

# Lattice field theory and BSM: the beginning of a beautiful friendship

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IP2I Lyon, France



Louis, I think this is the beginning of a  
beautiful friendship.

Lattice 2021  
July 27

# Strong coupling in BSM

- Composite Higgs models
- Composite Dark Matter
- SIMP
- Dark glueballs
- Composite axions
- ...



# Which theory to choose?

$SU(3)$  w.  
8F

$SU(3)$  w.  
2A

$SU(2)$  w.  
2F

$SU(3)$  w.  
10F



$SU(4)$  w.  
5A + 6F

...

$Sp(4)$  w.  
4F + 6A

# Which theory to choose?

SU(3) w.

SU(3)

SU(2) w.

F

Let's choose first the problem:  
what is inside the plush?

SU(3) w.  
10F



SU(4) w.  
+ 6F

...

4F + 6A

# Composite Higgs models 101



- Symmetry broken by a condensate (of TC-fermions)
- Higgs and longitudinal Z/W emerge as mesons (pions)



Scales:

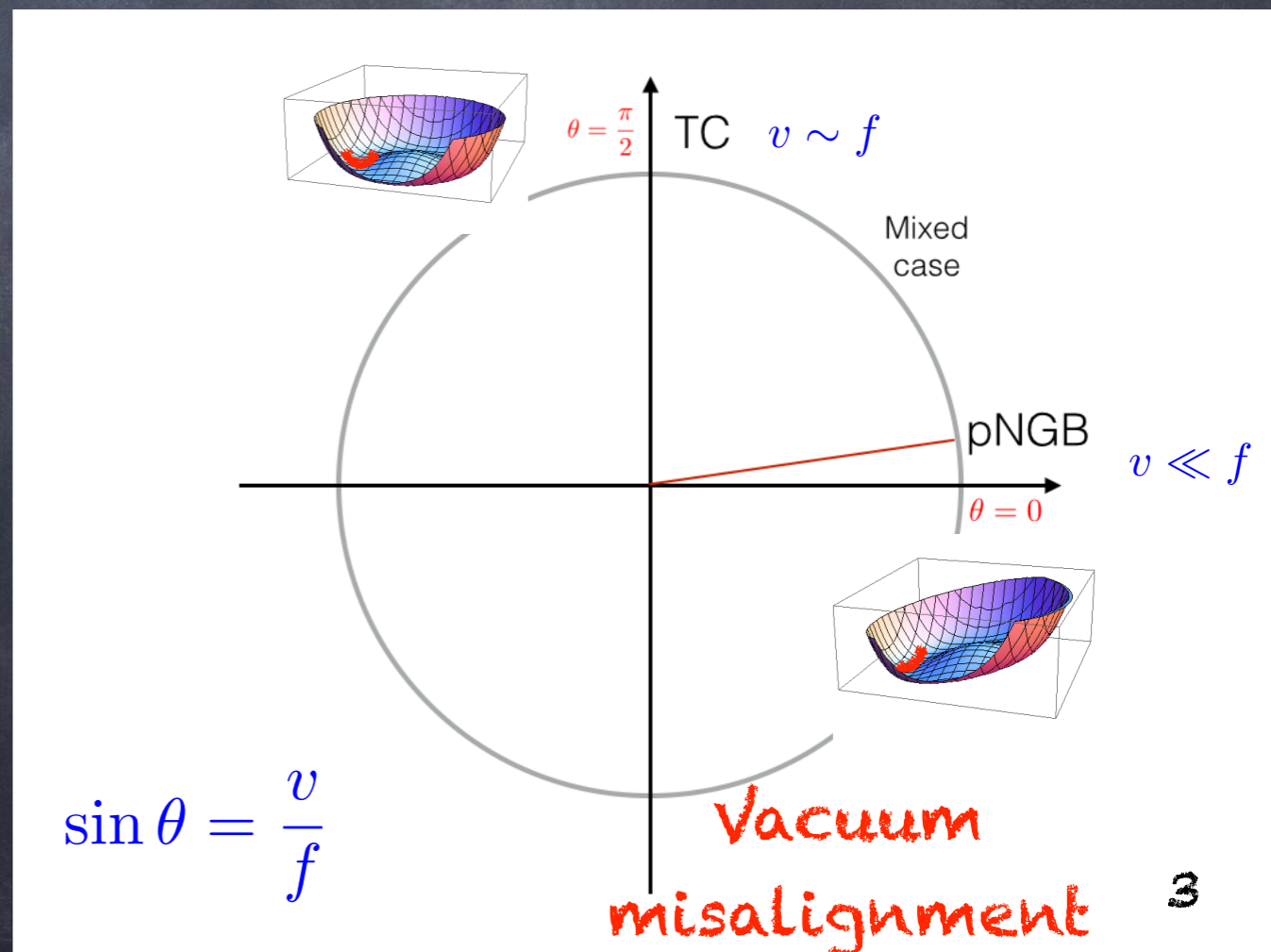
$f$  : Higgs decay constant

$v$  : EW scale

$$m_\rho \sim 4\pi f$$

EWPTs + Higgs coupl. limit:

$$f \gtrsim 4v \sim 1 \text{ TeV}$$



# Composite Higgs models 101



T. Rytov, F. Sannino 0809.0713  
Galloway, Evans, Luty, Tacchi 1001.1361

	$SU(2)_{TC}$	$SU(4)_\psi$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$	<input type="checkbox"/>		2	0
$\psi^3$	<input type="checkbox"/>	<input type="checkbox"/>	1	-1/2
$\psi^4$	<input type="checkbox"/>		1	1/2

The EW symmetry  
is embedded in the global  
flavour symmetry  
 $SU(4)$ !

- The global symmetry is broken:  $SU(4)/Sp(4)$   
Witten, Kosower
- 5 Goldstones (pions) arise:

$$5_{Sp(4)} \rightarrow (2, 2) \oplus (1, 1)$$

Higgs

additional singlet

# Composite Higgs models 101



The difficult parts:

- Generate the needed misalignment (via an effective potential)
- Generate couplings for the top (and other SM fermions)
- Correct Higgs mass and couplings
- Conformal window



# The partial compositeness paradigm

Kaplan Nucl.Phys. B366 (1991) 259

$$\frac{1}{\Lambda_{\text{fl.}}^{d-1}} \mathcal{O}_H q_L^c q_R \quad \Delta m_H^2 \sim \left( \frac{4\pi f}{\Lambda_{\text{fl.}}} \right)^{d-4} f^2 \quad \text{Both irrelevant if}$$

we assume:  $d_H > 1$   $d_{H^2} > 4$

Let's postulate the existence of fermionic operators:

$$\frac{1}{\Lambda_{\text{fl.}}^{d_F-5/2}} (\tilde{y}_L q_L \mathcal{F}_L + \tilde{y}_R q_R \mathcal{F}_R)$$

This dimension is not related to the Higgs!

$$f(y_L q_L Q_L + y_R q_R Q_R) \quad \text{with} \quad y_{L/R} f \sim \left( \frac{4\pi f}{\Lambda_{\text{fl.}}} \right)^{d_F-5/2} 4\pi f$$



# Top partners as baryons

Gauge-fermion underlying theory

$$\frac{1}{\Lambda_{\text{fl.}}} \underbrace{q \sigma^{\mu\nu} \psi G_{\mu\nu}}_{\text{T}}$$

$$d_T^{\text{naive}} = 7/2$$

- typically loop-suppressed
- psi need to carry QCD colour and flavour quantum numbers: too many!
- too many adjoint fermions!

# Top partners as baryons

Gauge-fermion underlying theory

$$\frac{1}{\Lambda_{\text{fl.}}^2} q \underbrace{\psi\psi\psi}_{\text{T}}$$

$$d_T^{\text{naive}} = 9/2$$

- higher dimension, but easier to generate
- More freedom in choosing the fermion representations



# Top partners as baryons

Gauge-fermion underlying theory

$$\frac{1}{\Lambda_{\text{fl.}}^2} q \underbrace{\psi\psi\psi}_T$$

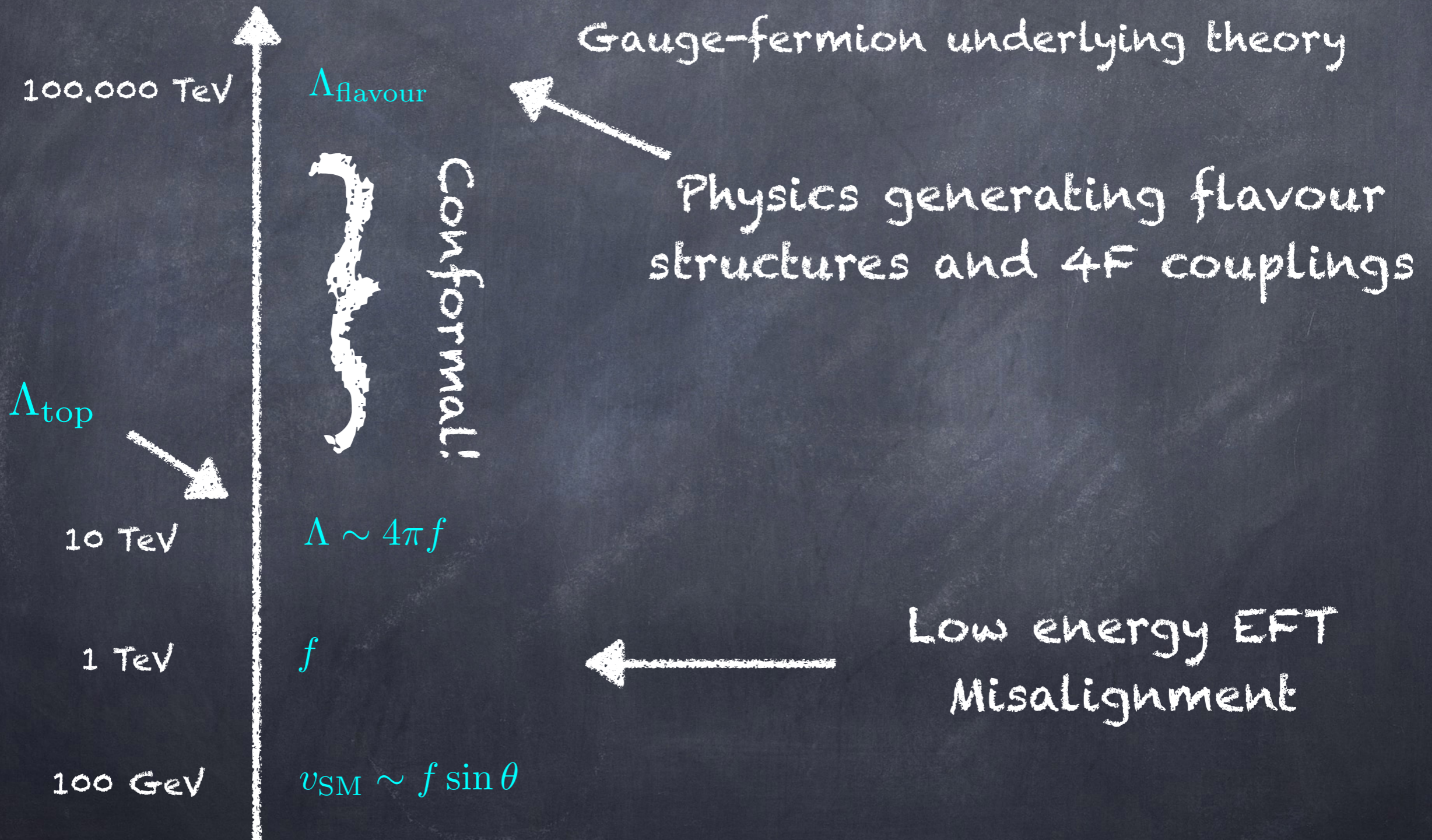
$$d_T^{\text{naive}} = 9/2$$

- higher dimension, but easier to generate
- More freedom in choosing the fermion representations

- What generated the 4-F interactions?
- We need large anomalous dimensions: strongly coupled conformal phase!

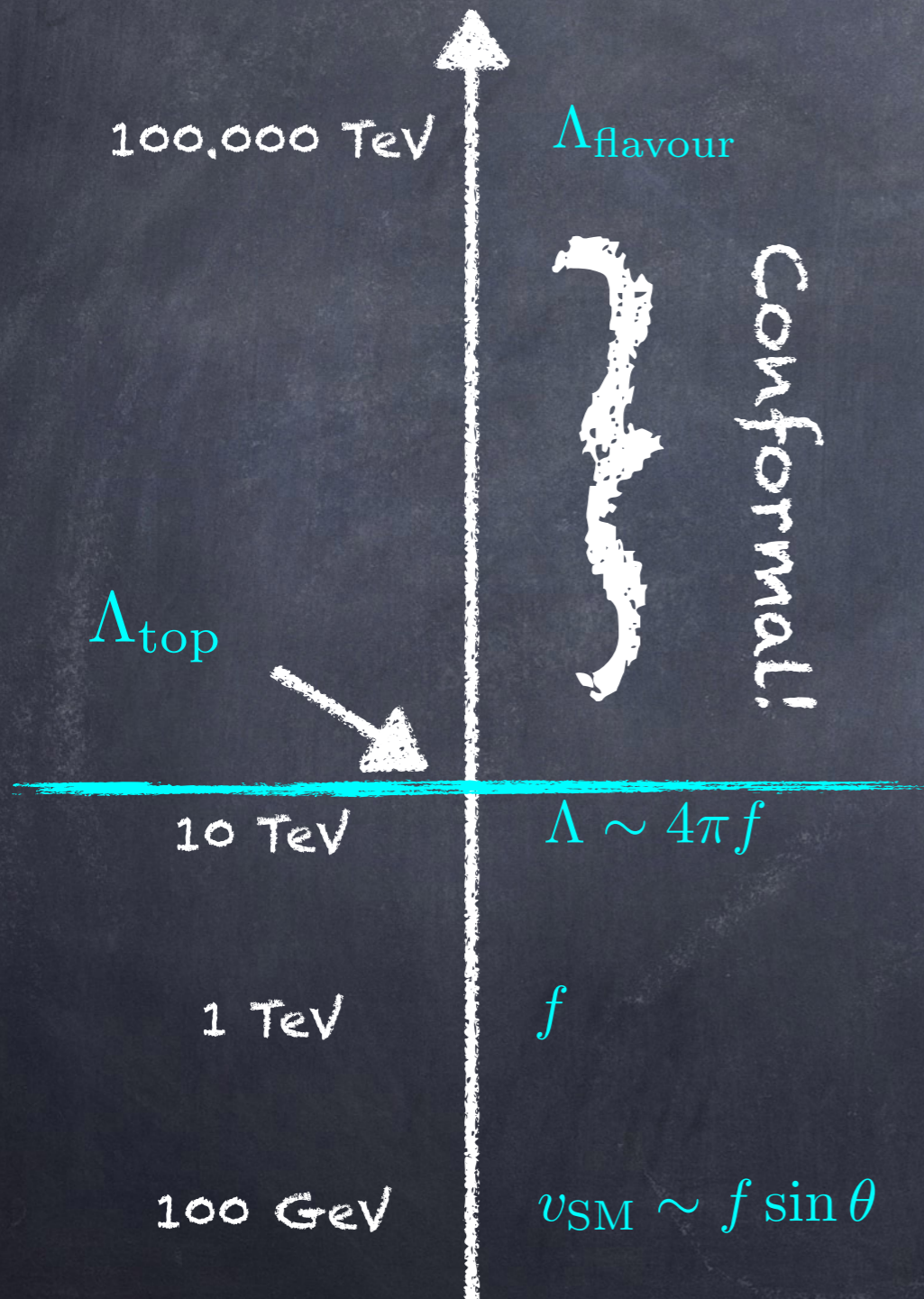


# Top partners as baryons



# Top partners as baryons

Gauge-fermion underlying theory



The theory needs to lie just below the conformal window

# IR Model zoology

$G_{\text{HC}}$	$\psi$	$\chi$	Restrictions	$-q_\chi/q_\psi$	$Y_\chi$	Non Conformal	Model Name
Real		Real	$SU(5)/SO(5) \times SU(6)/SO(6)$				
$SO(N_{\text{HC}})$	$5 \times \mathbf{S}_2$	$6 \times \mathbf{F}$	$N_{\text{HC}} \geq 55$	$\frac{5(N_{\text{HC}}+2)}{6}$	1/3	/	
$SO(N_{\text{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$N_{\text{HC}} \geq 15$	$\frac{5(N_{\text{HC}}-2)}{6}$	1/3	/	
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\text{HC}} = 7, 9$	$\frac{5}{6}, \frac{5}{12}$	1/3	$N_{\text{HC}} = 7, 9$	M1, M2
$SO(N_{\text{HC}})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 7, 9$	$\frac{5}{6}, \frac{5}{3}$	2/3	$N_{\text{HC}} = 7, 9$	M3, M4
Real		Pseudo-Real	$SU(5)/SO(5) \times SU(6)/Sp(6)$				
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 12$	$\frac{5(N_{\text{HC}}+1)}{3}$	1/3	/	
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 4$	$\frac{5(N_{\text{HC}}-1)}{3}$	1/3	$2N_{\text{HC}} = 4$	M5
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\text{HC}} = 11, 13$	$\frac{5}{24}, \frac{5}{48}$	1/3	/	
Real		Complex	$SU(5)/SO(5) \times SU(3)^2/SU(3)$				
$SU(N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \bar{\mathbf{F}})$	$N_{\text{HC}} = 4$	$\frac{5}{3}$	1/3	$N_{\text{HC}} = 4$	M6
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$N_{\text{HC}} = 10, 14$	$\frac{5}{12}, \frac{5}{48}$	1/3	$N_{\text{HC}} = 10$	M7
Pseudo-Real		Real	$SU(4)/Sp(4) \times SU(6)/SO(6)$				
$Sp(2N_{\text{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\text{HC}} \leq 36$	$\frac{1}{3(N_{\text{HC}}-1)}$	2/3	$2N_{\text{HC}} = 4$	M8
$SO(N_{\text{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 11, 13$	$\frac{8}{3}, \frac{16}{3}$	2/3	$N_{\text{HC}} = 11$	M9
Complex		Real	$SU(4)^2/SU(4) \times SU(6)/SO(6)$				
$SO(N_{\text{HC}})$	$4 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 10$	$\frac{8}{3}$	2/3	$N_{\text{HC}} = 10$	M10
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\text{HC}} = 4$	$\frac{2}{3}$	2/3	$N_{\text{HC}} = 4$	M11
Complex		Complex	$SU(4)^2/SU(4) \times SU(3)^2/SU(3)$				
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \bar{\mathbf{A}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}-2)}$	2/3	$N_{\text{HC}} = 5$	M12
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$3 \times (\mathbf{S}_2, \bar{\mathbf{S}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}+2)}$	2/3	/	
$SU(N_{\text{HC}})$	$4 \times (\mathbf{A}_2, \bar{\mathbf{A}}_2)$	$3 \times (\mathbf{F}, \bar{\mathbf{F}})$	$N_{\text{HC}} = 5$	4	2/3	/	

Ferretti  
1604.06467

	Real	Pseudo-Real	SU(5)/SO(5) × SU(6)/Sp(6)				
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 12$	$\frac{5(N_{\text{HC}}+1)}{3}$	$1/3$	/	
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 4$	$\frac{5(N_{\text{HC}}-1)}{3}$	$1/3$	$2N_{\text{HC}} = 4$	M5
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\text{HC}} = 11, 13$	$\frac{5}{24}, \frac{5}{48}$	$1/3$	/	


	Real	Complex	SU(5)/SO(5) × SU(3) <sup>2</sup> /SU(3)				
$SU(N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \bar{\mathbf{F}})$	$N_{\text{HC}} = 4$	$\frac{5}{3}$	$1/3$	$N_{\text{HC}} = 4$	M6
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$N_{\text{HC}} = 10, 14$	$\frac{5}{12}, \frac{5}{48}$	$1/3$	$N_{\text{HC}} = 10$	M7

	Pseudo-Real	Real	SU(4)/Sp(4) × SU(6)/SO(6)				
$Sp(2N_{\text{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\text{HC}} \leq 36$	$\frac{1}{3(N_{\text{HC}}-1)}$	$2/3$	$2N_{\text{HC}} = 4$	M8
$SO(N_{\text{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 11, 13$	$\frac{8}{3}, \frac{16}{3}$	$2/3$	$N_{\text{HC}} = 11$	M9

	Complex	Real	SU(4) <sup>2</sup> /SU(4) × SU(6)/SO(6)				
$SO(N_{\text{HC}})$	$4 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 10$	$\frac{8}{3}$	$2/3$	$N_{\text{HC}} = 10$	M10
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\text{HC}} = 4$	$\frac{2}{3}$	$2/3$	$N_{\text{HC}} = 4$	M11

	Complex	Complex	SU(4) <sup>2</sup> /SU(4) × SU(3) <sup>2</sup> /SU(3)				
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \bar{\mathbf{A}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}-2)}$	$2/3$	$N_{\text{HC}} = 5$	M12
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$3 \times (\mathbf{S}_2, \bar{\mathbf{S}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}+2)}$	$2/3$	/	
$SU(N_{\text{HC}})$	$4 \times (\mathbf{A}_2, \bar{\mathbf{A}}_2)$	$3 \times (\mathbf{F}, \bar{\mathbf{F}})$	$N_{\text{HC}} = 5$	4	$2/3$	/	

	Real	Pseudo-Real	SU(5)/SO(5) × SU(6)/Sp(6)				
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 12$	$\frac{5(N_{\text{HC}}+1)}{3}$	$1/3$	/	
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\text{HC}} \geq 4$	$\frac{5(N_{\text{HC}}-1)}{3}$	$1/3$	$2N_{\text{HC}} = 4$	M5
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\text{HC}} = 11, 13$	$\frac{5}{24}, \frac{5}{48}$	$1/3$	/	

	Real	Complex	SU(5)/SO(5) × SU(3) <sup>2</sup> /SU(3)					
$SU(N_{\text{HC}})$	$5 \times \mathbf{A}_2$					$N_{\text{HC}} = 4$	M6	
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	3				$N_{\text{HC}} = 10$	M7	
	Pseudo-Real							
$Sp(2N_{\text{HC}})$	$4 \times \mathbf{F}$						$2N_{\text{HC}} = 4$	M8
$SO(N_{\text{HC}})$	$4 \times \mathbf{Spin}$						$N_{\text{HC}} = 11$	M9
	Complex							
$SO(N_{\text{HC}})$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$					$N_{\text{HC}} = 10$	M10	
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\text{HC}} = 4$	$\frac{2}{3}$	$2/3$	$N_{\text{HC}} = 4$	M11	

	Complex	Complex	SU(4) <sup>2</sup> /SU(4) × SU(3) <sup>2</sup> /SU(3)				
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}-2)}$	$2/3$	$N_{\text{HC}} = 5$	M12
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{S}_2, \overline{\mathbf{S}}_2)$	$N_{\text{HC}} \geq 5$	$\frac{4}{3(N_{\text{HC}}+2)}$	$2/3$	/	
$SU(N_{\text{HC}})$	$4 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\text{HC}} = 5$	4	$2/3$	/	



# Partially Unified Partial Compositeness (PUPC)

G.C., S.Vatani, C.Zhang  
1911.05454, 2005.12302

Planck scale



Condensation scale

Usual low energy description  
of composite Higgs models

Standard Model

One of Ferretti  
models

# Partially Unified Partial Compositeness (PUPC)

G.C., S.Vatani, C.Zhang  
1911.05454, 2005.12302

Planck scale



Conformal window  
(large scaling dimensions)

One of Ferretti  
models +  
additional fermions

Condensation scale

Usual low energy description  
of composite Higgs models

One of Ferretti  
models

Standard Model

# Partially Unified Partial Compositeness (PUPC)

G.C., S.Vatani, C.Zhang  
1911.05454, 2005.12302

Planck scale

← HC and SM gauge groups partially unified

← Symmetry breaking by scalars ——— 4-fermion Ops generated!

← Conformal window (large scaling dimensions) ——— One of Ferretti models + additional fermions

← Condensation scale

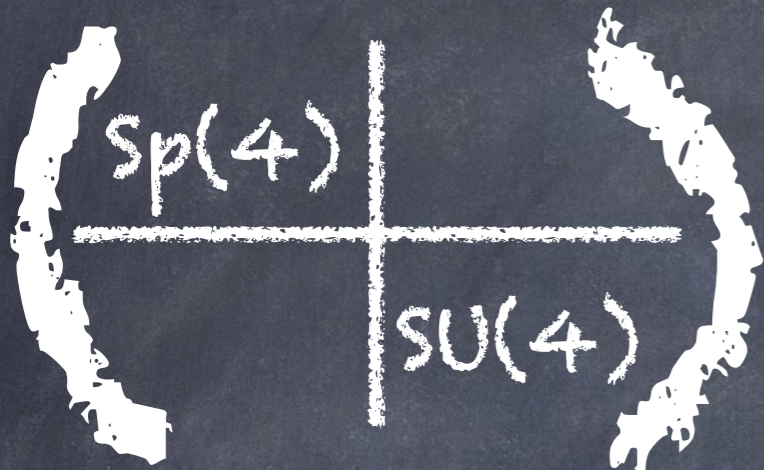
← Usual low energy description of composite Higgs models ——— One of Ferretti models

← Standard Model



# Techni-Pati-Salam

Parallel talk:  
Friday

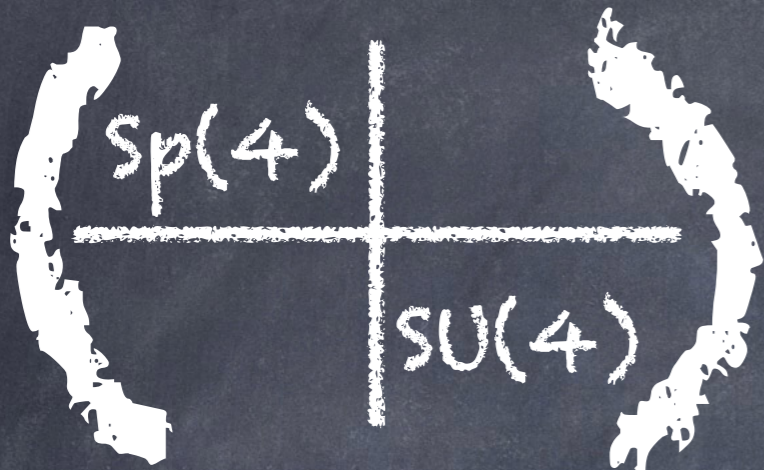


- Simplest model embeds an  $Sp(4)$  TC with  $SU(4)$  Pati-Salam in  $SU(8)$

$$\Omega = \begin{pmatrix} \psi_d \\ q \\ l \end{pmatrix}$$

# Techni-Pati-Salam

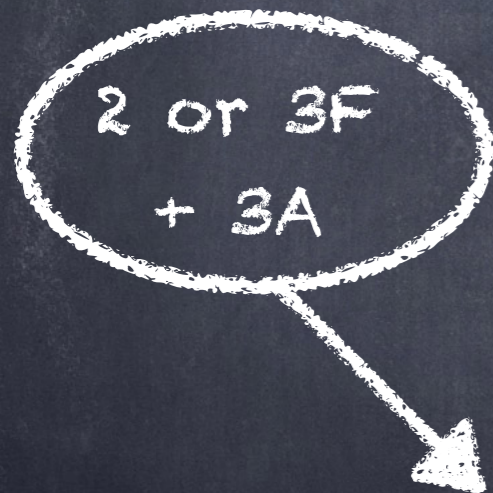
Parallel talk:  
Thursday



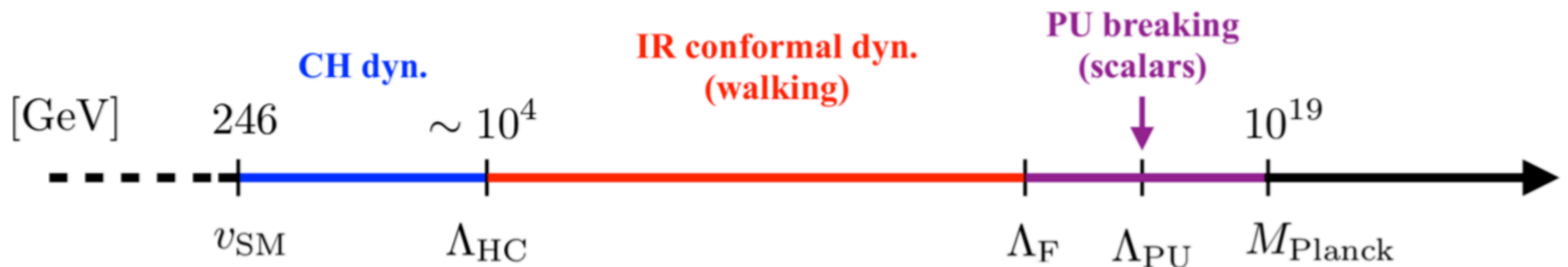
- Simplest model embeds an  $Sp(4)$  TC with  $SU(4)$  Pati-Salam in  $SU(8)$

$$\Omega = \begin{pmatrix} \psi_d \\ q \\ l \end{pmatrix}$$

$Sp(4)$  strong interactions emerge



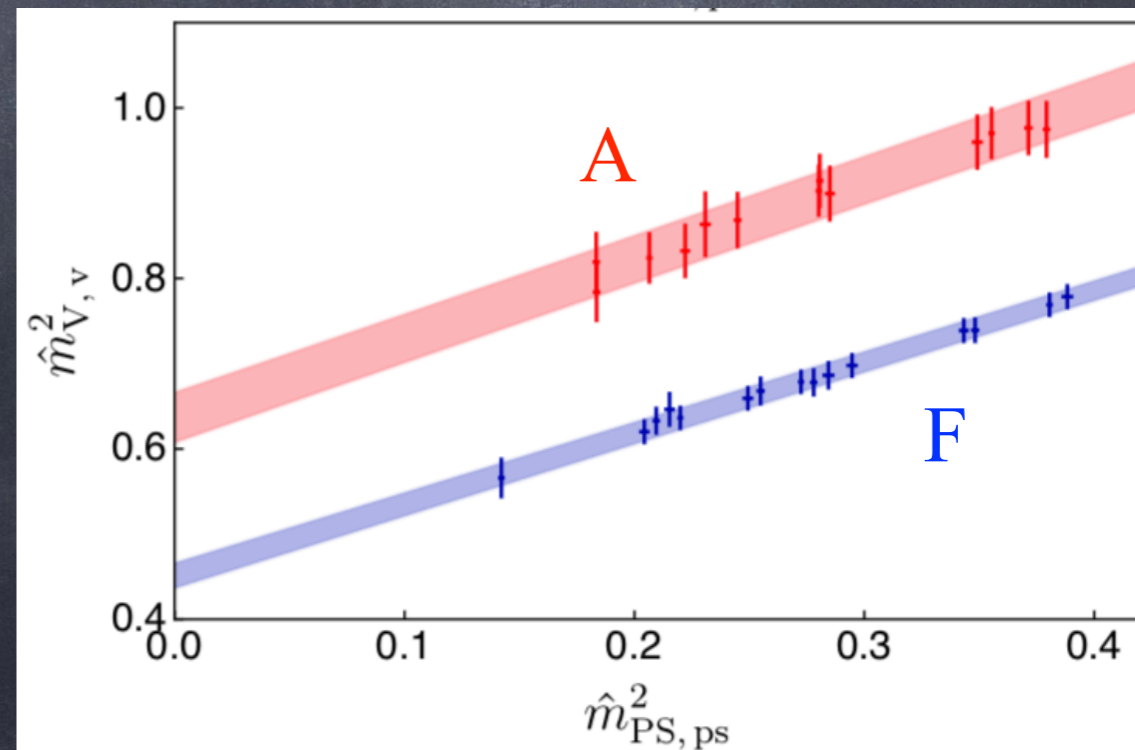
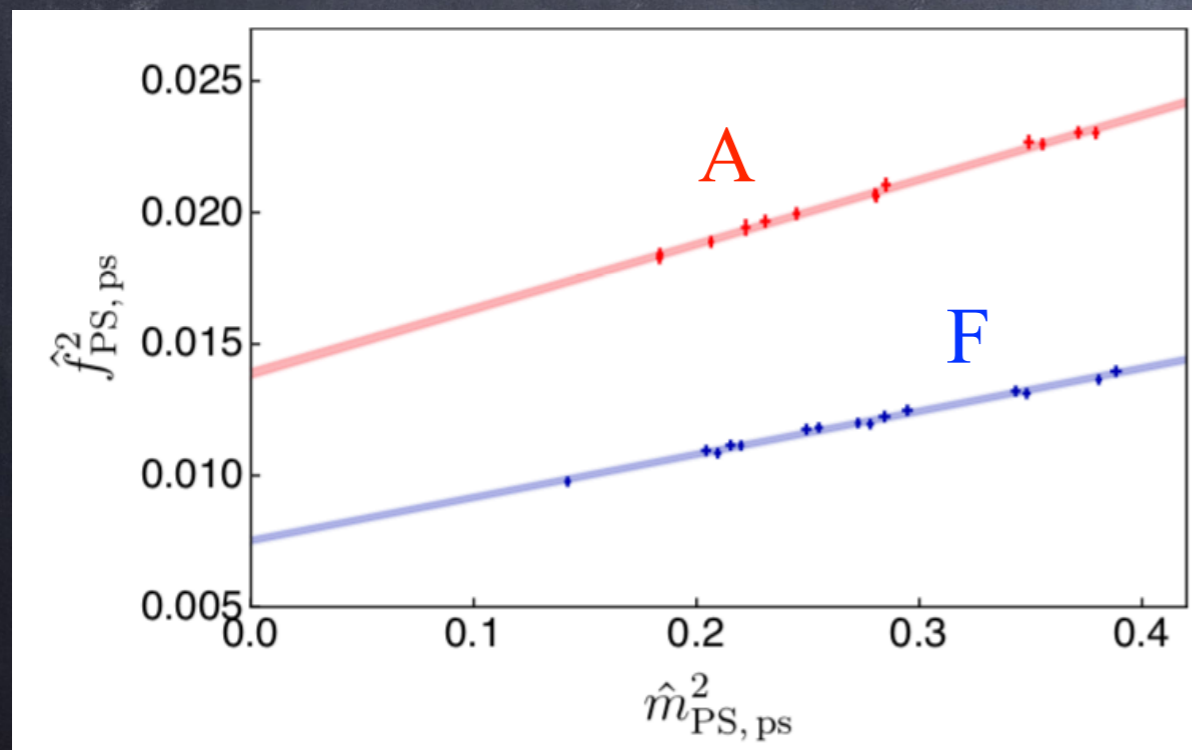
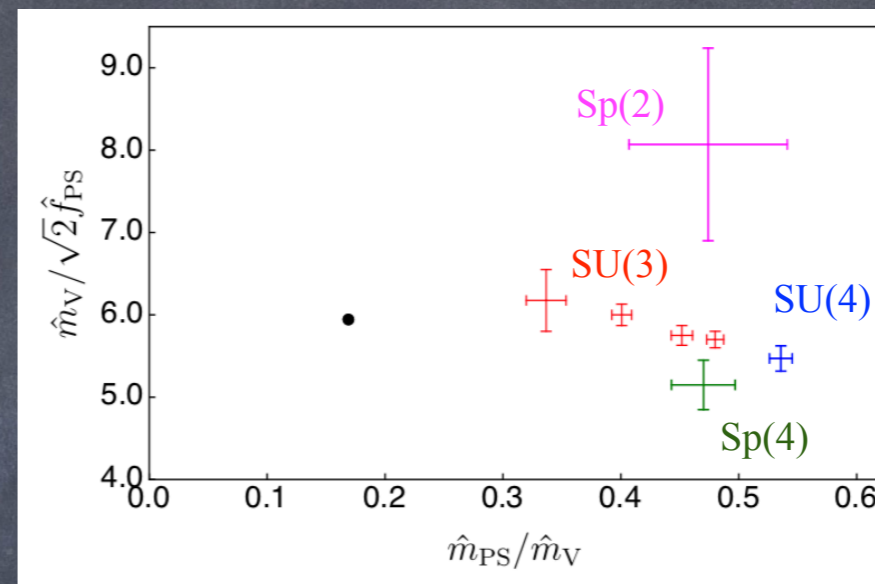
- Is this theory conformal?
- What are the anomalous dimensions?



# Sp(4) on the Lattice

E. Bennet et al  
1911.00437, 1912.06505

- This slide: 2F + 3A with quenched fermions (M8)
- Thursday parallel talks will give more updates
- Biagio Lucini, Jong-Wan Lee, Ho Hsiao, Jack Holligan



# Other theories

- SU(4) w. 2A + 2F (rel. for models M6 and M11)

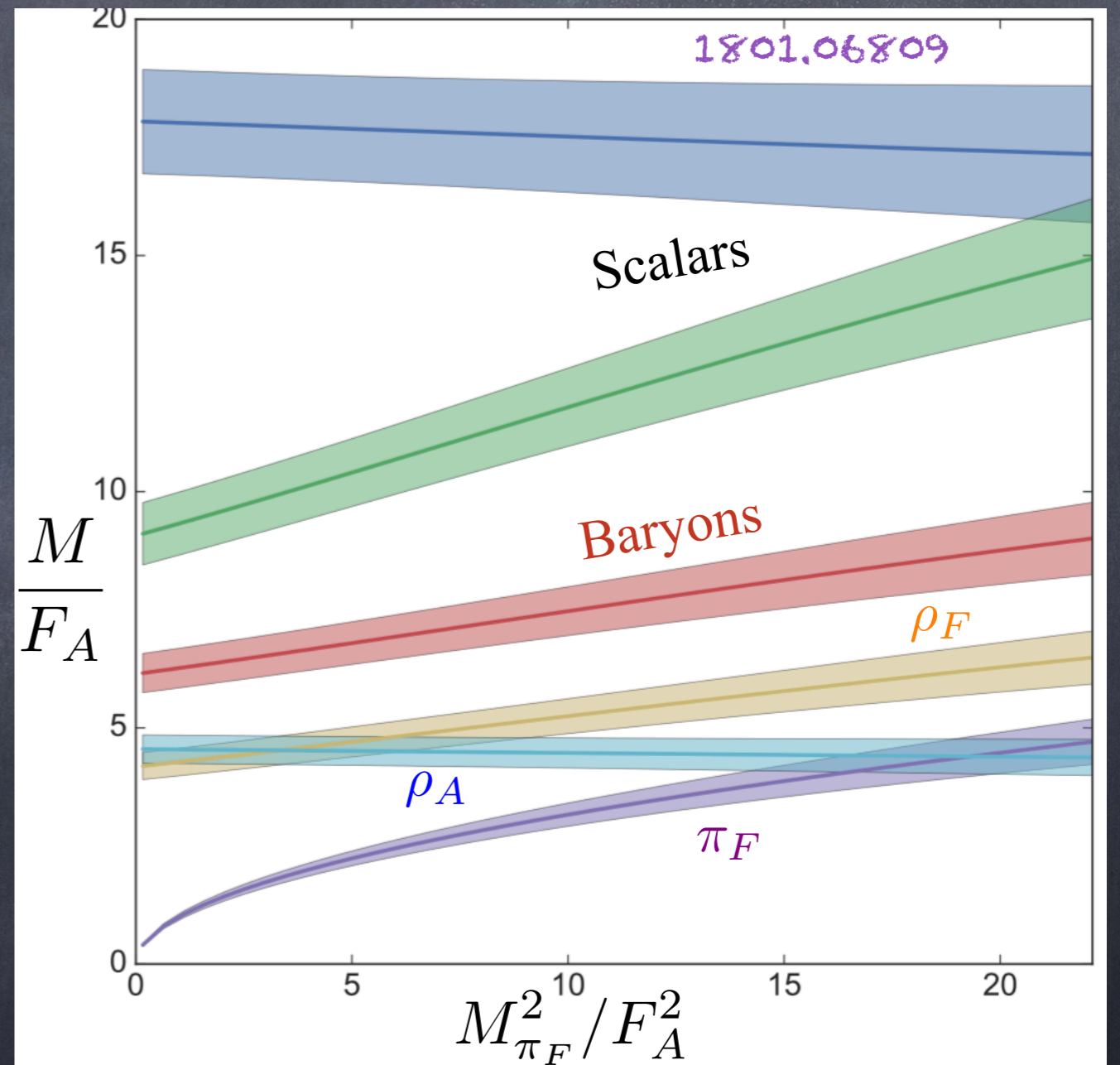
Thursday: Yigal Shamir  
Friday: Alessandro Lupo

First computation  
of baryon masses!

$$\frac{M_B}{F_A} \approx 6$$

Note:  $\frac{F_F}{F_A} \approx 0.67$

Tension  
reduced  
for M11



# Other theories

- $SU(4)$  w.  $2A + 2F$  (rel. for models M5 and M11)

Thursday: Yigal Shamir

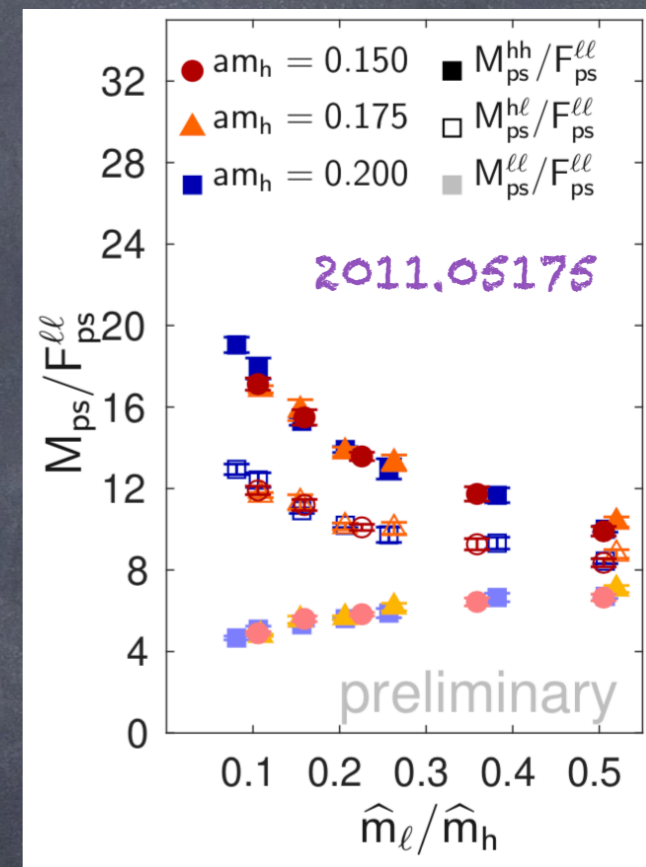
Friday: Alessandro Lupo

- $SU(3)$  w.  $8F$  or  $4+6F$

Thursday: Oliver Witzel,  
James Ingolby

- $SU(2)$  w.  $2F$  (minimal template without PC)

Friday: Vincent Drach





# Other theories

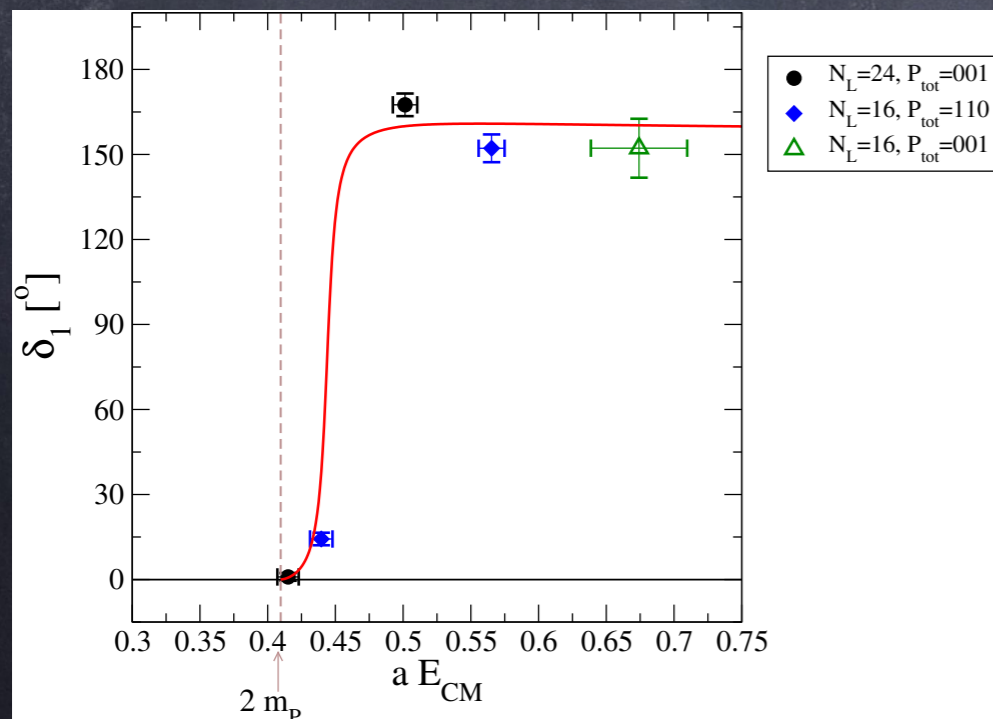
- SU(2) w. 2F (minimal template without PC) [see slide 4]

Friday: Vincent Drach

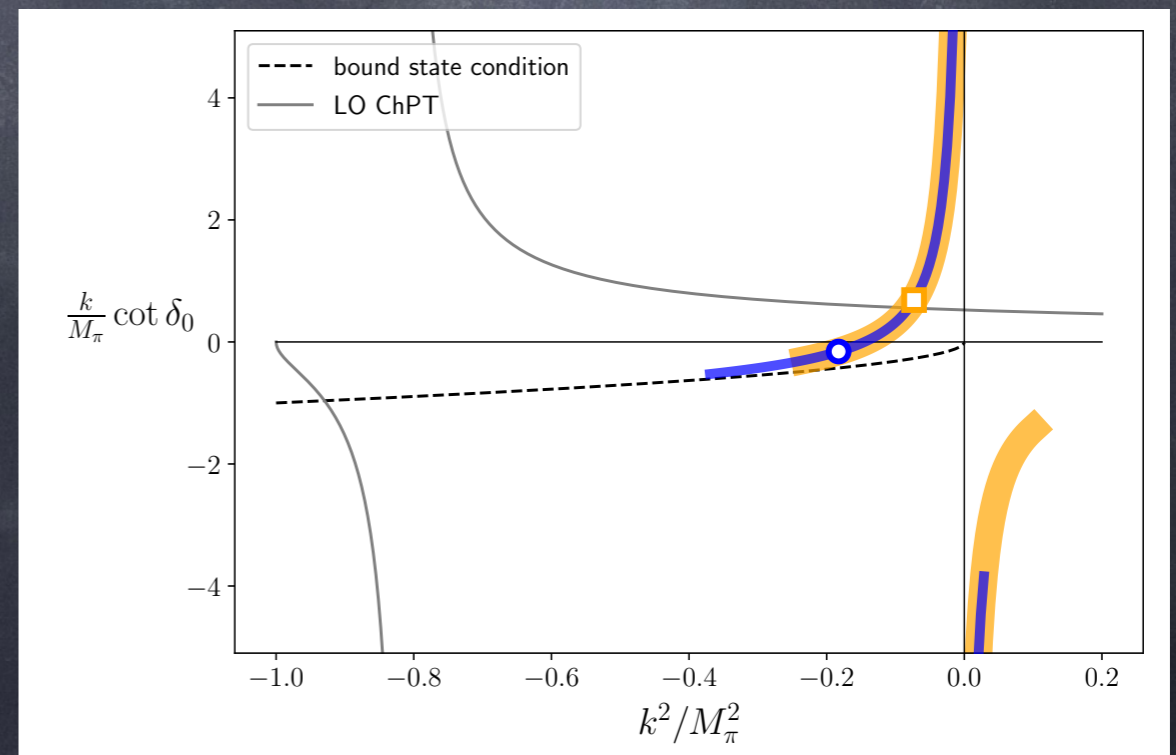
Scalar channel ( $0^{++}$ ):

- Determination of the flavour singlet coupling: the  $0^{++}$  mixes with the would be Higgs boson altering its physical properties (see [1809.09146](#)) and can be produced at the LHC.
- Results strongly suggest that in the explored region of fermion masses the sigma is a bound state, however more phenomenologically relevant regions (non stable sigma) will be soon investigated

Phase-shift in the vector channel - from [2012.09761](#)



Phase-shift in the flavour singlet channel - from [2107.09974](#)



# A closer look at the top and Higgs masses

- At the EFT level, the computation can be done in two ways:
  - 1) Integrating out the massive Baryons that mix with the top quark.
  - 2) Introducing EFT operators in terms of the spurion couplings of the top to the composite operators (that generate Baryons)
- Are they equivalent?

# A closer look at the top and Higgs masses

- Are they equivalent? No!

Consider a generic Ferretti model, with a light ALP coming from the spontaneous breaking of a global U(1) symmetry.

Baryon mixing, Q and S

$$\begin{aligned}
 -\mathcal{L}_{\text{PC}} &= y_L f e^{i\xi_Q \frac{a}{f_a}} \bar{Q} P_L q + y_R f e^{i\xi_S \frac{a}{f_a}} \bar{t} P_L S \\
 &- y'_L H e^{-i\xi_S \frac{a}{f_a}} \bar{S} P_L q - y'_R H e^{-i\xi_Q \frac{a}{f_a}} \bar{t} P_L Q
 \end{aligned}
 \quad
 -i \frac{m_t}{f_a} \left( \xi_Q \frac{y_L^2 f^2}{M_Q^2} + \xi_S \frac{y_R^2 f^2}{M_S^2} \right) \sim \mathcal{O}(y^4)$$

Top mass operator

$$-\mathcal{L}_{m_t} = y_L y_R H e^{i(\xi_Q + \xi_S) \frac{a}{f_a}} \bar{t} P_L q
 \quad
 -i \frac{m_t}{f_a} (\xi_Q + \xi_S) \sim \mathcal{O}(y^2)$$

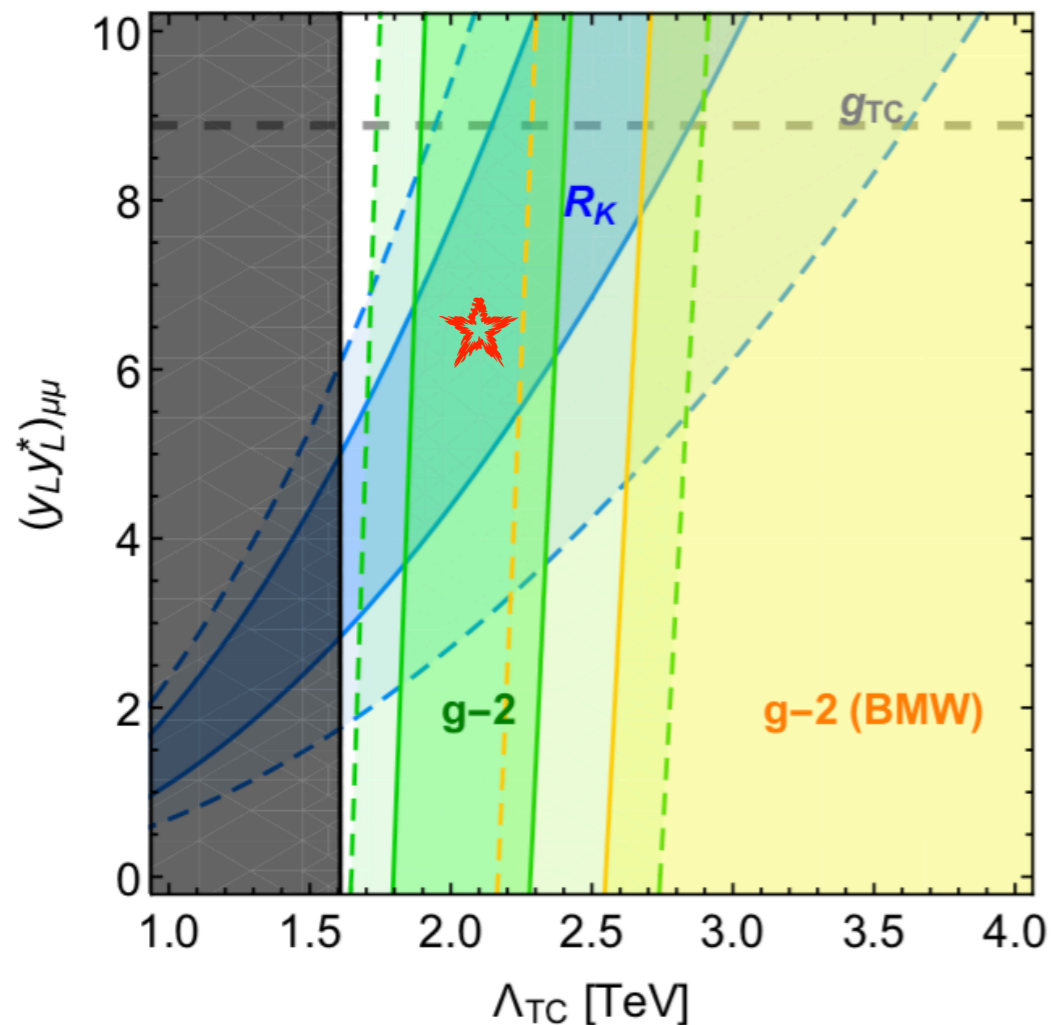
The results are parametrically different.  
 What is the impact on the Higgs mass calculation?  
 Heavy Baryons may not be disfavoured!

# There's something about Muons

Technicolor strikes back?



$N_{TC}=2, (y_Q y_Q^*)_{bs}=0.035$



$$R_K = \frac{\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\text{BR}(B^+ \rightarrow K^+ e^+ e^-)} = 0.846_{-0.041}^{+0.044}$$

- g-2 fixes the scale of new physics
- natural values for TC-like theories!

$$\Delta a_\mu|_{BSM} \approx \frac{m_\mu^2}{\Lambda^2}$$



$$\Lambda \approx 2 \text{ TeV} \approx 4\pi v$$

- R\_K requires large muon couplings (attainable in strong dynamics)

# There's something about Muons

## Technicolor strikes back?

- If this scenario is confirmed by the anomalies, the Higgs must be a dilaton-like light scalar.
- Lattice crucial in computing its mass and couplings!
- Which theory? [Back to slide 10]

Thursday parallel session:

Maarten Golterman, Chih Him Wong

# Outlook

- Composite (Higgs or Dark Matter or...) models are a feasible route for New Physics
- Lattice input is dearly needed to establish the feasibility of these scenarios
- Intriguing hint: muon  $g-2$  and  $R_K$  explainable via TC-like theories!
- Lots of useful results already available, and much more to come (stay tuned to the parallel sessions)