudo [54,90]	130	1.38	6%
pseudo [87]	172	2.10	8%
current-current [93]	278	1.65	19%*
quasi [67]	300	1.72	6%†
quasi/pseudo [72]	300	2.45	8%†
quasi/pseudo [65]	310	1.84	3%†
twist-3 [143]	260	1.67	15%
s-quark quasi [108]	260	1.24	31%
s-quark quasi [107]	310	1.30	43%**
gluon pseudo [129]	310	1.73	39%
quasi-GPDs [163] $-t$=0.69GeV^2	260	1.67	23%
quasi-GPDs [162] $-t$=0.92GeV^2	310	1.74	59%

† At $T_{\rm sink} < 1$ fm.
⋆ At smallest z value used, $z = 2$.
⋆⋆ At maximum value of imaginary part, $z = 4$.

M. Constantinou, EPJA57(2021)77
proceedings of would-be plenary talk of LATTICE 2020

Distillation for PDFs

Very recent work applied distillation first time for PDFs

C. Egerer et al. (HadStruc), 2107.05199

PSEUDO	clover	$m_\pi = 358$ MeV	$a = 0.094$ fm

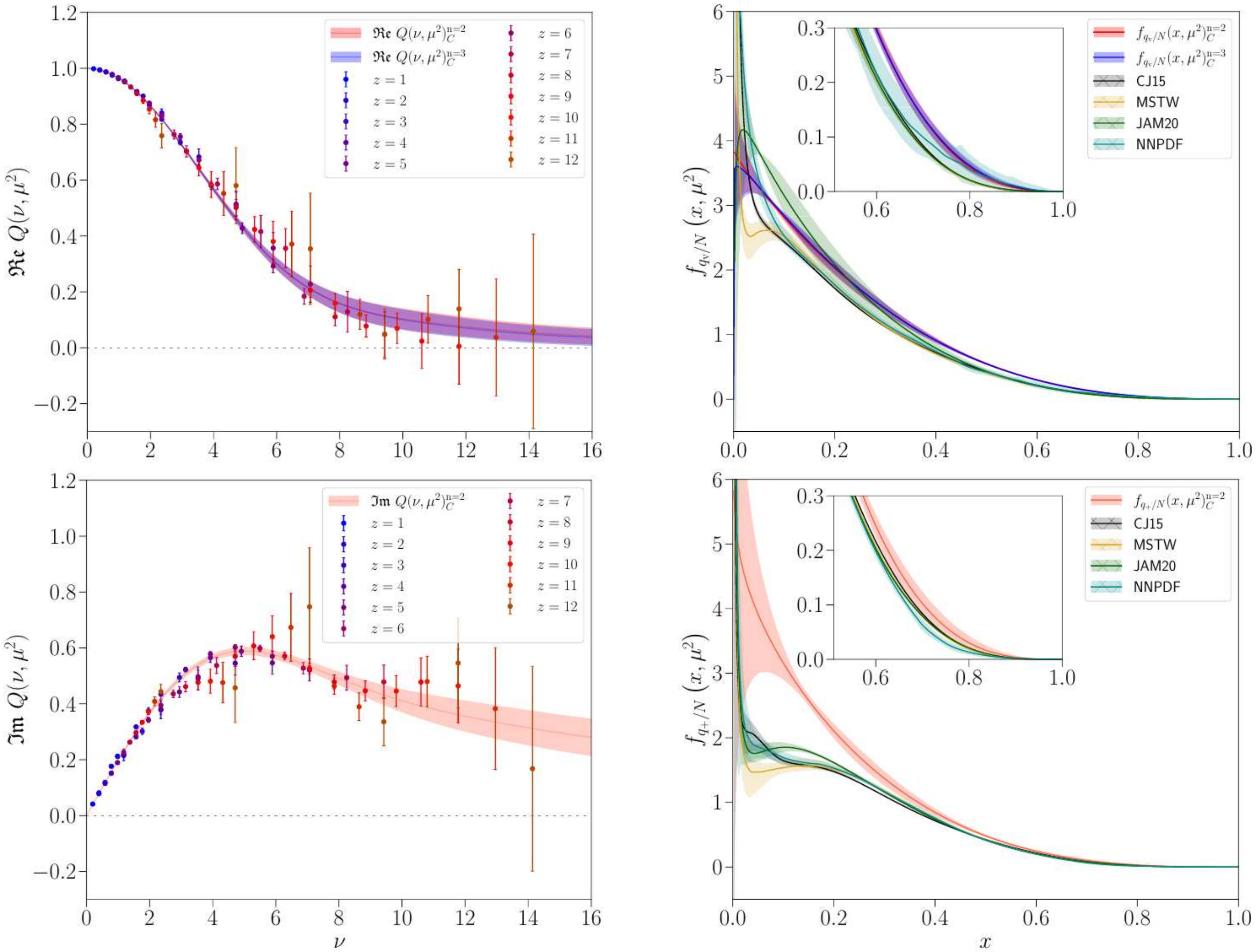
- distillation combined with momentum smearing
  C. Egerer et al. (HadStruc), PRD103(2021)034502
- other important ingredients: summation method, Jacobi polynomials, Bayesian fits
- found inconsistency with DGLAP unless allowing for discretization effect in fits

C. Egerer Wed 06:00

R. Sufian Tue 14:45

T. Khan Tue 21:00

N. Karthik Wed 14:30



Gluon PDFs (pseudo)

Another brand new work where distillation is crucial

T. Khan et al. (HadStruc), 2107.08960

PSEUDO	clover	$m_\pi = 358$ MeV	$a = 0.094$ fm

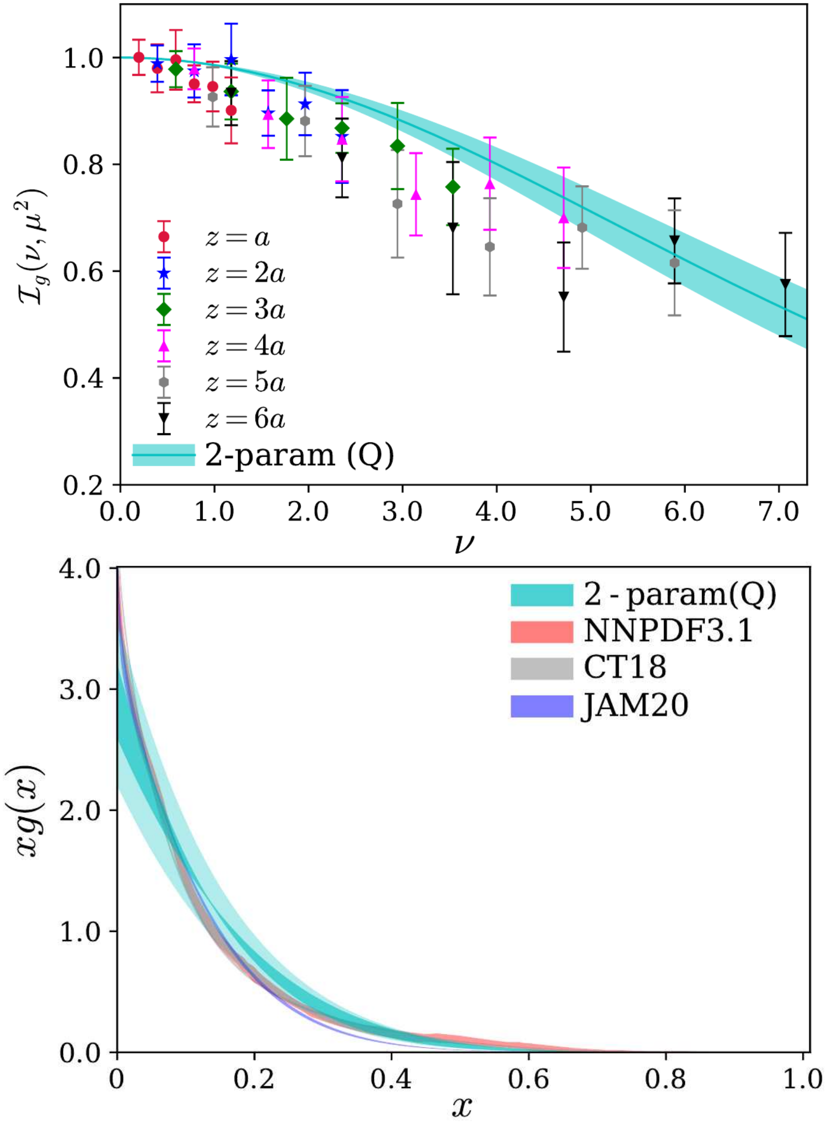
Key aspects of the calculation:

- distillation combined with momentum smearing
- gradient flow to improve signal (extrapolate to $\tau = 0$)
- summed GEVP to access smaller temp. sep.

T. Khan Tue 21:00

W. Morris Tue 21:30

R. Sufian Tue 14:45



Matching – take only gg part
(mixing with singlet quark neglected)

Fit to pheno-inspired ansatz
(2 or 3-param., incl./excl. cutoff effects term)

Gluon PDFs/ITDs model motivated by counting rules based on pQCD analyses at large-x and pheno. behavior at low-x

R. Sufian, T. Liu, A. Paul, Phys. Rev. D103 (2021) 036007

Gluon PDFs (pseudo)

PSEUDO	clover on HISQ	$m_\pi^{\rm sea} = 310$ MeV, $m_\pi^{\rm val} = 310, 690$ MeV	$a = 0.12$ fm

Earlier work for nucleon & pion gluon PDFs

Z. Fan et al. (MSULat), Int. J. Mod. Phys. A36(2021)13

Z. Fan et al. (MSULat), 2104.06372

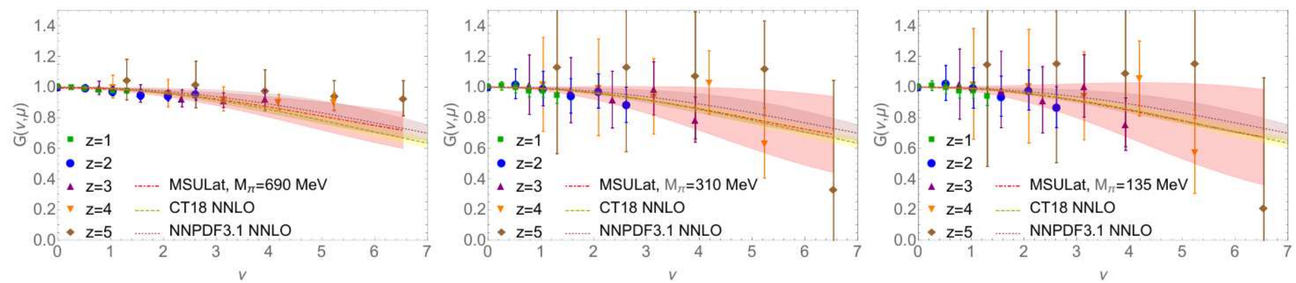
Z. Fan Tue 21:15

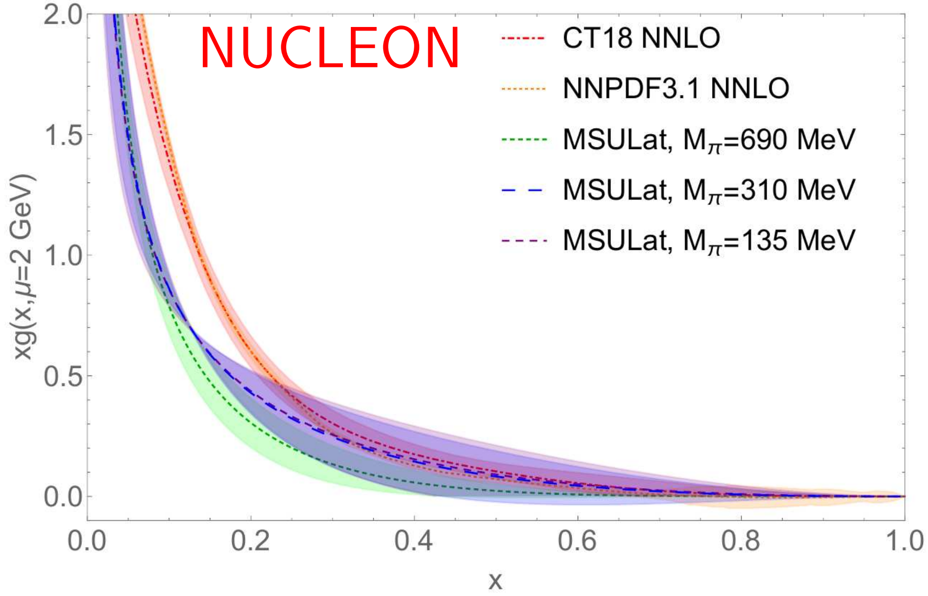
H.-W. Lin Wed 14:45

Matching – gg part

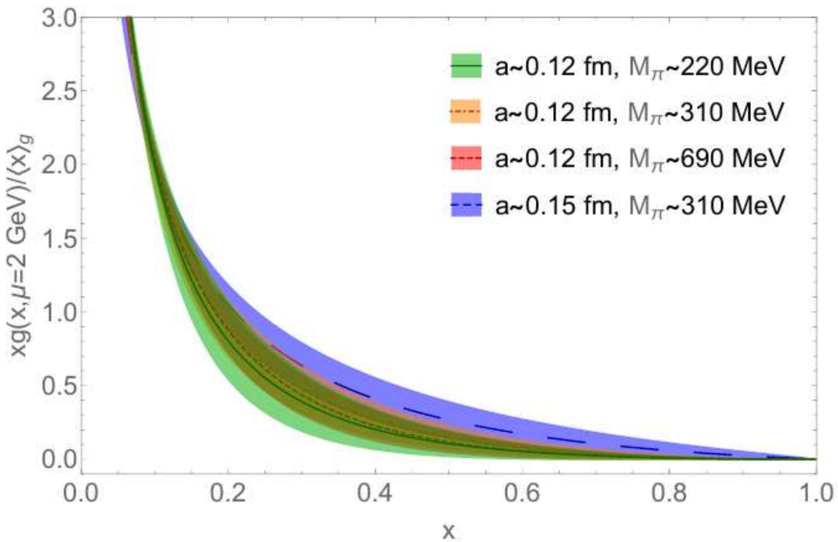
(check inclusion of CT18 singlet quark – 4% effect)

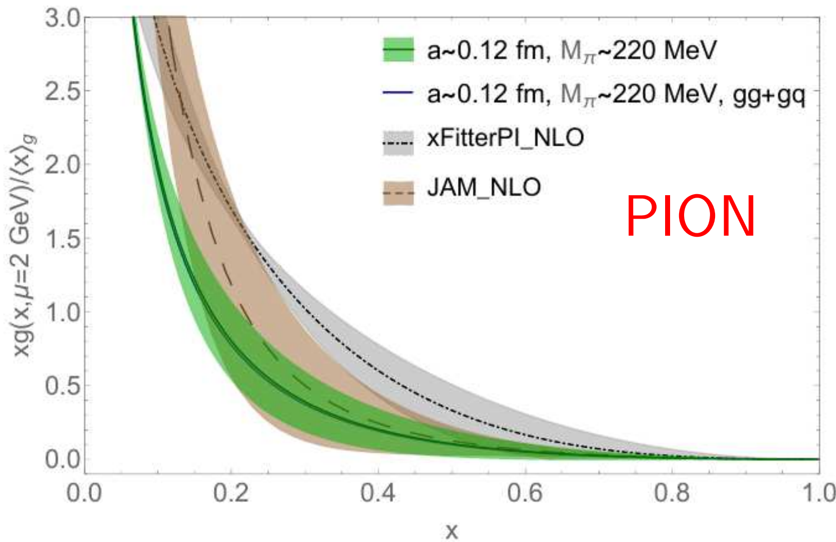
Fit to pheno-inspired 2-param. ansatz





PSEUDO
clover on HISQ
$m_\pi^{\rm sea} = 310$ MeV
$m_\pi^{\rm val} = 220, 310, 690$ MeV
$a = 0.12, 0.15$ fm





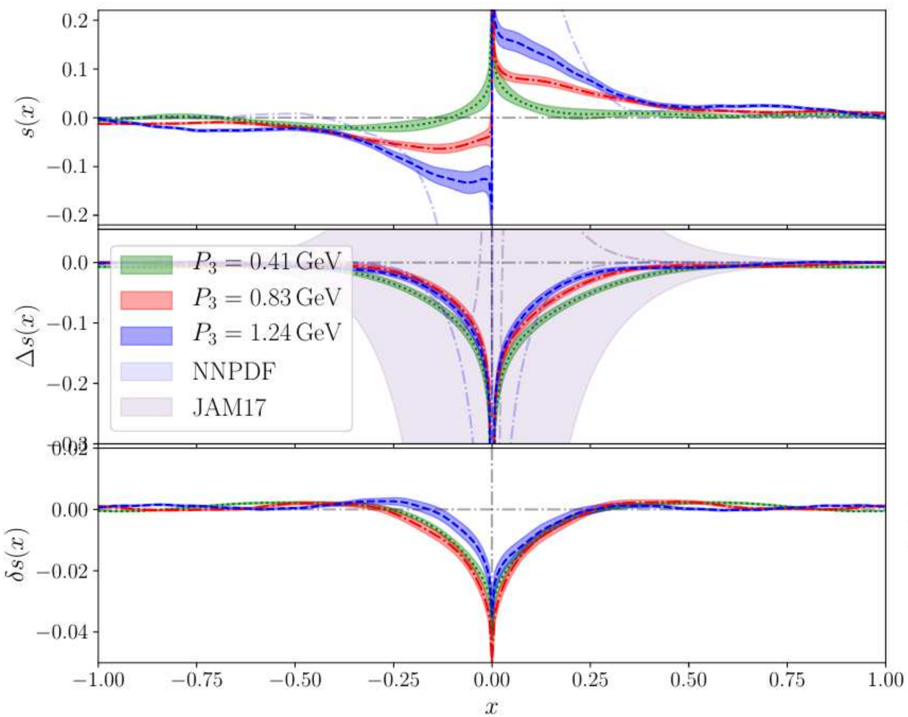
Flavor decomposition

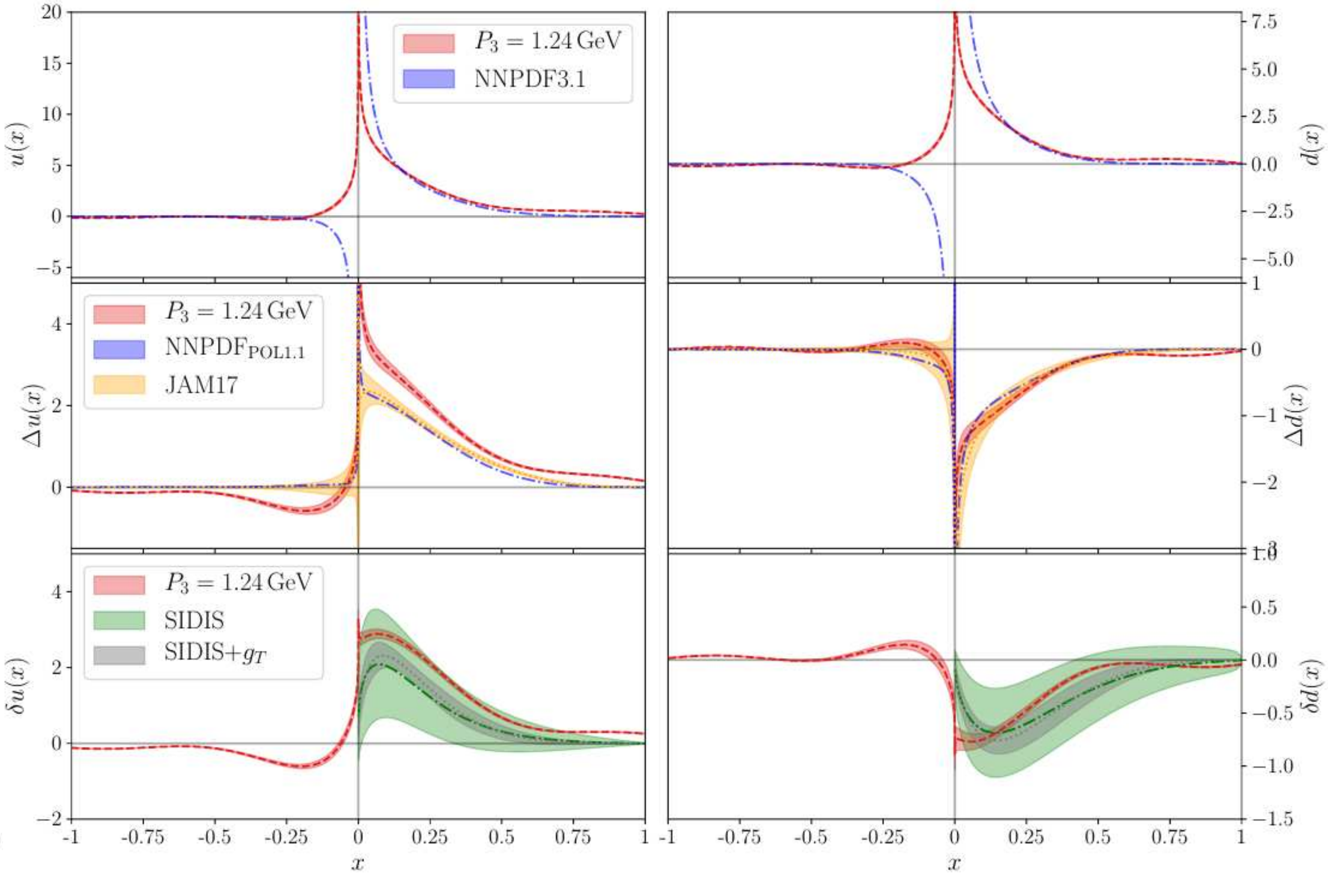
Most studies up to date were for the flavor non-singlet $u - d$ combination.
Important direction: flavor decomposition.
C. Alexandrou et al. (ETMC), Phys. Rev. Lett. 126 (2021) 102003; 2106.16065

F. Manigrasso Tue 14:30

- disconnected diagrams (hierarchical probing, one-end trick)
- mixing with gluon PDFs neglected
- justifiable approximation for u, d
- problematic for strange

QUASI	TMF	m_π = 260 MeV	a=0.093 fm





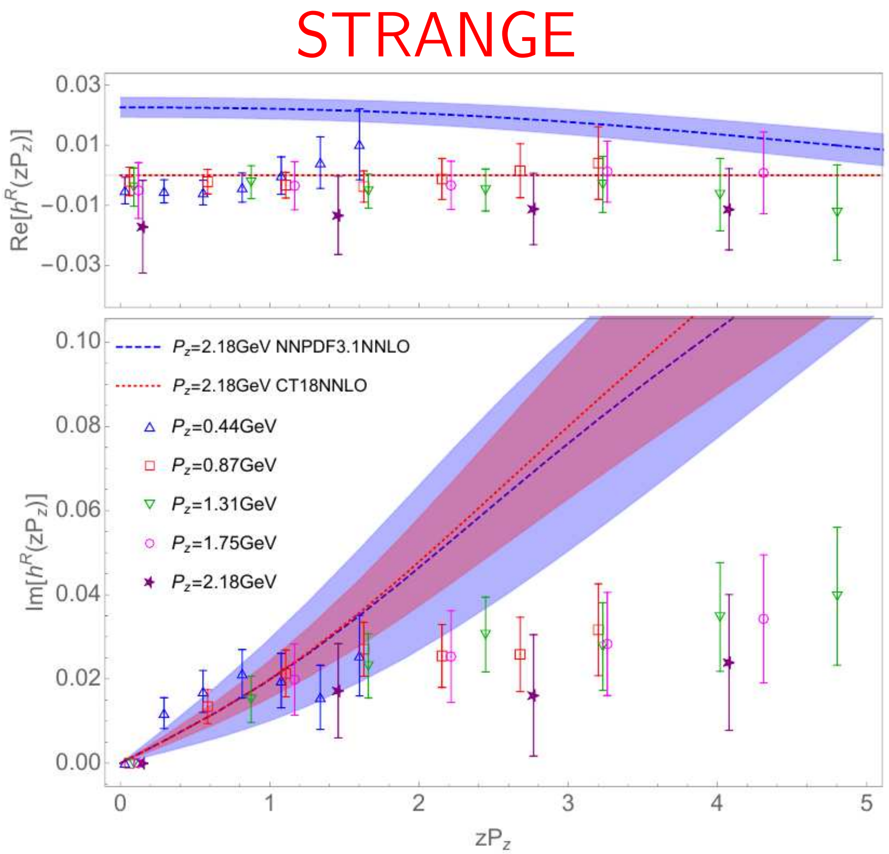
Strange and charm contributions

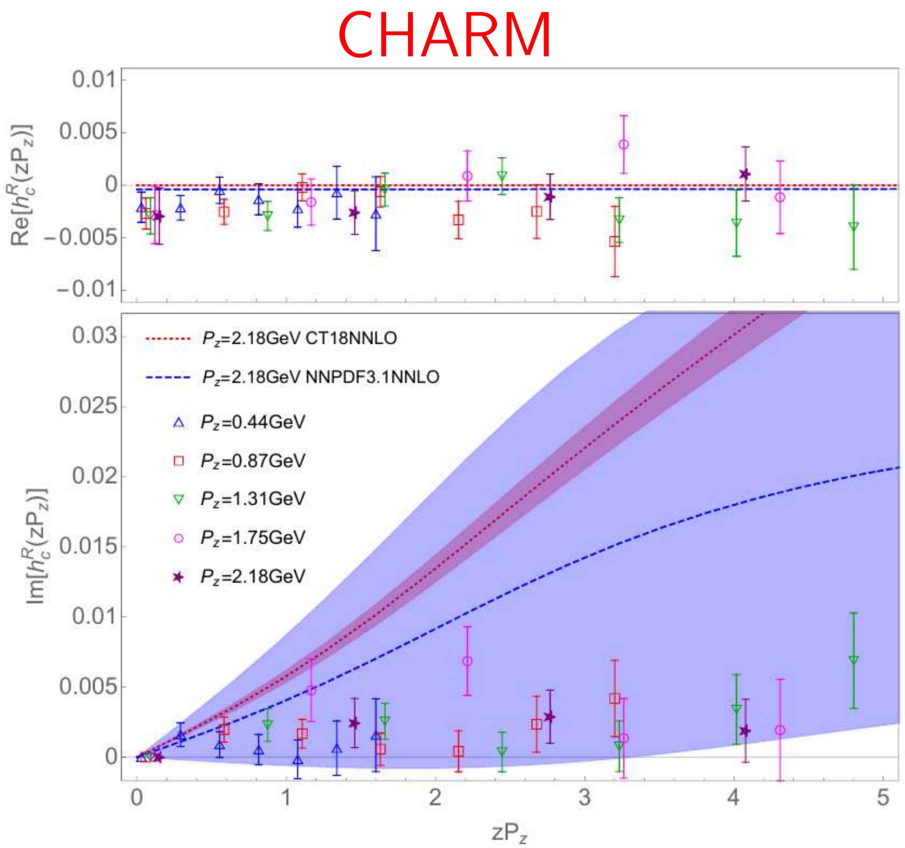
Another attempt: to determine the contribution of strange and charm quarks

R. Zhang, H.-W. Lin, B. Yoon, 2005.01124

R. Zhang Tue 21:45, H.-W. Lin Wed 14:45

QUASI	clover on HISQ	$m_\pi^{\rm sea} = 310$ MeV $m_\pi^{\rm val} = 310, 690$ MeV	$a = 0.12$ fm





Strange and charm are sea quarks, hence the ignored gluon splitting is hard to justify

Generalized parton distributions (GPDs)

First studies also for GPDs

C. Alexandrou et al. (ETMC), Phys. Rev. Lett. 125 (2020) 262001

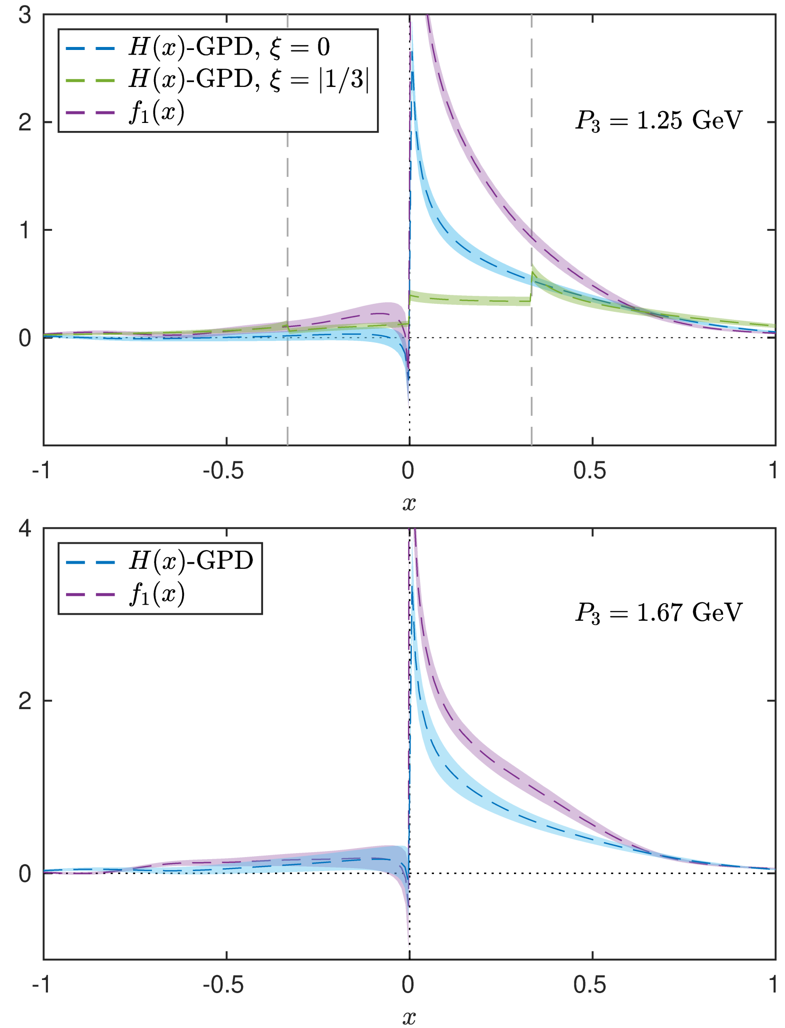
QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- nucleon boosts up to 1.67 GeV

Challenges:

- momentum transfer lowers the signal-to-noise ratio
- 2 or 4 GPDs $(H, E, \tilde{H}, \tilde{E})$ contribute to MEs at $Q^2 \neq 0$
  $\Rightarrow$ need to disentangle them using different projectors
- standard GPDs need Breit frame: $P_\perp^i = -P_\perp^f$
- needs optimization of momentum smearing for each $\vec{Q}$

A. Scapellato Thu 13:30



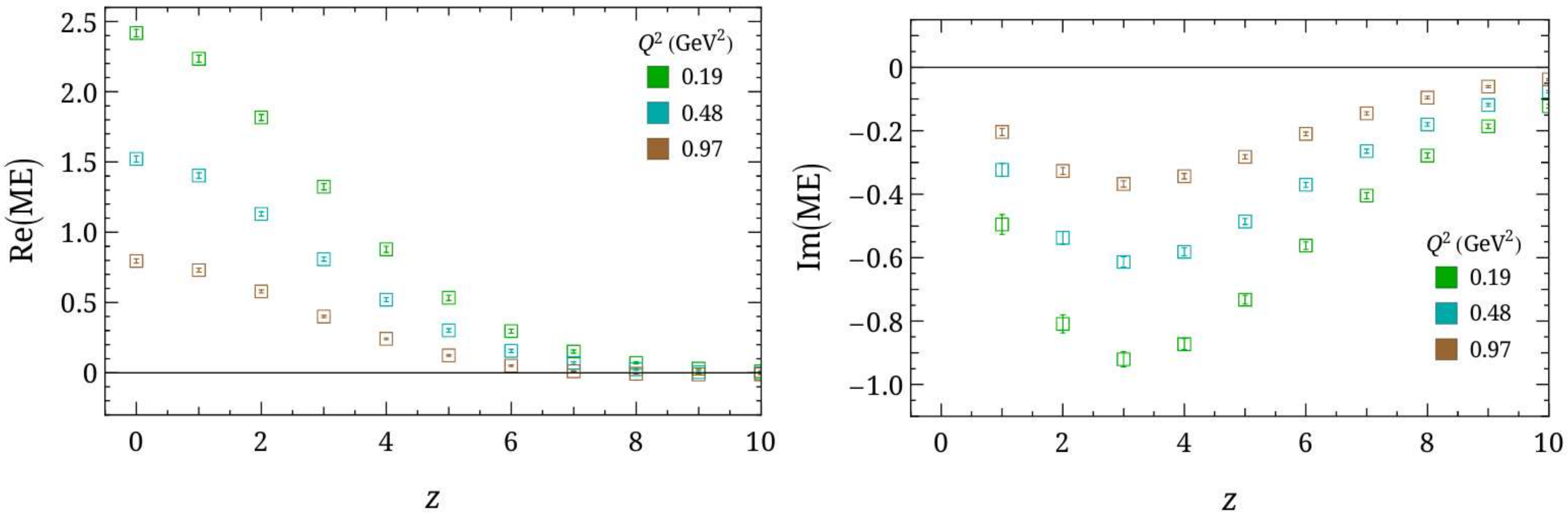
Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz, Phys. Lett. B788 (2019) 453
S. Bhattacharya, C. Cocuzza, A. Metz, Phys. Rev. D102 (2020) 054201

Independent work for GPDs

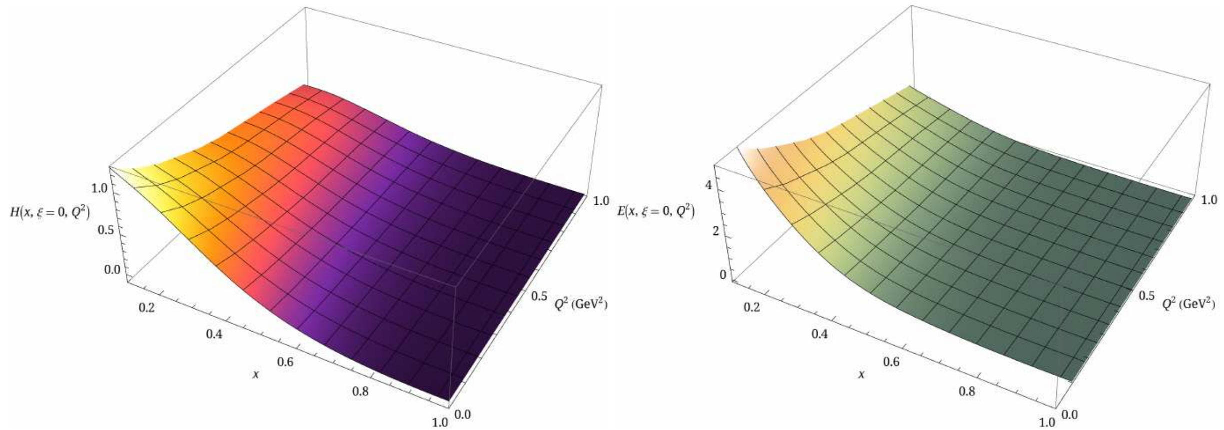
H.-W. Lin, 2008.12474 H.-W. Lin Wed 14:45

QUASI	clover on HISQ	$m_\pi = 135$ MeV	$a = 0.09$ fm



non-Breit frame

Again: how are such small errors possible at the physical point, here additionally with $Q^2 \neq 0$?



3D plot of matched non-Breit GPDs

left: H
right: E

v2 on arXiv this week now in Breit frame

Twist-3 PDFs

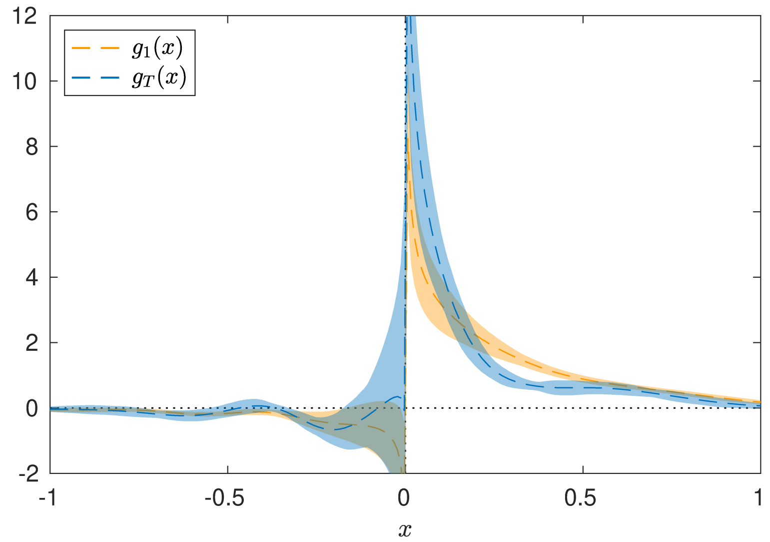
PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.



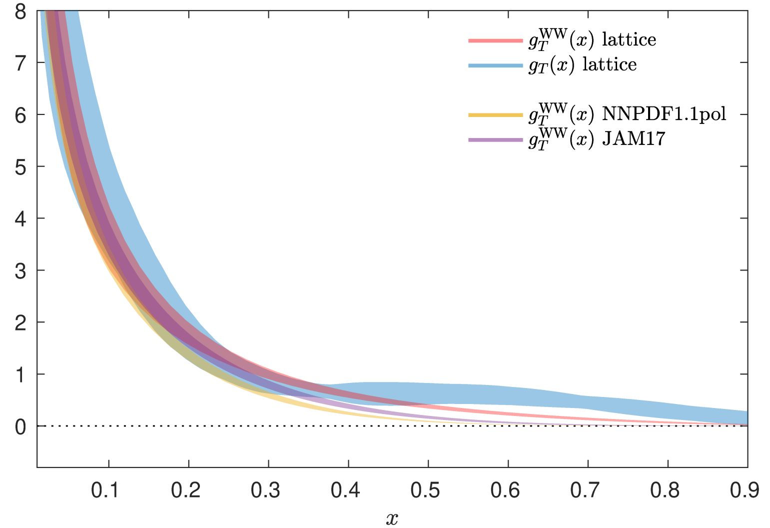
Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282
  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, 2103.12105

A. Metz Thu 13:45

- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
  + test of Wandzura-Wilczek approximation
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
  S. Bhattacharya et al., 2107.02574

M. Constantinou Thu 14:00



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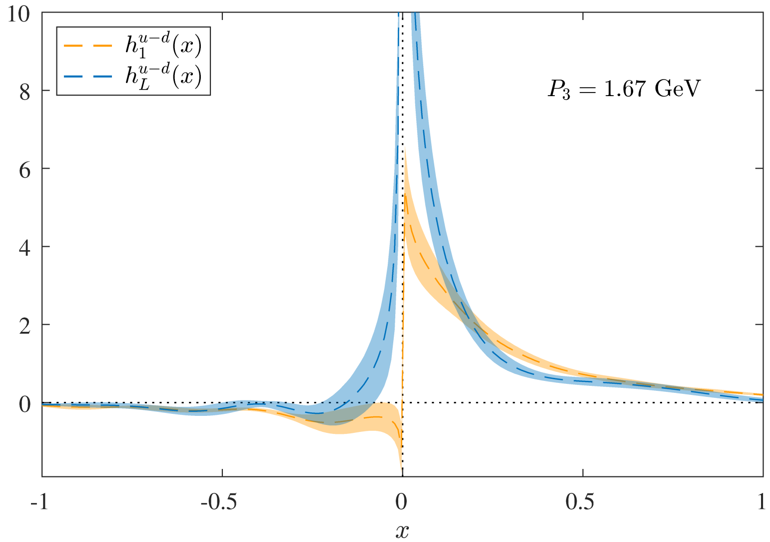

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  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
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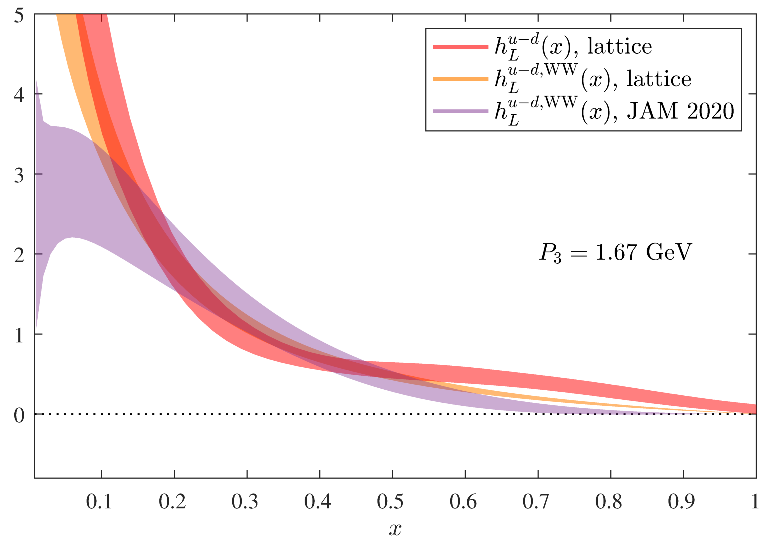
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
  S. Bhattacharya et al., 2107.02574

M. Constantinou Thu 14:00

- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2107.12818

J. Dodson Wed 15:00 (poster)

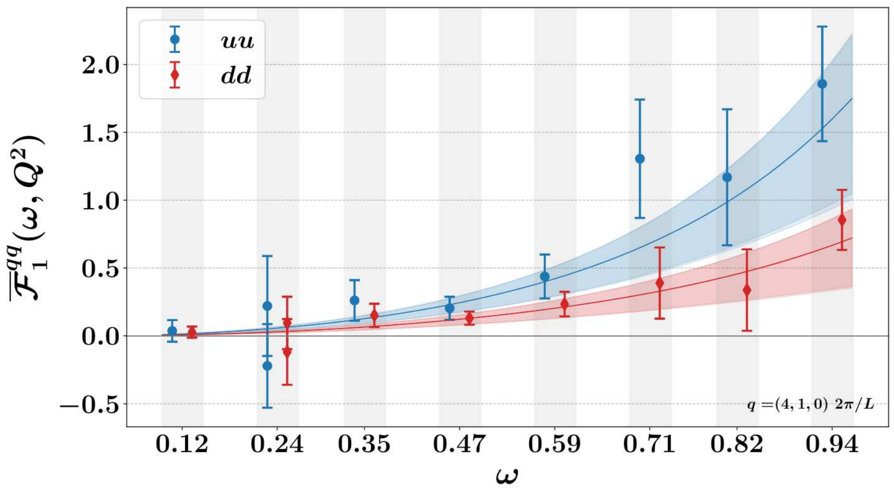


Hadronic tensor & forward Compton amplitude

Another way of approaching partonic distributions: hadronic tensor / Compton amplitude.
Unpolarized: can be factorized into F_1/F_2 or $\mathcal{F}_1/\mathcal{F}_2$ (Compton) structure functions
(optical theorem: Im of $\mathcal{F}_i$ related to F_i).

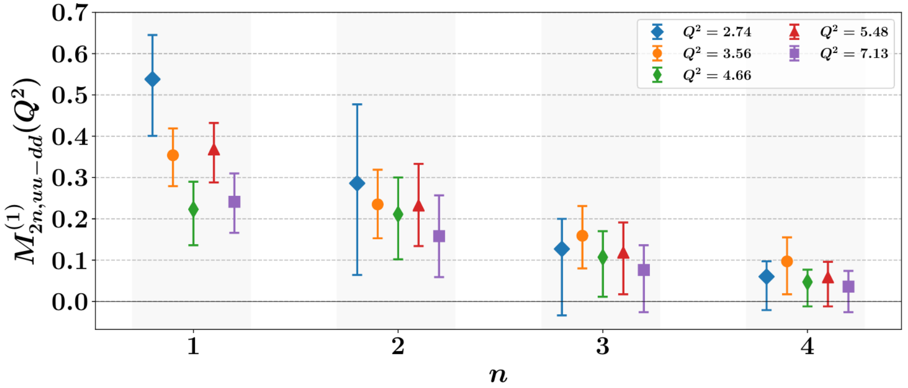
- hadronic tensor K.-F. Liu, S.-J. Dong, Phys. Rev. Lett. 72 (1994) 1790
  lattice-computed Euclidean HT $\xrightarrow{\text{inverse Laplace transform}}$ Minkowski HT
  main obstacle: inverse problem J. Liang et al. (χQCD), Phys. Rev. D101 (2020) 114503
- forward Compton amplitude A. J. Chambers et al. (QCDSF), Phys. Rev. Lett. 118 (2017) 242001
  spatial component T_{33} can be used to extract Compton SF $\mathcal{F}_1$ and Mellin moments
  i.e. Euclidean=Minkowski (as long as $|\omega| < 1$)
  effective computation: Feynman-Hellmann theorem

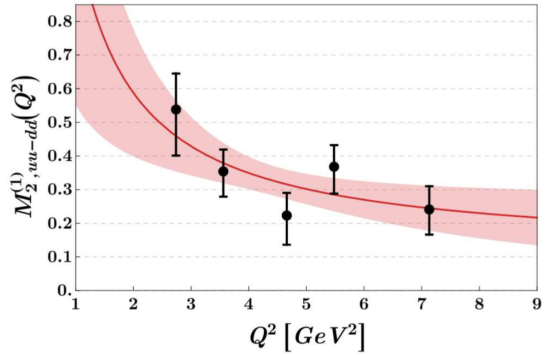
K. U. Can et al. (QCDSF/UKQCD/CSSM), Phys. Rev. D102 (2020) 114505



K. Can Mon 21:15

Compton
clover
$m_\pi = 470$ MeV
$a = 0.074$ fm





E. Sankey Wed 22:30

A. Hannaford-Gunn Mon 21:45

$$\mathcal{F}_1(\omega, Q^2) - \mathcal{F}_1(0, Q^2) = 4(\omega^2 M_2^{(1)}(Q^2) + \omega^4 M_4^{(1)}(Q^2) + \ldots)$$

Heavy OPE approach

Yet another method: Compton tensor with an auxiliary heavy quark + OPE to relate to Mellin/Gegenbauer moments of PDFs/LCDAs.
W. Detmold, C.-J. D. Lin, Phys. Rev. D73 (2006) 014501

- flavor-changing axial vector current: $J_A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$,
  $\Psi\ (\psi)$ – heavy (light) quark
- physics independent of the auxiliary quark mass as long as $\Lambda_{\rm QCD} \ll m_\Psi \sim \sqrt{Q^2}$
  (and $m_\Psi \ll a^{-1}$ for control of discretization effects)
- all effects of the heavy quark in Wilson coefficients
- no power-divergent mixings, suppressed HTE

Method recently dubbed **HOPE** (heavy OPE)
W. Detmold, A. Grebe, I. Kanamori, C.-J. D. Lin, R. Perry, Y. Zhao (HOPE), 2103.09529

Recent development:

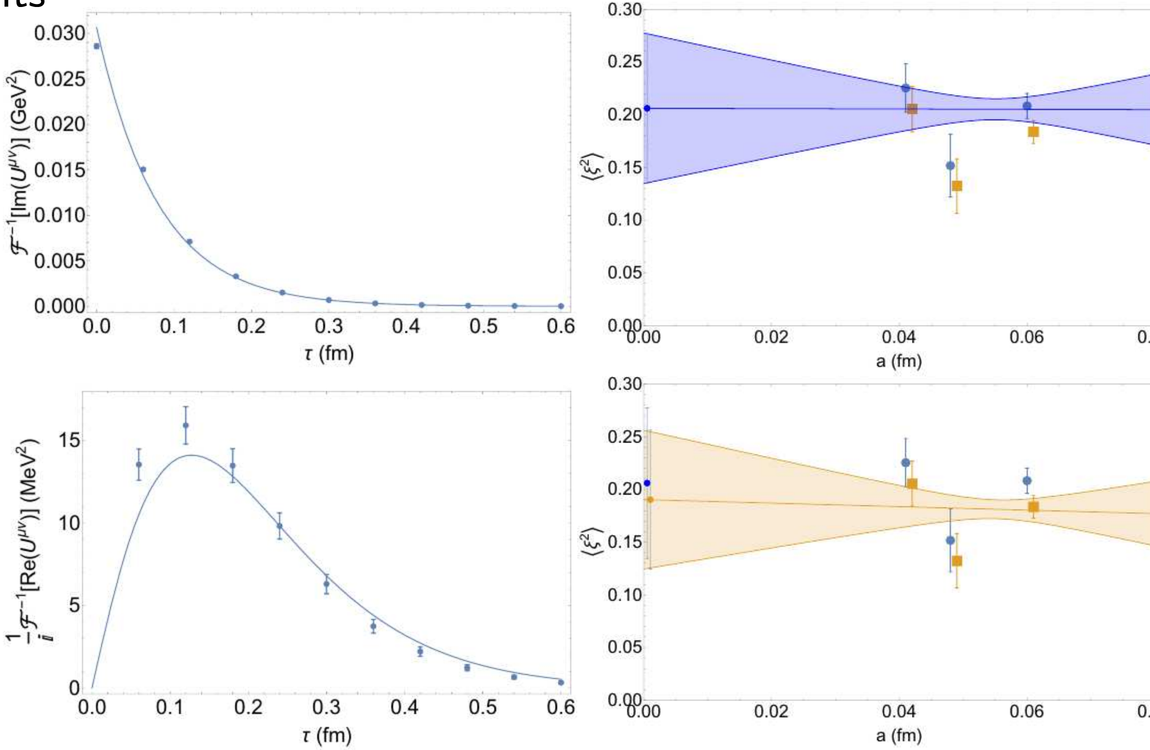
- relation to other approaches
- analytic structure of HOPE amplitudes, convergence radius
- calculation of 1-loop Wilson coefficients for unpolarized/helicity PDFs and LCDA

A. Grebe Thu 21:00

R. Perry Thu 21:15

HOPE	clover	$m_\pi = 560$ MeV	$a = 0.041$-0.06 fm

W. Detmold et al. (HOPE), 2009.09473



Pion PDFs

Interest also in pion PDFs, using several approaches.
Question: large-x behavior $(1-x)^{-1}$ vs. $(1-x)^{-2}$ decay.

C. Lauer Wed 21:00

C. Alexandrou et al. (ETMC), 2104.02247

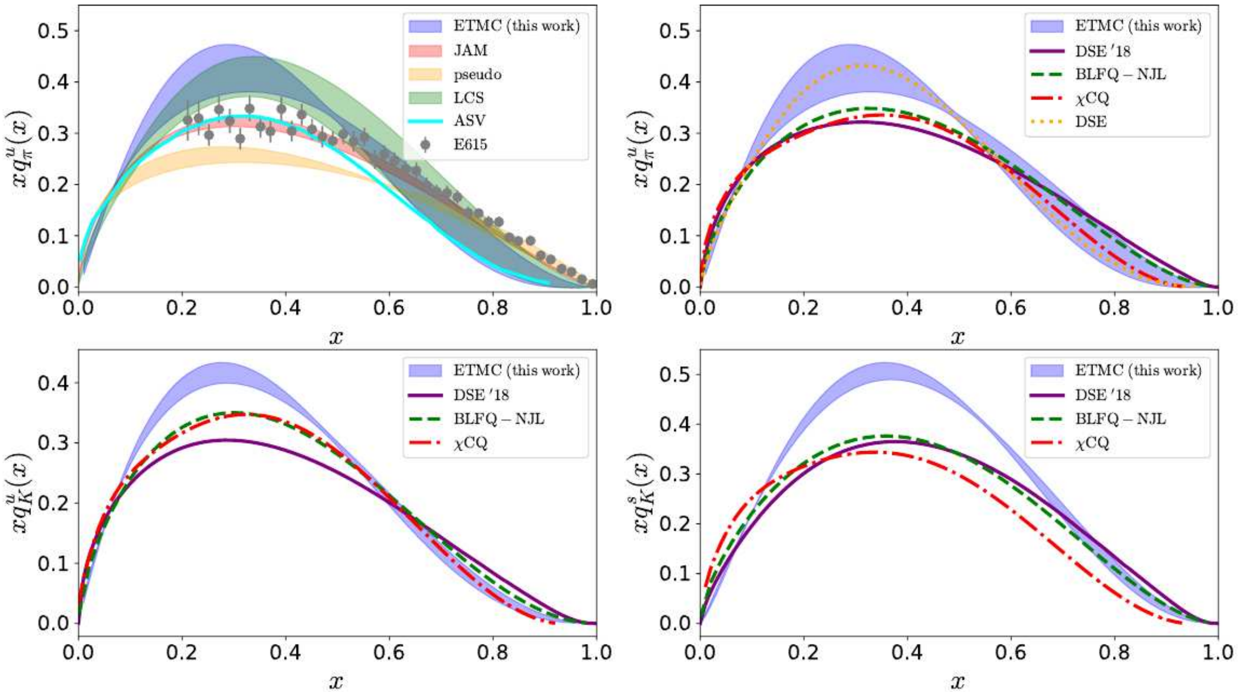
moments	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

B. Joo et al., Phys. Rev. D100 (2019) 114512

PSEUDO	clover	$m_\pi = 415$ MeV	$a = 0.127$ fm

R. Sufian et al., Phys. Rev. D102 (2020) 054508

current-current (LCS)	clover	$m_\pi = 415$, 358,278 MeV	$a = 0.127$, 0.094 fm

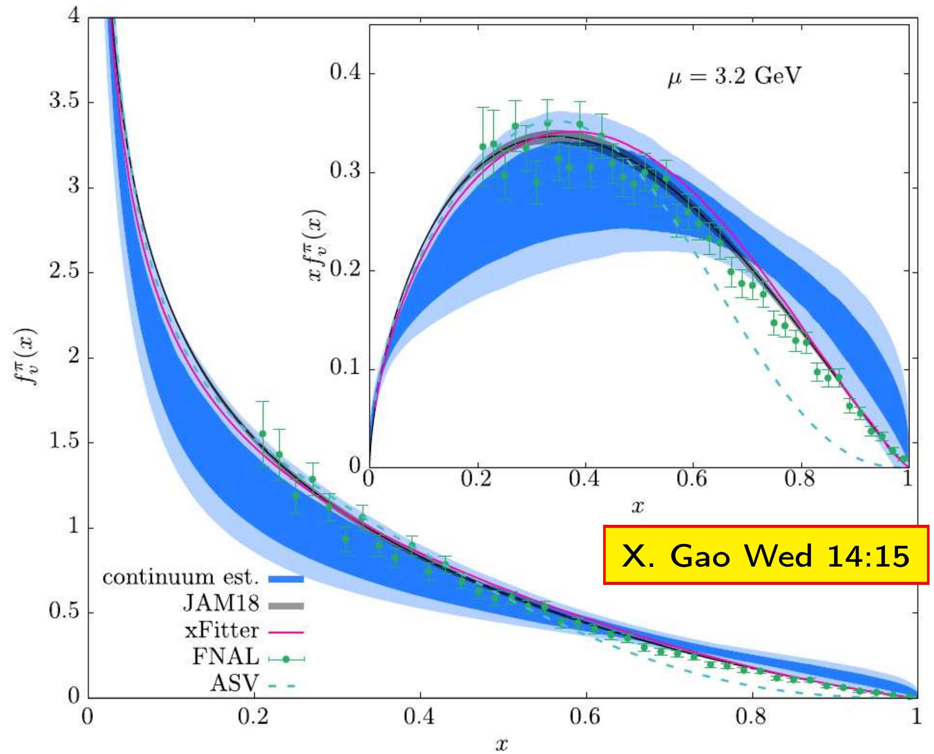


BNL – different analyses:
β from $0.66(34)(22)$ to $1.55(34)(27)$

Pheno analyses:
FNAL E615: $\beta \approx 1$ ASV: $\beta \approx 2$
JAM: $\beta \approx 1.2$ DSE: $\beta \approx 2$
xFitter: $\beta \approx 1$ NJL: $\beta \approx 1$

X. Gao et al. (BNL), Phys. Rev. D102 (2020) 094513

PSEUDO (SDF)	clover on HISQ	$m_\pi^{\rm sea} = 160$ MeV, $m_\pi^{\rm val} = 300$ MeV	$a = 0.06$, 0.04 fm



X. Gao Wed 14:15

ETMC: $\beta \approx 2$, pseudo: $\beta = 1.1(4)$, LCS: 2-param.: $\beta = 1.24(22)(7)$, 3-param.: $\beta = 2.12(56)(14)$

Meson PDFs/DAs

Pion (pseudo-)PDFs also investigated in QCD_3 with 0,2,4,8 flavors
N. Karthik, Phys. Rev. D103 (2021) 074512

Meson distribution amplitudes (DAs) important for many exclusive decays.

DAs represent momentum distribution of quarks/antiquarks in the leading $q\bar{q}$ Fock state of the meson's wave function.

Computed rather early with x-dependent approaches.
Recent work:

- K^* and ϕ mesons with quasi (physical point, continuum limit) J. Hua Thu 21:30
  J. Hua et al. (LPC), 2011.09788
- π and K mesons P. Scior Thu 21:45
- π and K mesons with quasi (continuum limit) N. Juliano Thu 22:00
  R. Zhang et al., Phys. Rev. D102 (2020) 094519
- B meson DA formalism with
  pseudo: S. Zhao, A. Radyushkin, Phys. Rev. D103 (2021) 054022
  quasi: W. Wang, Y.-M. Wang, J. Xu, S. Zhao, Phys. Rev. D102 (2020) 011502

2-loop matching

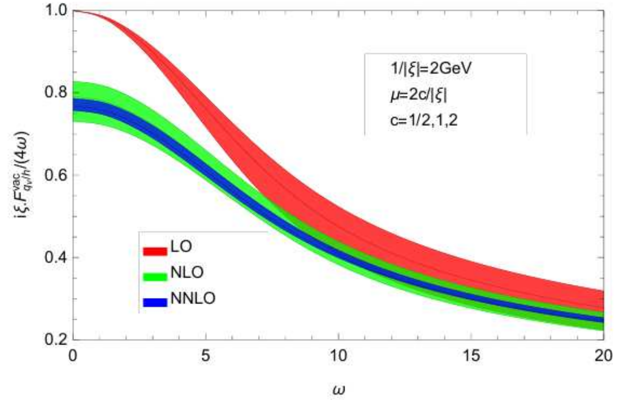
One of the important systematics of PDF computations is the truncation effect in the matching.

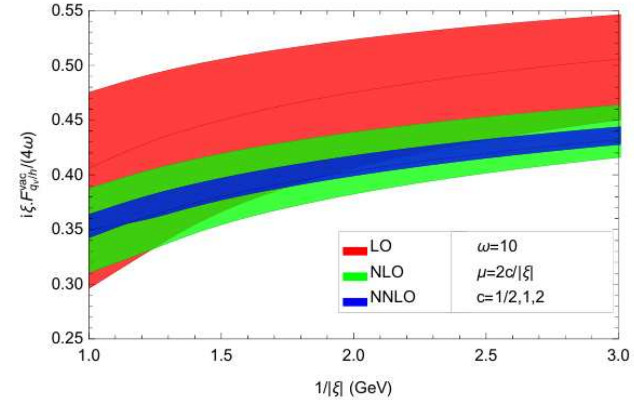
Recent 2-loop analyses:

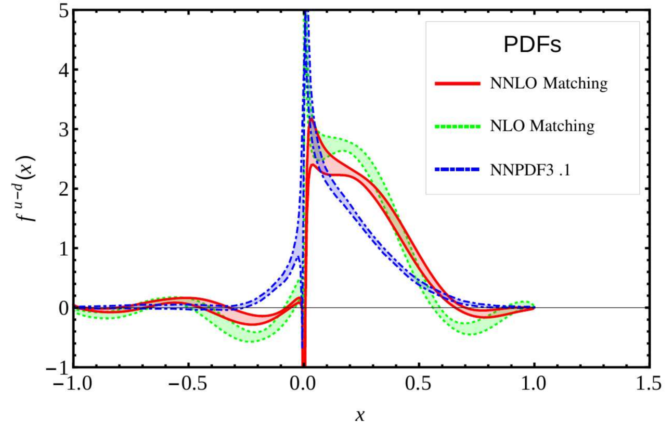
- renormalization and VEVs of non-local off-light-cone $\bar{q}q$ or two $F^{\mu\nu}$'s
  V. Braun, K. Chetyrkin, B. Kniehl, JHEP 07 (2020) 161
  nucleon MEs of these are quasi-PDFs at space-like separations
  similar, but time-like MEs in HQET
  showed that renorm. is the same for space-like and time-like
  extracted $\overline{\mathrm{MS}}$ anom. dimensions and renorm. factors
- perturbative results for QCFs (quark correlation functions), $\overline{\mathrm{MS}}$ renorm. factors and conversion factors to a vacuum scheme
  Z.-Y. Li, Y.-Q. Ma, J.-W. Qiu, Phys. Rev.Lett. 126 (2021) 072001
  FT of QCFs gives either quasi-PDFs or pseudo-PDFs
  matching coefficients given
- matching coefficients for quasi-PDFs in the modified $\overline{\mathrm{MS}}$ and RI/MOM schemes
  L.-B. Chen, W. Wang, R. Zhu, Phys. Rev. D102 (2020) 011503
  L.-B. Chen, W. Wang, R. Zhu, JHEP 10 (2020) 079
  L.-B. Chen, W. Wang, R. Zhu, Phys. Rev.Lett. 126 (2021) 072002
  (lattice data from Y.-S. Liu et al. (LPC), Phys. Rev. D101 (2020) 034020)

R. Zhu Wed 6:30

Y. Zhao Wed 14:00, J.-H. Zhang Wed 6:15







Hybrid renormalization

The standard procedure of quasi-PDF MEs renormalization:

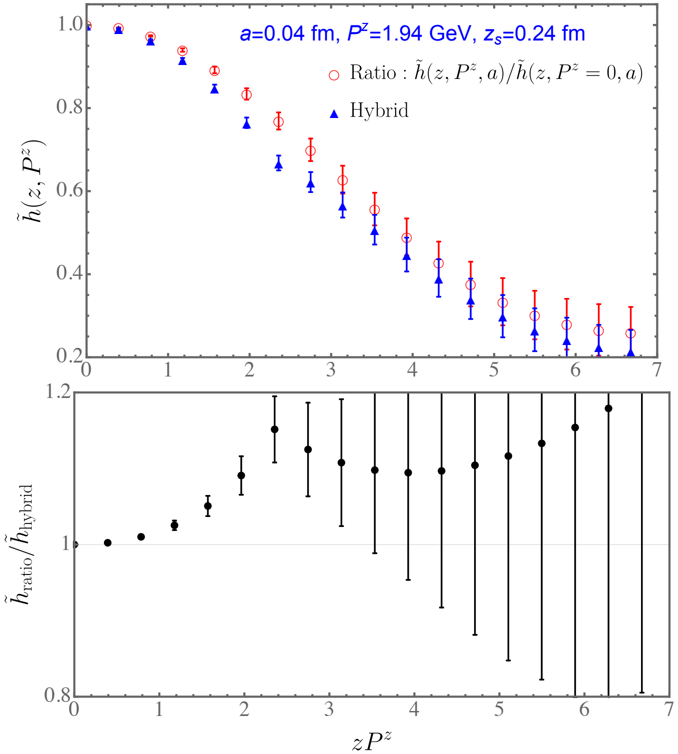
$$O_{\overline{\rm MS}}(z,\mu) = Z_{\overline{\rm MS}}(z,-p^2,\mu)\frac{O(z,a)}{Z(z,-p^2,a)}$$

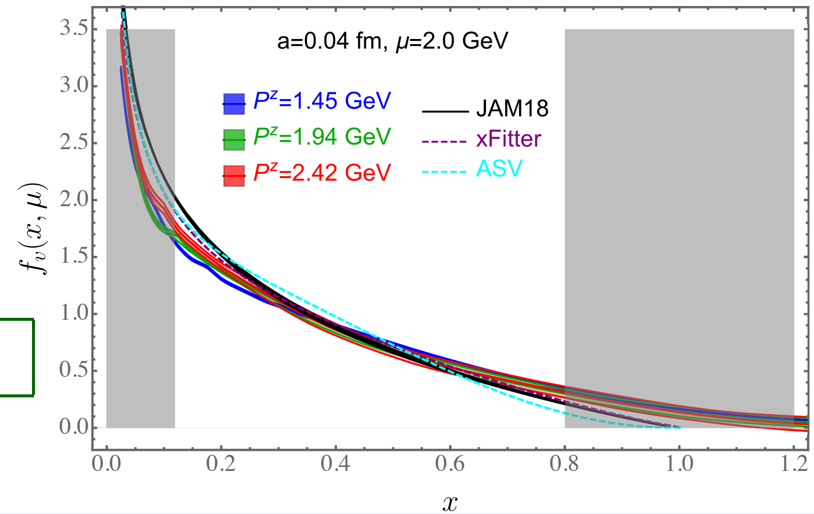
is argued to contain non-perturbative effects at large-z.

X. Ji et al., Nucl. Phys. B964 (2021) 115311

Proposed way out: **hybrid renormalization**.

- short distance $z \leq z_S \approx 0.3$ fm
  – ratio scheme / RI-MOM,
- intermediate distance 0.3 fm $\approx z_S \geq z \leq z_L \approx \Lambda_{\rm QCD}^{-1}$
  – separate renormalization of log and linear divergences:
  $Z(a,\mu)\exp(-\delta m|z|)O(z,a)$,
  δm – Wilson line mass renormalization, e.g. from the static potential or from fitting MEs at large-z
- large distance $(z_L \approx \Lambda_{\rm QCD}^{-1} \approx z_L \geq z)$
  – exponential/algebraic extrapolation (Regge-based),
- matching the different procedures at z_S and z_L.





Y. Zhao Wed 14:00

QUASI	clover on HISQ	$m_\pi = 300$ MeV	$a = 0.04$ fm

J.-H. Zhang Wed 6:15

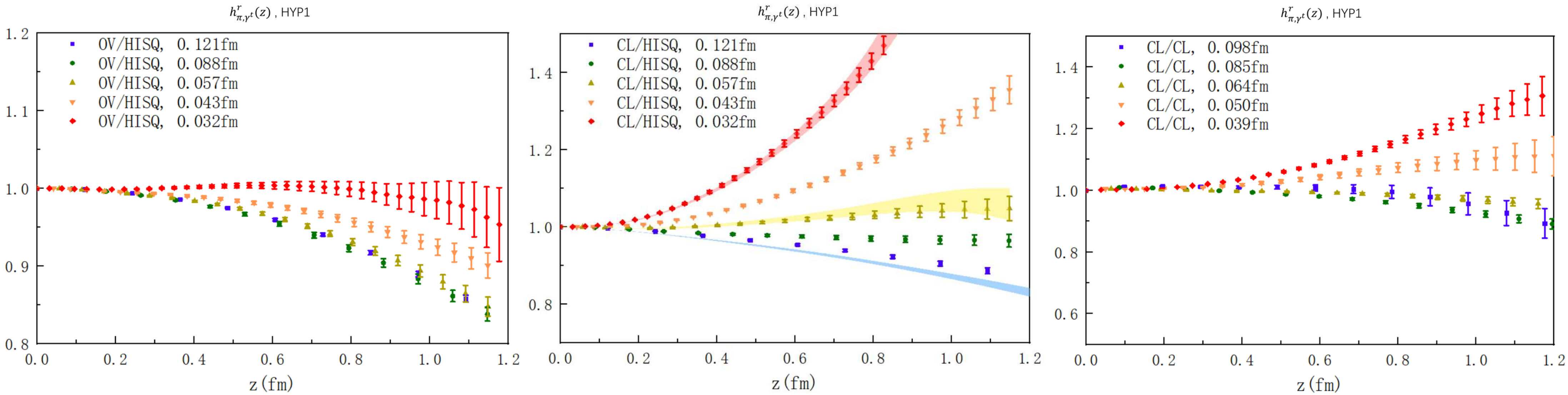
J. Hua Thu 21:30

Residual divergence

There is also numerical evidence pointing to the possible contamination of RI/MOM-renormalized MEs with a residual linear divergence.
K. Zhang et al. (χQCD), 2012.05448

- investigated several types of MEs
- pion in the rest frame to achieve good statistical precision
- different lattice setups:
  - ⋆ overlap on HISQ, DWF
  - ⋆ clover on HISQ, unitary clover
  - ⋆ TMF on HISQ
  - ⋆ lattice spacings from 0.12 to 0.032 fm

OV on HISQ clover on clover/HISQ	m_π = 310-360 MeV	clover: 0.039-0.098 fm HISQ: 0.032-0.121 fm



Other developments

Y. Su Wed 21:30

J.-H. Zhang Wed 6:15

- Self-renormalization Y.-K. Huo et al. (LPC), Nucl. Phys. B969 (2021) 115443
  disentangle Z-factors directly from MEs at several lattice spacings
- Bayesian determination of OPE Wilson coefficients from lattice and pheno data
  N. Karthik, R. Sufian, 2106.03875
- Bayes-Gauss-Fourier transform for PDF reconstruction
  C. Alexandrou et al. (ETMC), Phys. Rev. D102 (2020) 094508
- Renormalon effects in quasi- and pseudo-distributions
  V. Braun, A. Vladimirov, J.-H. Zhang, Phys. Rev. D99 (2019) 014013; W.-Y. Liu, J.-W. Chen, 2010.06623
  enhanced power corrections at small- and large-x
- Chiral perturbation theory for LaMET W.-Y. Liu, J.-W. Chen, 2011.13536
  FVE for $m_\pi L \geq 3$ below 1% (smaller than in rest frame)
  determined also leading pion mass dependence
- Origin and resummation of threshold logarithms X. Gao et al., Phys. Rev. D103 (2021) 094504
- FVE for non-local current-current operators
  R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511
  R. Briceño, C. Monahan, Phys. Rev. D103 (2021) 094521
  FVE can depend on $m_\pi(L-z)$ and, thus, be enhanced at large separations
- Parton distributions in nongauge theories L. Del Debbio, T. Giani, C. Monahan, JHEP 09(2020)021
  formal equivalence of quasi- and pseudo-distributions
  but: different systematics and hence complementary
  insights also for new class of factorizable observables based on gradient flow
  (earlier work: C. Monahan, K. Orginos, JHEP 03 (2017) 116)
  advocating global analyses of lattice data to extract PDFs

L. Del Debbio Thu 13:15

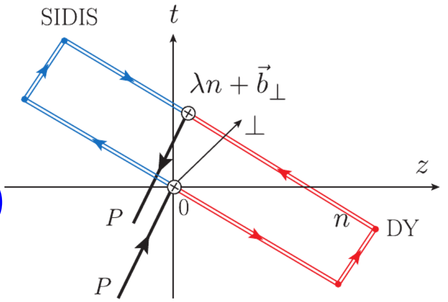
Transverse momentum dependent PDFs

PDFs provide information only on the longitudinal momentum distributions, while in many cases important effects also from transverse momentum.

I. Stewart Thu 10:30

- Important for wide kinematical ranges in Drell-Yan, e^+e^- annihilation, SIDIS
- Example: unpolarized

$$f(x,\vec{k}_\perp) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi}\frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\lambda x + i\vec{k}_perp\cdot\vec{b}_\perp}\langle P|\bar{\psi}(\lambda n/2+\vec{b}_\perp)\gamma^+\mathcal{W}_n(\lambda n/2+\vec{b}_\perp)\psi(-\lambda n/2)|P\rangle$$



- Crucial new aspect: rapidity divergences from soft gluon radiation
  $\Rightarrow$ rapidity regulator δ + UV renormalization scale μ
- Rapidity divergences can be incorporated in the soft function $S(b_\perp, \mu, \delta^+, \delta^-)$
  represents soft gluon radiation effects of a fast-moving charged particle
- Physical renormalized TMD: $f^{\rm TMD} = f/\sqrt{S}$
- Soft function:
  - ⋆ intrinsic part (rapidity-independent)
  - ⋆ rapidity-dependent part defining Collins-Soper kernel $K(b_\perp, \mu)$ – log-derivative of $f^{\rm TMD}$.

From: X. Ji et al., 2004.03543

- $f^{\rm TMD}(x, b_\perp, \mu, \zeta)$ – final desired object with evolution in the 2 last arguments governed by:
  - ⋆ CS kernel for rapidity ζ
  - ⋆ γ_μ anomalous dimension (consisting of cusp and hard anomalous dimension) for renormalization scale μ
- also: single transverse-spin asymmetry & Sivers Function from LaMET
  X. Ji, Y. Liu, A. Schäfer, F. Yuan, Phys. Rev. D103 (2021) 074005
  light-front wave functions from LaMET X. Ji, Y. Liu, 2106.05310

Intrinsic soft function

The soft function can be extracted from a pseudoscalar meson form factor

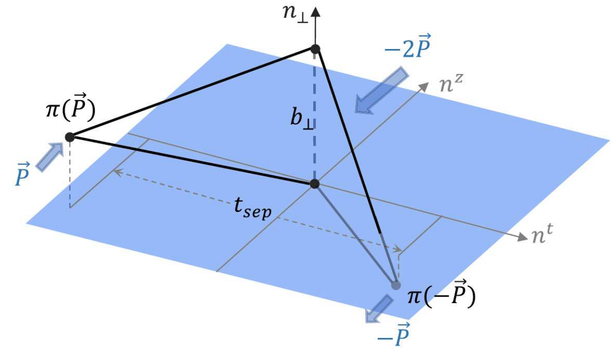
X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. B955 (2020) 115054, Phys. Lett. B811 (2020) 135946

$$F_\Gamma(b_\perp, P^z) = \langle \pi(-P^z)|\bar{u}\Gamma u(t, b_\perp)\, \bar{d}\Gamma d(t,0)|\pi(P^z)\rangle$$

$F_\Gamma(b_\perp, P^z)$ can be factorized into:

Y. Liu Wed 7:15

- intrinsic soft function
- quasi-TMDWF ≈ pion LCDA with a staple-shaped operator



From: LPC, PRL125(2020)192001

2 groups followed this strategy

Q.-A. Zhang et al. (LPC), Phys. Rev. Lett. 125 (2020) 192001

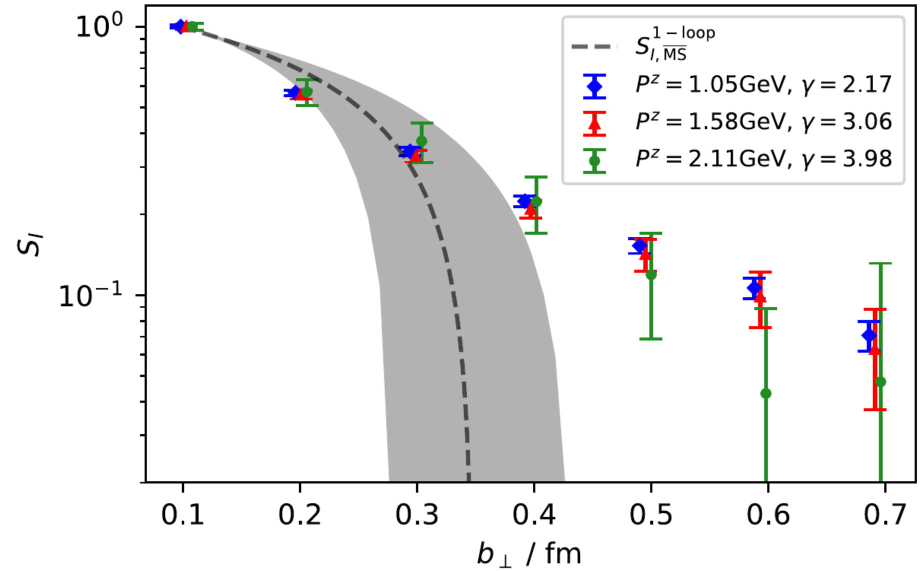
Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

QUASI	clover	$m_\pi^{\rm sea} = 333$ MeV $m_\pi^{\rm val} = 547$ MeV	$a = 0.098$ fm

LPC calculation:

Q.-A. Zhang Wed 7:30

- $\Gamma = I$ – best signal, leading-twist
- renormalization of bare $S_I(b_\perp, 1/a)$:
  $S_I^{\overline{\rm MS}}(b_\perp, 1/a) = \frac{S_I(b_\perp, 1/a)}{S_I(b_{\perp,0}, 1/a)} S_I^{\overline{\rm MS}}(b_{\perp,0}, \mu)$
  ($S_I^{\overline{\rm MS}}(b_{\perp,0}, \mu)$ from 1-loop PT)
- leading-order matching: $1/2N_c + O(\alpha_s)$



Intrinsic soft function

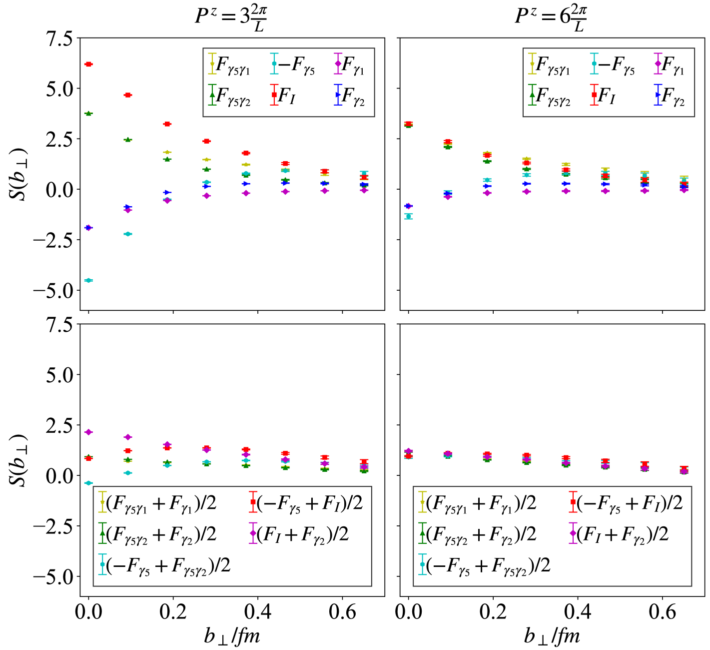
Beijing+ETMC calculation

Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

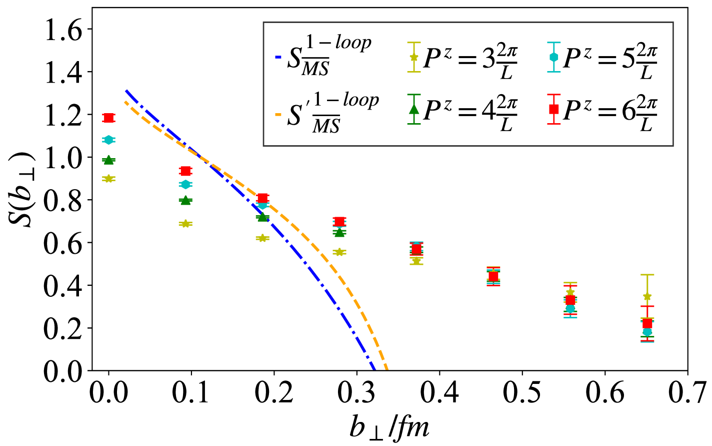
S.-C. Xia Wed 6:45

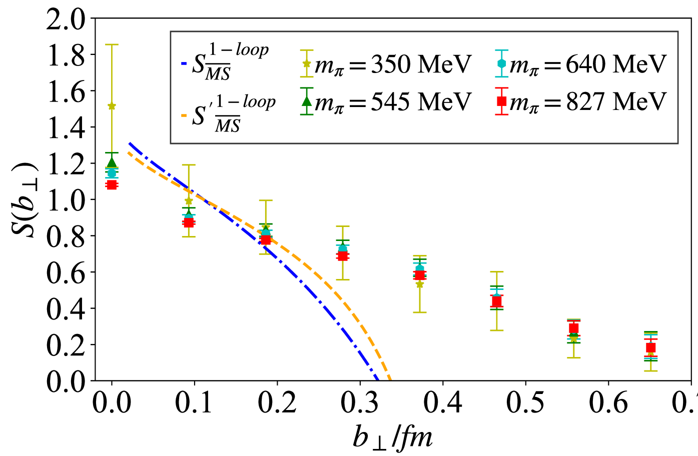
QUASI	TMF	$m_\pi^{\rm sea} = 350$ MeV $m_\pi^{\rm val} = 350$-827 MeV	$a = 0.093$ fm

- $\Gamma = I, \gamma_1, \gamma_2, \gamma_5, \gamma_5\gamma_1, \gamma_5\gamma_2$
  found significant higher-twist contamination!
- considered combinations to reduce HTE
  using Fierz identities
- ratio scheme renormalization:
  $C^{\rm ratio}(b_\perp, l, P_3) = \frac{C(b_\perp, l, P_3)}{C(b_\perp, l, 0)} C^{\overline{\rm MS}}(0,0,0), \mu)$
  ($C^{\overline{\rm MS}}(0,0,0)$ – standard local RI' renormalization)
- leading-order matching: $1/2N_c + O(\alpha_s)$
- test of convergence in hadron boost P_3



test of pion mass effects





Collins-Soper kernel

The CS kernel governs the rapidity evolution of TMDs
Two approaches:

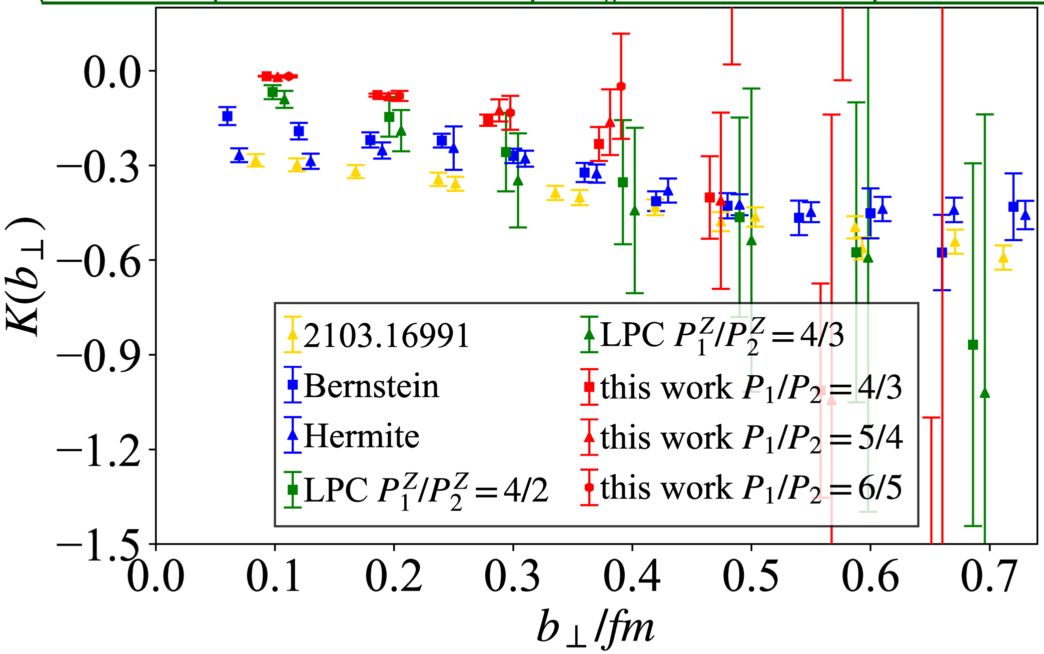
- ratio of TMDs at different rapidities
  M. Ebert, I. Stewart, Y. Zhao, PRD99(2019)034505
- ratios of first Mellin moments of TMDs
  M. Schlemmer et al., 2103.16991

M. Schlemmer Wed 7:00

MOMENTS	clover	$m_\pi = 422$ MeV	$a = 0.085$ fm

P. Shanahan, M. Wagman, Y. Zhao, PRD102(2020)014511; 2107.11930

QUASI	clover (quench.) clover on HISQ	$m_\pi^{\text{val}} = 1.2$ GeV $m_\pi^{\text{val}} = 538$ MeV	$a = 0.06$ fm $a = 0.12$ fm



Q.-A. Zhang Wed 7:30

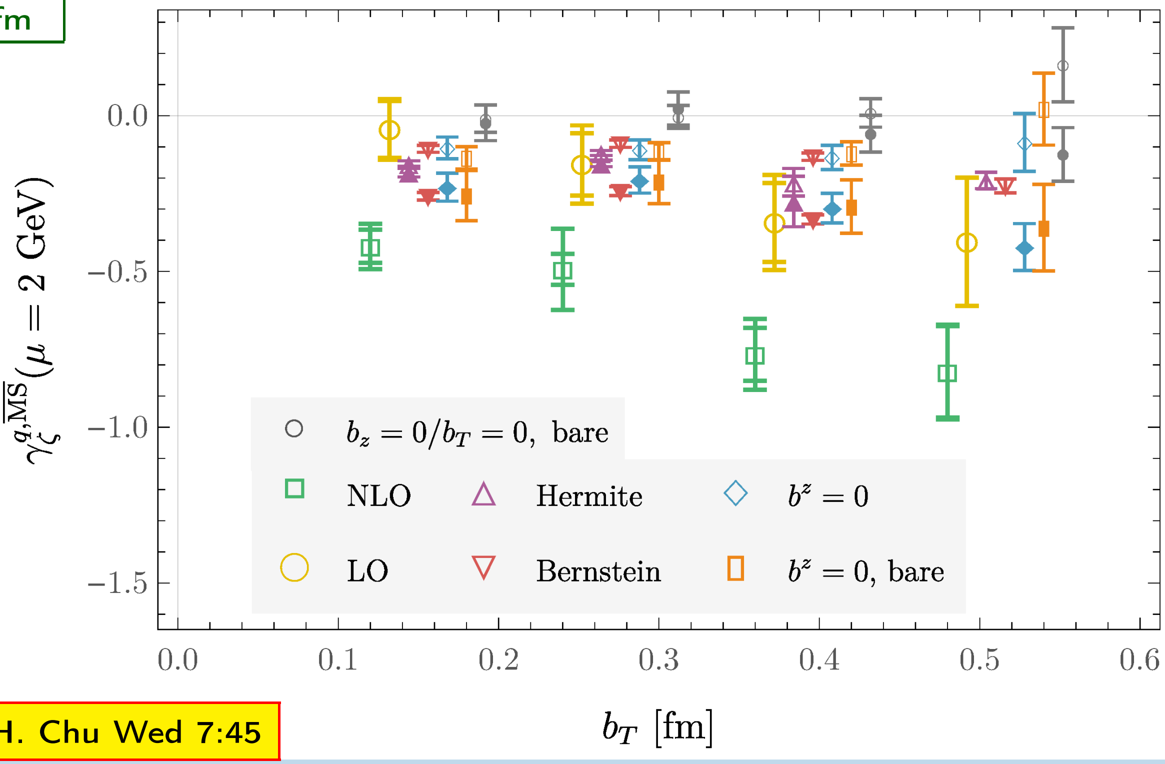
Q.-A. Zhang et al. (LPC), Phys. Rev. Lett. 125 (2020) 192001

QUASI	clover	$m_\pi^{\text{sea}} = 333$ MeV $m_\pi^{\text{val}} = 547$ MeV	$a = 0.098$ fm

S.-C. Xia Wed 6:45

Y. Li, S.-C. Xia et al. (Beijing+ETMC), 2106.13027

QUASI	TMF	$m_\pi^{\text{sea}} = 350$ MeV $m_\pi^{\text{val}} = 350\text{-}827$ MeV	$a = 0.093$ fm



M.-H. Chu Wed 7:45

Key prospects for the future

1. Robustness and reliability of the lattice extraction of x-dependent distributions
   $\Rightarrow$ **towards precision studies**
   improvements of lattice techniques
   study and removal of systematic effects

2. Exploration of new directions
   new kinds of distributions higher-twist, GPDs, TMDs, LFWFs
   other hadrons?
   can be phenomenologically relevant, e.g. K^*, ϕ J. Hua et al. (LPC), 2011.09788 J. Hua Thu 21:30
   can shed light on the nucleon, e.g. Δ^+ Y. Chai et al. (Beijing+ETMC), PRD102(2020)014508

3. Synergy between lattice and phenomenology
   unpolarized PDFs – benchmark A. Deshpande Thu 9:00, I. Stewart Thu 10:30
   other distributions – potentially crucial impact

Robustness/reliability of lattice extraction

- Lattice-specific systematics:
  - ⋆ isolation of the ground state hadron
  - ⋆ discretization effects
  - ⋆ finite volume effects
  - ⋆ pion mass dependence (if not working at the physical point)

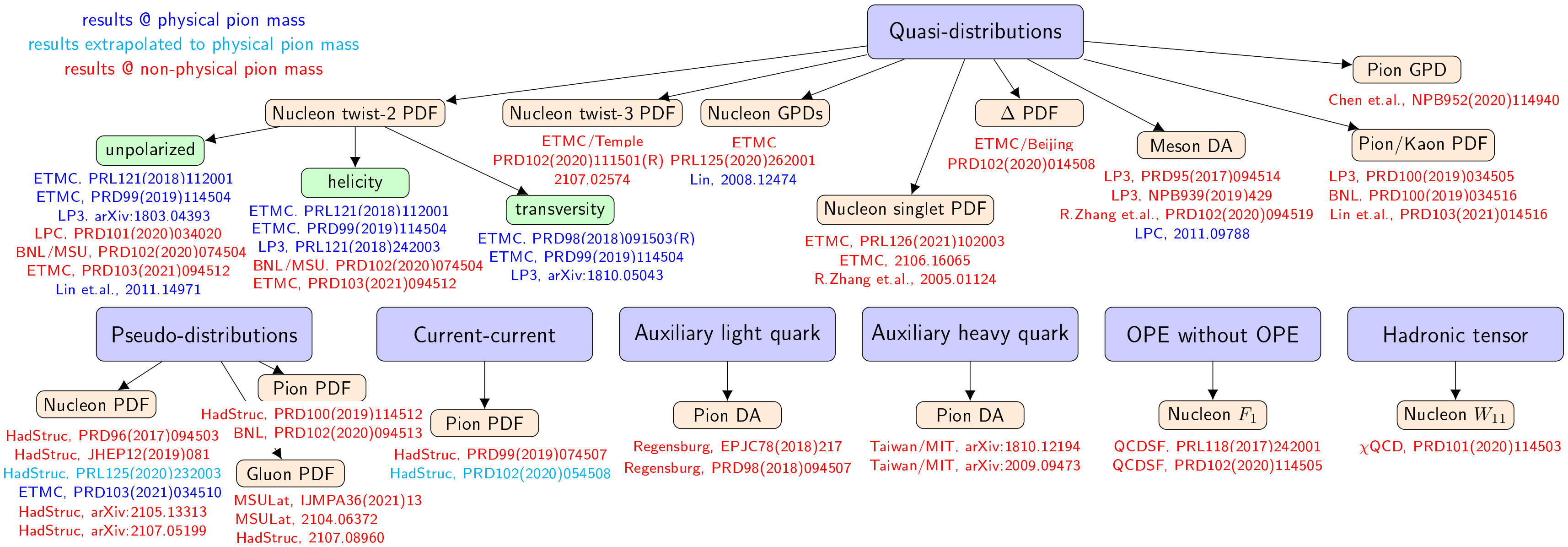
  Note: hierarchy of systematics needs to be observed
- Broader systematics of the lattice calculation:
  - ⋆ reconstruction of the x-dependence
  - ⋆ non-perturbative renormalization
  - ⋆ truncation effects: conversion, evolution, matching
  - ⋆ higher-twist effects

Key challenges:

- **lattice: reliably reach large hadron boosts**
- **lattice: control all lattice-specific systematics**
- **pheno: insights into HTE?**

Conclusions

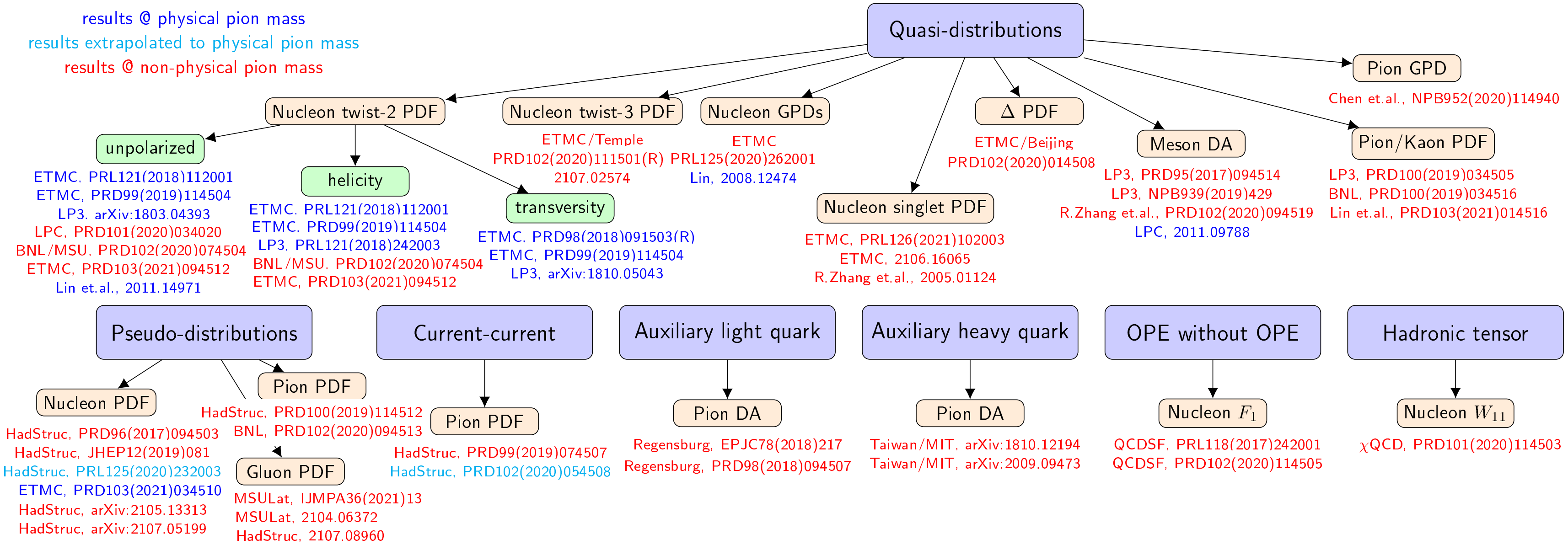
- Message of the talk: **enormous progress in lattice calculations of x-dependent distributions with very encouraging results!**



- Increasing number of distribution types accessible for lattice.
- However, there are still major challenges related to control of several sources of systematics.
- Expect:
  - ⋆ slow, but consistent progress,
  - ⋆ complementary role of LQCD and phenomenology.

Conclusions

- Message of the talk: **enormous progress in lattice calculations of x-dependent distributions with very encouraging results!**



- Increasing number of distribution types accessible for lattice.
- However, there are still major challenges related to control of several sources of systematics.
- Expect:
  - ⋆ slow, but consistent progress,
  - ⋆ complementary role of LQCD and phenomenology.

Thank you for your attention!