Aspects of QCD towards the chiral limit

Anirban Lahiri

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CRC-TR 211
Strong-interaction matter under extreme conditions

Overview

1. Why this talk?
2. Results - a few mentions
3. Take home messages

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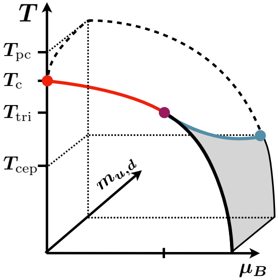
Why the heck the chiral limit?

- The chiral transition temperature defines a scale itself.
  - Interplay between the chiral symmetry breaking and confinement. [A. Casher, Phys. Lett. B83 395 (1979).]
  - Comparative study of the energy (or length) scales of the chiral symmetry breaking and confinement. [J. Kogut *et. al.* Phys. Rev. Lett. 48, 1140 (1982).]
- The chiral transition temperature has a phenomenological importance establishing the global phase structure of the strongly interacting matter.



[F. Karsch. arXiv:1905.03936.]

- The phase transition at $\mu_B = 0 = m_{u,d}$, is expected to be of second order belonging to $SU(2) \times SU(2) \simeq O(4)$ universality class. [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]
- CEP shifts to larger μ_B and smaller T with increasing mass. [Hatta and Ikeda. Phys. Rev. D67, 014028, 2003.]
- $T_{\text{CEP}} < T_{\text{Tri}} < T_c \Rightarrow T_c$ puts an upper bound on T_{CEP}.

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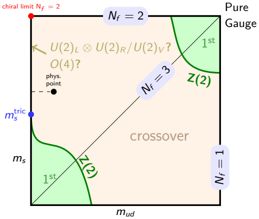
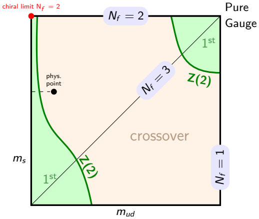
Life is not easy!

- In the case effective restoration of anomalous $U_A(1)$, the chiral transition with two light flavors can be of first order. [Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.]
- Questions to be answered:
  - What is the chiral transition temperature, T_c?
  - What is the nature of the chiral phase transition?
- Two possible scenarios: [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]





- Few other possible scenarios proposed. [Gupta, J. Phys. G: Nucl. Part. Phys. 35 (2008) 104018.]

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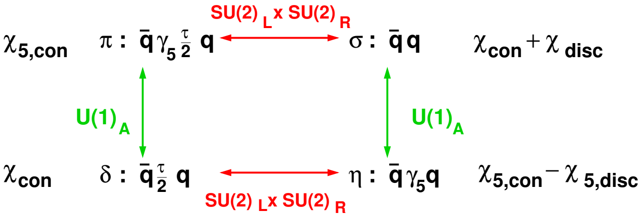
Questions to be answered

- What is the chiral transition temperature, T_c?
  - (2+1)-flavor HISQ: $T_c = 132^{+3}_{-6}$ MeV. [HotQCD, Phys. Rev. Lett. 123, 062002 (2019).]
  - (2+1+1)-flavor twisted mass Wilson fermion: $T_c = 134^{+6}_{-4}$ MeV. [Kotov *et. al.*, arXiv:2105.09842 [hep-lat].] Talk by Andrey Kotov; Tue 27^{th}, 1:15 PM.
- What is the nature of the chiral phase transition?
  - Chiral observables and study with variable N_f favor a second order transition in the chiral limit. [Kaczmarek *et. al.*, arXiv:2010.15593 [hep-lat]. Cuteri *et. al.*, arXiv:1811.03840 [hep-lat].]
  - Is $U_A(1)$ broken or effectively restored?



[HotQCD, Phys. Rev. D86, 094503 (2012).]

"Order parameter" for effective restoration of $U_A(1)$.

$$\chi_H = \int d^4x \left[\langle O_H(x) O_H(0) \rangle - \langle O_H(x) \rangle \langle O_H(0) \rangle \right]$$

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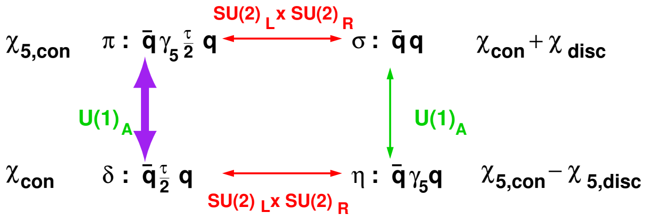

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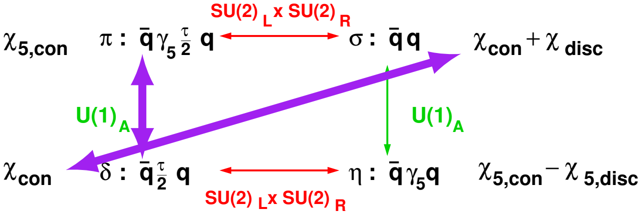

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- $m_\pi \overset{?}{=} m_{a_0}$ or more generally $\chi_\pi \overset{?}{=} \chi_{a_0}$.
- In the chirally symmetric phase, $\chi^{\text{disc}}_{\bar{\psi}\psi} \overset{?}{=} 0$.

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Effective restoration of $U_A(1)$

- For temperatures significantly higher than the transition temperature, $U_A(1)$ breaking is small, which is consistent with dilute instanton gas picture.

$U_A(1)$ broken	$U_A(1)$ restored
DWF $N_\tau = 8,\ 16$ HotQCD, Phys. Rev. D86, 094503, (2012).	Overlap $N_\tau = 8$ Cossu *et. al.*, Phys. Rev. D87, 114514, (2013). Erratum: Phys. Rev. D88, 019901(2013).
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- Effective restoration of $U_A(1)$ needs more attention near the chiral transition temperature!
- For temperatures higher but close to the transition temperature, the chiral extrapolation is more subtle and yet inconclusive.

Kaczmarek *et. al.*, arXiv:2003.07920 [hep-lat].
Dentinger *et. al.*, arXiv:2102.09916 [hep-lat].

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Effective restoration of $U_A(1)$ in the chiral limit

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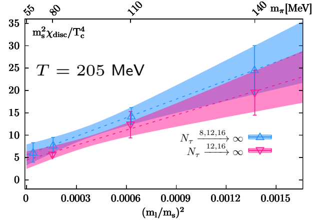

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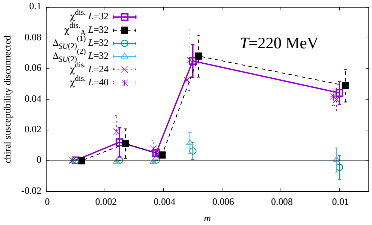
Even using same observable, contradictory findings between different fermion formulation, for high T.

Möbius DWF: $U_A(1)$ restored.



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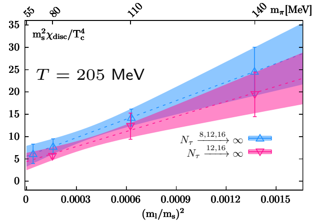

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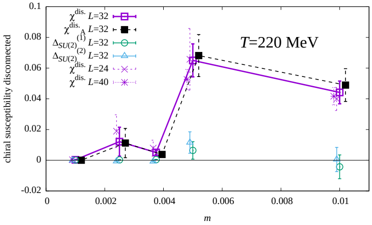

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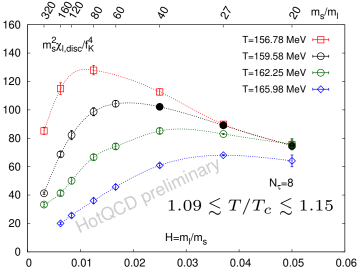

- For $T \gtrsim T_c$, the approach to the chiral limit is highly non-monotonic.
- Towards the chiral limit the aspect ratio has to be increased to keep $m_\pi L$ roughly constant.
- Continuum extrapolation for $T \gtrsim T_c$ is tricky, although necessary.

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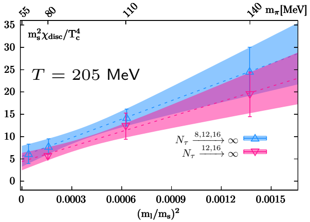

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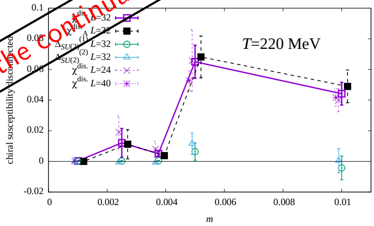

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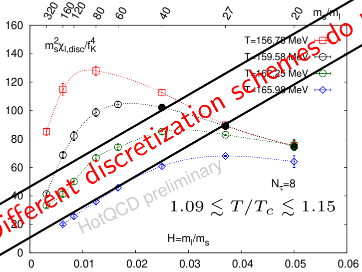
Even using same observable, contradictory findings between different fermion formulation, for high T.

Different discretization schemes do not yet reach same conclusions in the continuum limit.



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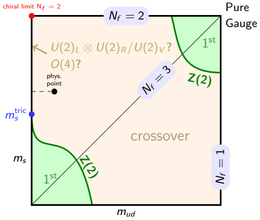
Chiral transition for $N_f = 3$



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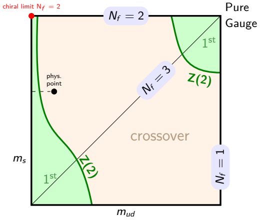

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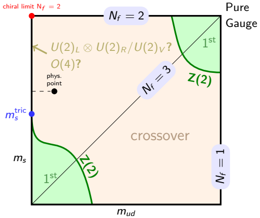

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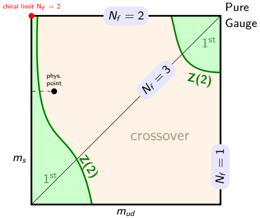

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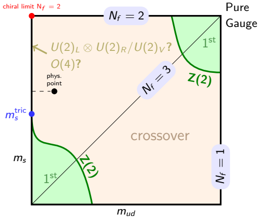

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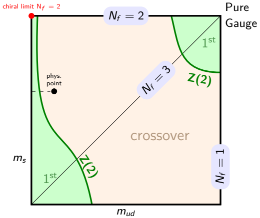

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  Talk by Sipaz Sharma; Thu 29^{th}, 7:00 AM.
- Möbius DWF, $N_\tau = 6,\ 8$: Crossover for $m/T \sim 1$. [Nakamura *et. al.*]
  Poster session A.

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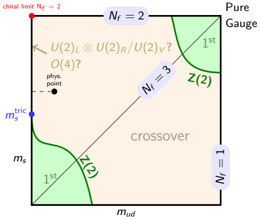

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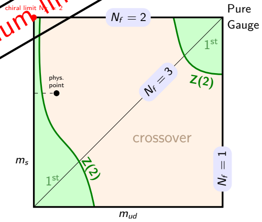

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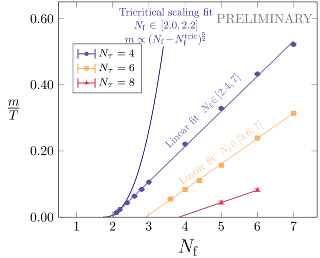
Chiral transition with $N_f > 3$

- $N_f = 4$ with standard staggered: strong cutoff dependence of $m_\pi^{Z_2}$ and in the continuum limit the $m_\pi^{Z_2}$ is going to be extremely small. [de Forcrand and D'Elia, PoS LATTICE2016 (2017) 081.]
- $N_f = 4$ with Wilson-Clover: 1^{st}-order chiral transition in the chiral limit.
  Talk by Hiroshi Ohno; Mon 26^{th}, 10:00 PM.



- Many (even fractional) N_f with standard staggered: a sizeable shift of the critical line to smaller $m_\pi^{Z_2}$/larger N_f. [Cuteri *et. al.*, arXiv:1811.03840 [hep-lat].]
- 1^{st}-order chiral transition seems to be unlikely for $N_f = 2$, towards continuum.
  Talk by Owe Philipsen; Thu 29^{th}, 7:15 AM.

- Many flavors: in search of a conformal window. [Lombardo *et. al.*, PoS CPOD2014 (2015) 059.]

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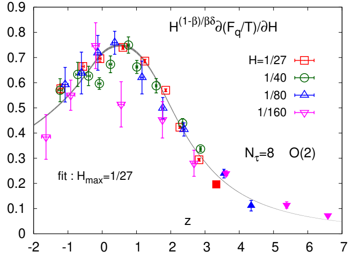
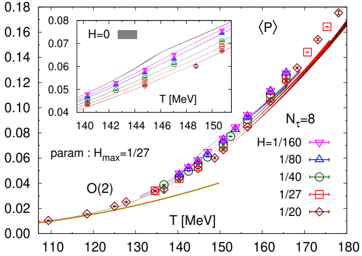
Gluonic observables towards chiral limit

- Being gluonic, Polyakov loop and heavy quark free energy (HQFE) are both expected to be energy-like operators w.r.t. the chiral phase transition.

- Scaling expectation: HQFE is finite in the chiral limit.
- Mixed susceptibility: $\frac{\partial F_q(T,H)/T}{\partial H}$, divergent $(H \propto m_l)$.
- Determined singular part parameters compares well with other determinations. [Clarke *et. al.*, Phys. Rev. D103, L011501 (2021).]





- No inflection point for $\langle P \rangle$ in the chiral crossover region.
- Relation of Polyakov loop and HQFE to deconfinement, even at physical mass, is questionable.

Talk by David Clarke; Thu 29th, 5:45 AM.

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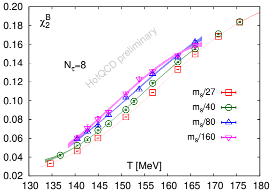
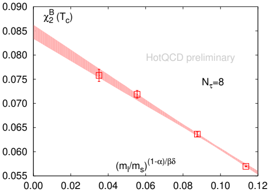
Some more energy-like observables

- μ_B does not break chiral symmetry explicitly $\Rightarrow$ μ_B appears in the T-like scaling field.
- $t \propto \left(\frac{T-T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right)$ $\Rightarrow$ Close to chiral limit: $\frac{\partial}{\partial T} \sim \frac{\partial^2}{\partial \mu_B^2}$.
- χ_2^B is energy-like observable $\Rightarrow$ $\chi_2^B(T,H) \propto \kappa_2^B H^{(1-\alpha)/\beta\delta}$ + regular terms.





- Linear fit in $H^{(1-\alpha)/\beta\delta}$ works quite well.
- $\chi_2^B(T_c,0) - \chi_2^B(T_c,H)$ gives the singular part for any finite H. [Sarkar *et. al.*, arXiv:2011.00240.]

Talk by Mugdha Sarkar; Thu 29th, 6:00 AM.

- Ratio of singular parts of conserved charge X and Y is same as κ_2^X/κ_2^Y.
- Consistent with physical mass results for ratios of κs. [HotQCD; PLB 795 15 (2019)].
- Curvature does not change much towards chiral limit.
- m_s does not break chiral symmetry explicitly $\Rightarrow$ $\langle\bar{\psi}\psi\rangle_s$ is also energy-like.

Overview

1. Why this talk?
2. Results - a few mentions
3. Take home messages

Take home

- There has been some progress ..
  - The chiral transition temperature for $N_f = 2$ has been found to be around 130 MeV.
  - CEP for physical world has to be searched for $T < 130$ MeV and correspondingly for $\mu_B > 400$ MeV.
  - The curvature of (pseudo-)critical lines remains almost unchanged towards the chiral plane.
  - Polyakov loop behaves as an energy-like observable w.r.t. the chiral phase transition putting a question on its relation with deconfinement, even at physical mass.
  - Various conserved charge fluctuations and the strange condensate also behave as energy-like observables.

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  - Effective restoration of $U_A(1)$ is yet to be settled.
  - Significantly more attention needed for temperatures higher but close to T_c.
  - 1^{st}-order region in the $N_f = 3$ corner, if it exists at all, seems to be limited to quite small m_π.

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