Review of muon g-2

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LATTICE 21
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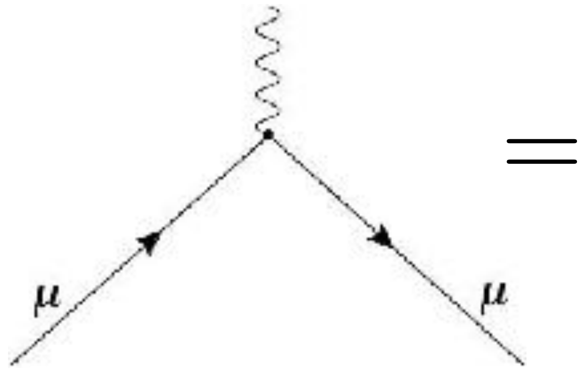
Outline

- Introduction
- Theory vs experiment
- Muon g-2 Theory Initiative
- g-2 SM contributions
- Dispersive, data driven methods for
  - Hadronic Vacuum Polarization (HVP)
  - Hadronic Light-by-Light scattering (HLbL)
- Lattice HVP
- Lattice HLbL
- Summary and Outlook

Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

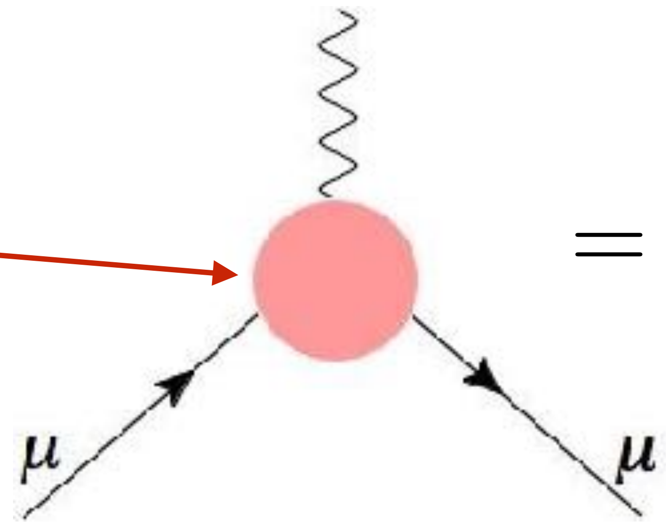
Dirac (leading order): $g = 2$



$$= (-ie)\, \bar{u}(p')\gamma^\mu u(p)$$

Quantum effects (loops):

All SM particles contribute

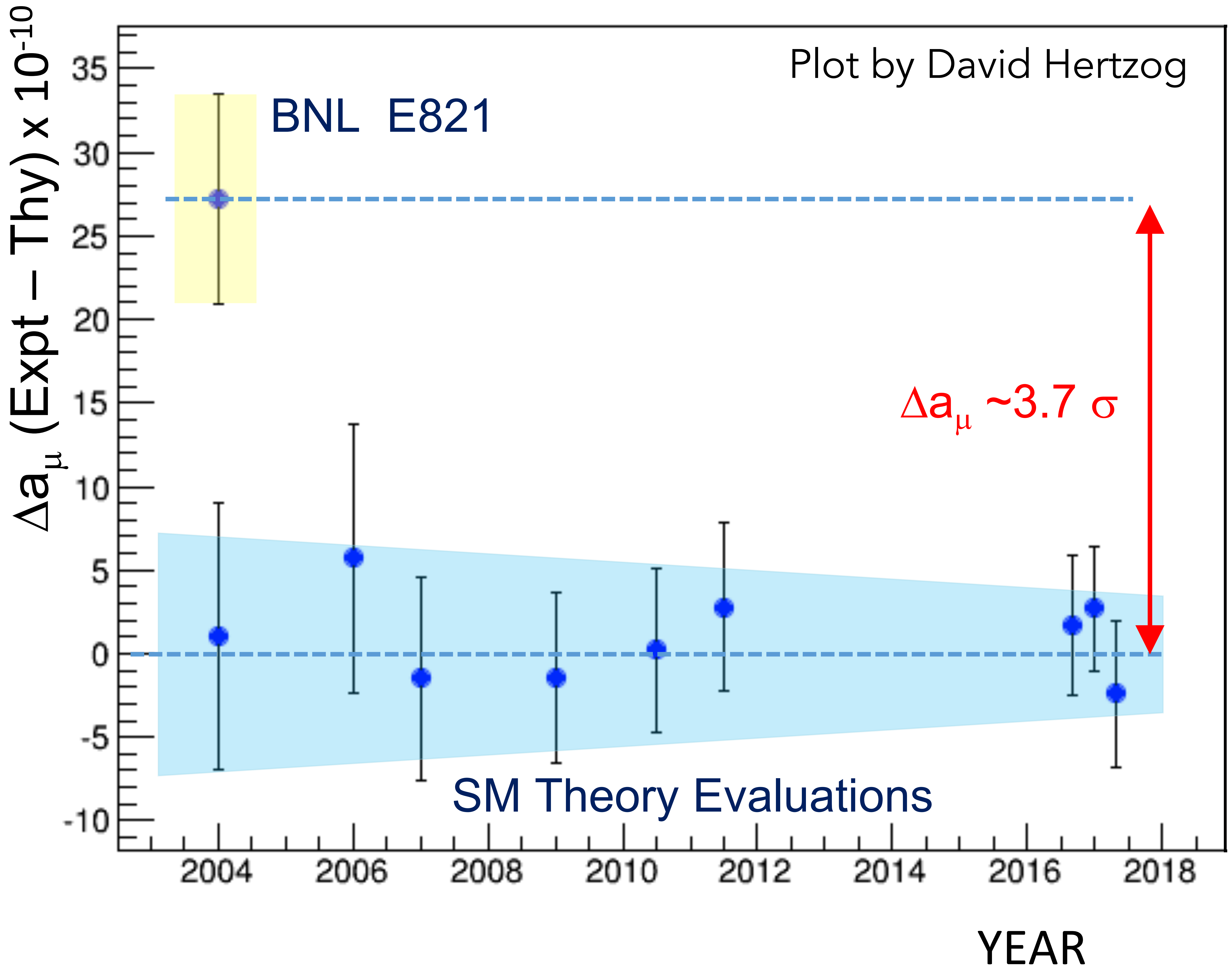


$$= (-i\, e)\, \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\, F_2(0)$

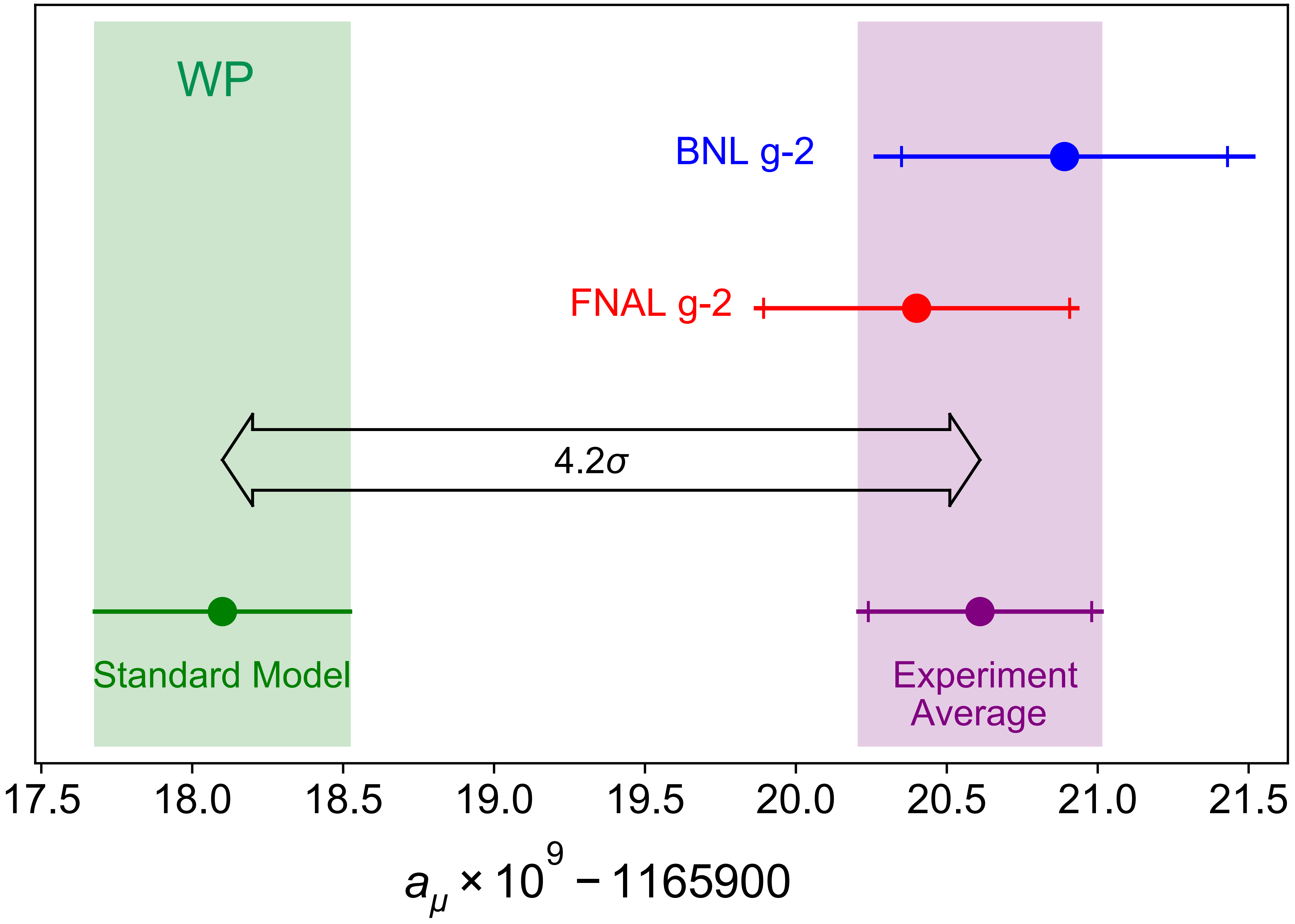
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$

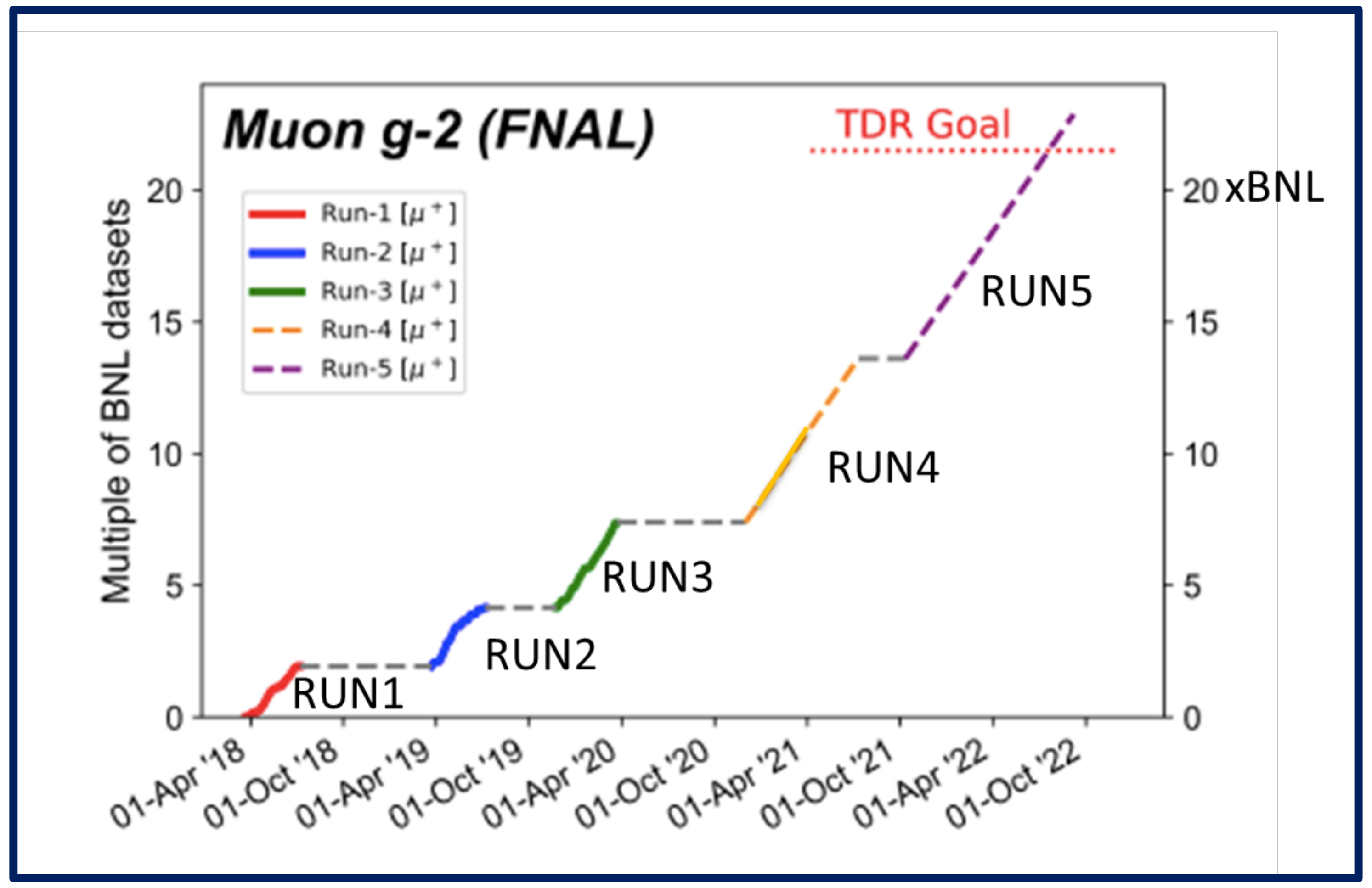
Muon g-2: history of experiment vs theory



Muon g-2: experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021. [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Analysis of runs 2 and 3 is now underway.





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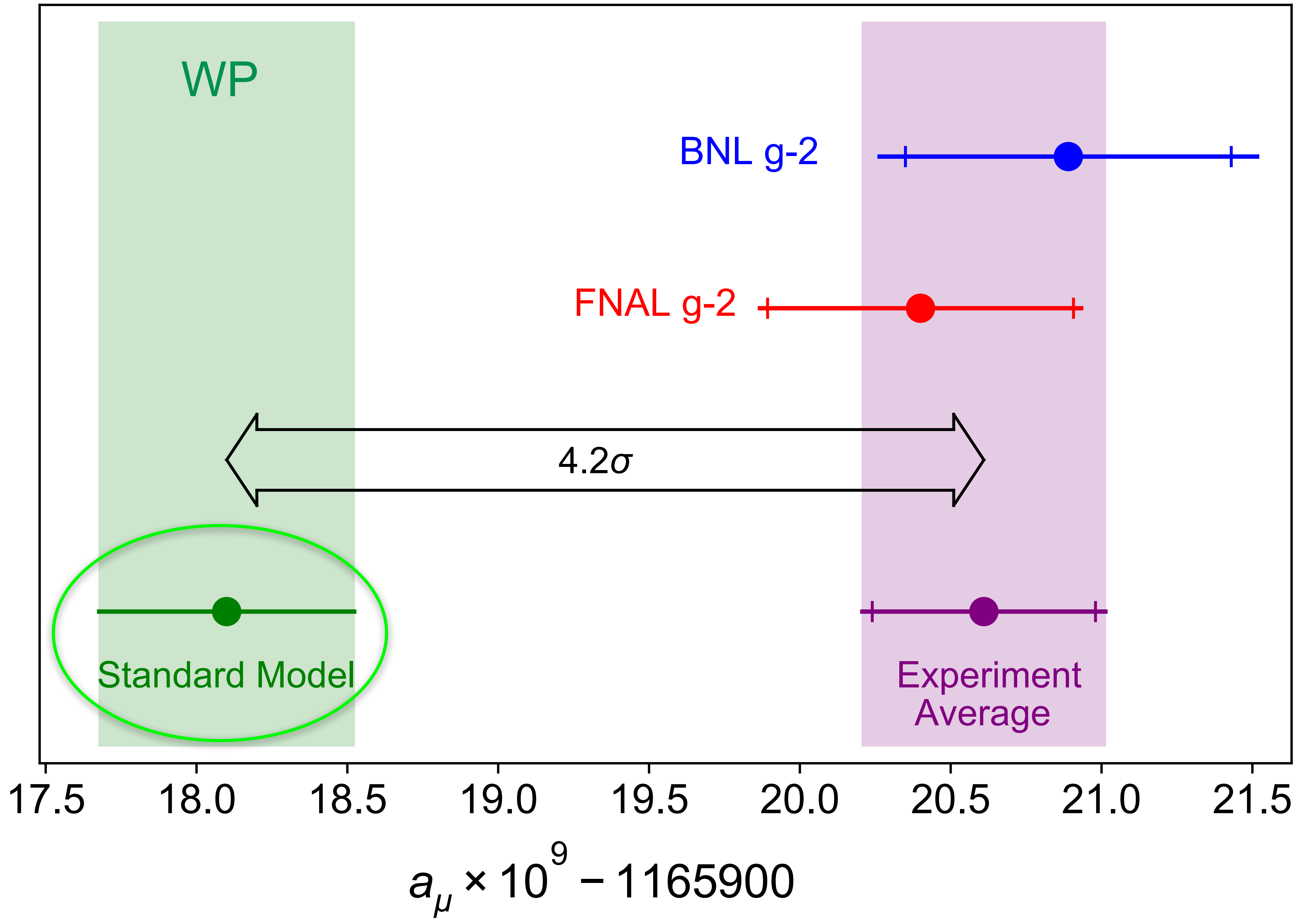
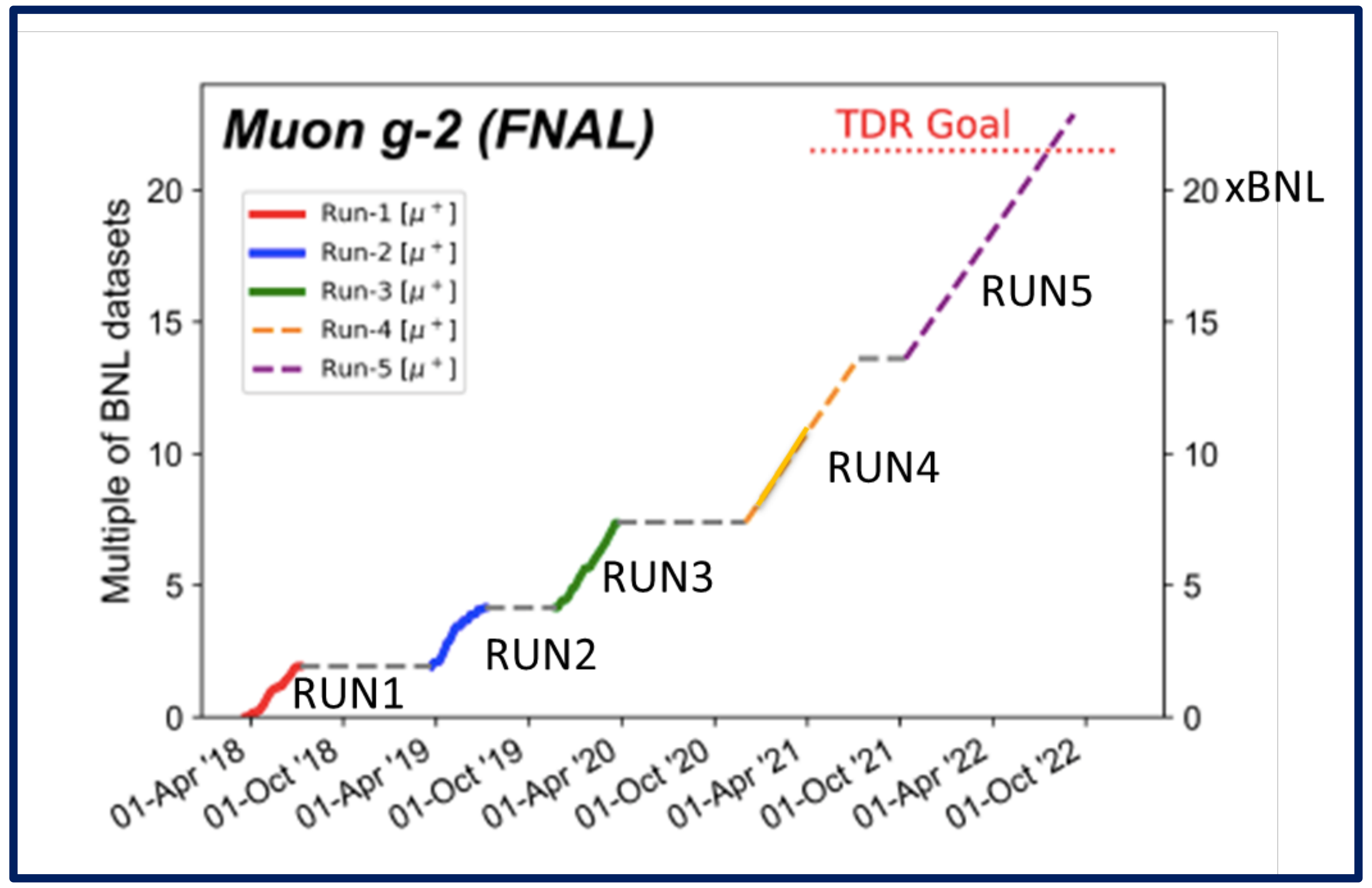
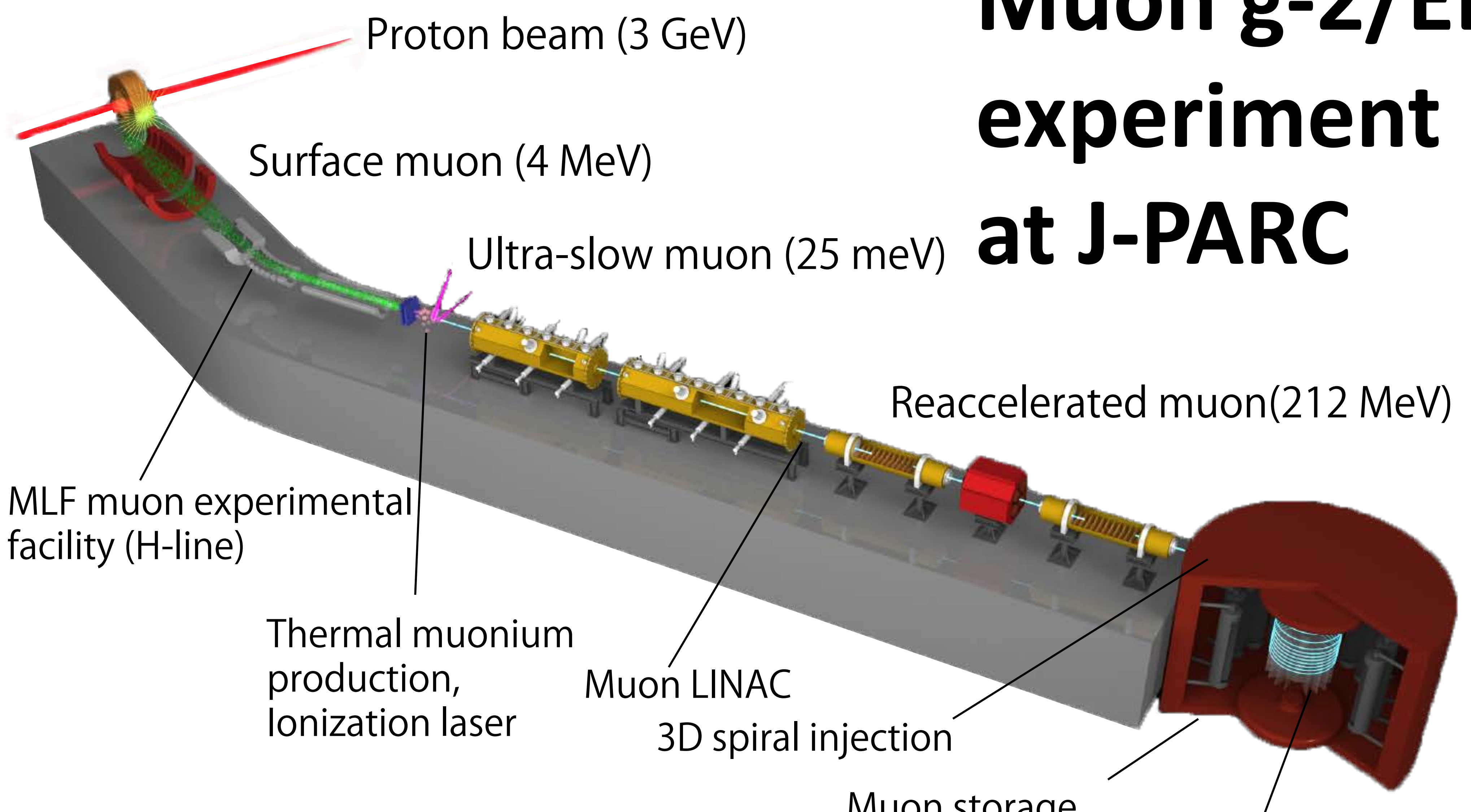




Muon g-2: experiment

T. Mibe for E34 @ INT g-2 workshop



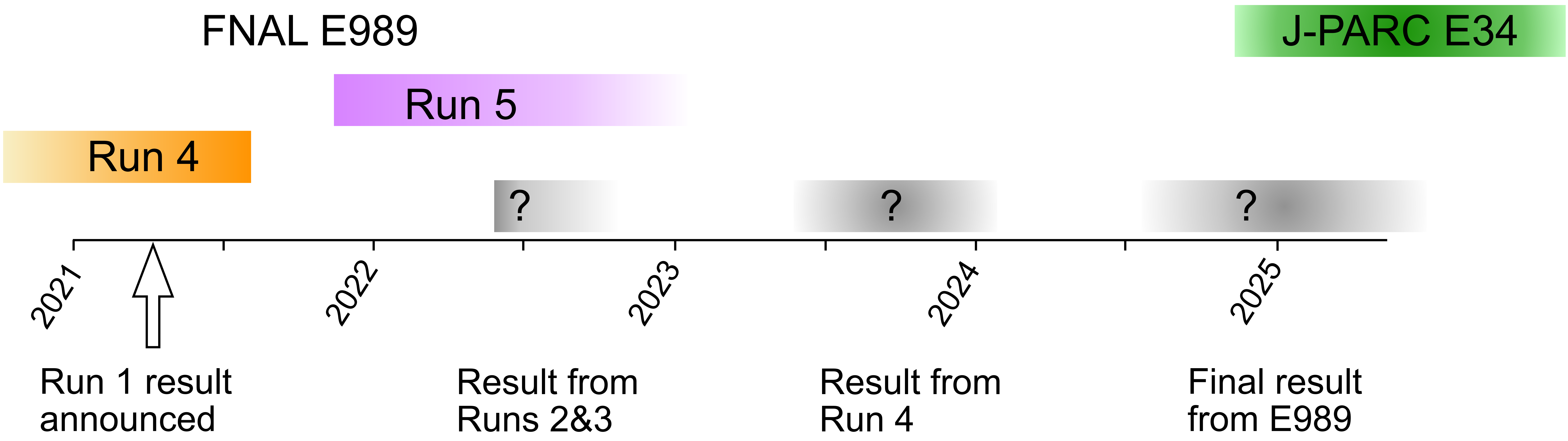
Muon g-2/EDM experiment at J-PARC

Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

- 2018:
  Stage II approval by IPNS and IMSS directors.
- March 2019:
  Endorsed by KEK-SAC as a near-term priority
- 2020:
  Funding request
- 2024+:
  data taking runs

Timeline of g-2 experiments



- The Fermilab and J-PARC g-2 experiments will yield increasingly precise measurements of a_μ through the rest of this decade.
- In addition, future efforts may include negative muon runs at Fermilab and new muon EDM experiments.

➠ Continue efforts to improve on the precision of the SM predictions.

➠ Theory Initiative: plan to continue efforts and update WP with new SM predictions

Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): early September 2022 (tentative)
- 1st White Paper published in 2020 (132 authors, 82 institutions, 21 countries)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- 2nd White Paper: First discussions @ KEK meeting

Muon g-2 Theory Initiative

Steering Committee

Simon Eidelman

[photo by Hartmut Wittig]

(1948-2021)

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay)
- Aida El-Khadra (UIUC & Fermilab)
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL)
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK) J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston) Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

QED

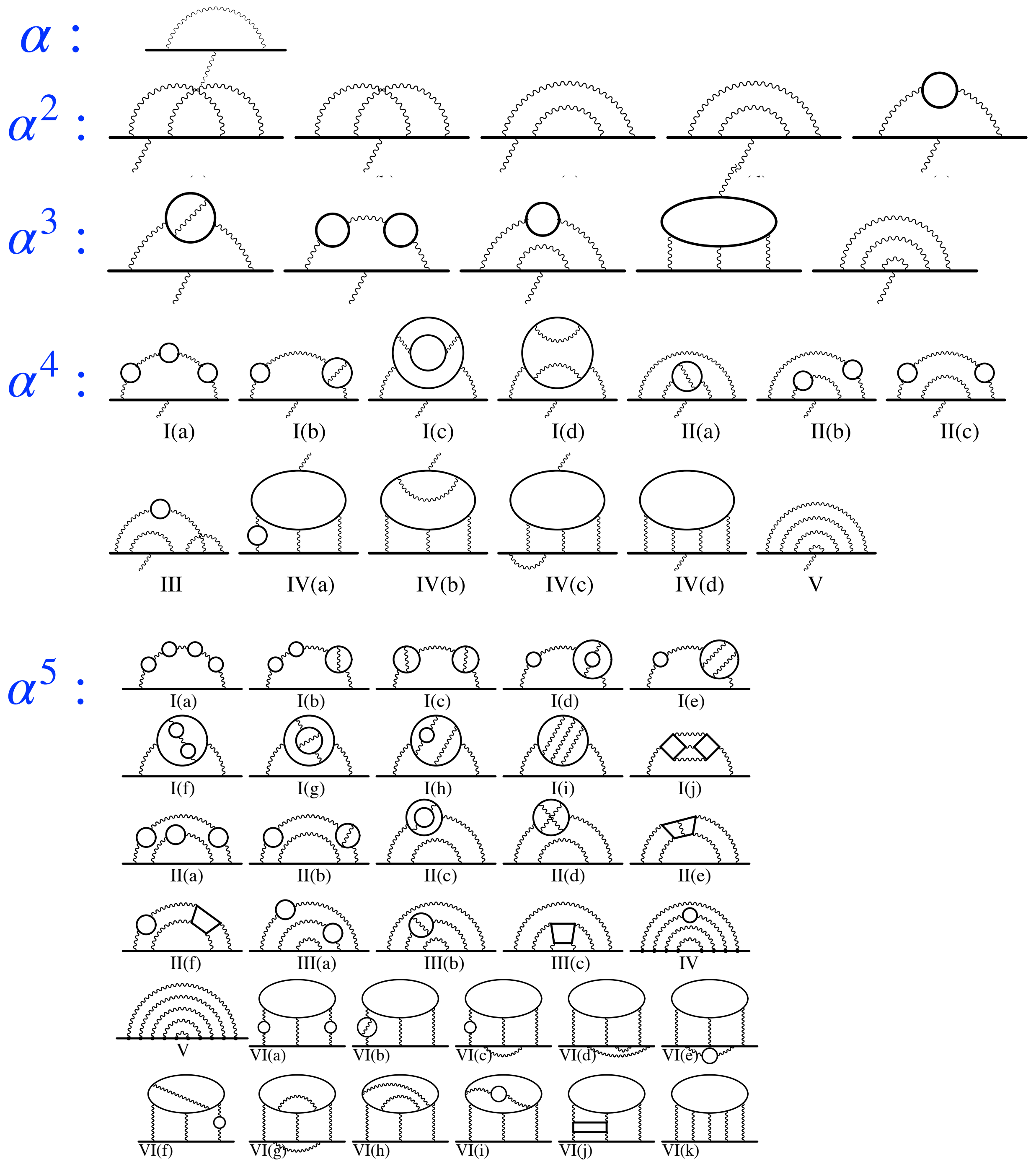
$$a_\mu(\mathrm{QED}) = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0}\left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\mathrm{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]

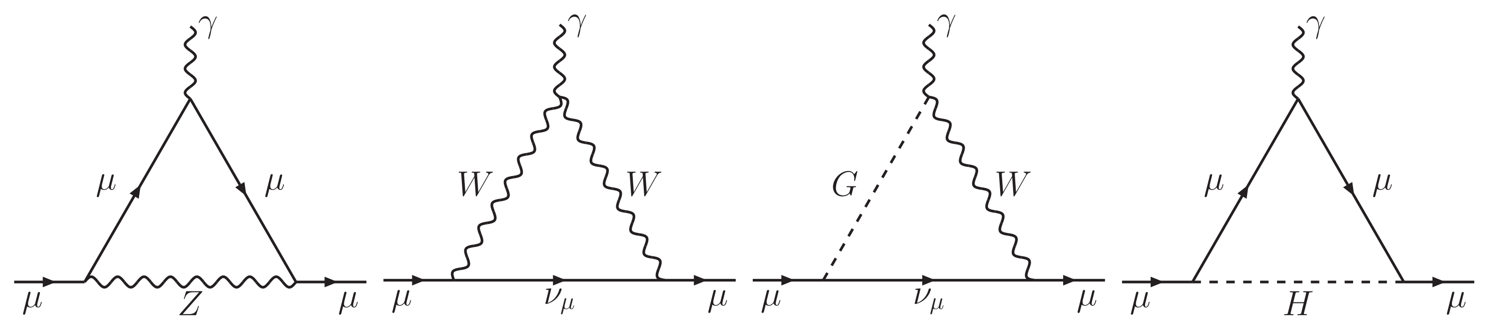


Muon g-2: SM contributions

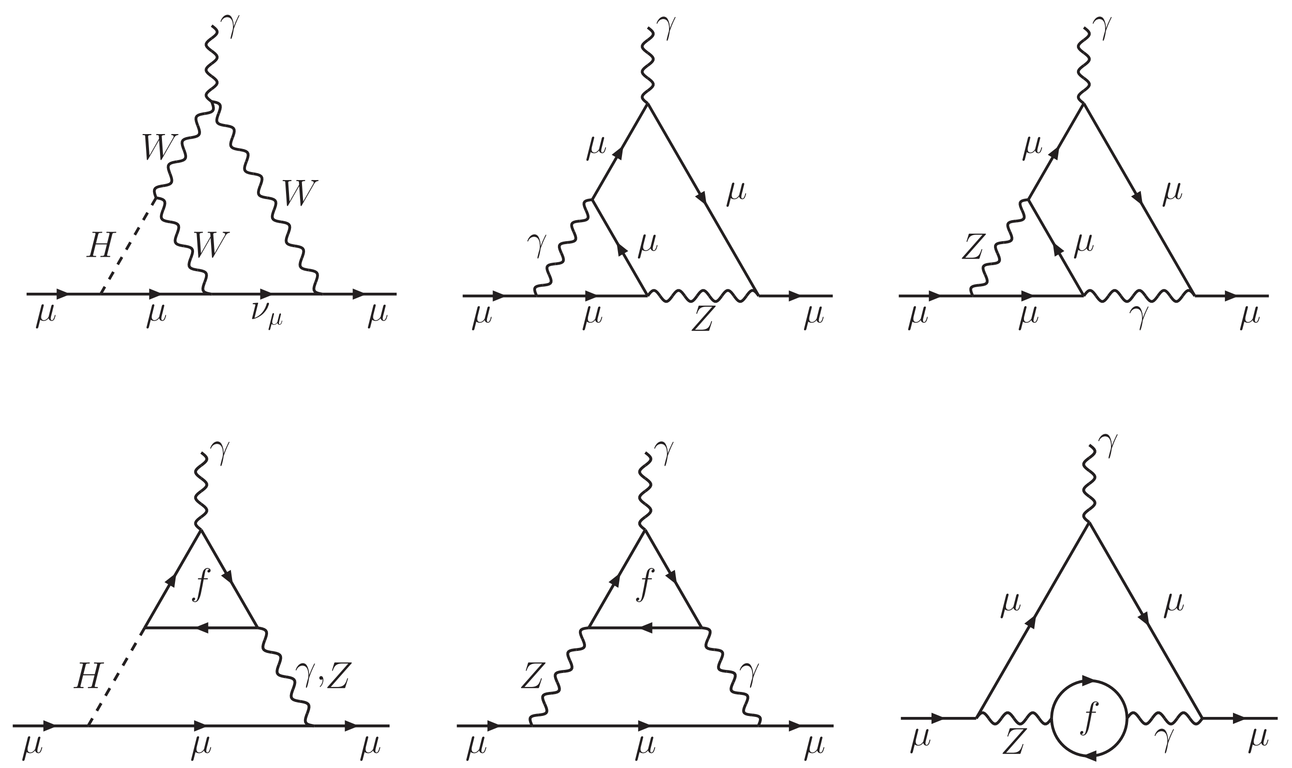
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

Electroweak
(contributions from W,Z,H,.. bosons)

1-loop



2-loop



$$a_\mu(\text{EW}) = 153.6\,(1.0) \times 10^{-11}$$

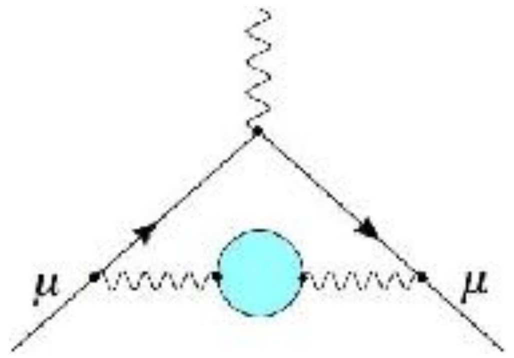
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

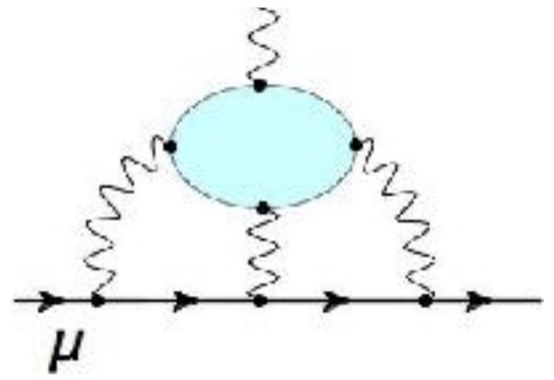
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

leading hadronic

α^2



α^3



✦ The hadronic contributions are written as:

$$a_\ell(\text{hadronic}) = a_\ell^{\text{HVP, LO}} + a_\ell^{\text{HVP, NLO}} + a_\ell^{\text{HVP, NNLO}} + \ldots$$
$$+ a_\ell^{\text{HLbL}} + a_\ell^{\text{HLbL, NLO}} + \ldots$$

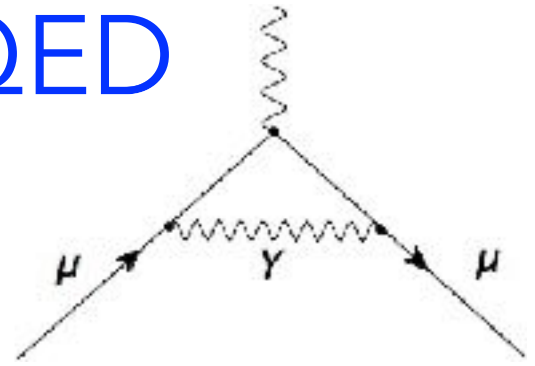
α^2 α^3 α^4

$\sim \quad 10^{-7}$

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

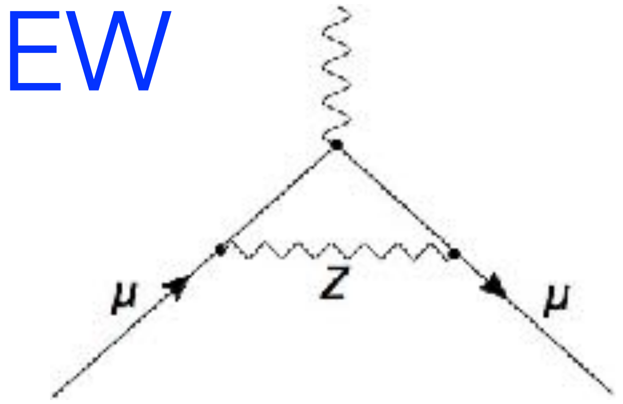
QED



+…

$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

EW



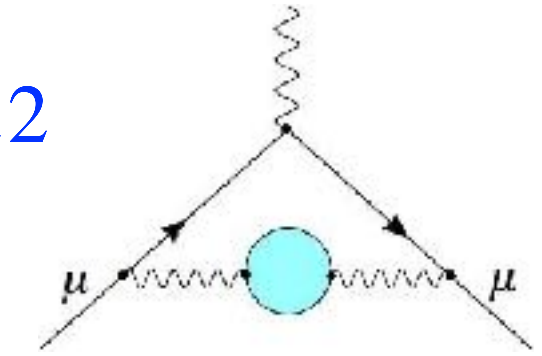
+…

$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

α^2

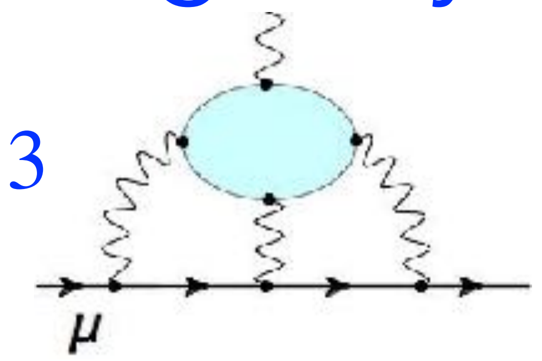


+…

$6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

…Light-by-Light (HLbL)

α^3



+…

$92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

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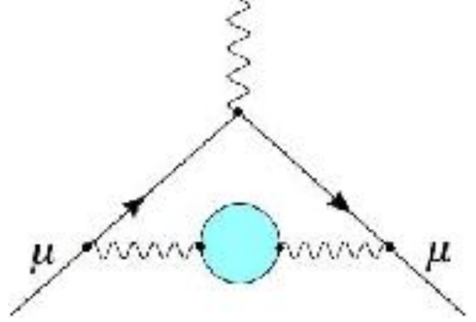
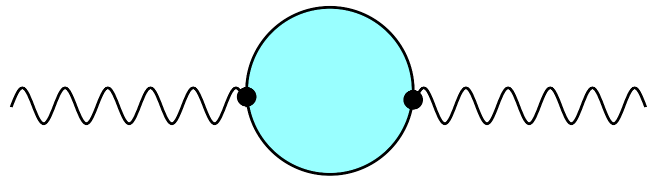


Hadronic vacuum polarization

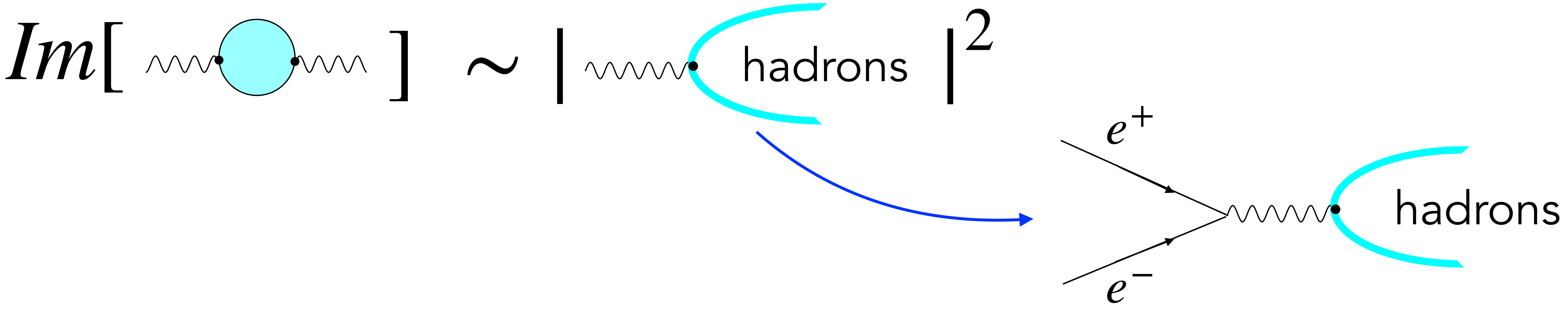


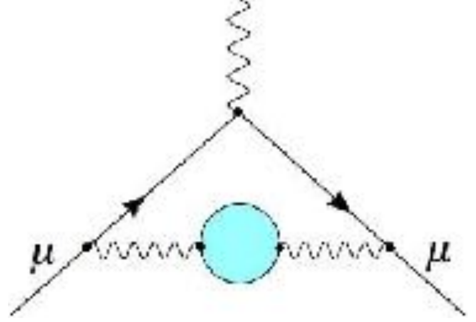
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

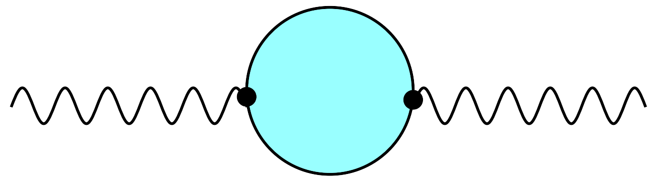
Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:





Hadronic vacuum polarization



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- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

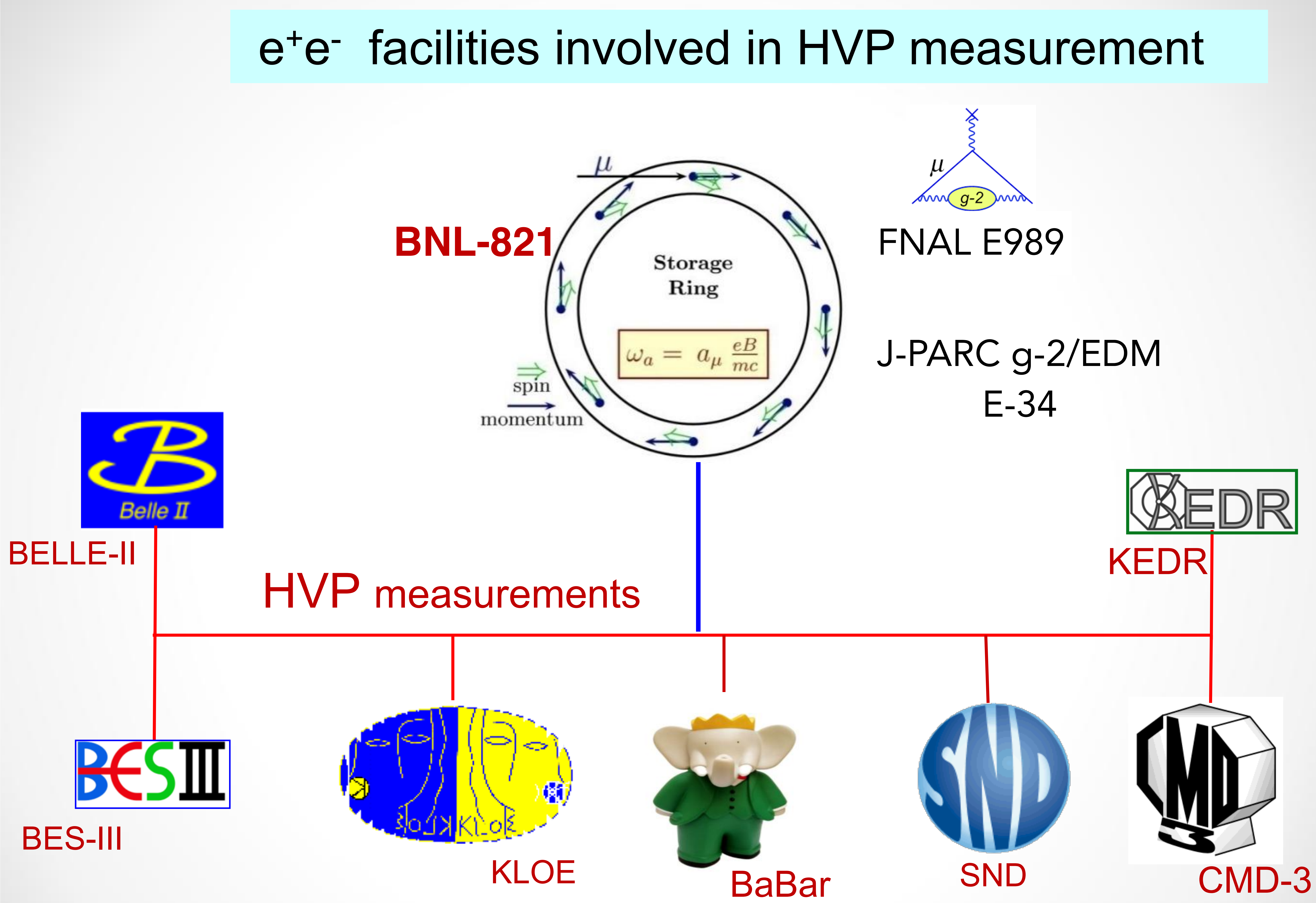
$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$

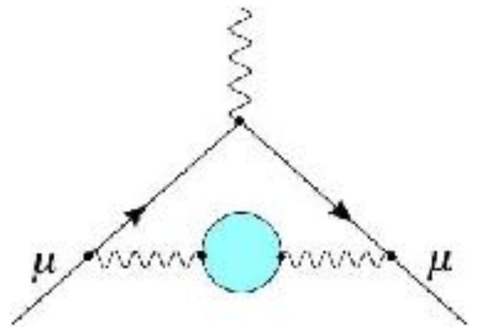
Dominant contributions from low energies
$\pi^+\pi^-$ channel: 73% of total

- Use direct integration method, summing up cross sections for all possible hadronic channels up to ~ 2 GeV

Experimental Inputs to HVP

S. Serednyakov (for SND) @ HVP KEK workshop





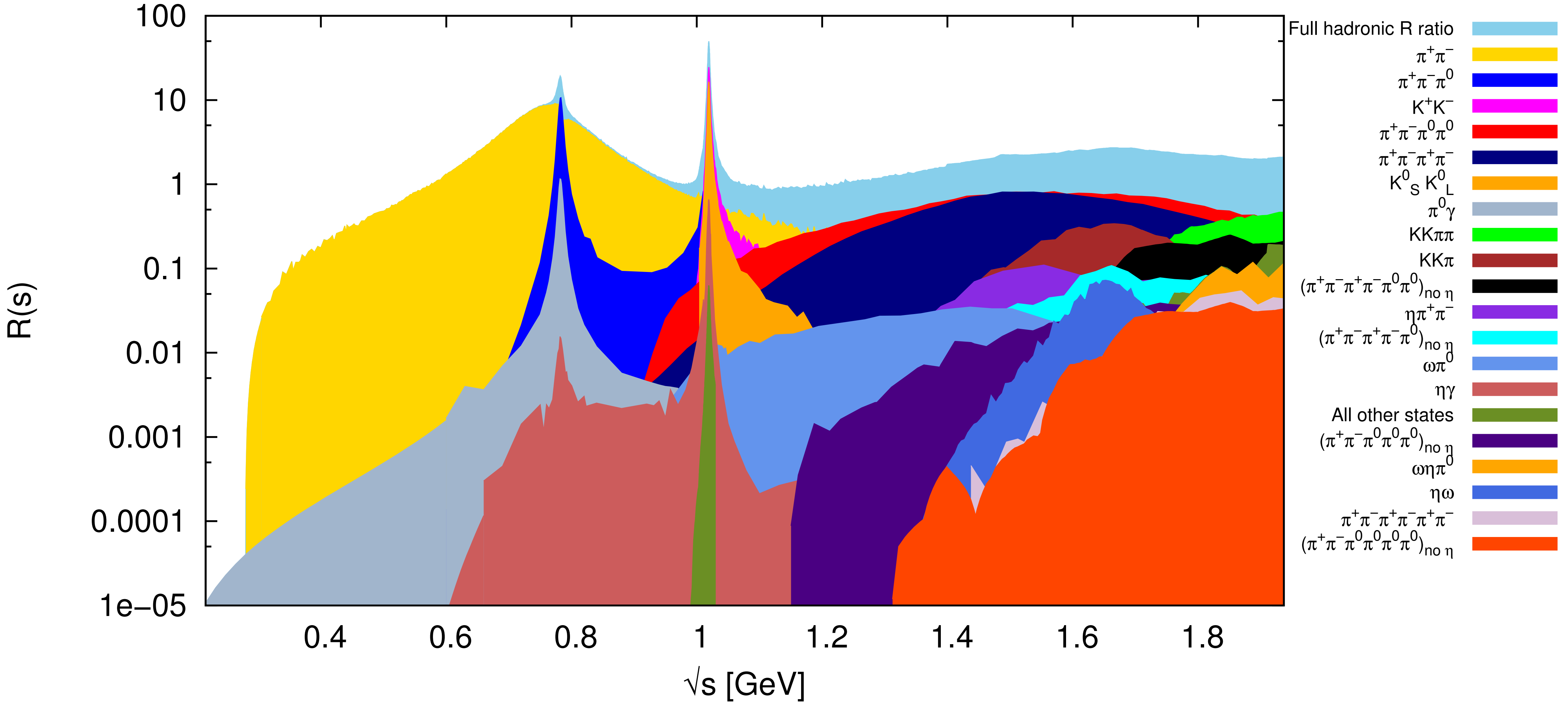
HVP: data-driven

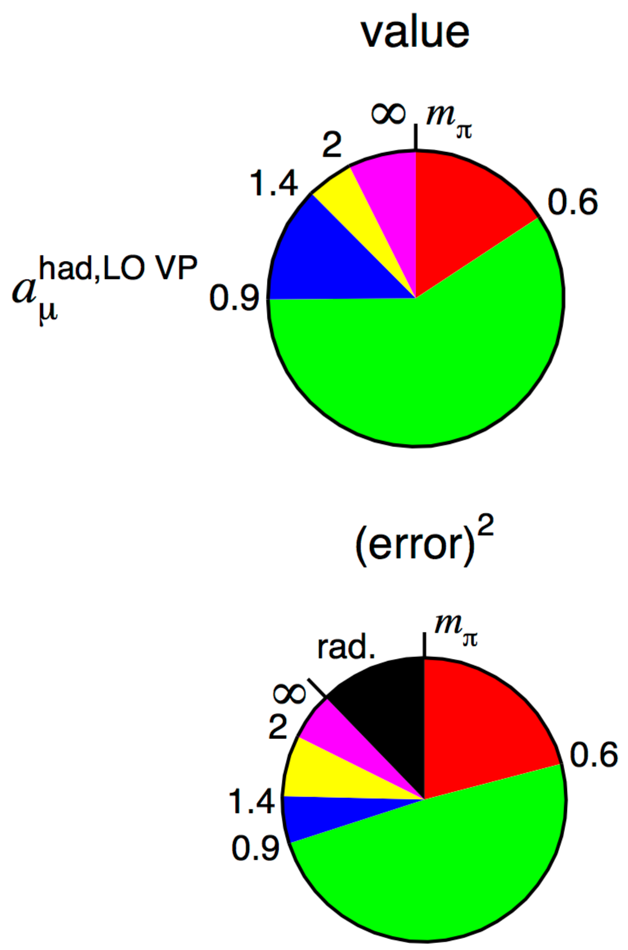
Z. Zhang for DHMZ @ INT g-2 workshop

[M. Davier et al, arXiv:1908.00921]

Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
R_{QCD} $[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01
R_{QCD} $[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$

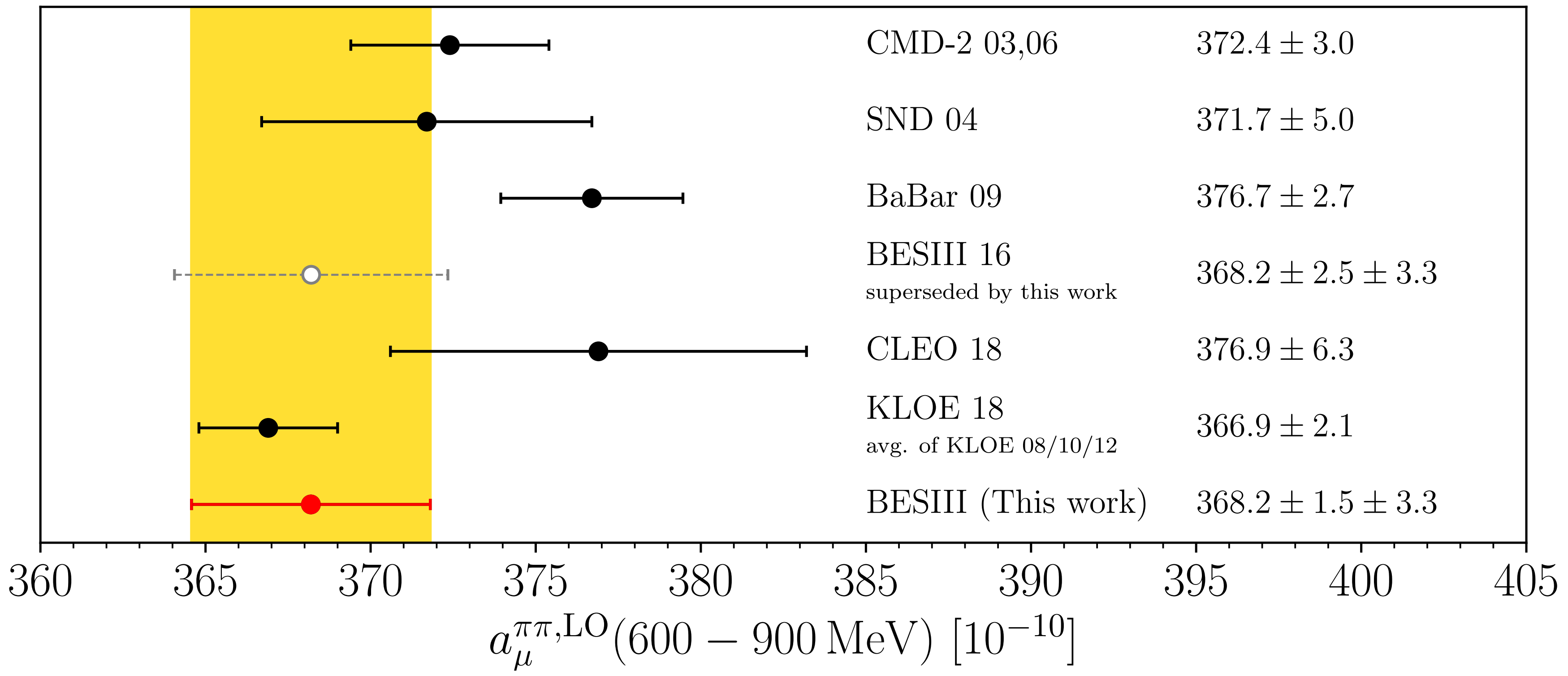
[A. Keshavarzi et al, arXiv:1802.02995]





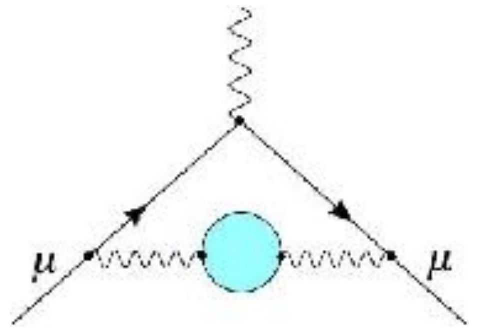
Tensions between BaBar and KLOE data sets:

[M. Ablikim et al (BES III), arXiv:2009.05011]



- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions

➠ conservative merging procedure



HVP: data-driven

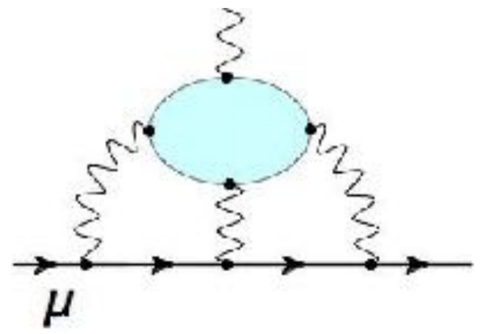
Conservative merging procedure

[B. Malaescu @ INT g-2 workshop]

to obtain a realistic assessment of the underlying uncertainties:

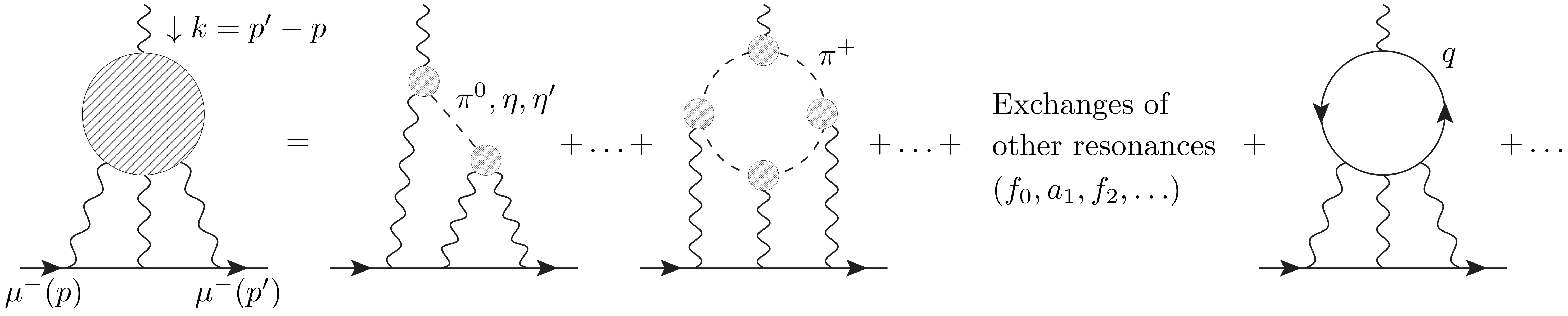
- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- constraints and cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

➠ $a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(2.8)_{\rm sys}\,(0.7)_{\rm DV+pQCD} \times 10^{-10} = 693.1\,(4.0) \times 10^{-10}$

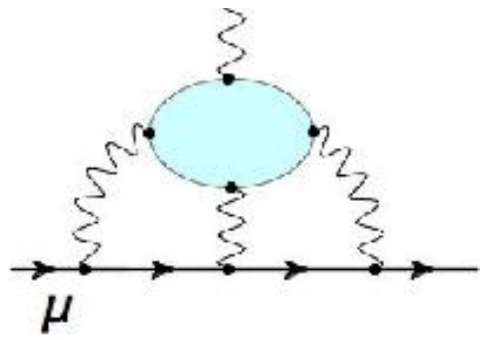


Hadronic Light-by-light

Hadronic light-by-light: Target: ≲ 10% total error



- ✦ previous estimates "Glasgow consensus" use models of QCD
- ✦ used to evaluate individual contributions to HLbL scattering tensor
- ✦ theory error not well determined and not improvable



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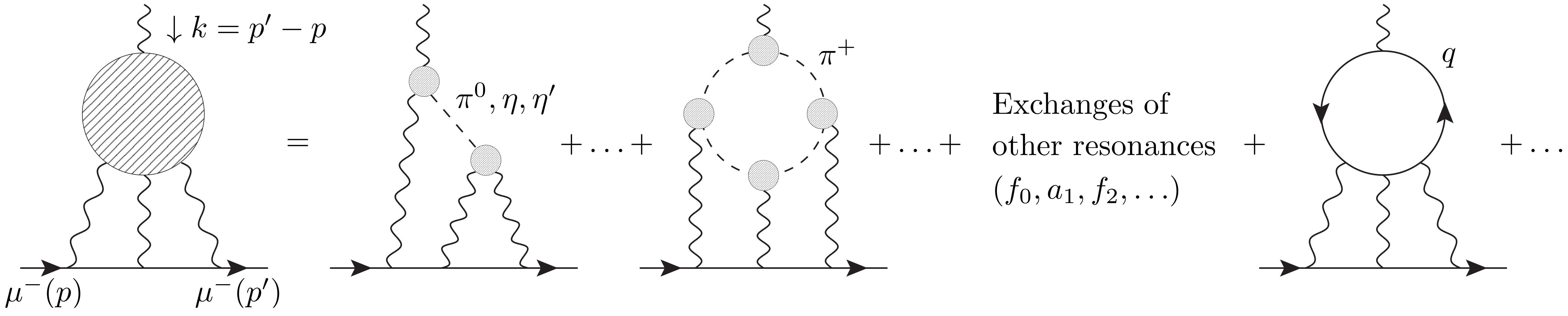
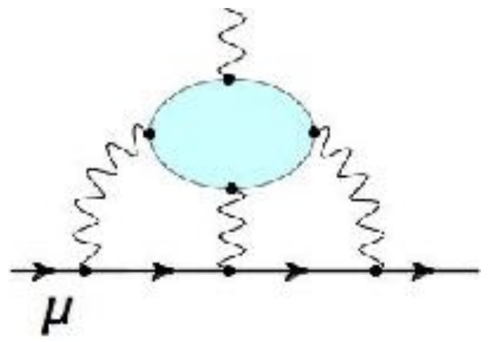


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

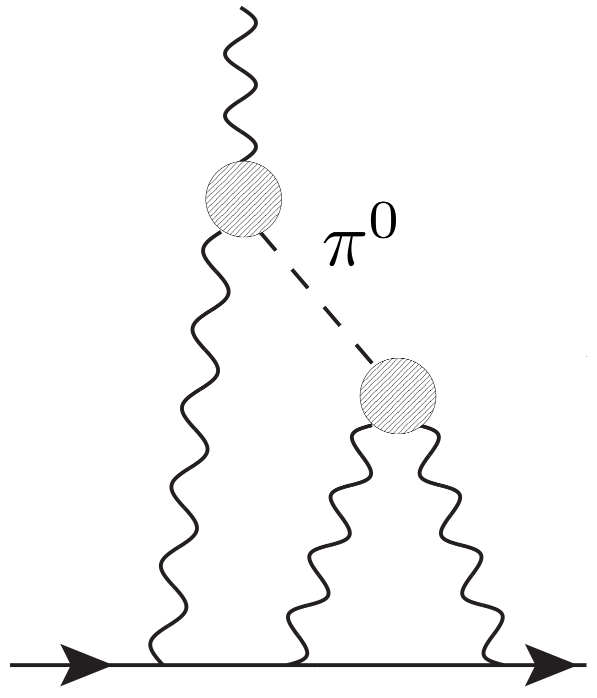
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ➠ ✦ can also use lattice results as inputs



HLbL: dispersive

Three independent results for the pion pole contribution:

[G. Colangelo @ INT g-2 workshop]



- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

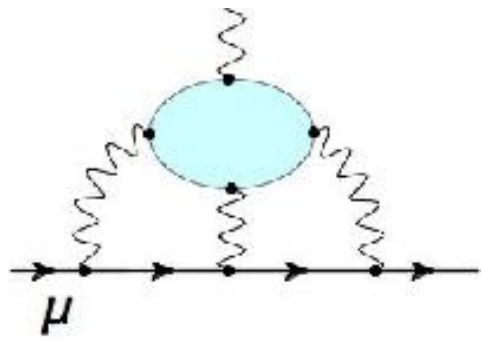
$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$



HLbL: dispersive

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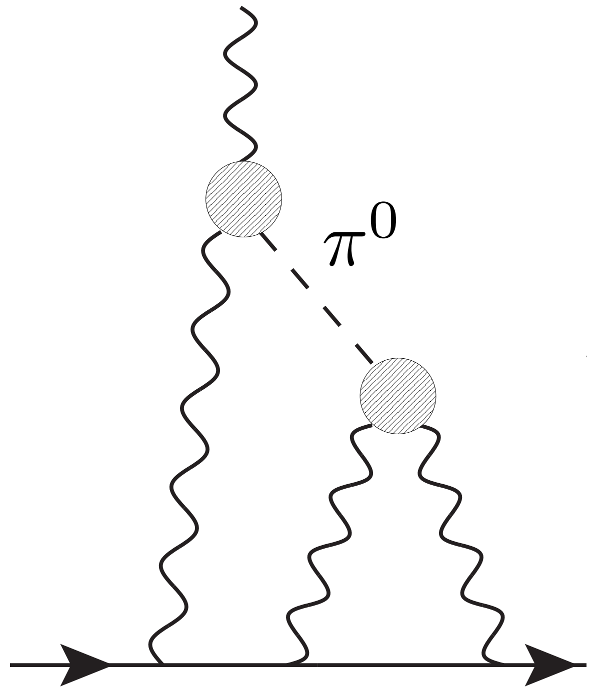

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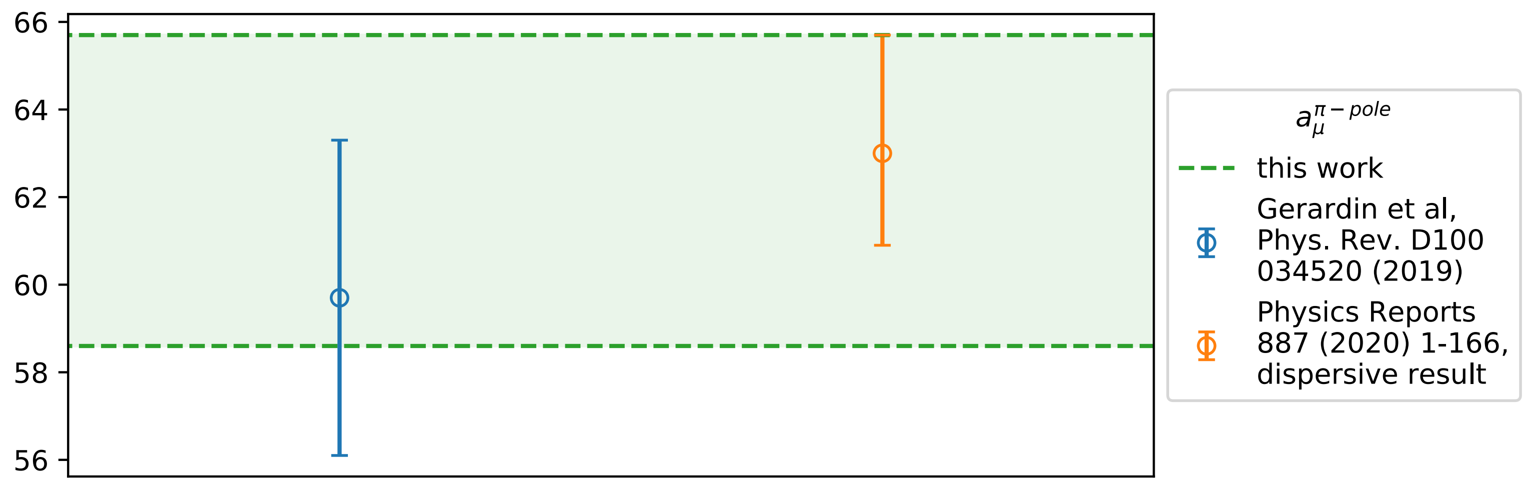
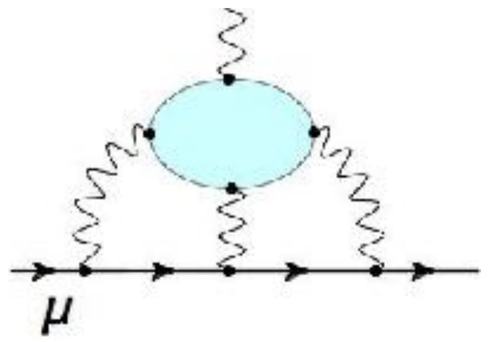
Tuesday, 5:00-8:00 US EDT
Sebastian Burri (ETMc)

New calculation of pion transition form factor on ETMc twisted-mass ensemble with phys. mass and a~ 0.08 fm

- From the data shown on the previous slide with $t_{cut} \in [0.88, 2.08]$ fm we get without considering the systematic error a preliminary

$a_\mu^{\pi-pole} \in [58.6, 65.7] \times 10^{-11}$ with a 2%-6% statistical error.





HLbL: dispersive

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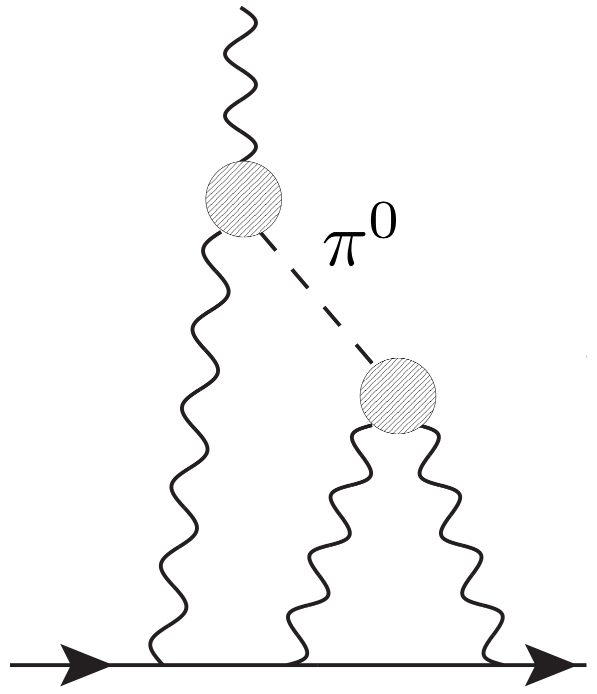
[G. Colangelo @ INT g-2 workshop]



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Tuesday, 5:00-8:00 US EDT
- Willem Verplanke (BMWc)

calculation of η, η' transition form factors on staggered ensembles (BMWc)

Tuesday, 5:00-8:00 US EDT
- Sebastian Burri (ETMc)

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- From the data shown on the previous slide with $t_{cut} \in [0.88, 2.08]$ fm we get without considering the systematic error a preliminary

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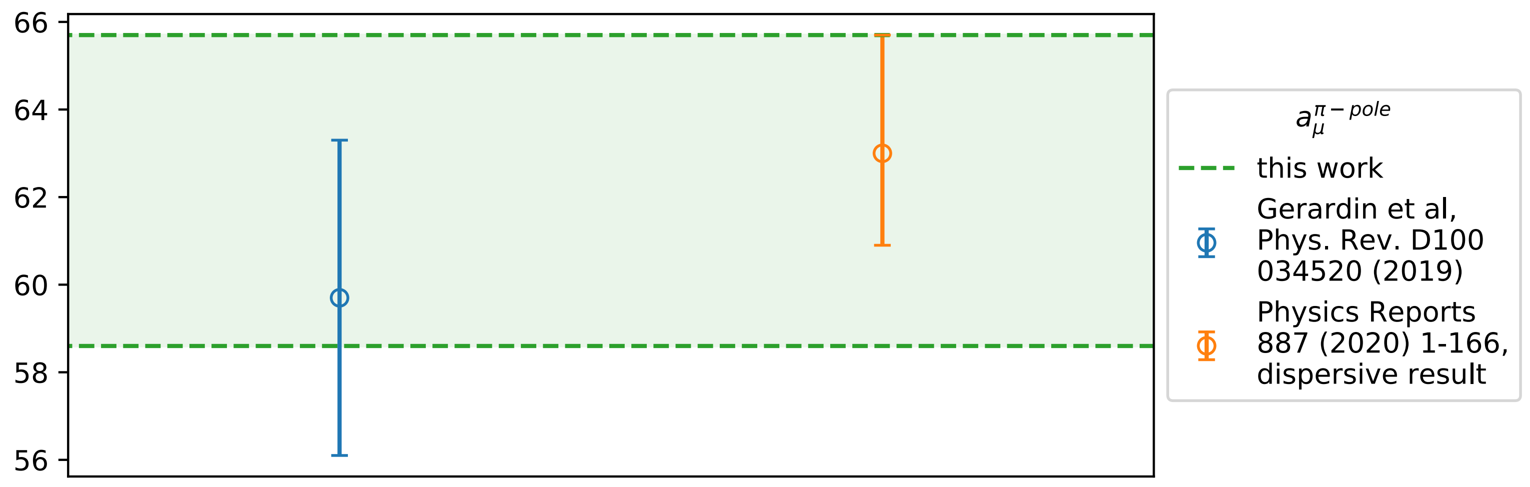
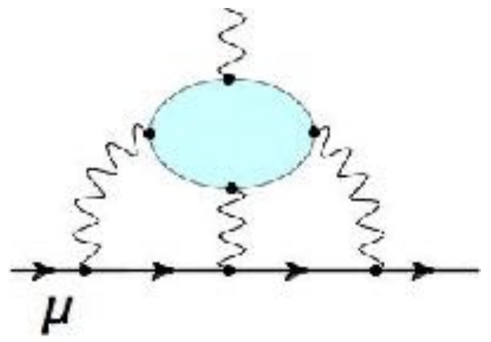
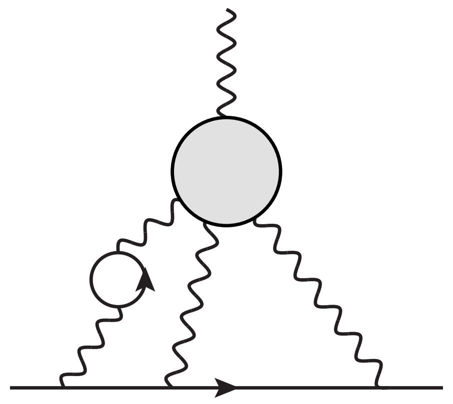




HLbL: dispersive

Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3) (scalars + tensors)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



NLO HLbL contribution:

$$a_\mu^{\mathrm{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

Outline

- Introduction
- Theory vs experiment
- Muon g-2 Theory Initiative
- g-2 SM contributions
- Dispersive, data driven methods for
  - Hadronic Vacuum Polarization (HVP)
  - Hadronic Light-by-Light scattering (HLbL)
- Lattice HLbL
- Lattice HVP
- Summary and Outlook

g-2 & related talks/posters @ Lattice 2021

QCD in searches for New Physics

(light-quark) connected HVP, windows

- Monday, 13:00-15:00 US EDT
  - Finn Stokes (BMWc) FV effects
  - Kalman Szabo (BMWc) cont. limit
  - Shaun Lahert (Fermilab-HPQCD-MILC)
  - Chris Aubin (Aubin et al)
- Tuesday, 5:00-8:00 US EDT
  - Hartmut Wittig (Mainz)
  - Christoph Lehner (RBC/UKQCD)
  - Davide Giusti (ETMc)

IB corrections, disc. HVP

- Tuesday, 5:00-8:00 US EDT
  - Andreas Risch (Mainz)
  - Letizia Parato (BMWc)
- Poster, Wednesday, 8:00-9:00 US EDT
  - C. McNeile (Fermilab-HPQCD-MILC)

$\Delta\alpha$ and $\Delta \sin^2\theta_W$

- Tuesday, 5:00-8:00 US EDT
  - Teseo San Jose (Mainz)
  - Kohtaroh Miura (Mainz)

HLbL contributions, PS transition form factors

- Tuesday, 5:00-8:00 US EDT
  - Willem Verplanke (BMWc)
  - Sebastian Burri (ETMc)
  - En-Hung Chao (Mainz) complete HLbL

Scale Setting

- Monday, 13:00-15:00 US EDT
  - Lukas Varnhorst (BMWc)
- Thursday 5:00-8:00 US EDT
  - Alexander Segner (Mainz)
- Friday, 5:00–8:00 US EDT
  - Ben Strassberger (Mainz)

Cut-off effects

- Tuesday, 5:00-8:00 US EDT
  - Tim Harris (NEPhEU QCD) thermal observables
- Friday, 5:00-8:00 US EDT
  - Nicolai Husung (DESY) log corrections

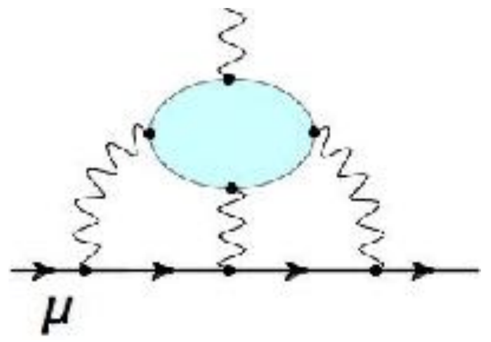
Variance reduction

- Tuesday, 5:00-8:00 US EDT
  - Leonardo Giusti (Milan) Multi-level integration
- Tuesday, 13:00-15:00 US EDT
  - Tej Kanwar (MIT) contour deformation

Hadron spect.

SM params

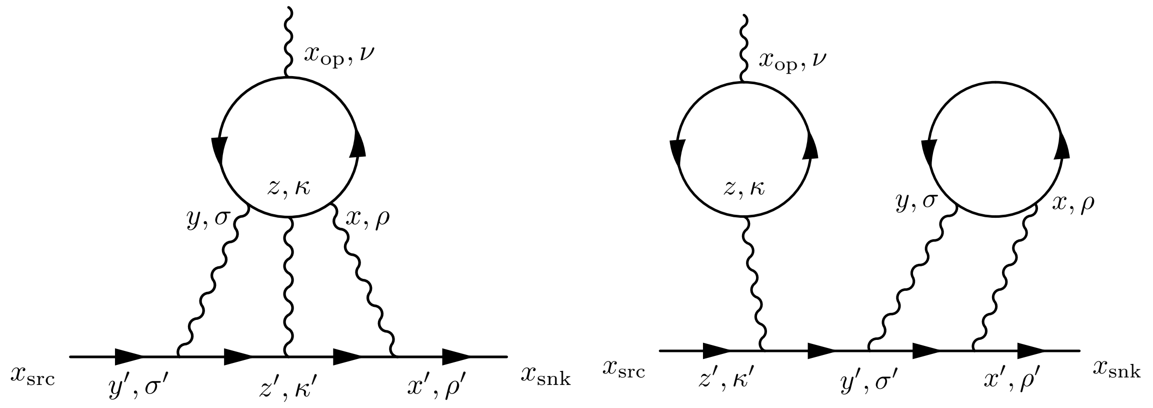
Algorithms

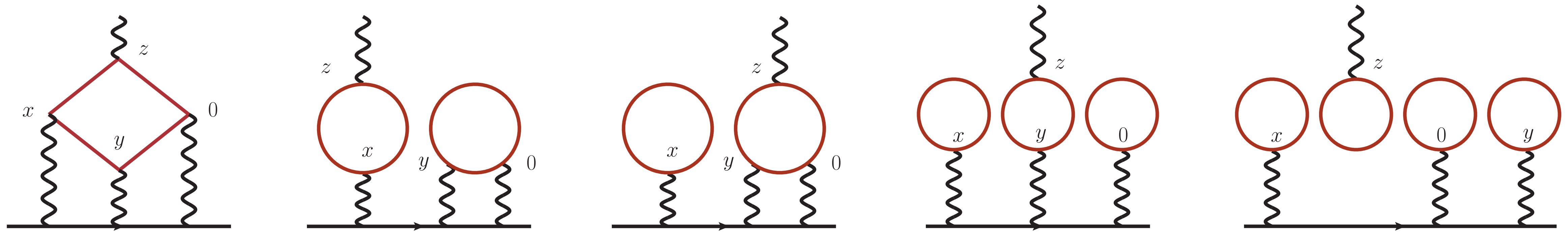


HLbL: lattice

Hadronic light-by-light: Target: ≲ 10% total error

Two independent and complete direct lattice calculations of a_μ^{HLbL}





✦ RBC/UKQCD
[T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]

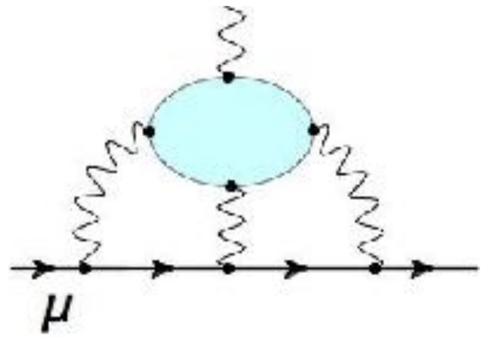
✦ QCD + $\mathrm{QED_L}$ (finite volume)
➠ $1/L^2$ FV effects
stochastic evaluation of position space sums
Feynman gauge photon propagators
DWF ensembles at/near phys mass,
$a \approx 0.08 - 0.2\,\mathrm{fm}$, $L \sim 4.5 - 9.3\,\mathrm{fm}$

✦ Mainz group
[E. Chao et al, arXiv:2104.02632]

✦ QCD + QED (infinite volume & continuum)
➠ $e^{-m_\pi L}$ FV effects
semi-analytic QED kernel function

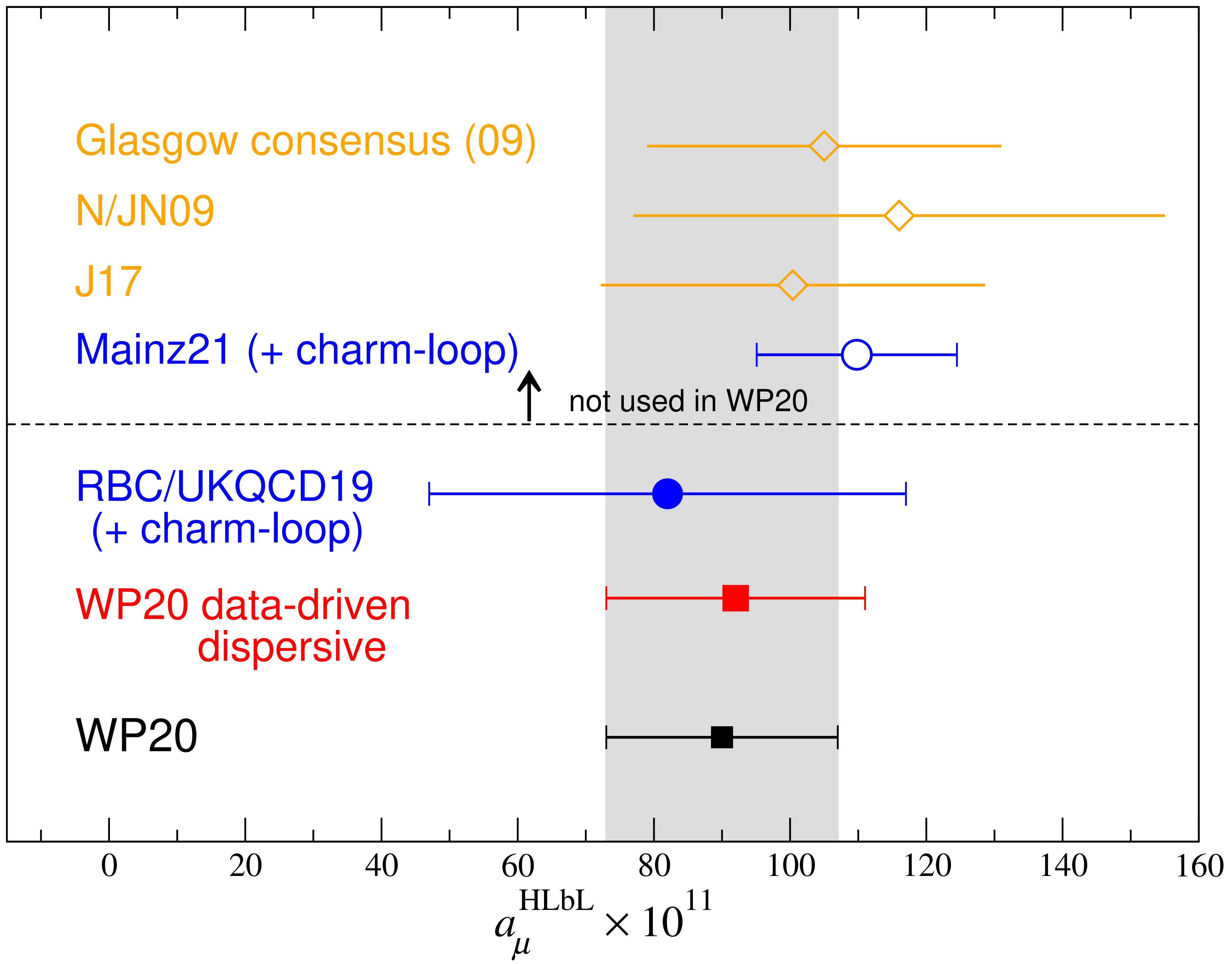
CLS (2+1 Wilson-clover) ensembles
$m_\pi \sim 200 - 430\,\mathrm{MeV}$, $a \approx 0.05 - 0.1\,\mathrm{fm}$, $m_\pi L > 4$

✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass



HLbL: Comparison

$$a_\mu^{\mathrm{HLbL}}$$

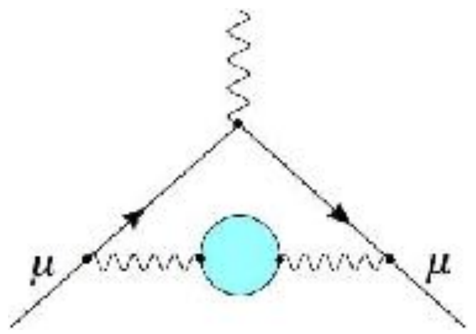


models of QCD +EFT, large N_c

Lattice QCD + QED

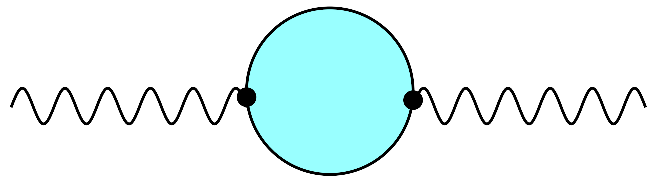
data + dispersive approach

Now well-determined in two independent approaches, systematically improvable



Lattice HVP: Introduction

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

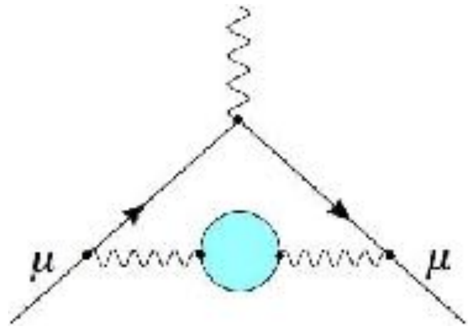
Compute correlation function: $$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$$

and $$\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$$

[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

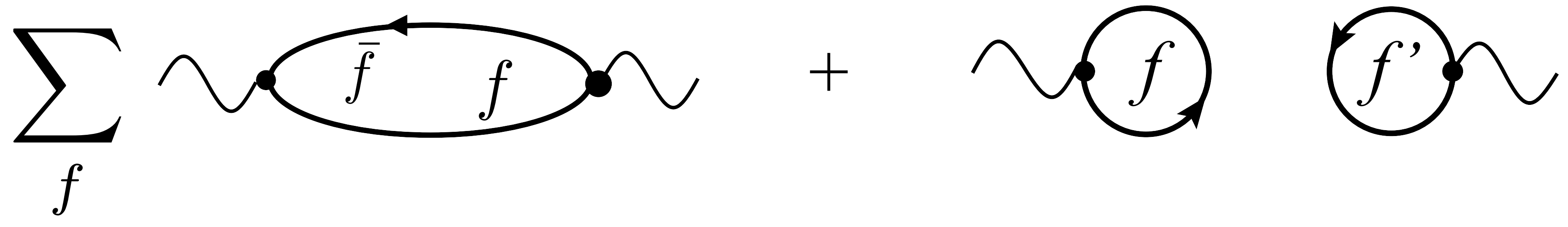


Lattice HVP: Introduction

Calculate a_μ^{HVP} in Lattice QCD:

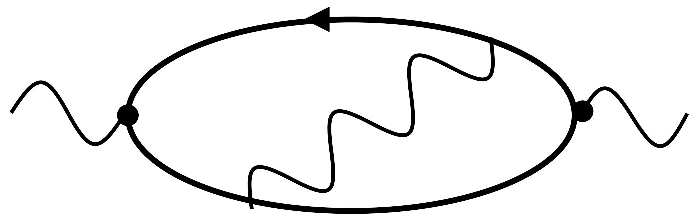
$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$



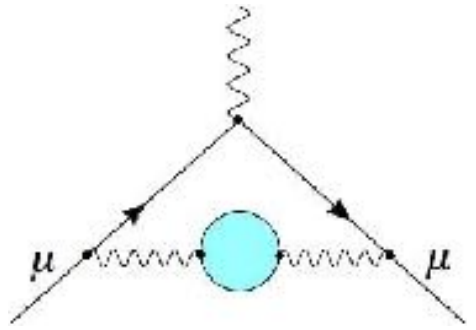
$f = ud, s, c, b$

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



$+ \ldots$

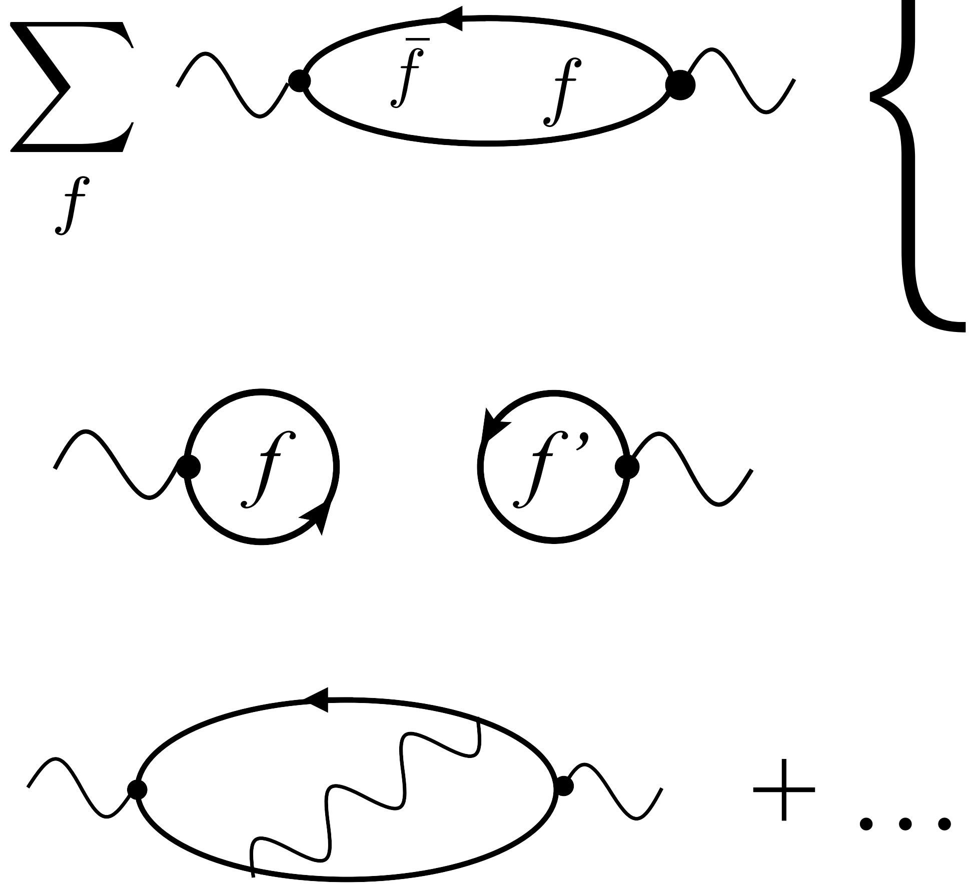
- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



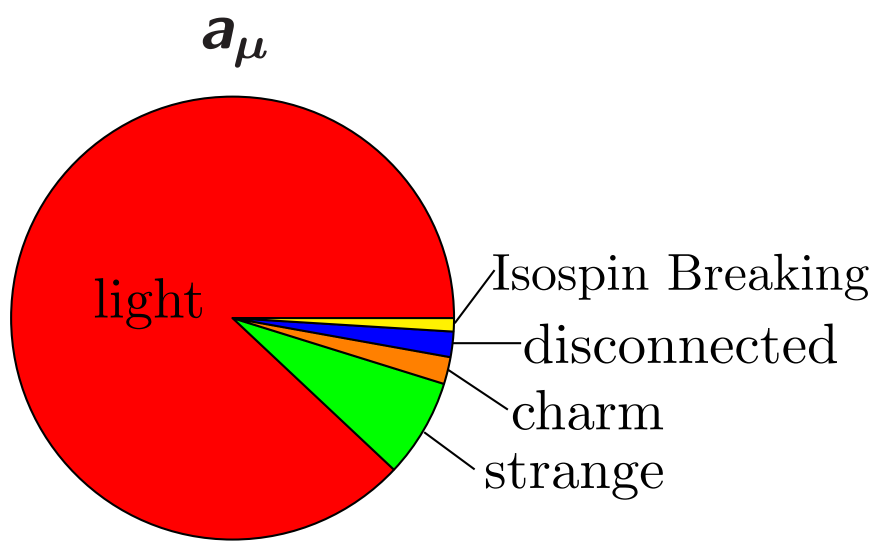
Lattice HVP: Introduction

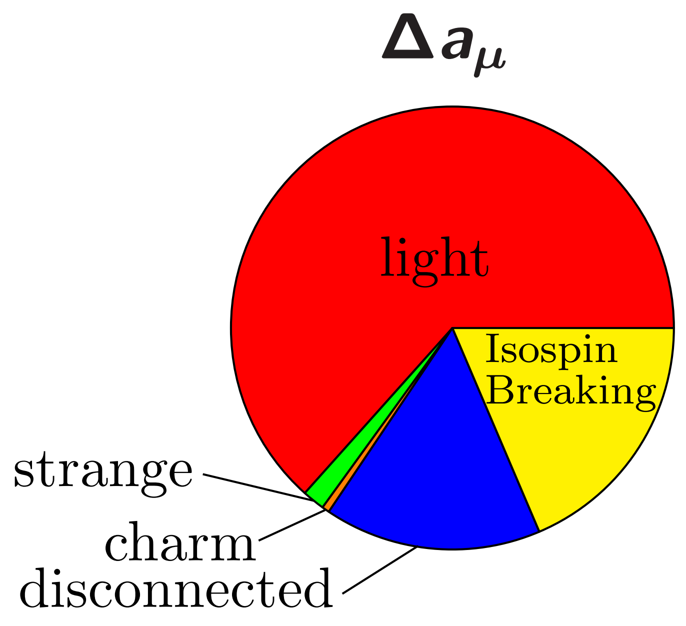
- Target: ~ 0.2% total error

V. Gülpers @ Lattice HVP workshop

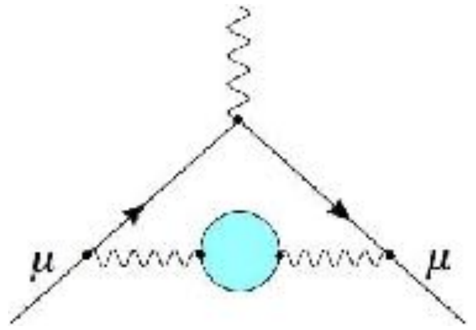


- light-quark connected contribution:
  $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\rm HVP,LO}$ ~1% of total



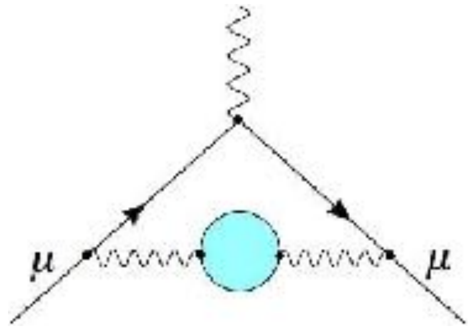


$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$



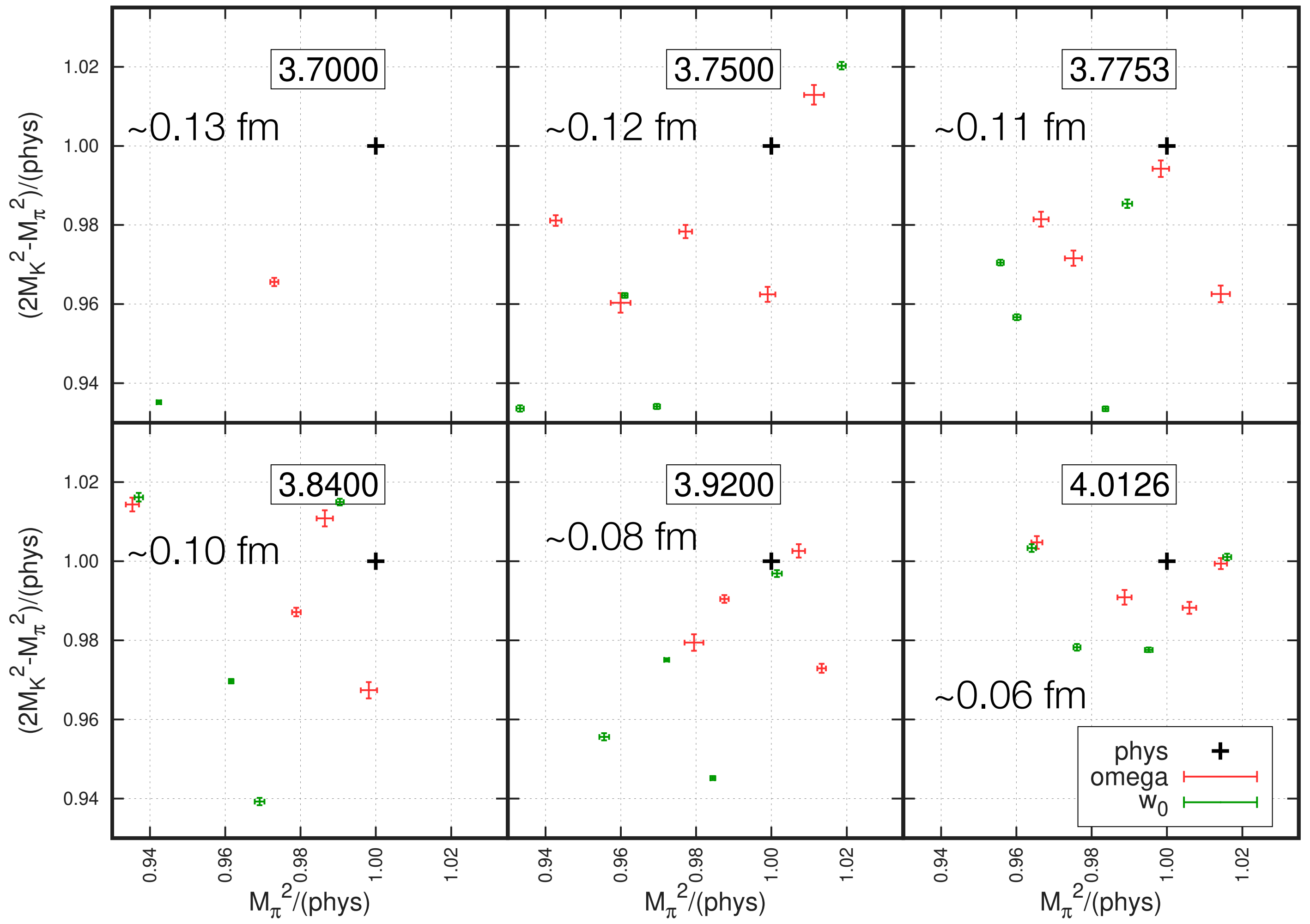
Lattice HVP: Introduction

- Target: ~ 0.2% total error
- Challenges:
  - ✓needs ensembles with (light sea) quark masses at their physical values
  - ✓ finite volume corrections
  - growth of statistical errors at long-distances
  - Continuum extrapolation
  - scale setting
  - disconnected contribution
  - QED and strong isospin breaking corrections ($m_u \neq m_d$)
- Focus on windows in Euclidean times [T. Blum et al, arXiv:1801.07224, 2018 PRL]
  - disentangle systematics/statistics from long distance/FV and discretization effects
    - ➠ valuable cross checks
  - intermediate window easy to compute & compare with disperse methods



Lattice HVP: Ensemble parameters

BMWc [K. Szabo, F. Stokes, L. Parato]



- Stout-smeared staggered fermions 2+1+1
- $L \sim 6 - 11\,\text{fm}$

ETMc [D. Giusti]

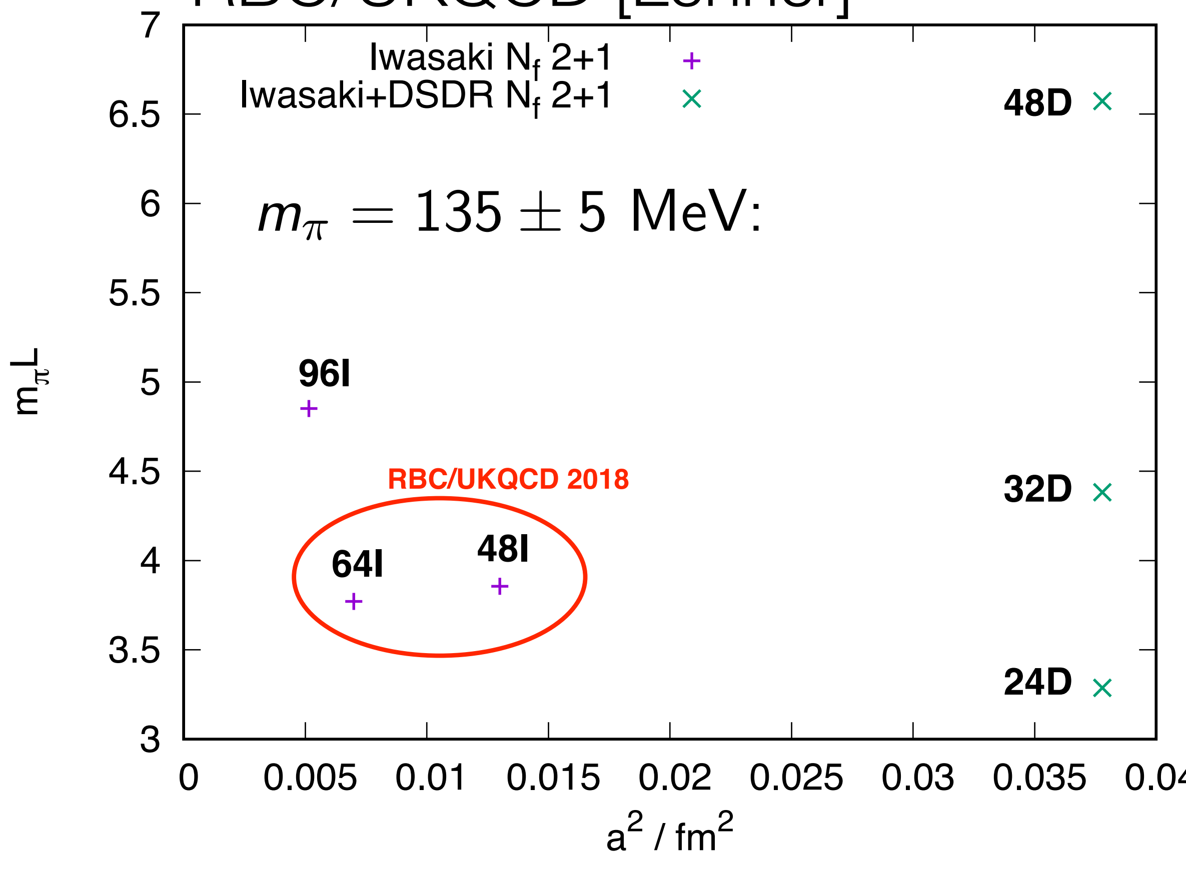
Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

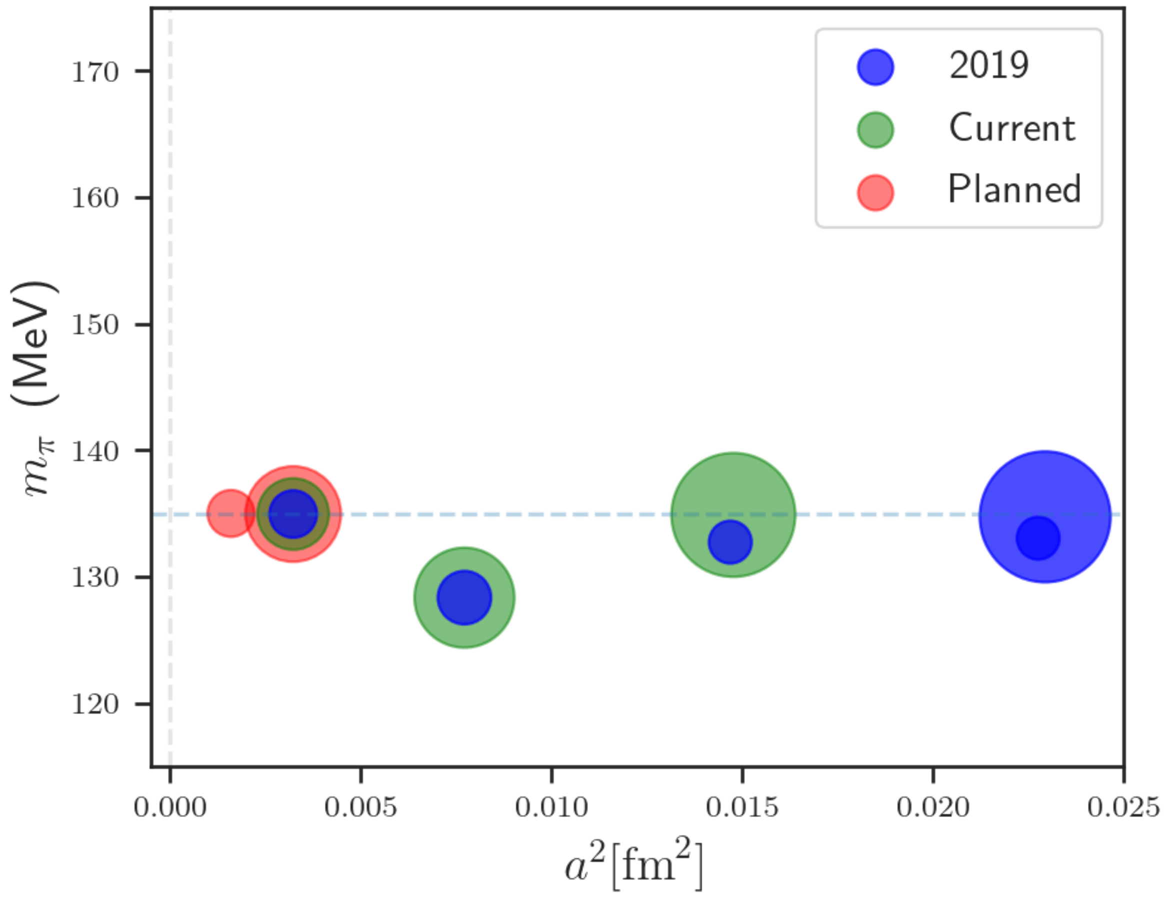
- Twisted-mass Wilson fermions, 2+1+1
- Plan to include phys. mass ensemble in future

RBC/UKQCD [Lehner]

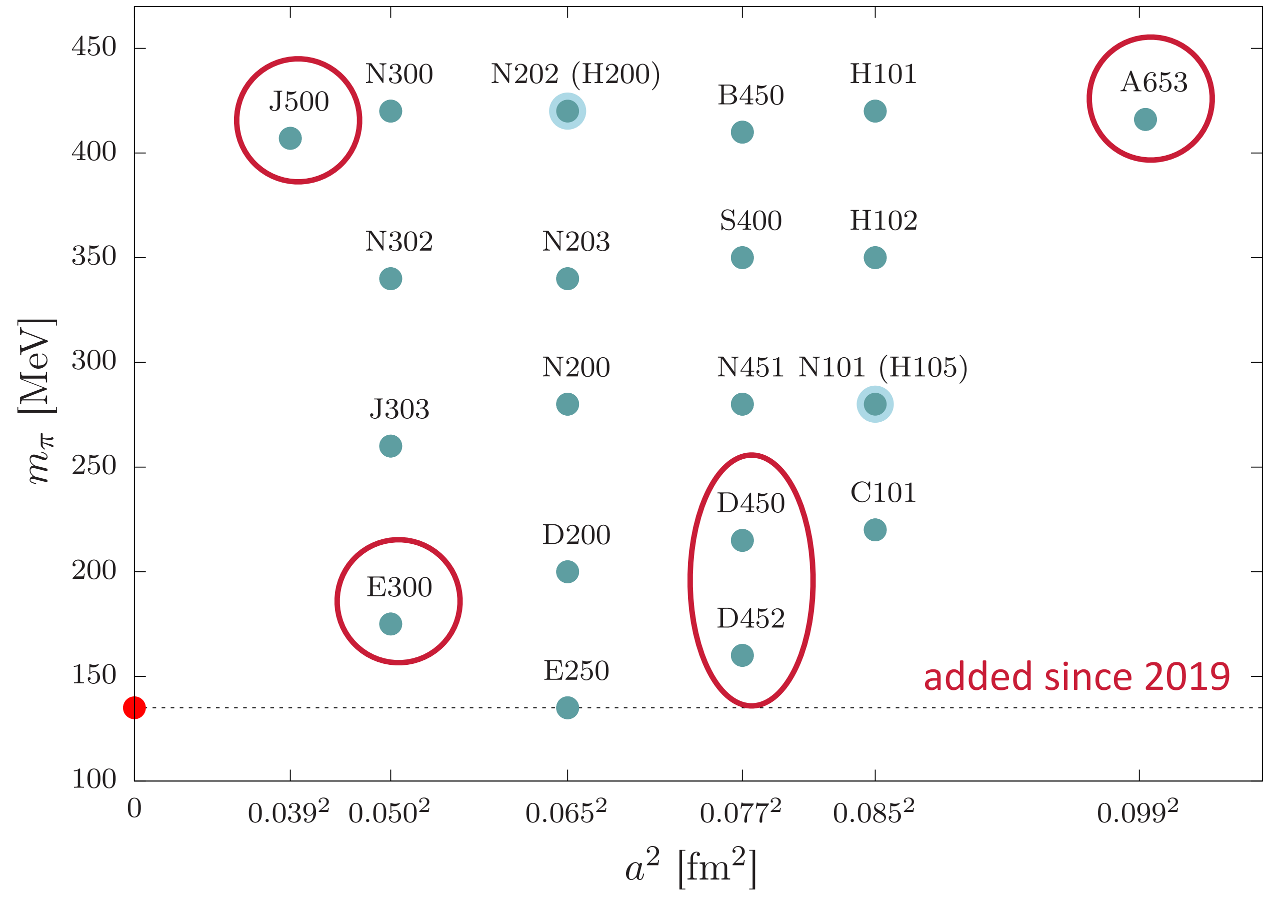


- Domain Wall fermions

Fermilab-HPQCD-MILC [Lahert, McNeile]

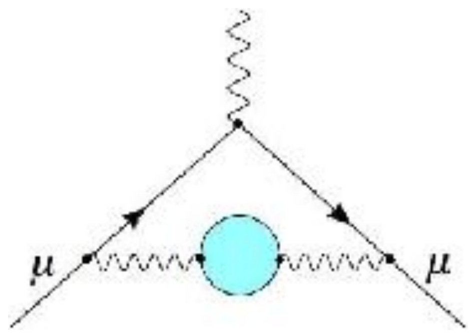


Mainz [Wittig, Risch, Miura, San Jose, Chao]



- Wilson-clover fermions, 2+1
- $L \sim 5 - 6\,\text{fm}$

- Highly Improved Staggered Quarks (HISQ) 2+1+1+1
- $L \sim 5 - 6\,\text{fm}$
- Subset also used by Aubin et al [Aubin]



Lattice HVP: Finite Volume

Monday, 13:00-15:00 US EDT
Finn Stokes (BMWc)

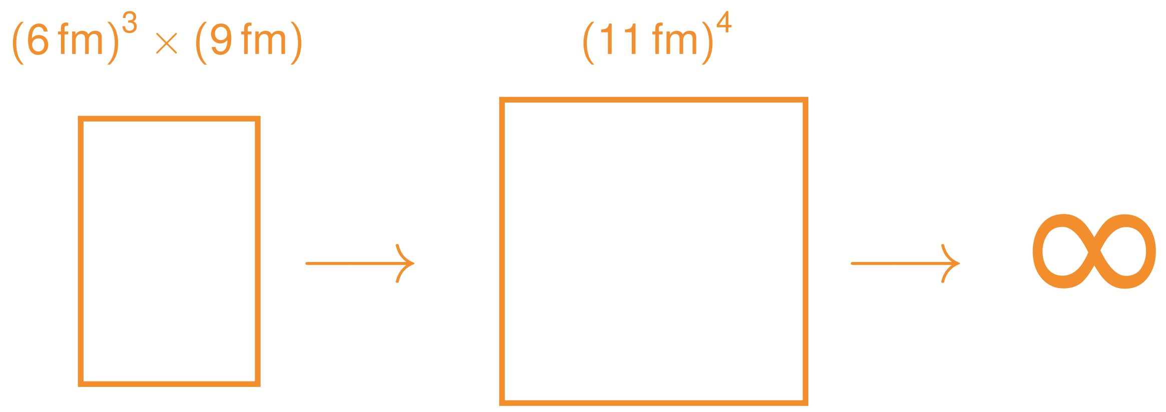
Tuesday, 5:00-8:00 US EDT
Hartmut Wittig (Mainz)

Model comparison

- Two more models for finite L (but not T)
  - Generic field-theory approach [Hansen & Patella '19, '20] (HP) relates the finite-size effect to $F_\pi(k)$
  - Rho-pion-gamma model [Chakraborty et al '17] (RHO) incorporates the $\rho(770)$ resonance directly into a χPT-like framework
- Compare finite L corrections for reference volume in infinite-T limit
- All four models agree within $\sim 2.5 \times 10^{-10}$

NNLO χPT	16.7	$\times 10^{-10}$
MLLGS	18.8	$\times 10^{-10}$
HP	17.7	$\times 10^{-10}$
RHO	16.2	$\times 10^{-10}$

Residual correction

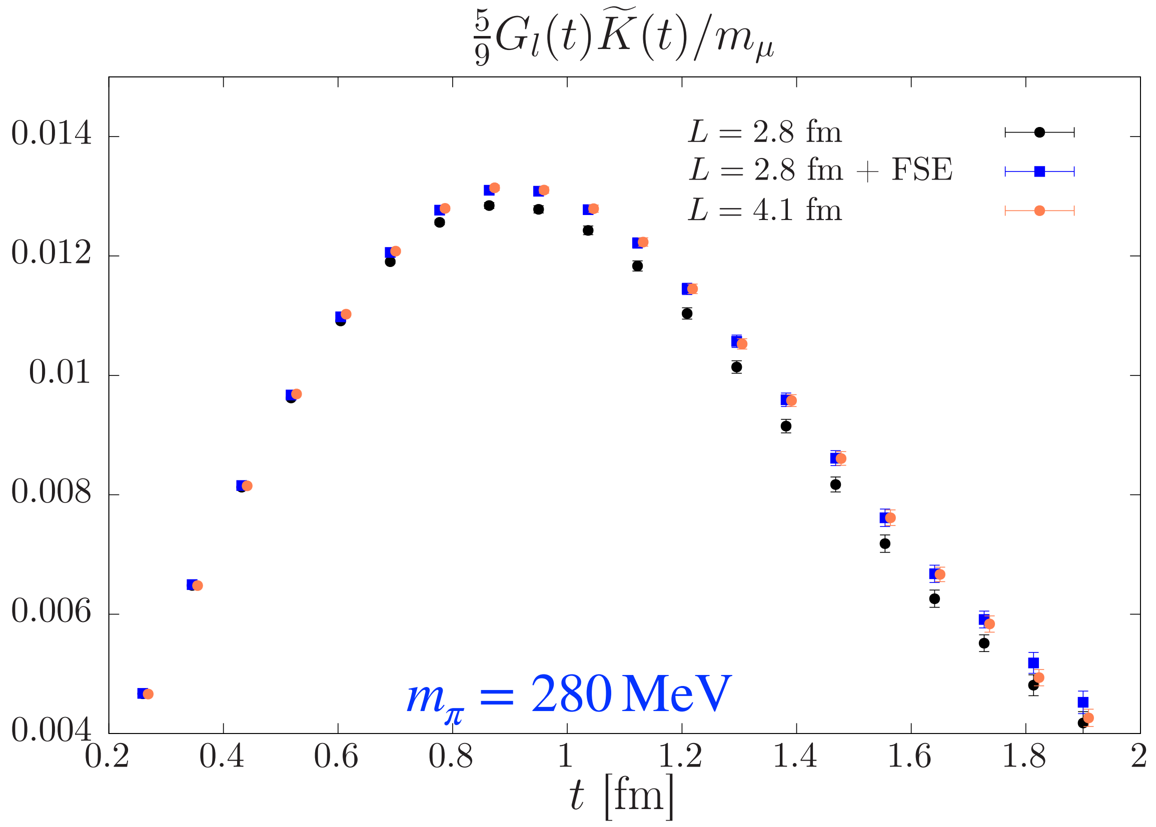


4HEX	18.1 ± 2.4		$\times 10^{-10}$
NNLO χPT	15.7	0.6 ± 0.3	$\times 10^{-10}$
MLLGS	17.8		$\times 10^{-10}$

Mainz method (aka MLL): $G(t,L) \overset{t\to\infty}{=} \sum_n |A_n|^2 \, e^{-\omega_n t}$ $\quad G(t,\infty) = \int_0^\infty d\omega\, \omega^2 \rho(\omega^2)\, e^{-\omega|t|}$

Both $|A_n|$ and $\rho(\omega^2)$ can be related to the pion form factor $F_\pi(\omega)$ $\Rightarrow$ $G(t,\infty) - G(t,L)$

[Meyer 2011, Francis et al. 2013, Della Morte et al. 2017; Lellouch & Lüscher 2001]



Full HVP contribution:

Explicit verification at $m_\pi = 280\,\mathrm{MeV}$:
→ method works remarkably well

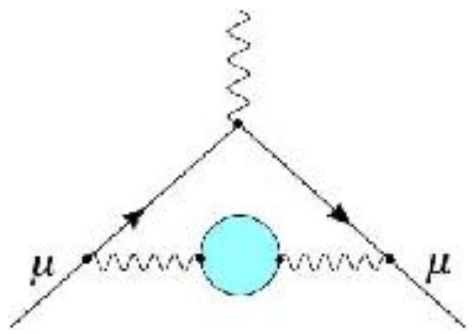
$m_\pi L = 4 \quad (L = 6.2\,\mathrm{fm})$

$\Rightarrow \quad \Delta_{\mathrm{FV}} a_\mu^{\mathrm{hvp}} = 22.6 \cdot 10^{-10} \quad (3\%)$

In agreement with direct lattice calculations: PACS, BMWc

[Shintani & Kuramashi 2019, BMWc 2020]

- Other direct calculations by RBC/UKQCD [Lehner @ 2019 INT workshop, Shintani & Kuramashi, PRD 2019] are consistent (but with larger errors).

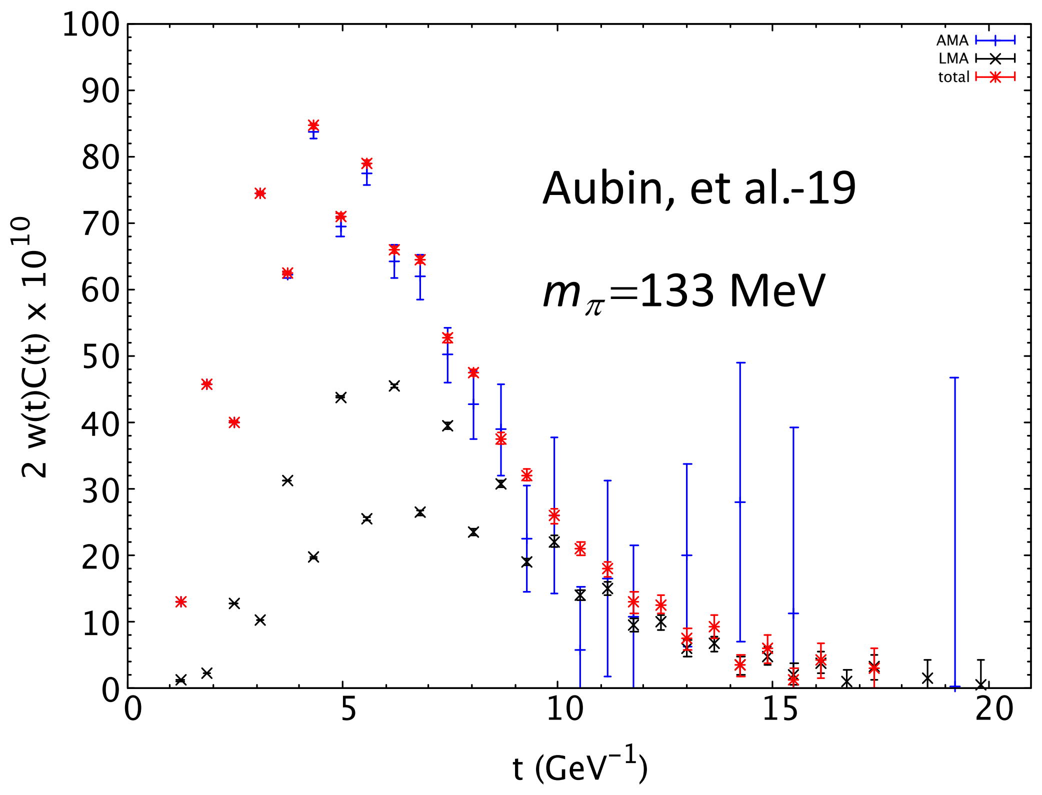


Lattice HVP: long-distance tail

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

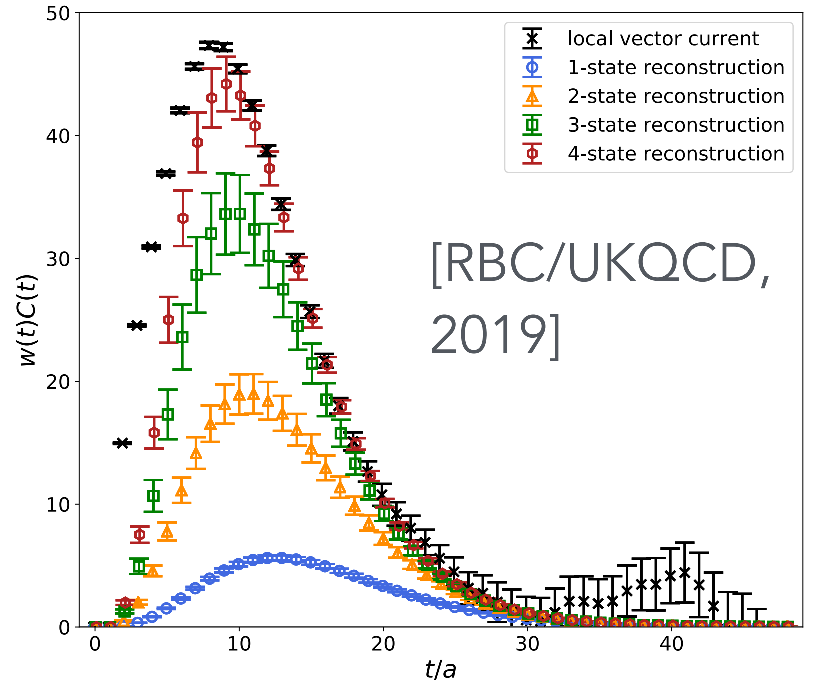
- Use noise reduction methods (AMA, LMA,…):

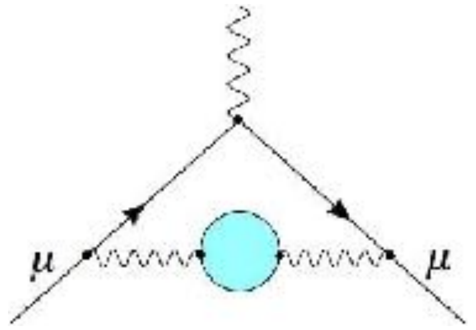
  Aubin et al, RBC/UKQCD, BMWc, Mainz, …



- Spectral reconstruction (RBC/UKQCD, Mainz):
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$



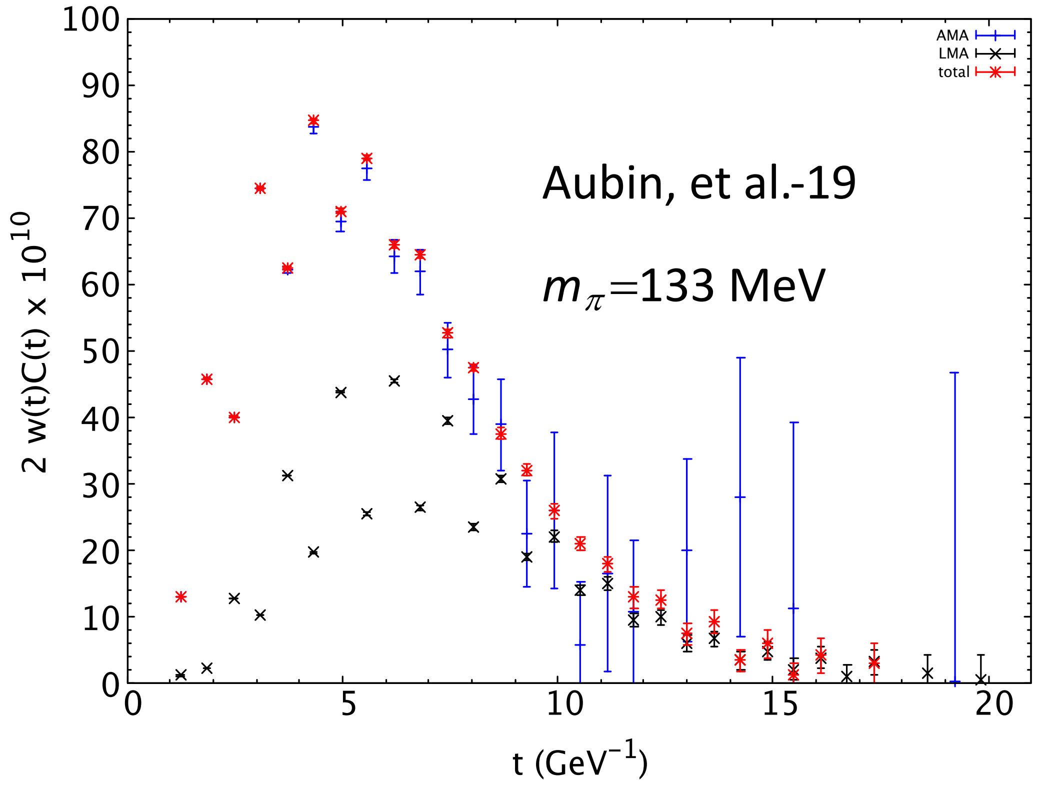


Lattice HVP: long-distance tail

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

- Use noise reduction methods (AMA, LMA,…):

  Aubin et al, RBC/UKQCD, BMWc, Mainz, …



- Spectral reconstruction (RBC/UKQCD, Mainz):
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
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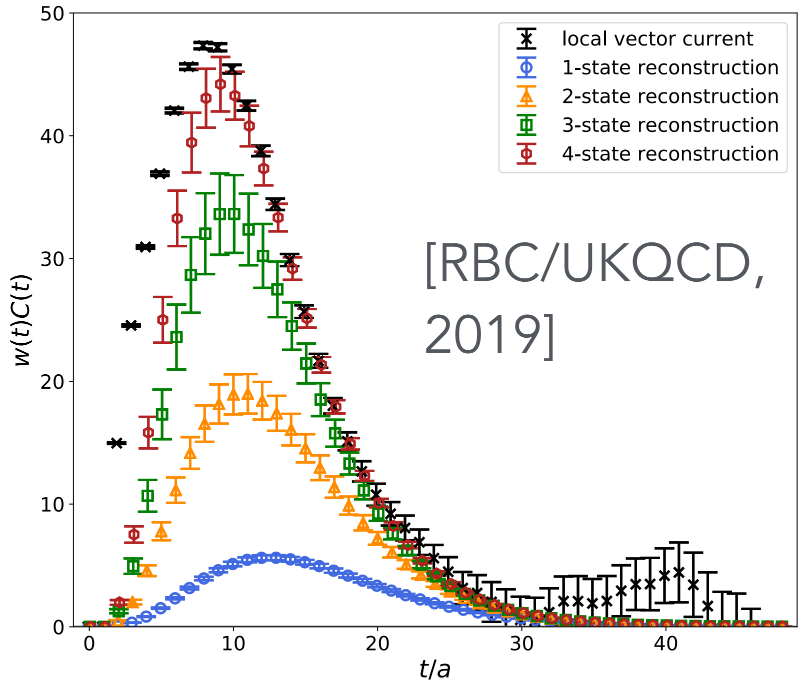
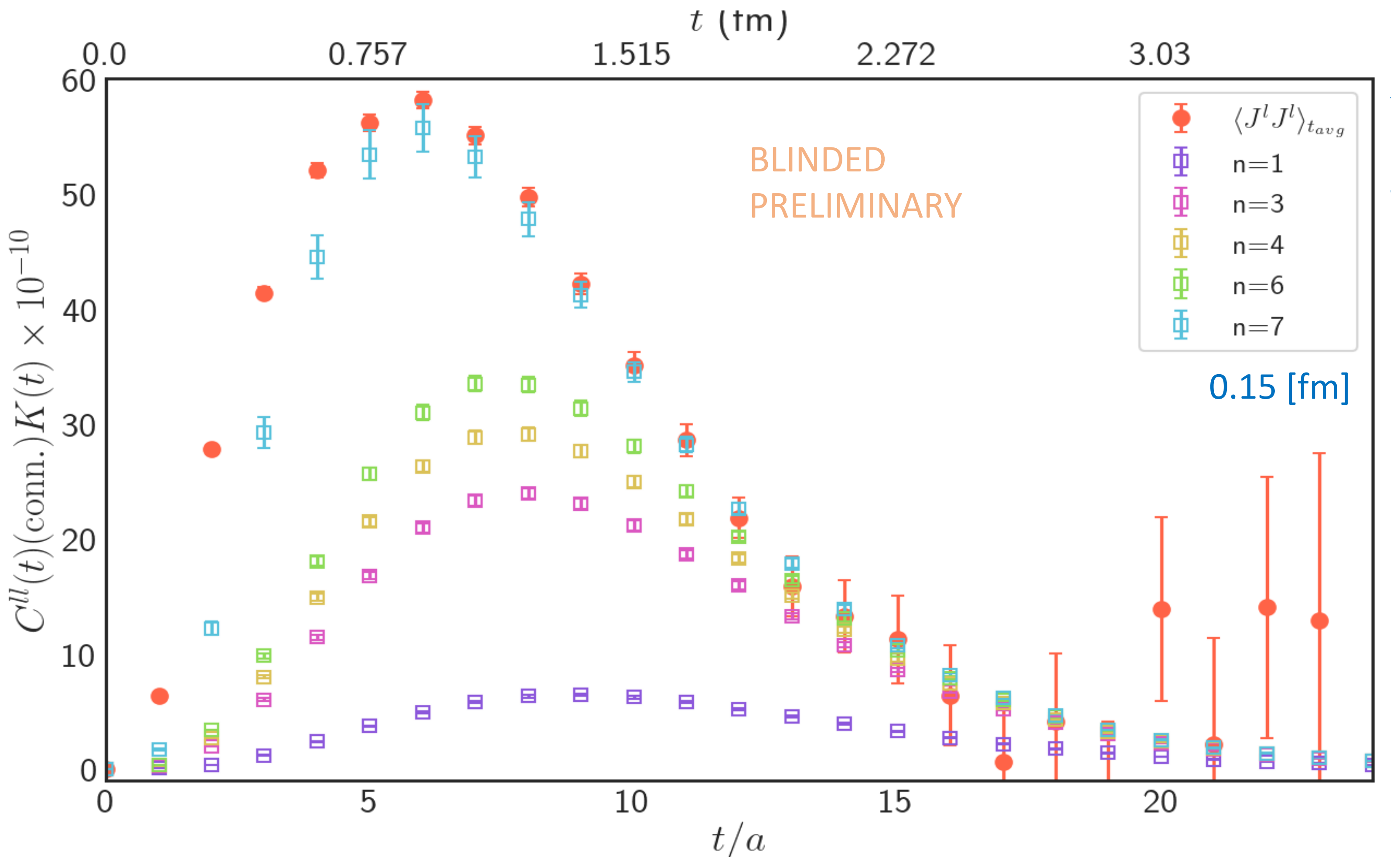
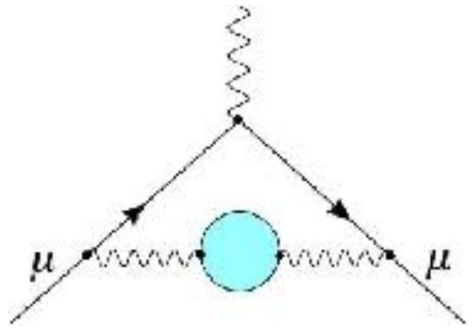


Monday, 13:00-15:00 US EDT
Shaun Lahert
(Fermilab-HPQCD-MILC)

- First calculation with staggered multi-pion operators





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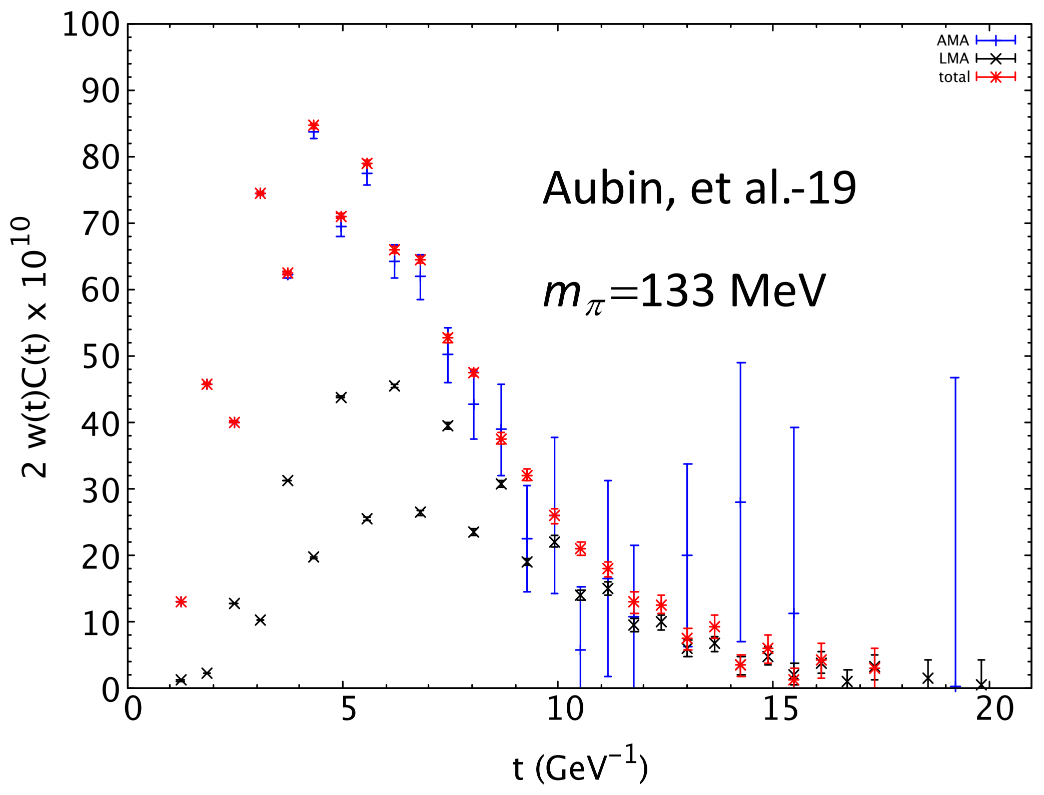

> Tuesday, 5:00-8:00 US EDT
> - Leonardo Giusti (Milan) Multi-level integration
>
> Tuesday, 13:00-15:00 US EDT
> - Tej Kanwar (MIT) contour deformation

- Spectral reconstruction (RBC/UKQCD, Mainz):
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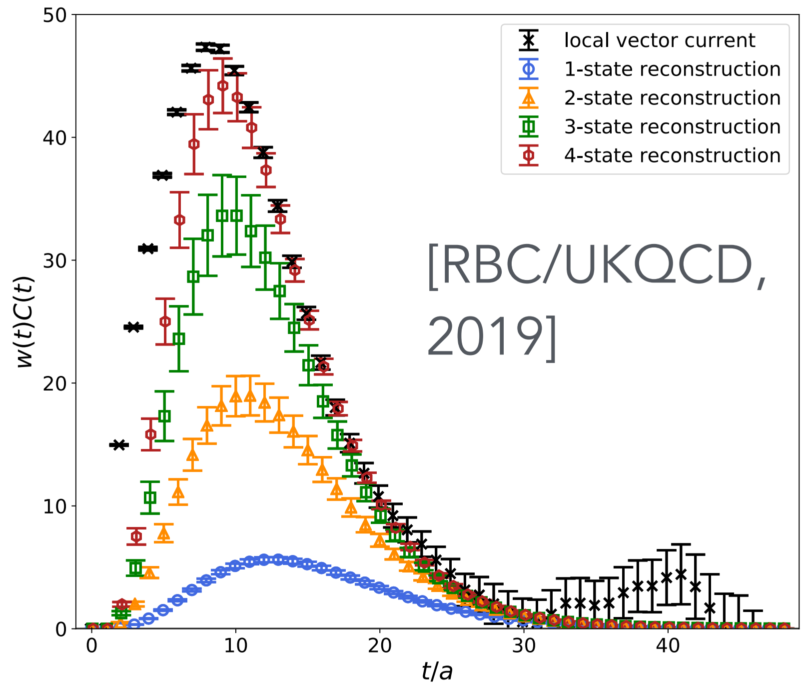

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> - Shaun Lahert (Fermilab-HPQCD-MILC)

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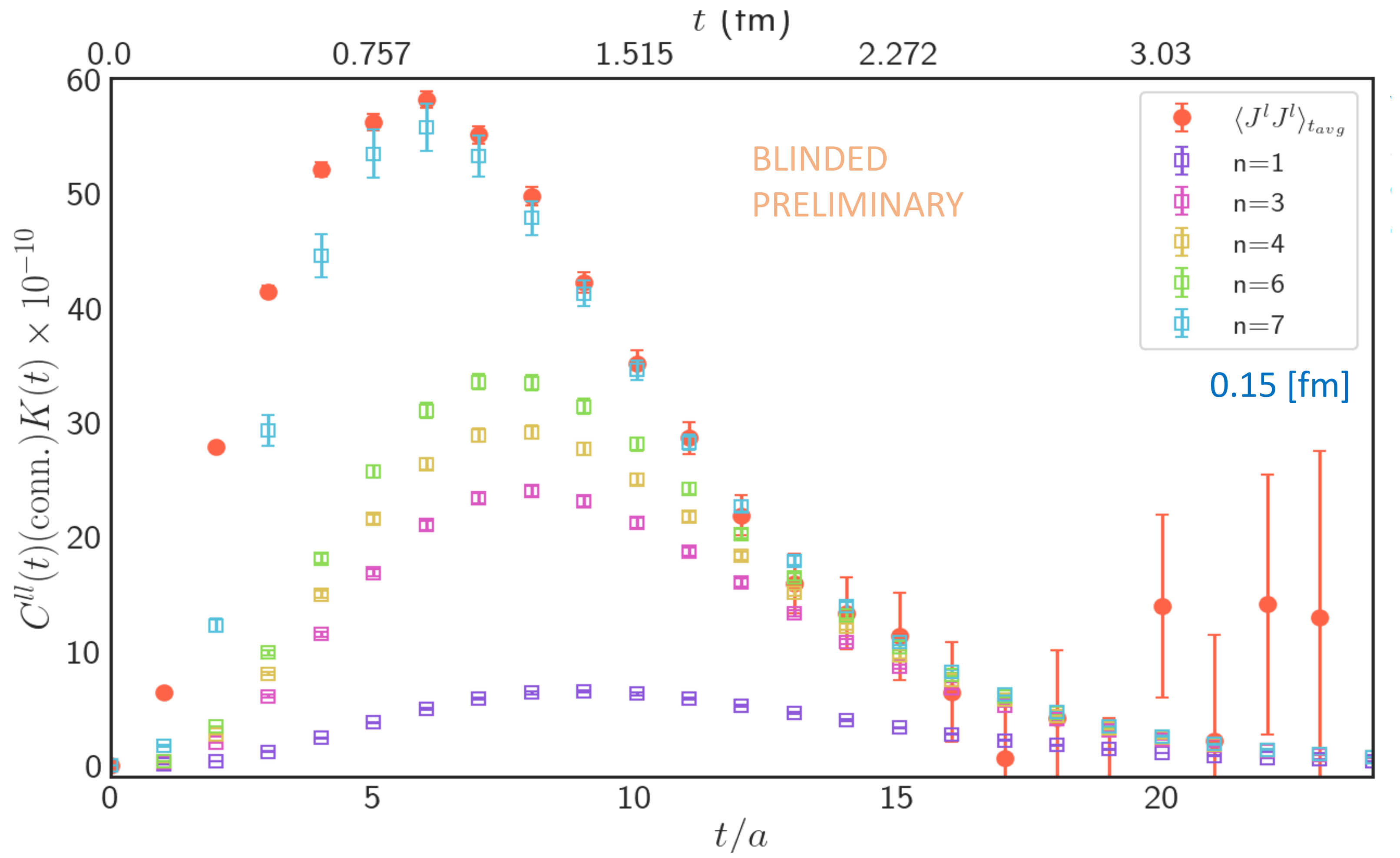
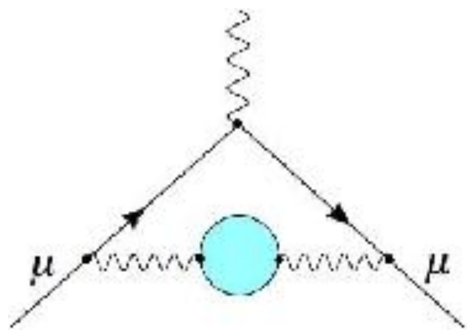


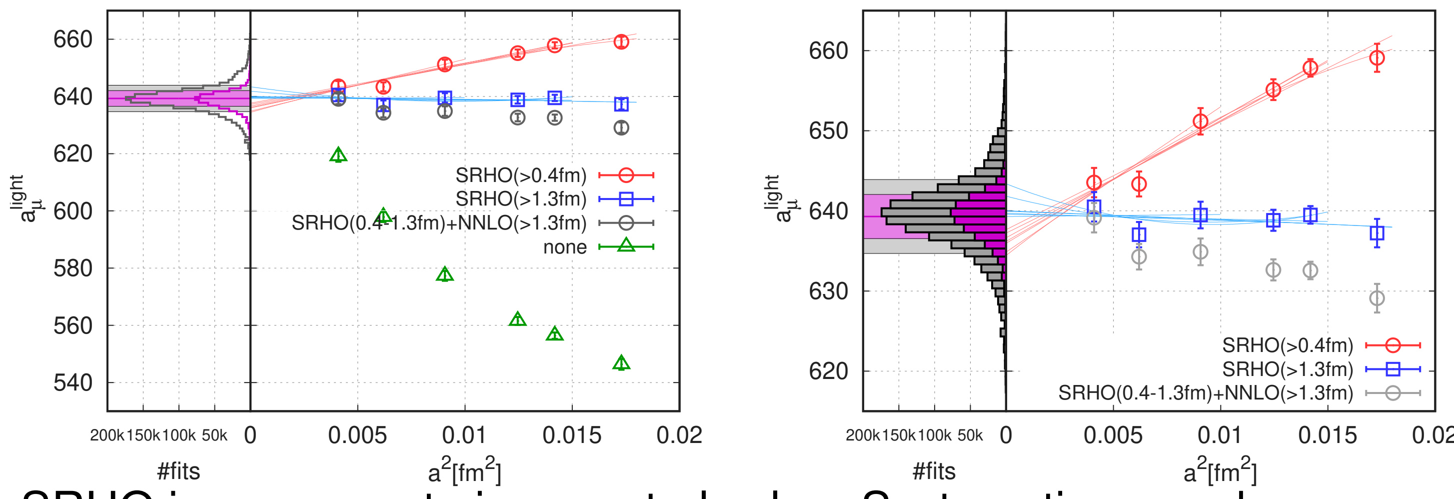


Lattice HVP: continuum extrapolation

Monday, 13:00-15:00 US EDT
Kalman Szabo (BMWc)

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\rm SRHO}(a) + a_\mu^{\rm RHO}$
- reduces lattice artefact, also makes a^2 dependence linear



SRHO improvement gives central value. Systematic errors by:

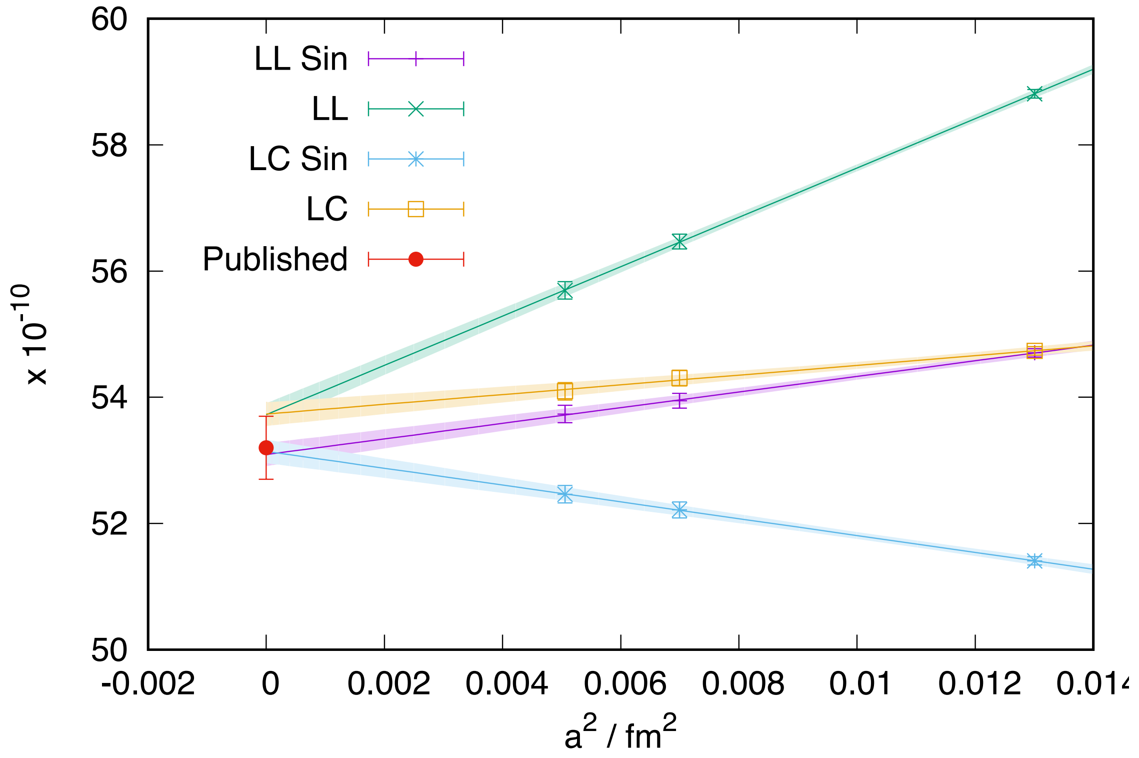
1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

- Large taste-breaking effects with BMW set-up ➠ uncorrected data not easily fit to power series, i.e.

1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\frac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\frac{1}{a})\right]^2$

Tuesday, 5:00-8:00 US EDT
Christoph Lehner (RBC/UKQCD)

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

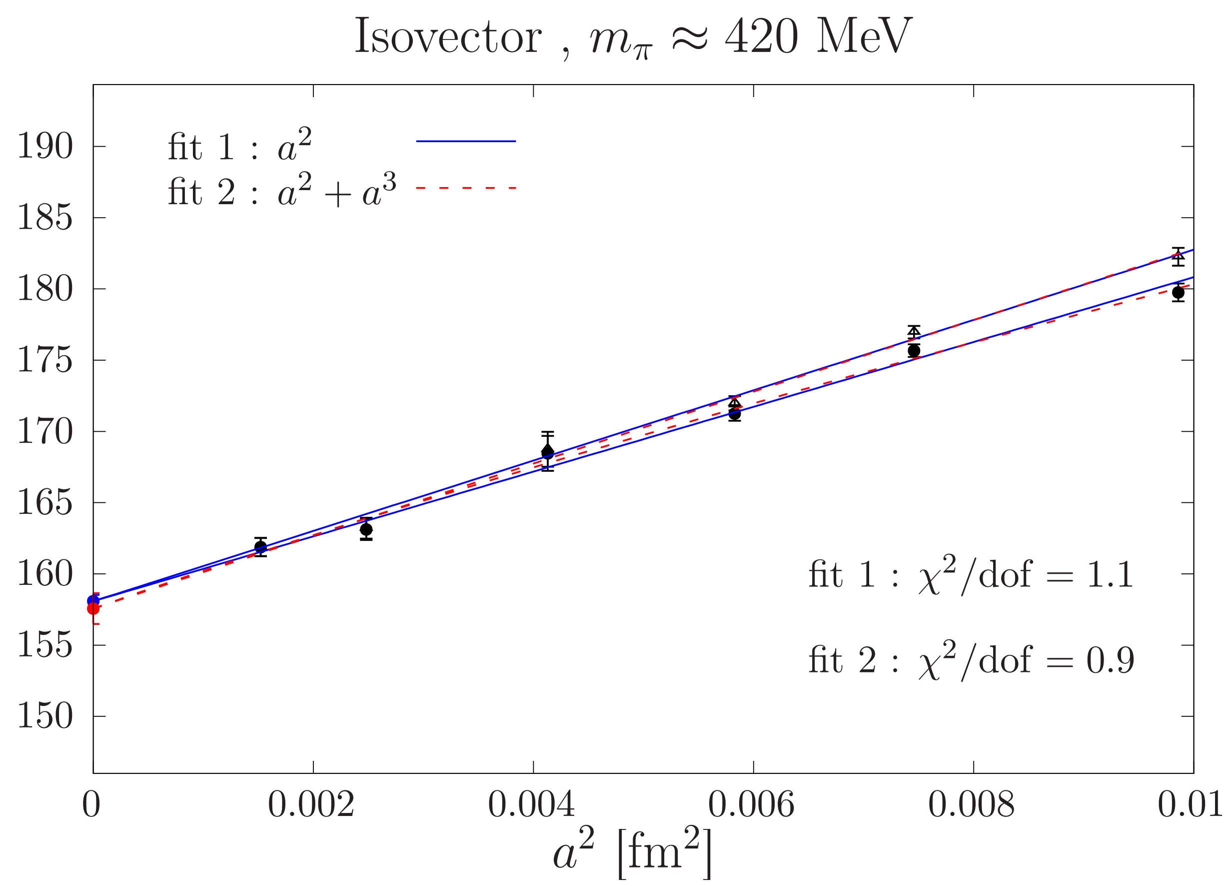


- For light quark use new 96I ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

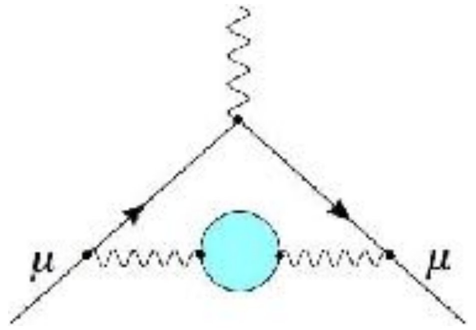
- RBC:
  Currently adding add a third lattice spacing

- Fermilab-HPQCD-MILC:
  planning to add a 5th lattice spacing (0.042 fm).

Tuesday, 5:00-8:00 US EDT
Hartmut Wittig (Mainz)



- Mainz and ETMc perform combined chair and continuum extrapolaiton



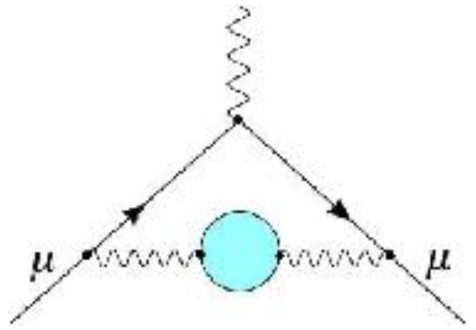
Lattice HVP: Scale Setting

- a_μ is dimensionless, but depends on the lattice indirectly, through masses in lattice units in the Kernel. In particular, am_μ:

$$\frac{\delta a_\mu^{\rm hvp}}{a_\mu^{\rm hvp}} = \underbrace{\frac{1}{a_\mu^{\rm hvp}} \left| a \frac{d a_\mu^{\rm hvp}}{da} \right|}_{\approx 1.8} \frac{\delta a}{a}$$

[H. Wittig @ 1st Muon g-2 Theory Initiative workshop; Della Morte et al, Lattice 2017]

- need a good physical quantity to determine lattice spacing to high precision (< 0.2%). Currently in use:
  - f_π — depends on V_{ud} and requires radiative QED corrections
  - Ω baryon mass (RBC/UKQCD, BMW) also being adopted by Mainz, Fermilab-HPQCD-MILC



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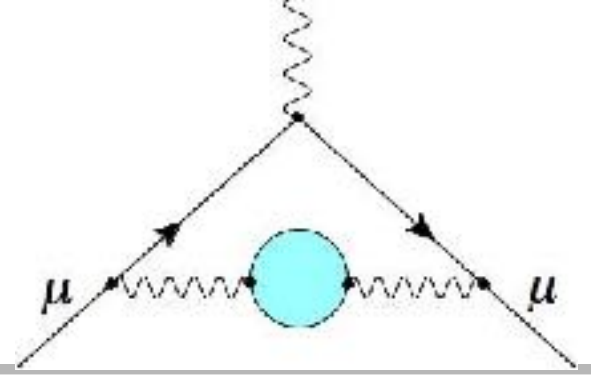
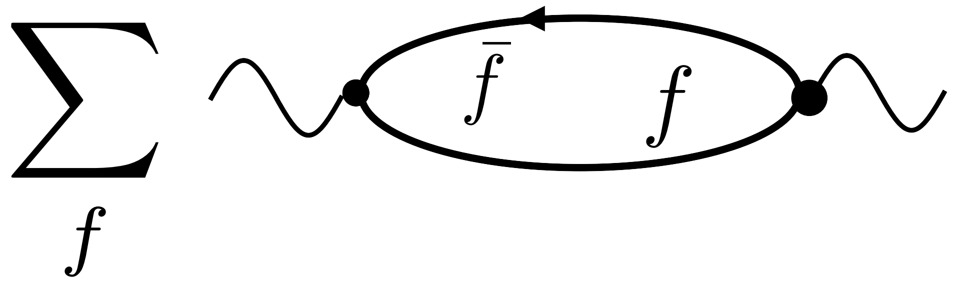
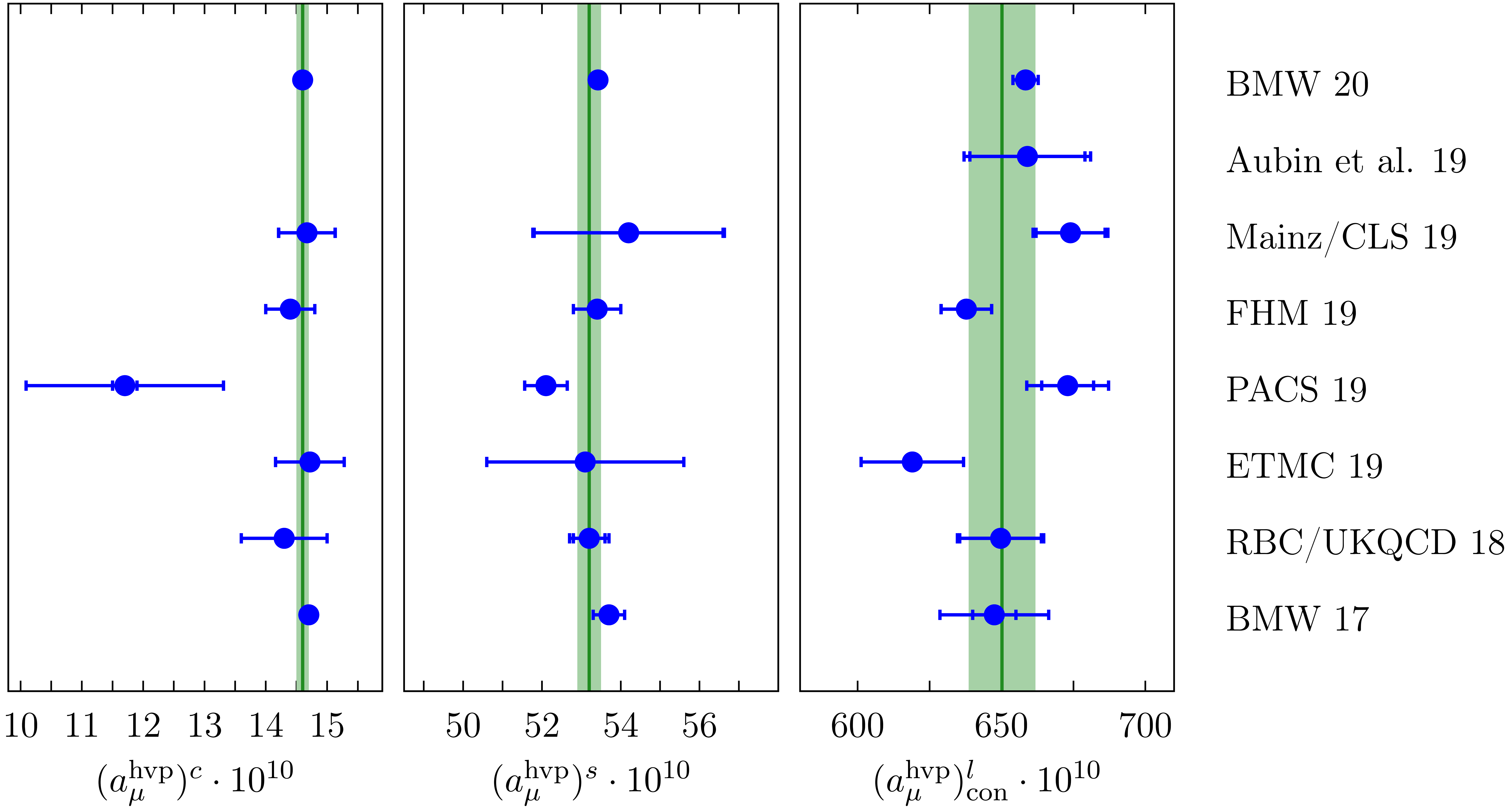
- Monday, 13:00-15:00 US EDT
  - Lukas Varnhorst (BMWc)
- Thursday 5:00-8:00 US EDT Friday, 5:00–8:00 US EDT
  - Alexander Segner (Mainz) Ben Strassberger (Mainz)

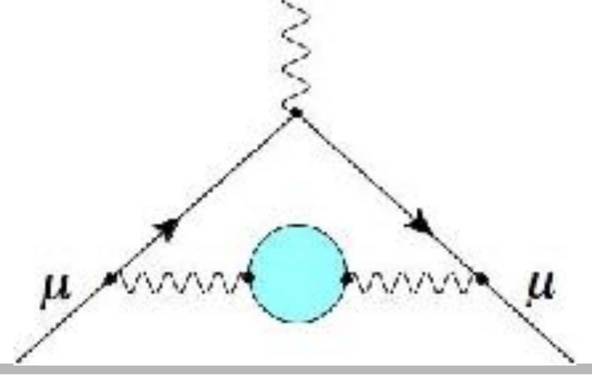


Lattice HVP: c, s, ℓ connected

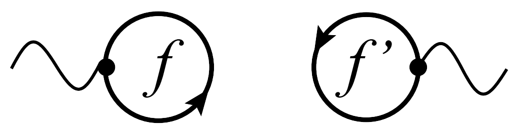
H. Wittig @ Lattice HVP workshop



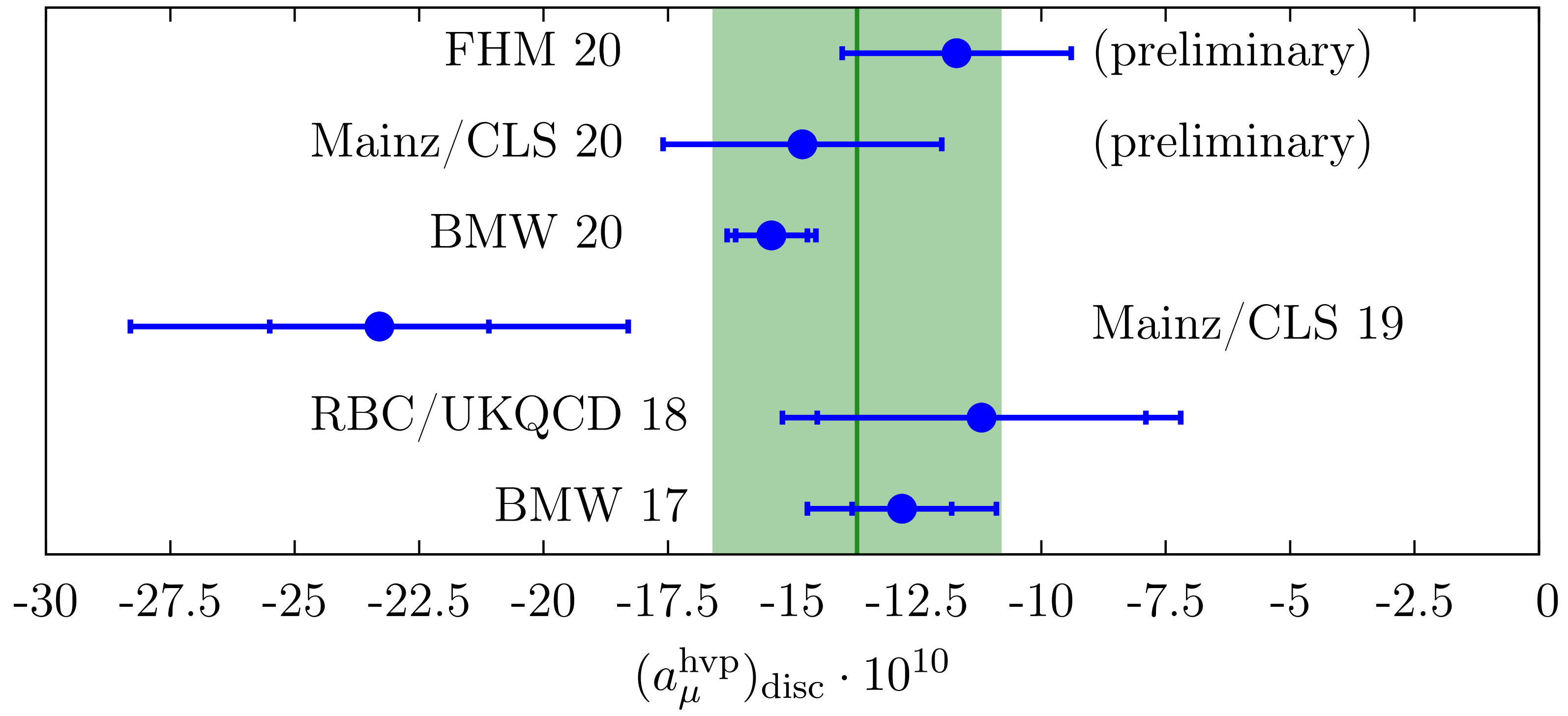




Lattice HVP: disconnected corrections

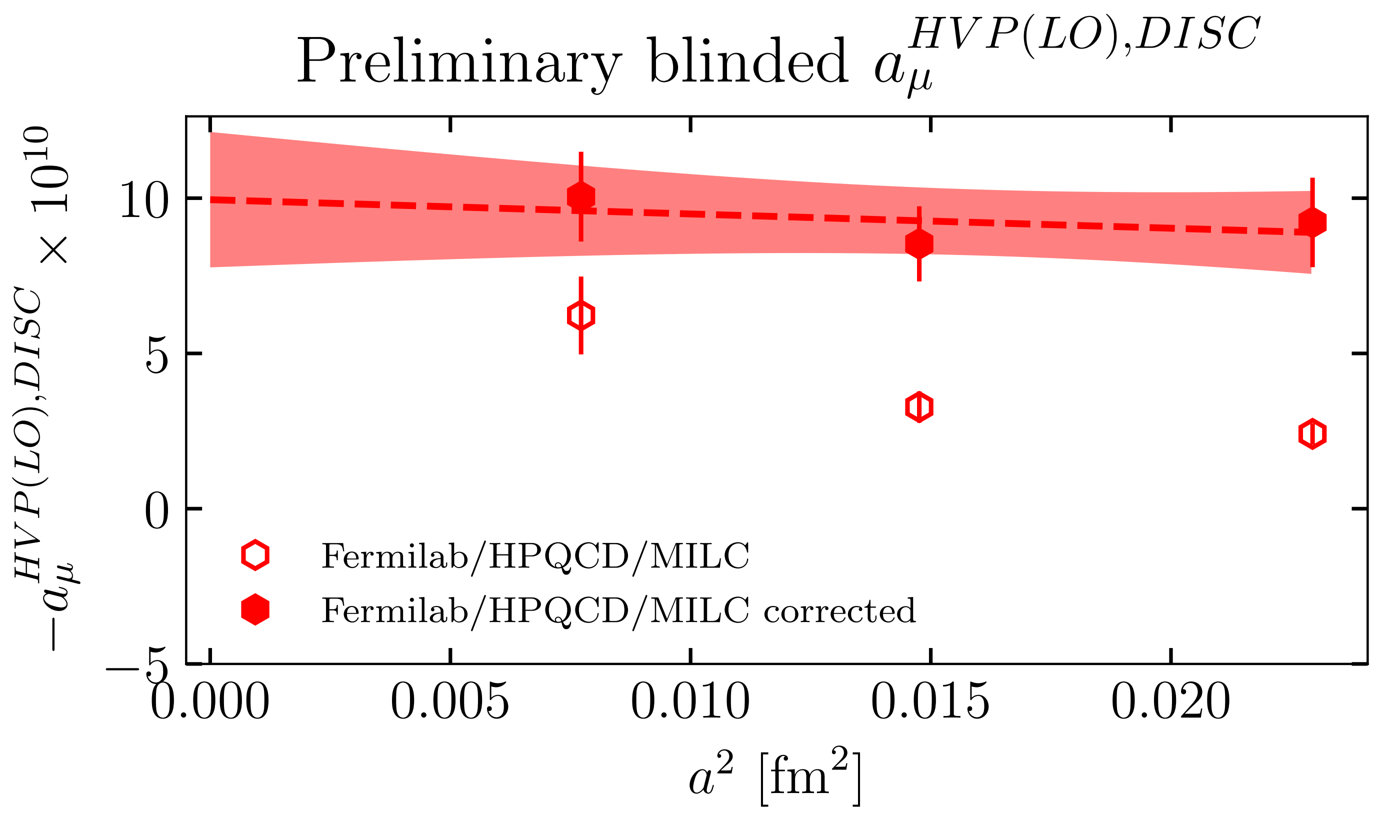


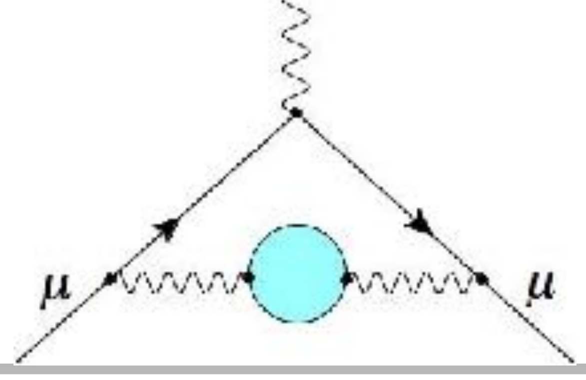
H. Wittig @ Lattice HVP workshop



Convergence towards precise & consistent results

- Poster, Wednesday, 8:00-9:00 US EDT
  - C. McNeile (Fermilab-HPQCD-MILC)

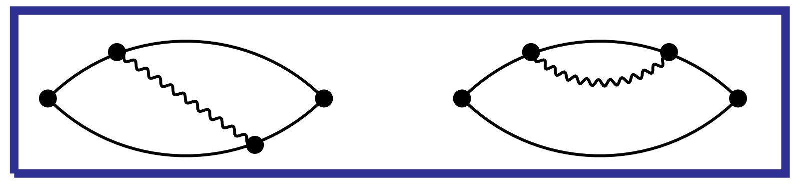




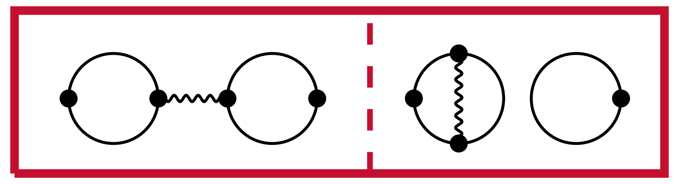
Lattice HVP: Isospin corrections

V. Gülpers @ Lattice HVP workshop

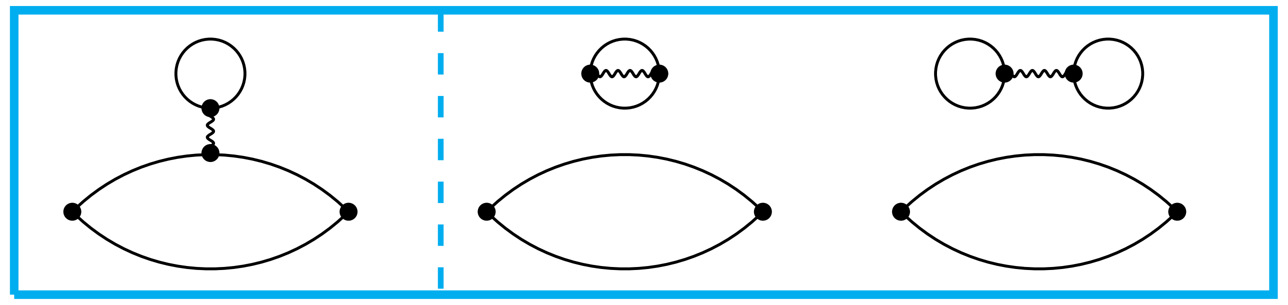
Overview of published results - contributions to $a_\mu \times 10^{10}$



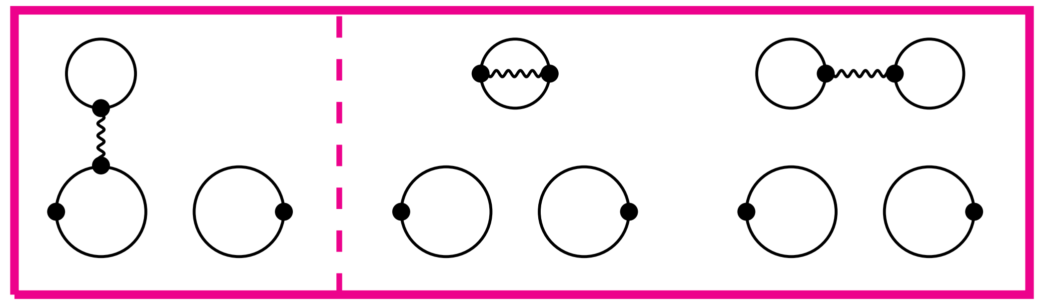
BMW	$-1.27(40)(33)$
RBC/UKQCD	$5.9(5.7)(1.7)$
ETM	$1.1(1.0)$



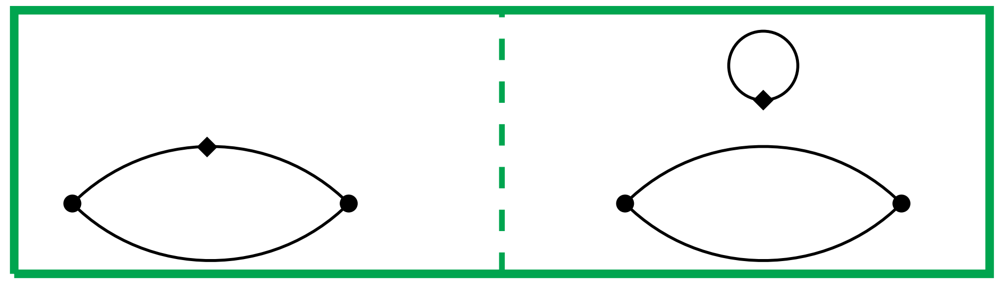
$-0.55(15)(11)$	BMW
$-6.9(2.1)(2.0)$	RBC/UKQCD



$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



$6.59(63)(53)$		BMW
$10.6(4.3)(6.8)$		RBC/UKQCD
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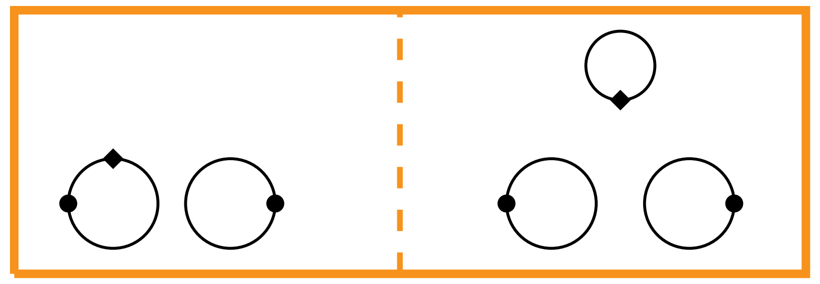
$7.7(3.7)$	$9.0(2.3)$	FHM
$9.0(0.8)(1.2)$		LM



$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
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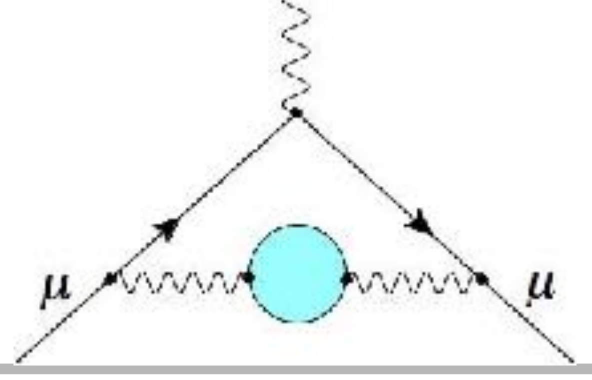
LM [Phys.Rev.D 101 (2020) 074515]

- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$
- Ongoing efforts presented by Mainz and Fermilab-HPQCD-MILC.



Lattice HVP: Isospin corrections

V. Gülpers @ Lattice HVP workshop

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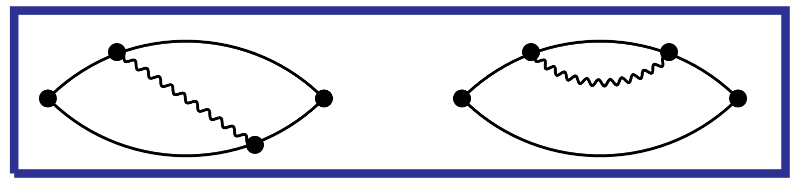

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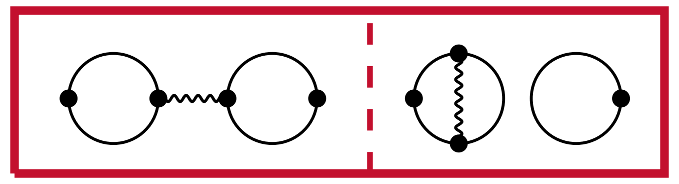

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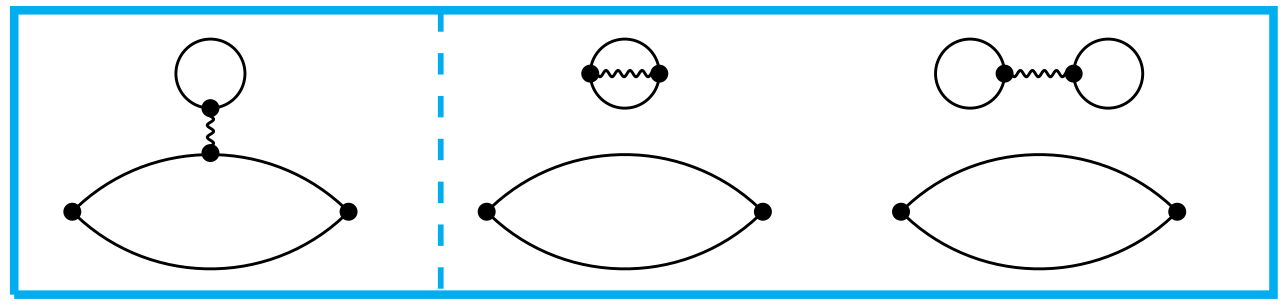

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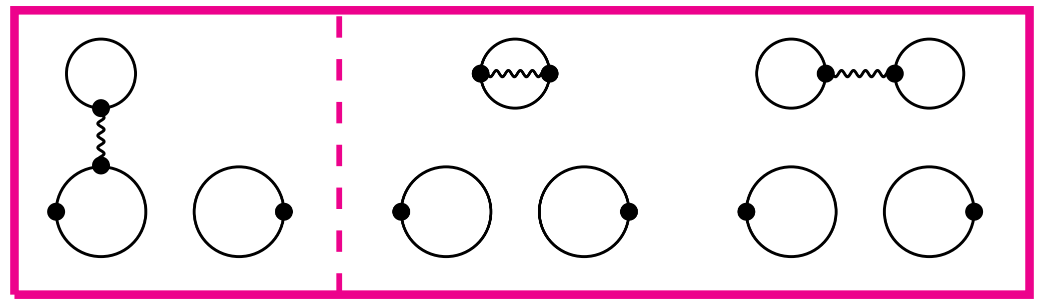

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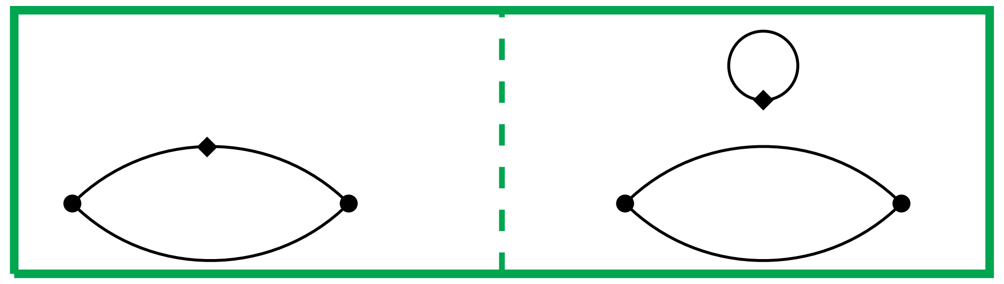

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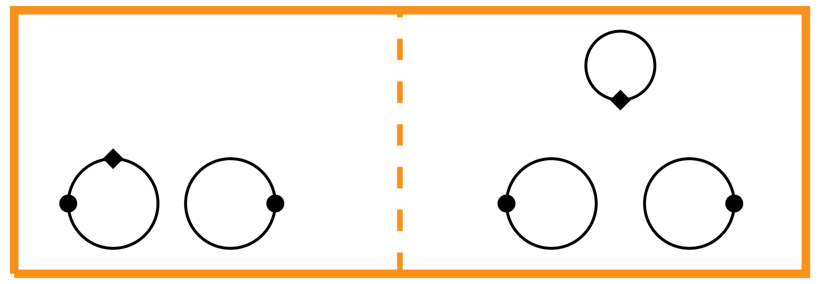

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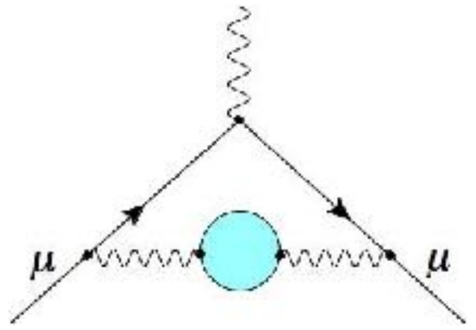


HVP: Comparison

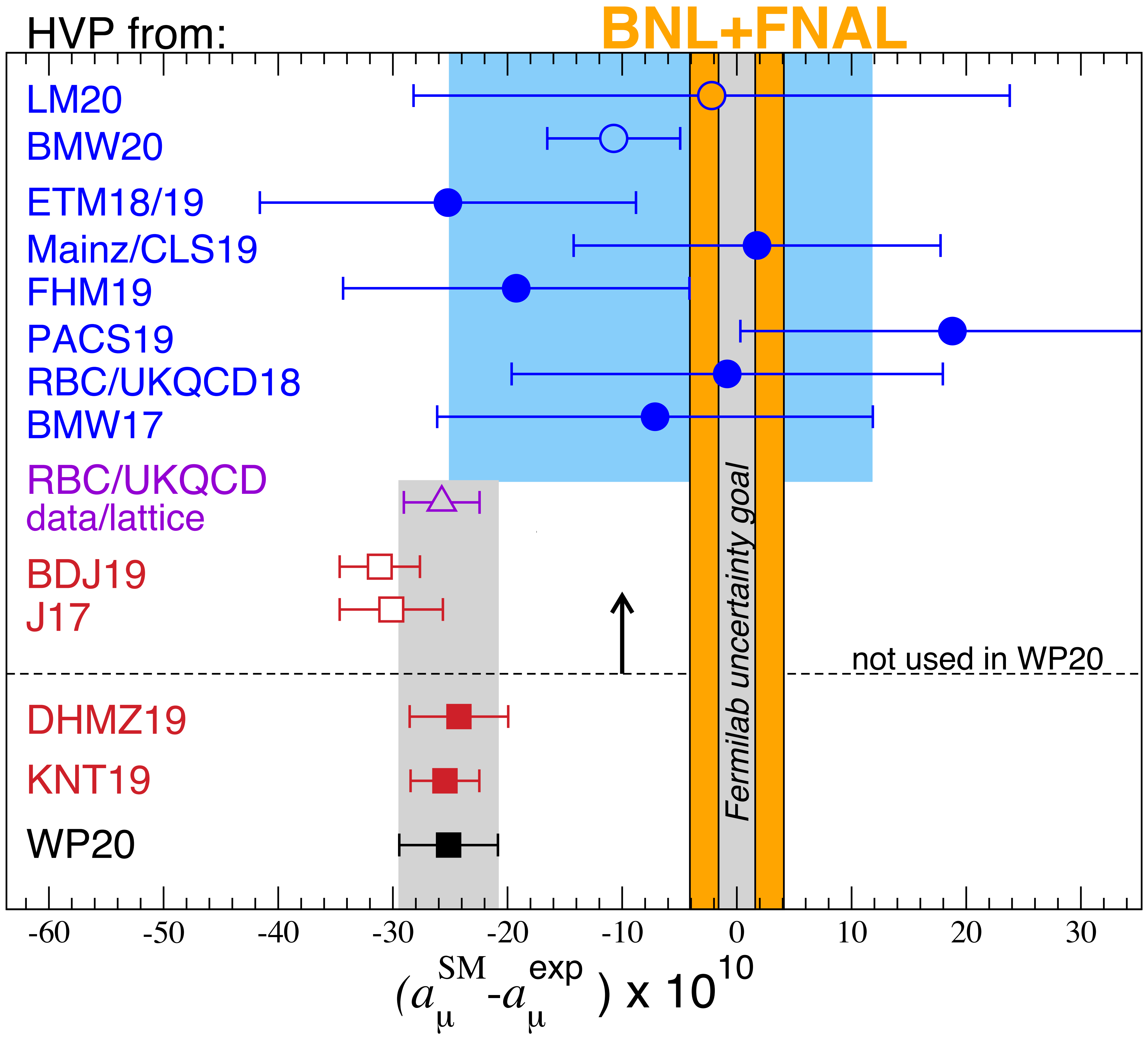
$$a_\mu^{\rm SM} \;\Big\langle\; a_\mu^{\rm HVP} + [a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}]$$

Lattice QCD + QED

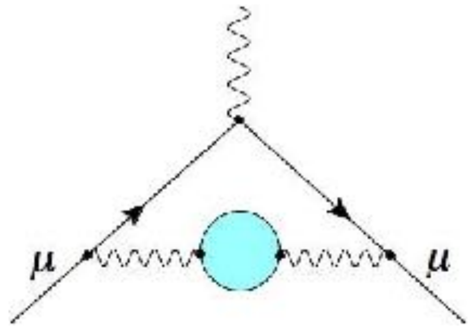
hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints



Where do we go from here?



Lattice HVP: windows in Euclidean time

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately.

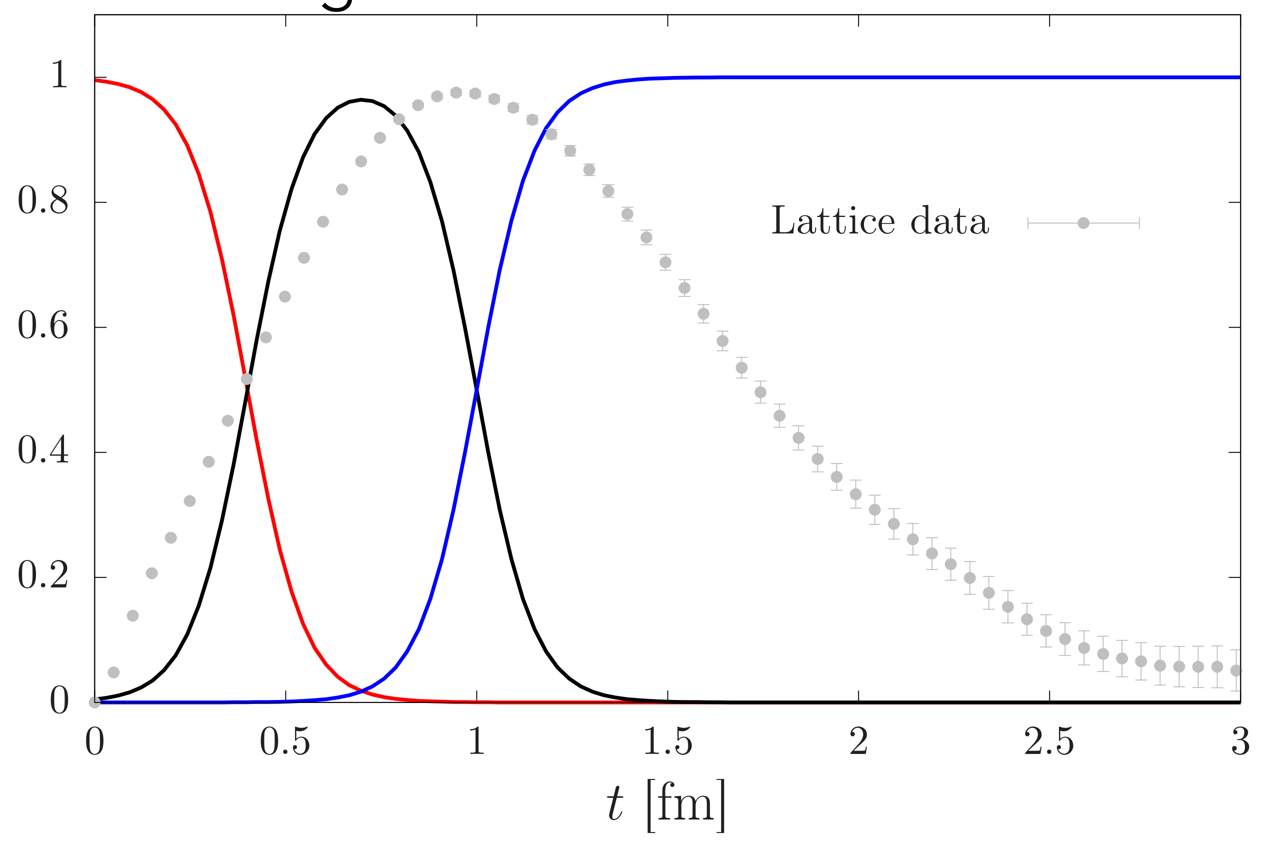
Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$

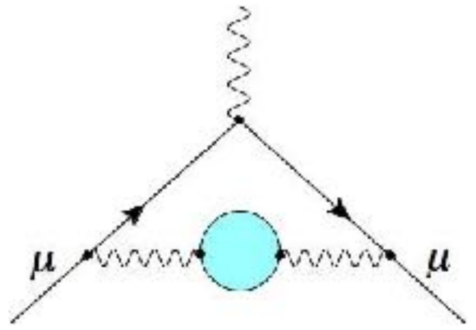
$$t_0 = 0.4\,\mathrm{fm},\, t_1 = 1.0\,\mathrm{fm}$$

H. Wittig @ Lattice 2021



- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$



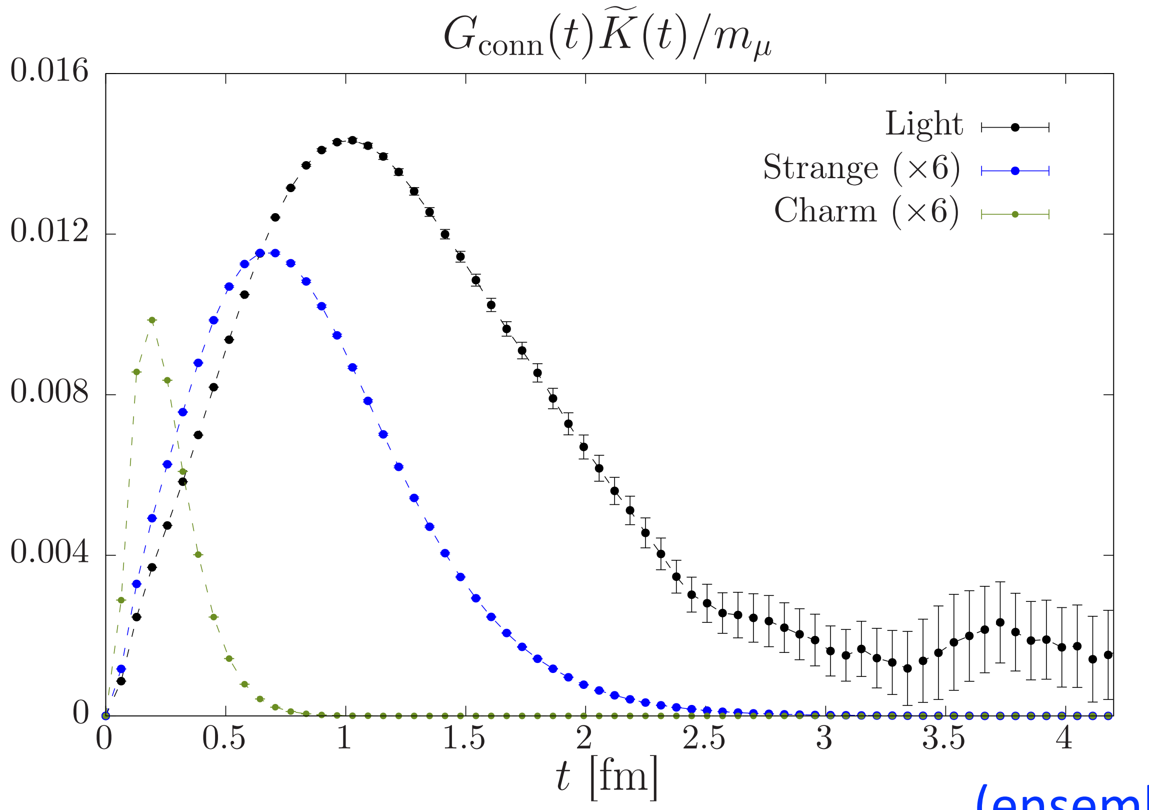
Lattice HVP: windows in Euclidean time

Intermediate window

Tuesday, 5:00-8:00 US EDT

Hartmut Wittig (Mainz)

Long-distance tail of the integrand is suppressed:



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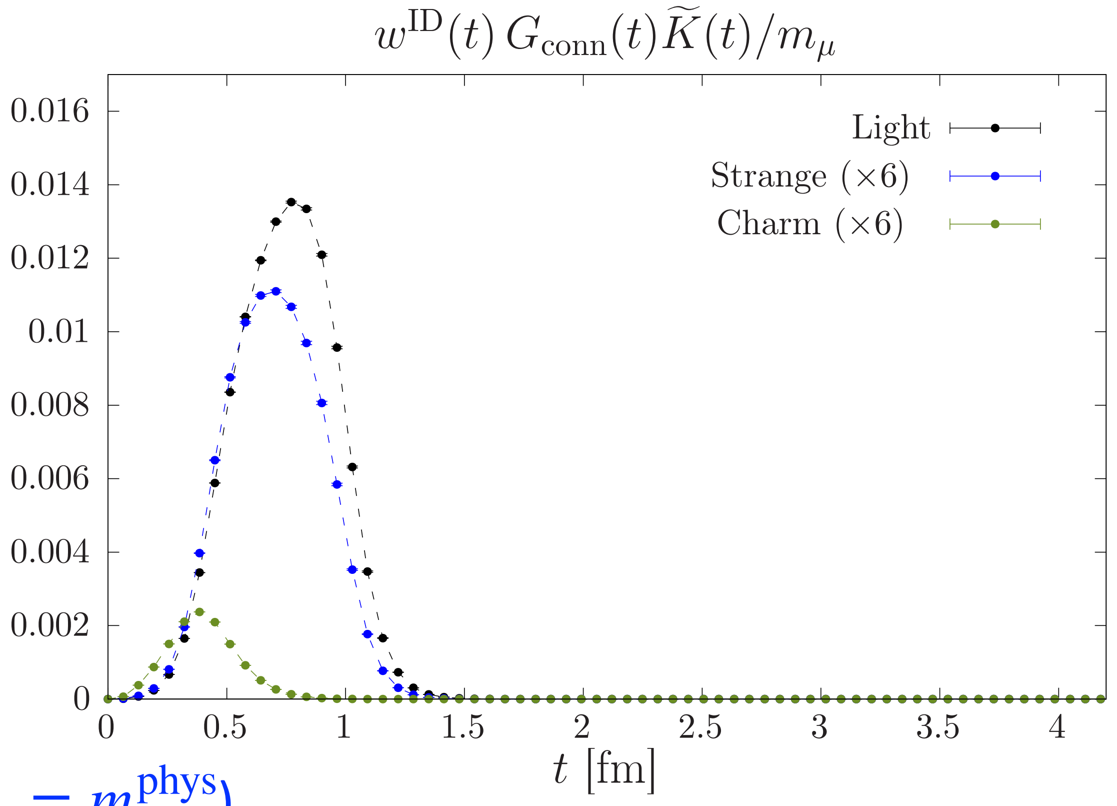
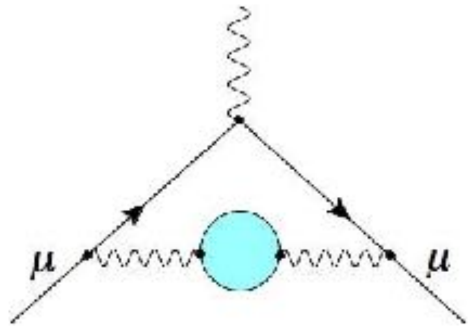


(ensemble E250: $m_\pi = m_\pi^{\rm phys}$)



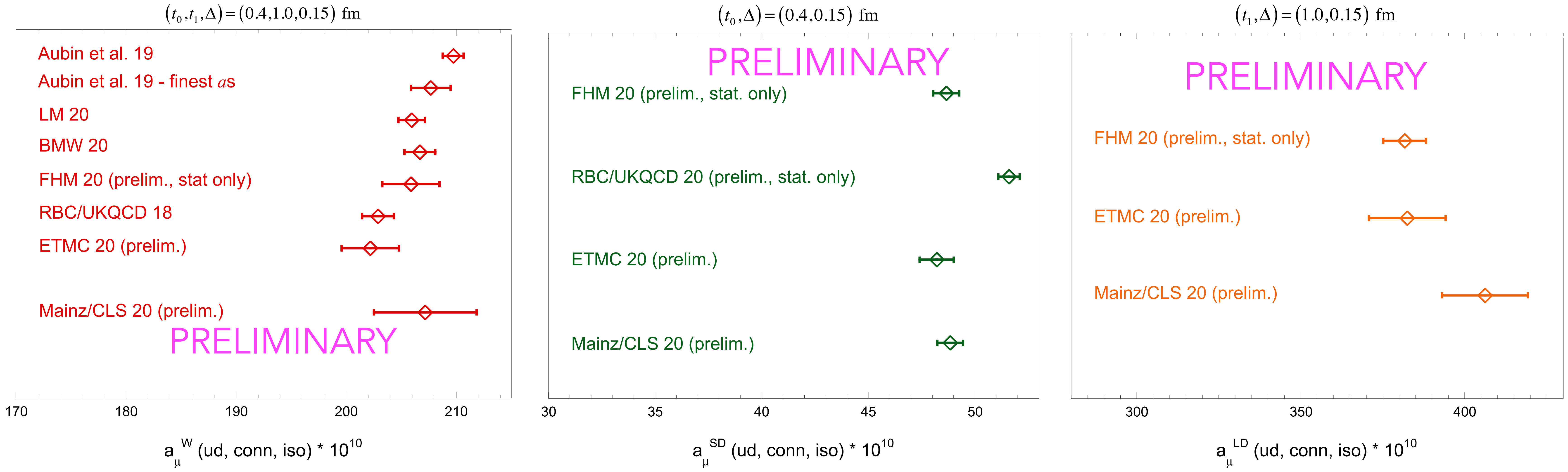
Lattice HVP: windows in Euclidean time

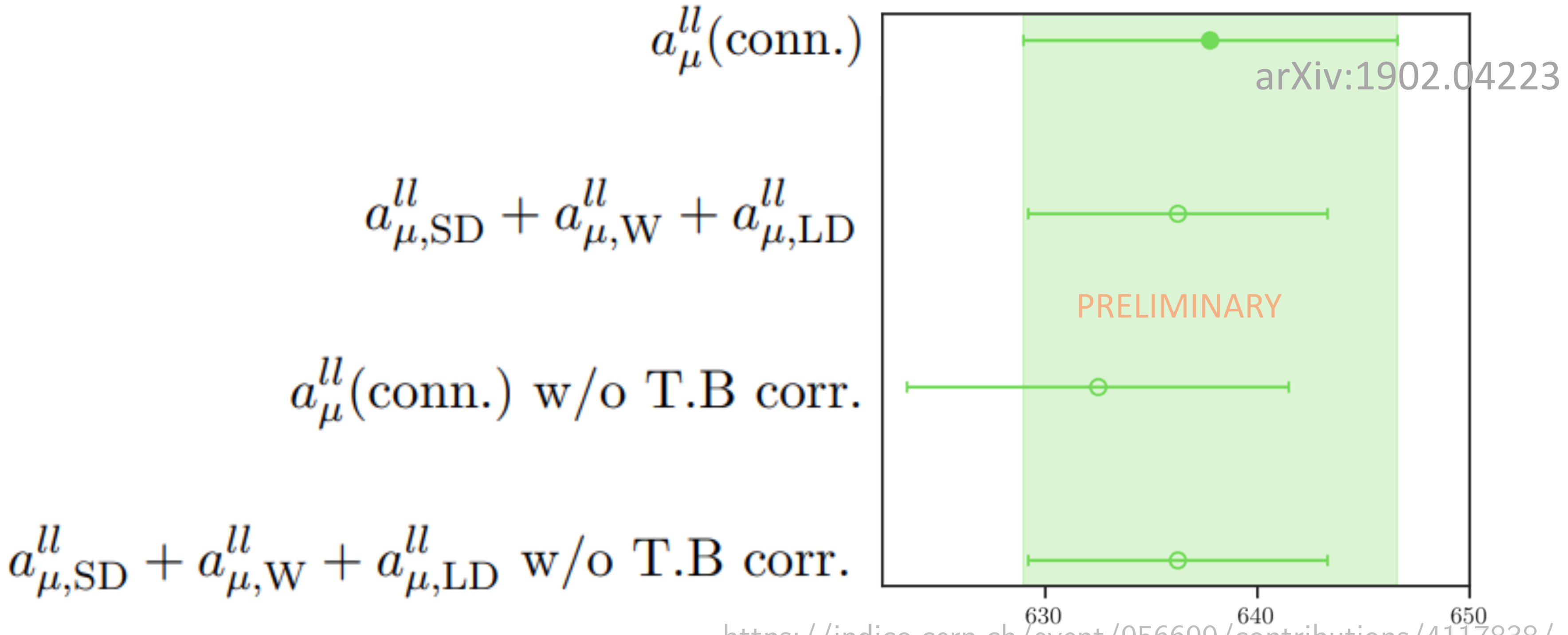
H. Wittig @ Lattice HVP workshop

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

“Window” quantities

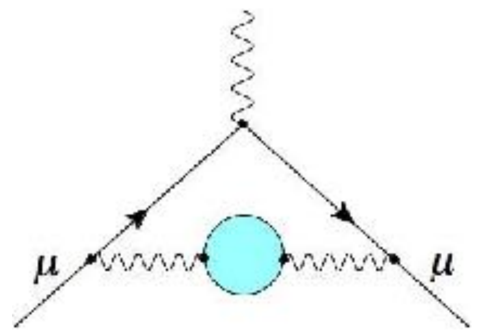
(Plots from Davide Giusti)





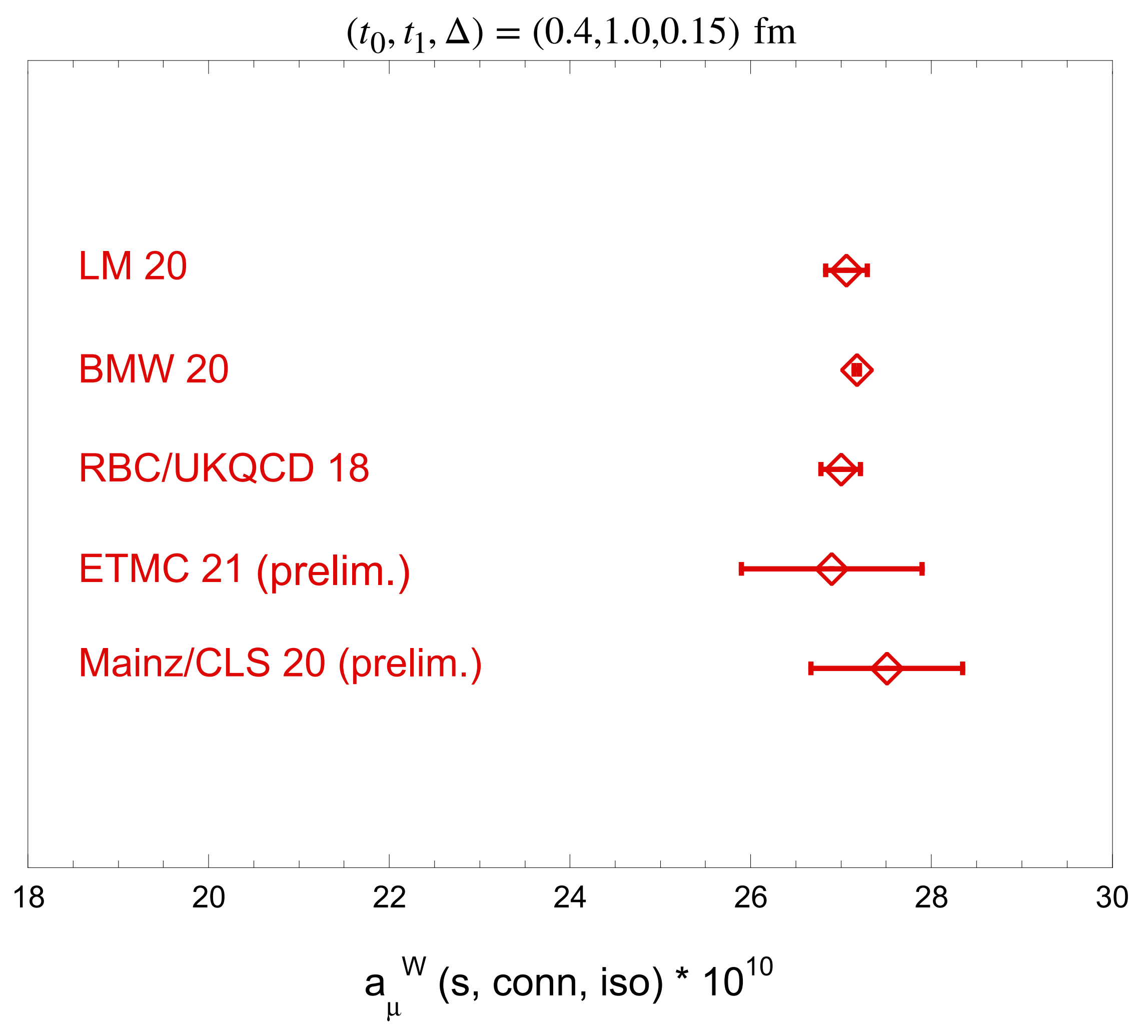
https://indico.cern.ch/event/956699/contributions/4117838/

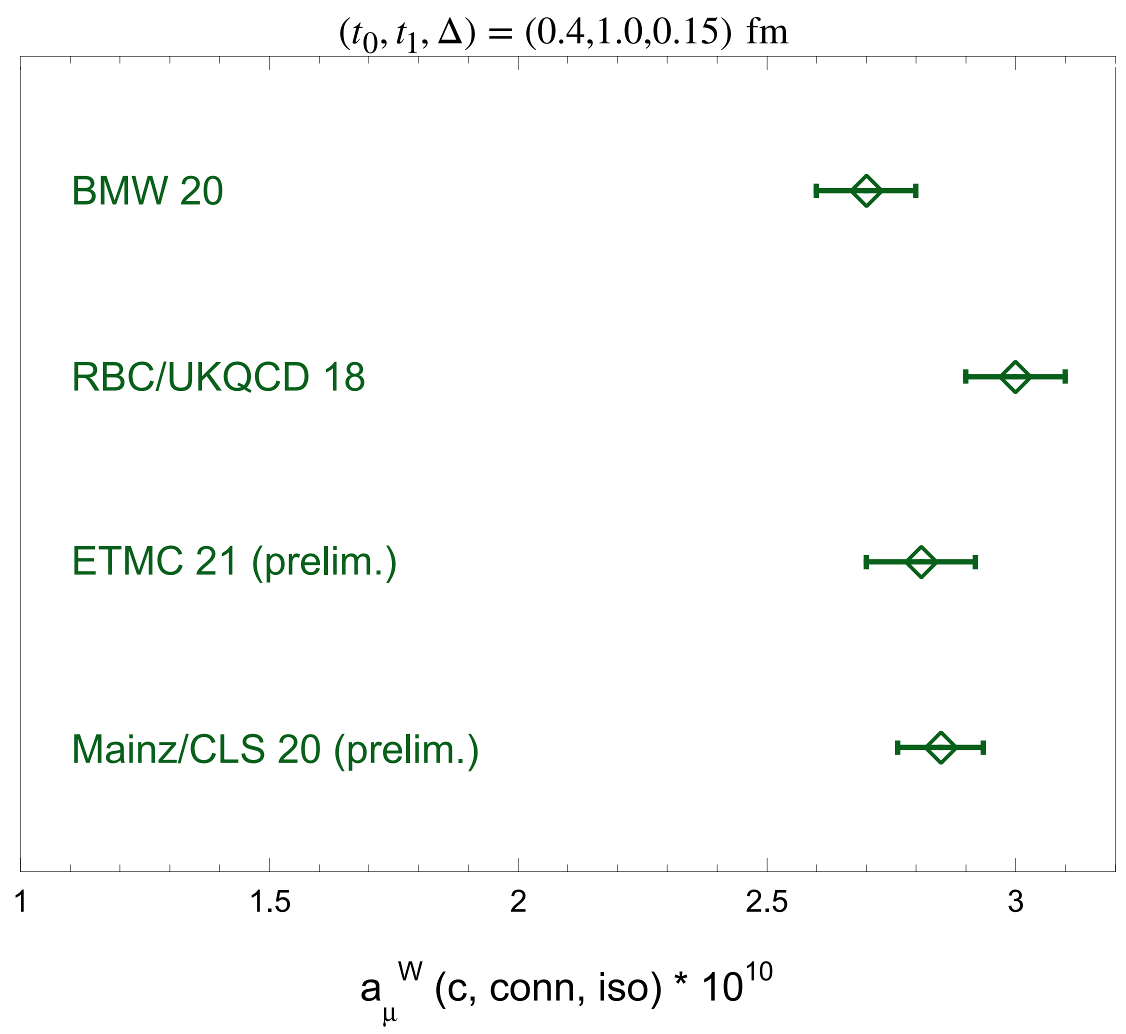
- Monday, 13:00-15:00 US EDT
  - Shaun Lahert (Fermilab-HPQCD-MILC)

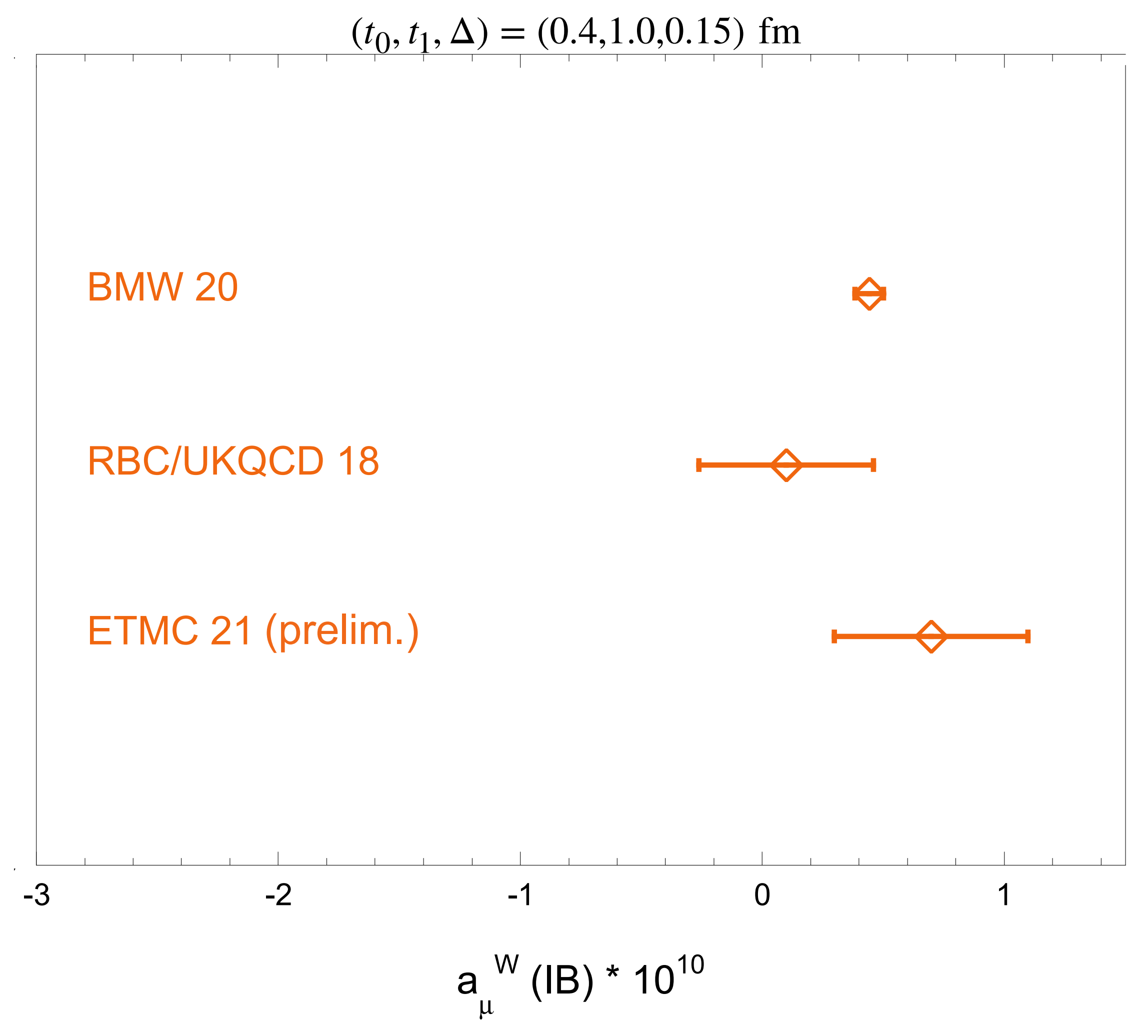


Lattice HVP: windows in Euclidean time

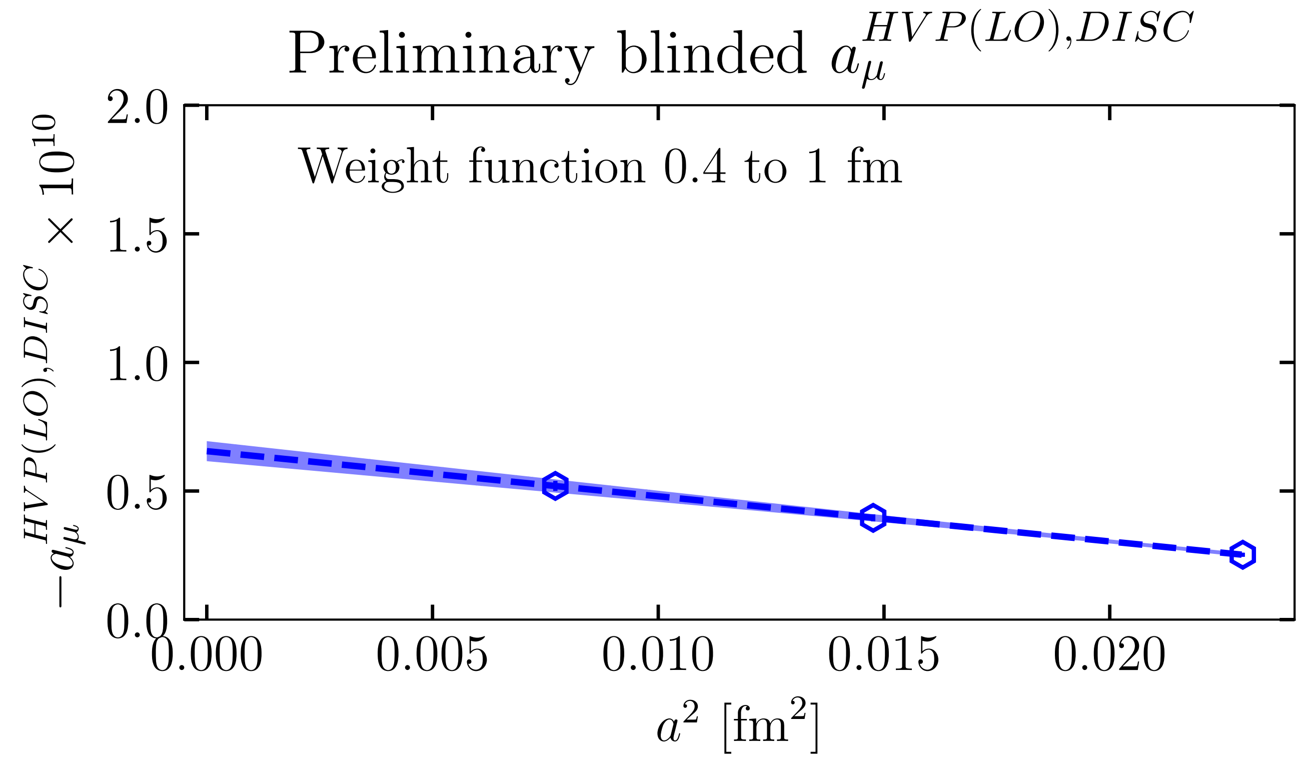
- Tuesday, 5:00-8:00 US EDT
  - Davide Giusti (ETMc)



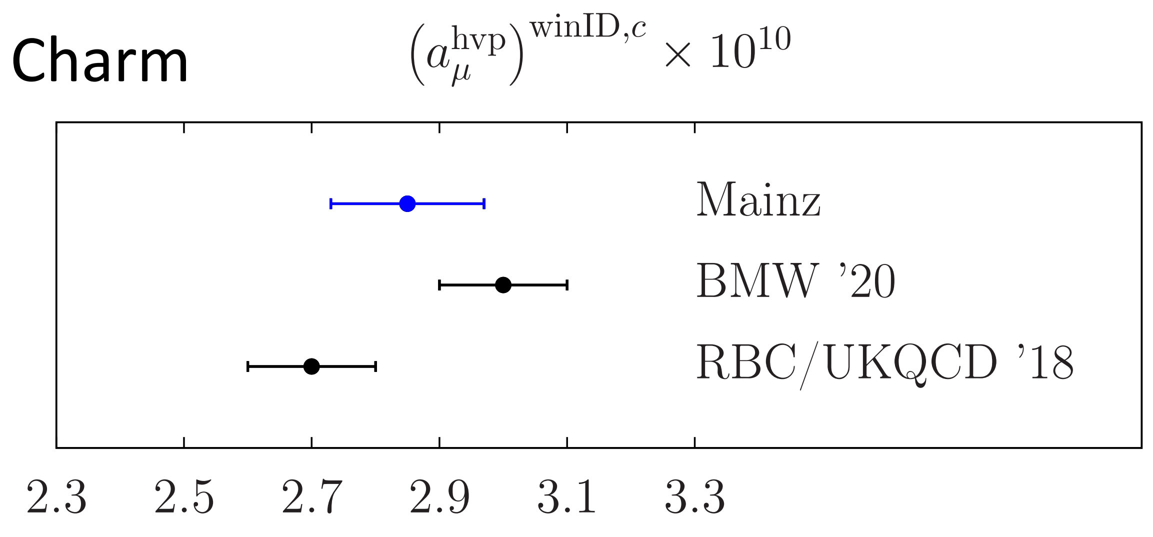


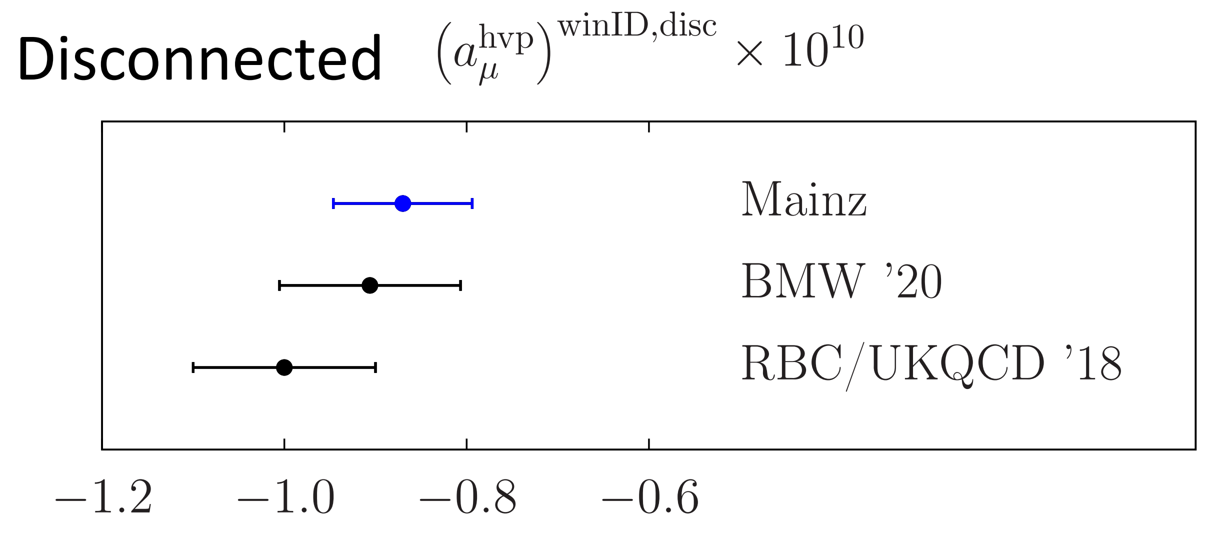


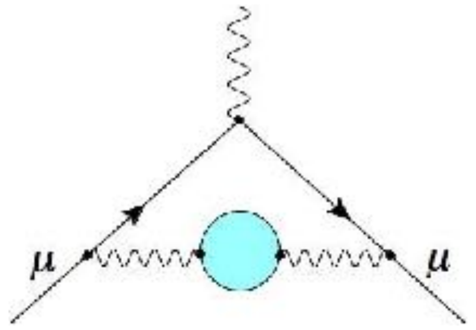
- Poster, Wednesday, 8:00-9:00 US EDT
  - C. McNeile (Fermilab-HPQCD-MILC)



- Tuesday, 5:00-8:00 US EDT
  - Hartmut Wittig (Mainz)

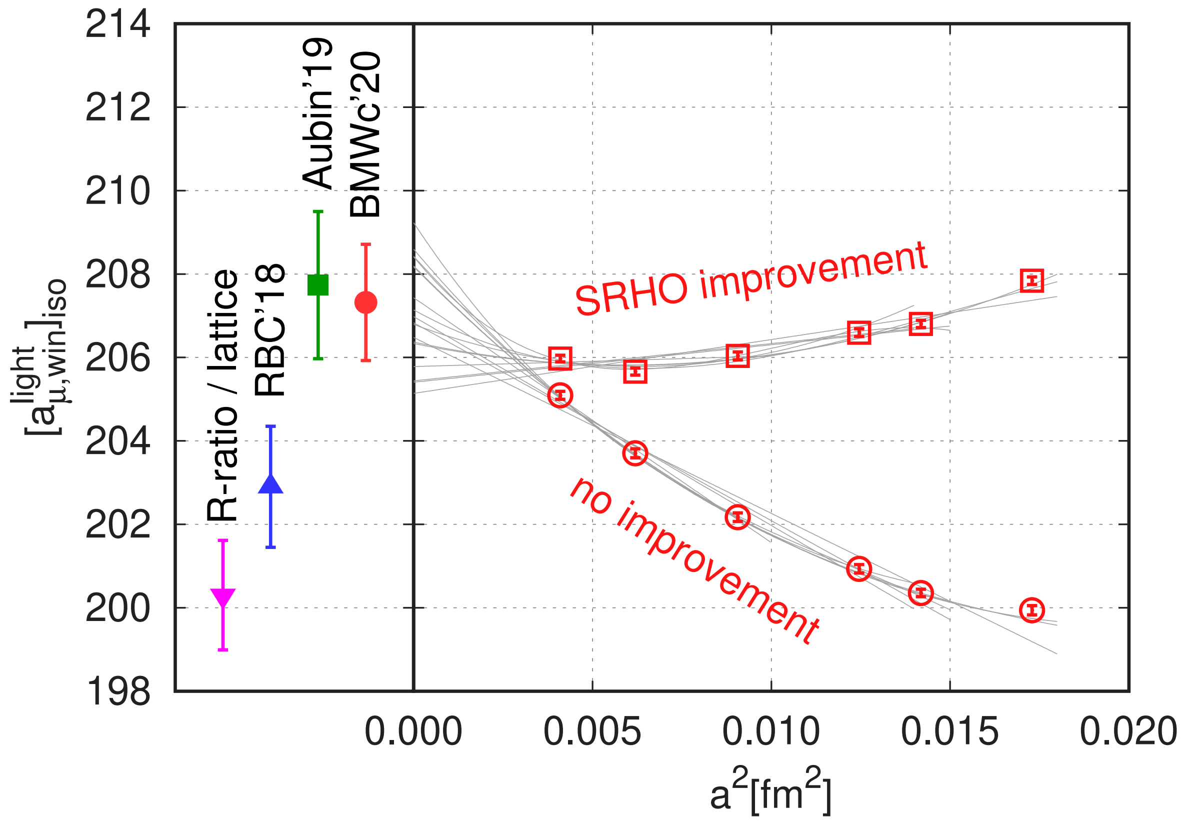






Lattice HVP: intermediate window (ud)

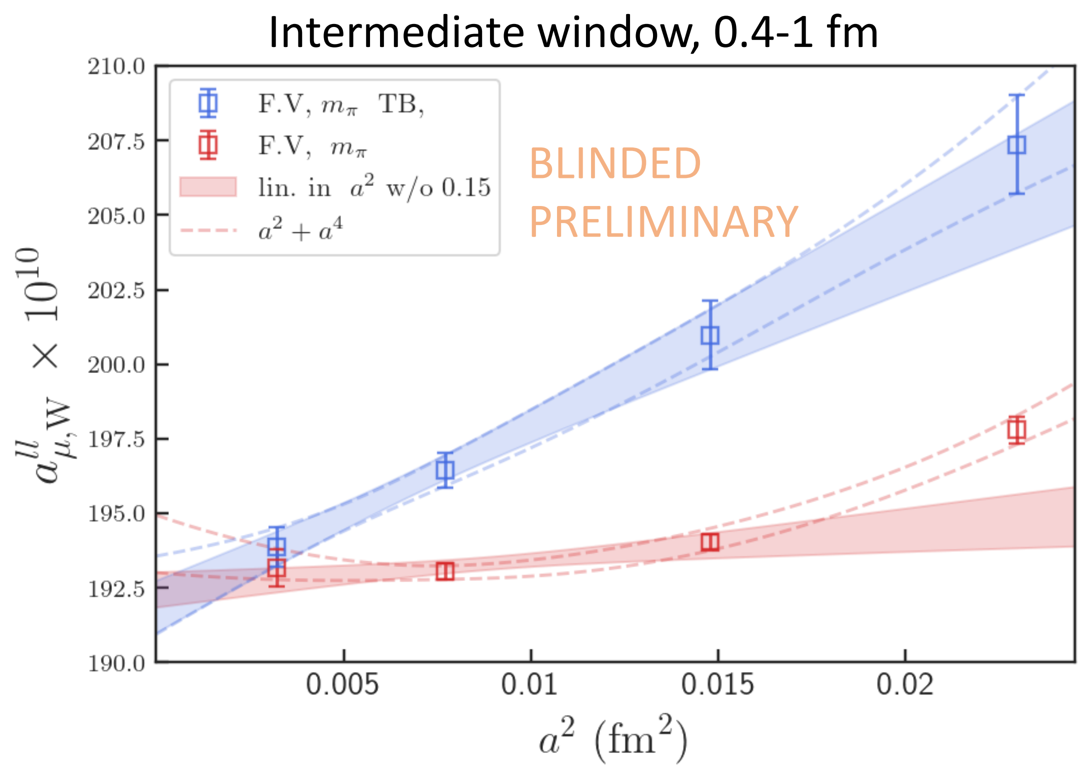
Monday, 13:00-15:00 US EDT
- Kalman Szabo (BMWc)



-3.7 σ tension with data-driven evaluation

-2.2 σ tension with RBC/UKQCD18

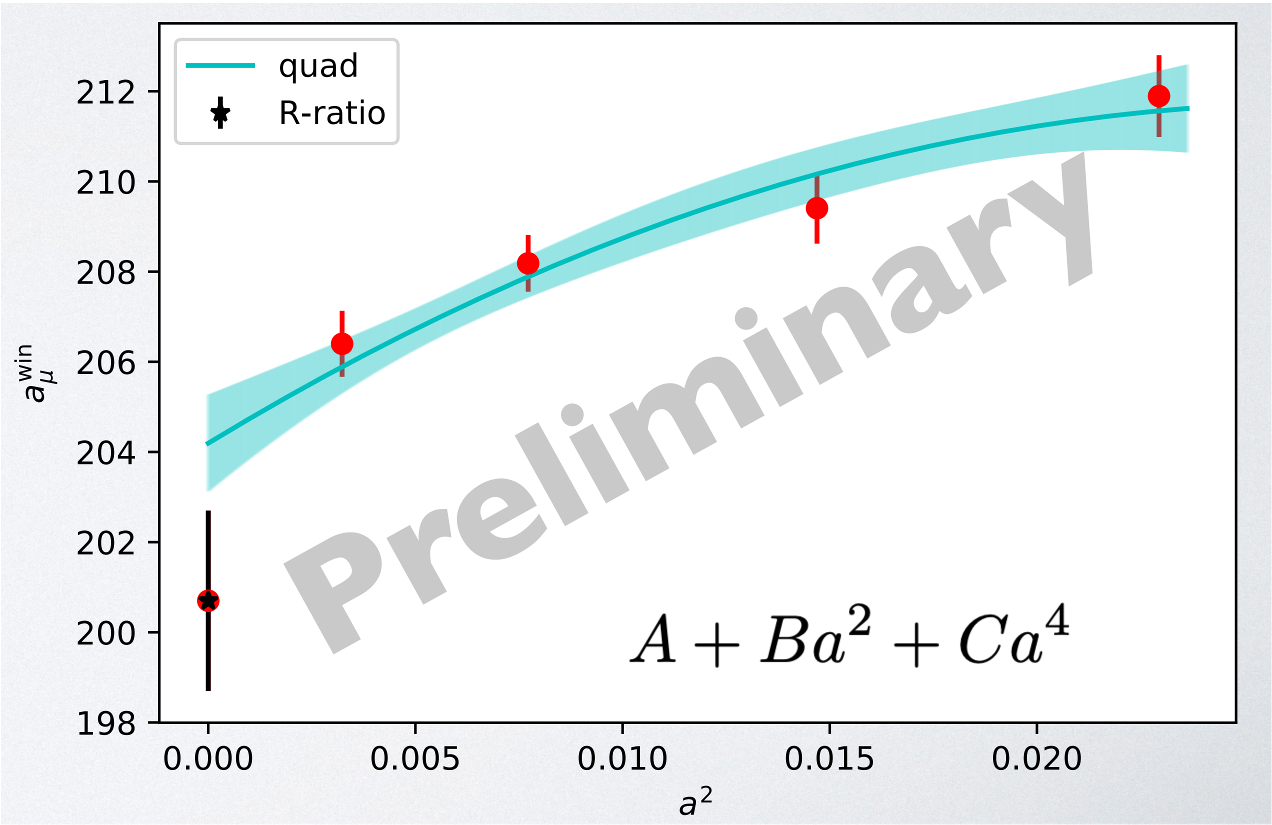
Monday, 13:00-15:00 US EDT
- Shaun Lahert (Fermilab-HPQCD-MILC)



❖ Corrections from $\rho - \gamma - \pi\pi$ model (leading order).

❖ Good consistency between extrapolations of data with(out) discretization effect corrections.

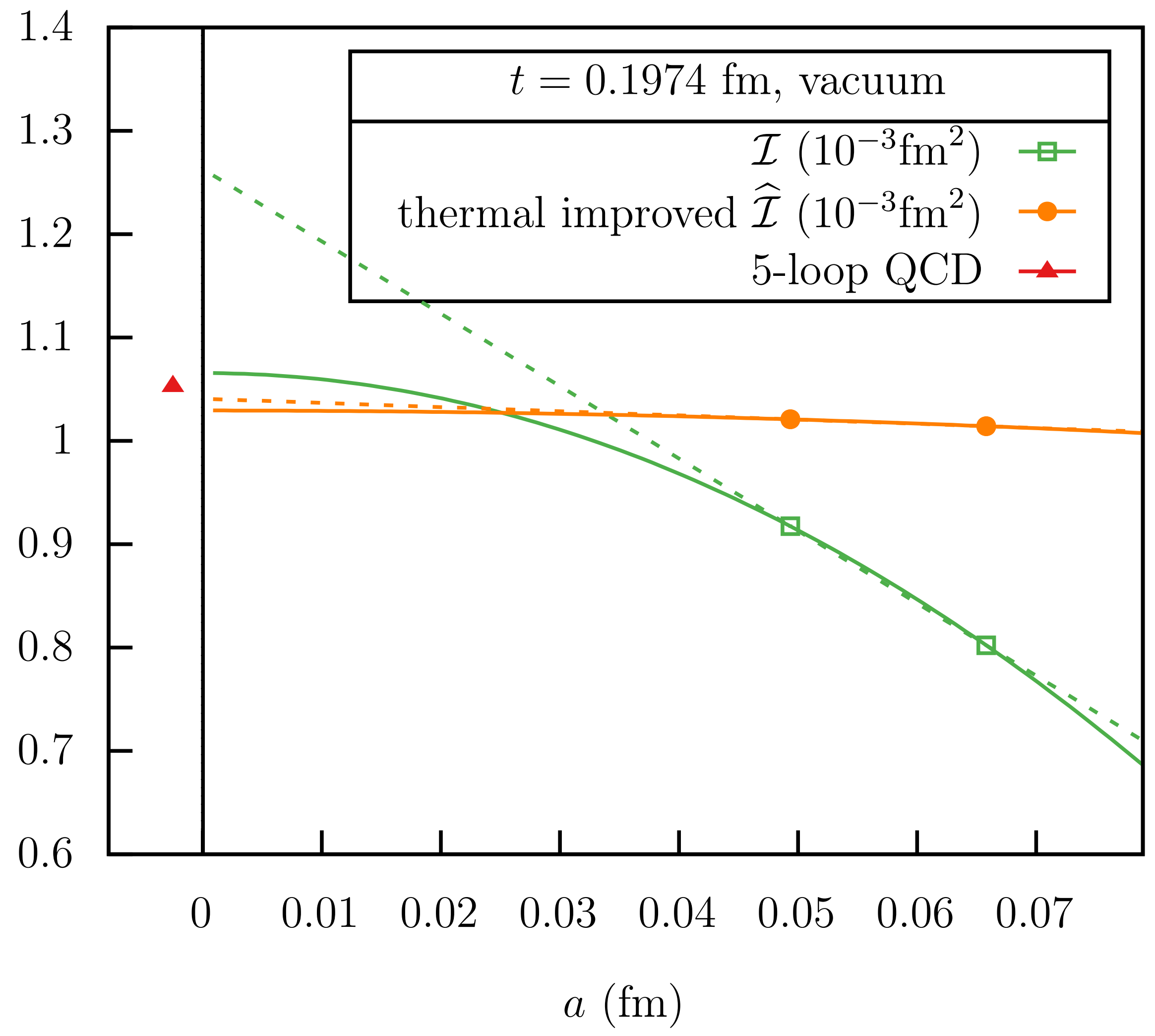
Monday, 13:00-15:00 US EDT
- Chris Aubin (Aubin et al)



Ongoing work:
- vary functional form of extrapolation, e.g., include $\alpha_s^m a^{2n}$ terms in expansion
- RBC/UKQCD adding a 3rd lattice spacing
- Blind analyses by Fermilab-HPQCD-MILC and RBC/UKQCD

Short-distance corrections

- Tuesday, 5:00-8:00 US EDT
  - Tim Harris (NEPhEU QCD)



- Use thermal observables and gauge ensembles at finite temperature and very fine lattice spacings to resolve discretization effects in calculations of short-distance quantities
- Construct improved observables with better smaller discretization effects
- Can be applied to SD window and $\Delta\alpha$
- New step-scaling method for computing $\Delta\alpha$ at large Q^2

- Friday, 5:00-8:00 US EDT
  - Nicolai Husung (DESY)

- Compute anomalous dimensions of higher dimensional operators in Symanzik EFT
- Use to guide continuum extrapolation
- In most cases studied: $\hat{\Gamma} \gtrsim 0$

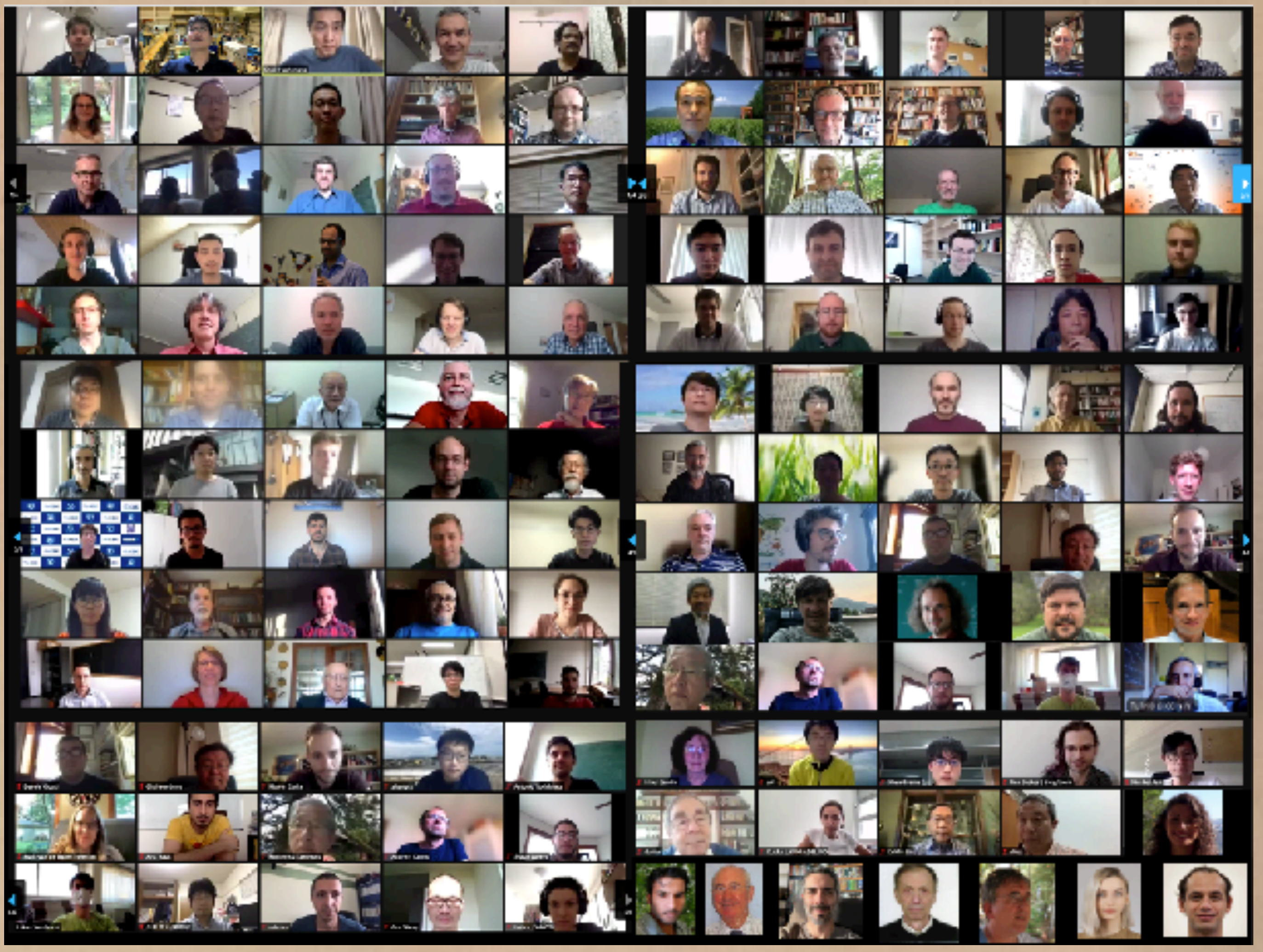
Summary

- The QED and EW contributions are known very precisely
  Hadronic contributions determine the uncertainty in the SM prediction.
- dispersive HVP: ~0.6% error [0.34ppm]
  based on well-tested experimental data, will be improved with new measurements (coming soon).
- lattice HVP: first LQCD calculation with sub-percent uncertainty by BMWc
  but in tension with data-driven approach
  a lot of activity and progress, expect more sub-precision results soon.
- dispersive HLBL: ~20% error [0.15ppm]
  newly developed dispersive approach with almost fully quantified errors
  systematically improvable
- lattice HLbL: two complete lattice calculations
  consistent with each other and with data-driven result
  systematically improvable

Outlook

- ☆ Theory Initiative:
  a "WP process" for Lattice HVP and HLbL results
- ☆ Starting to developing guidelines for assessing lattice HVP calculations
  panel discussion at the KEK workshop
- ☆ Quantities to calculate:
  windows for lattice-only cross checks for all components
  $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$
  I=1,0 (light quark)
  derivatives of a_μ (or windows) w.r.t. parameters
- ☆ Prescription for defining isospin limit and separating QED & SIB contributions
- ☆ Blinding lattice calculations is good practice to avoid unintended bias

Thank you!



Thank you!

Fermilab

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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

RIKEN BNL Research Center

PRISMA

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

UCONN DEPARTMENT OF PHYSICS

Appendix

Muon g-2 Theory Initiative

WP section authors:

- Section 2: Data-driven evaluations of HVP
  M. Benayoun, C. M. Carloni Calame, H. Czyz, M. Davier, S. I. Eidelman, M. Hoferichter, F. Jegerlehner, A. Keshavarzi, B. Malaescu, D. Nomura, M. Passera, T. Teubner, G. Venanzoni, Z. Zhang
- Section 3: Lattice QCD calculations of HVP
  T. Blum, M. Bruno, M. Ce, C. T. H. Davies, M. Della Morte, A. X. El-Khadra, D. Giusti, Steven Gottlieb, V. Guelpers, G. Herdoiza, T. Izubuchi, C. Lehner, L. Lellouch, M. K. Marinkovic, A. S. Meyer, K. Miura, A. Portelli, S. Simula, R. Van de Water, G. von Hippel, H. Wittig
- Section 4: Data-driven and dispersive approach to HLbL
  J. Bijnens, G. Colangelo, F. Curciarello, H. Czyz, I. Danilkin, F. Hagelstein, M. Hoferichter, B. Kubis, A. Kupsc, A. Nyffeler, V. Pascalutsa, E. Perez del Rio, M. Procura, C. F. Redmer, P. Sanchez-Puertas, P. Stoffer, M. Vanderhaeghen
- Section 5: Lattice approaches to HLbL
  N. Asmussen, T. Blum, A. Gerardin, M. Hayakawa, R. J. Hudspith, T. Izubuchi, L. Jin, C. Lehner, H. B. Meyer, A. Nyffeler
- Section 6: The QED contributions to a_μ
  T. Aoyama, T. Kinoshita, M. Nio
- Section 7: The electroweak contributions to a_μ
  D. Stoeckinger, H. Stoeckinger-Kim

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.

- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift is due to differences in the low ($\lesssim 2\,\text{GeV}$) energy region, the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies ➠ $\sigma(e^+e^- \to \pi\pi)$ must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$ can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer 2021]

Connections

Martin Hoferichter @ Lattice HVP workshop

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

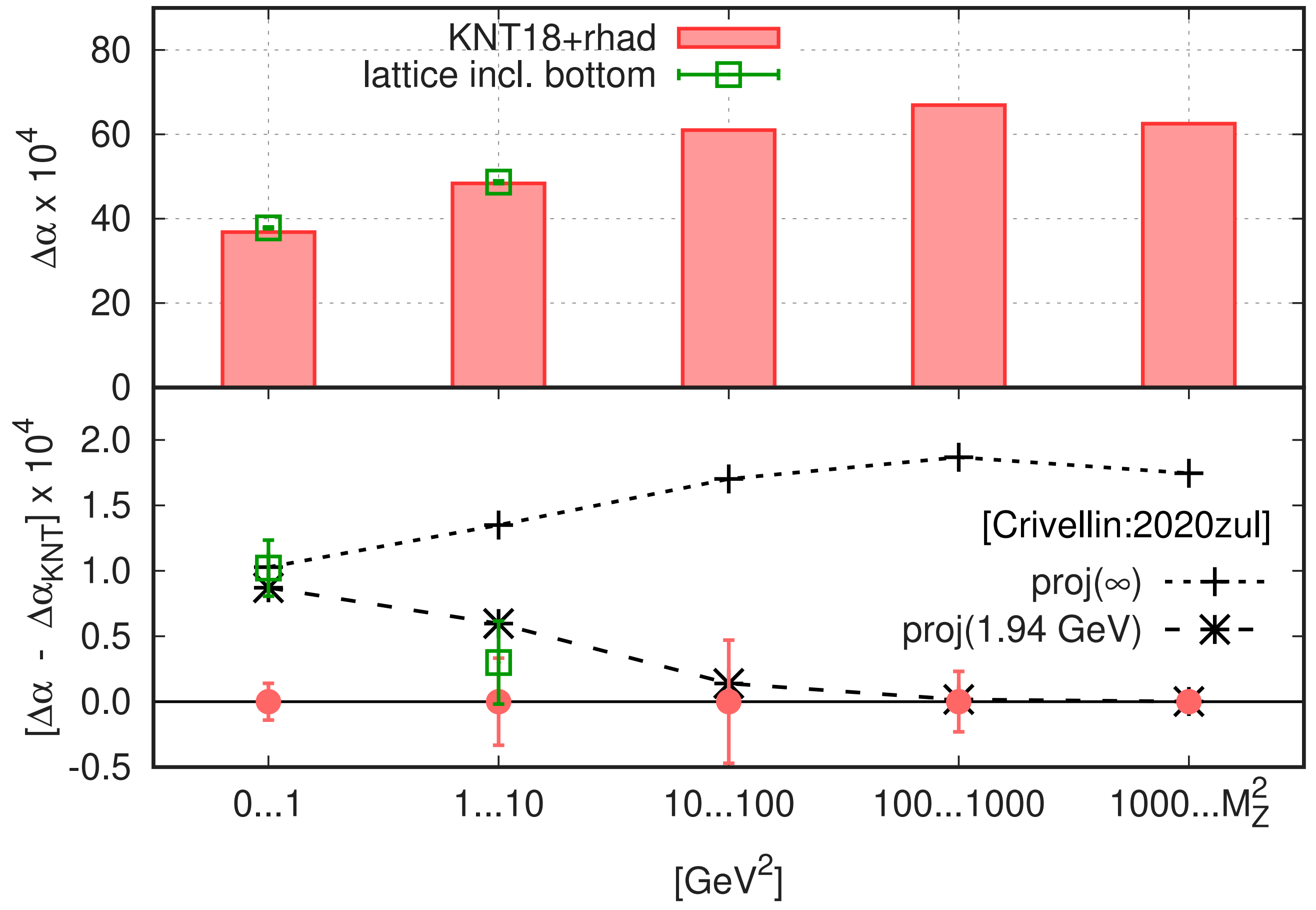
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

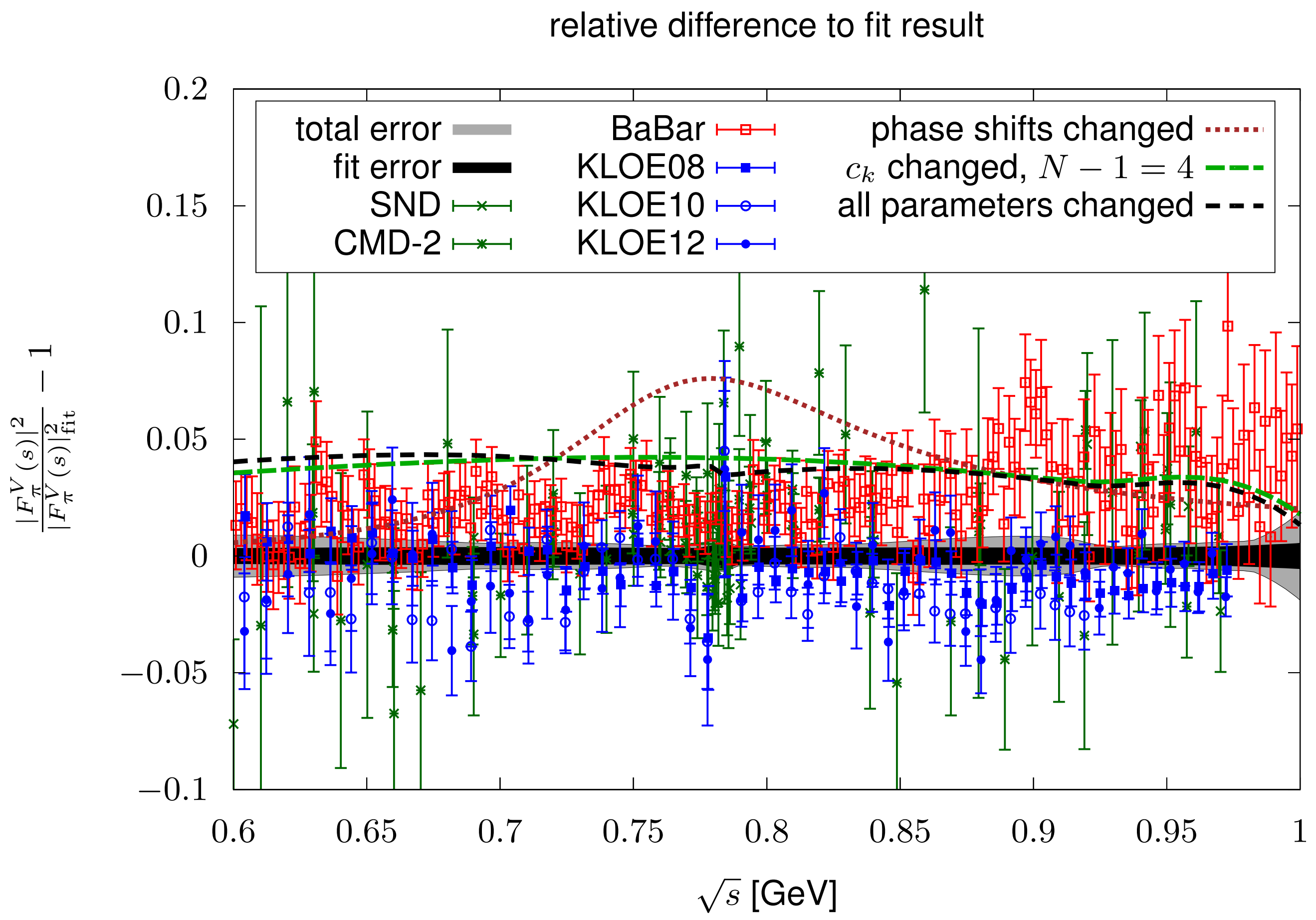
Connections

Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP   arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

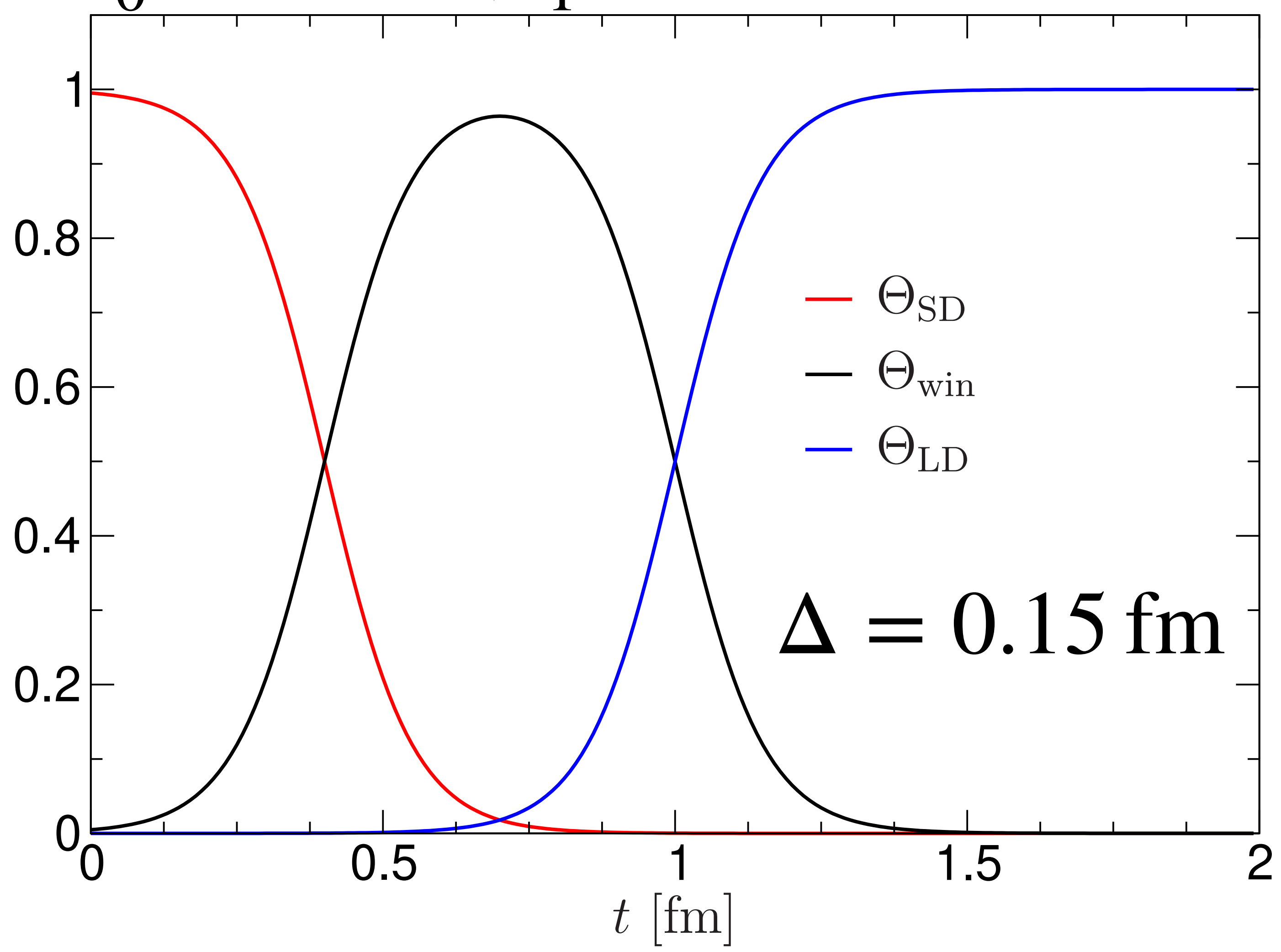
- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around** ρ
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor** ~ 3 **improvement** over χQCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  $\rightarrow$ arXiv:2006.05431 [hep-ph]

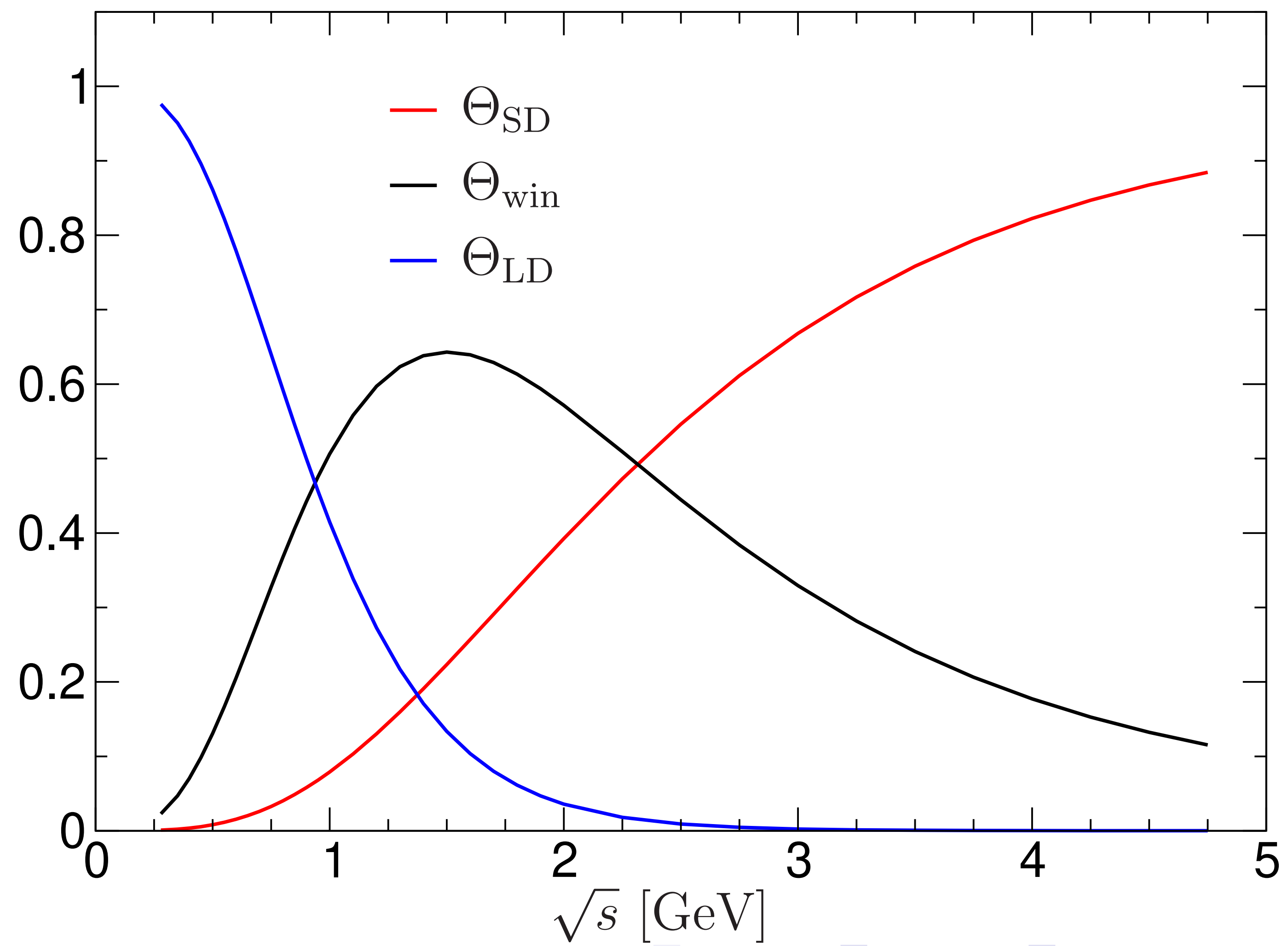


Windows: Euclidean time vs $\sqrt{s}$

Martin Hoferichter @ Lattice HVP workshop

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$





$[t_0, t_1]$ intermediate window	percentage captured of $\pi\pi$ channel $\leq$ 1 GeV SD	intermediate	LD
[0.4, 1.0] fm	3	28	69
[1.0, 2.0] fm	31	51	18
[1.0, 2.5] fm	31	61	9
[1.0, 3.0] fm	31	65	4

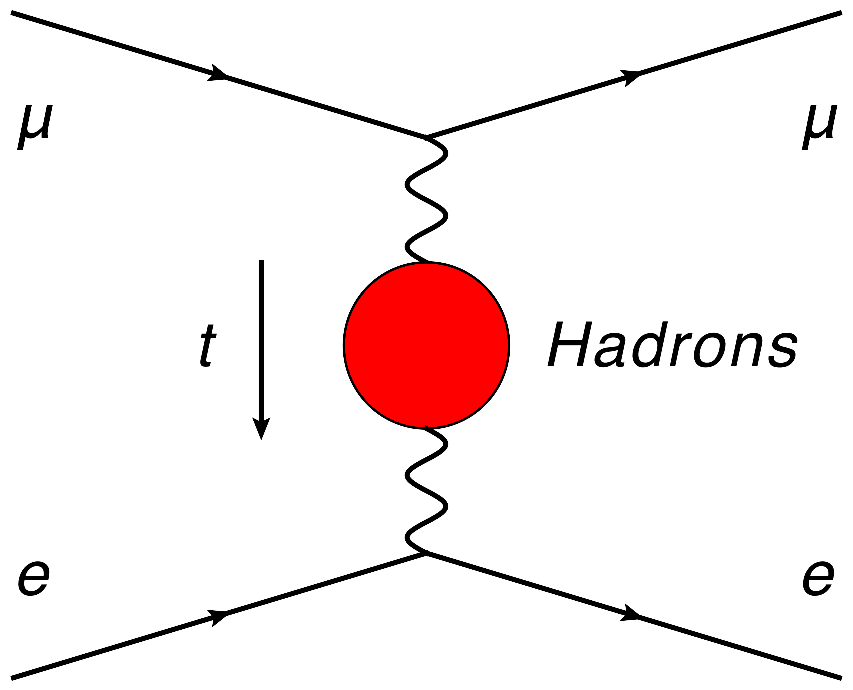
SD: $[0, t_0]$
LD: $[t_1, \infty]$
intermediate: $[t_0, t_1]$

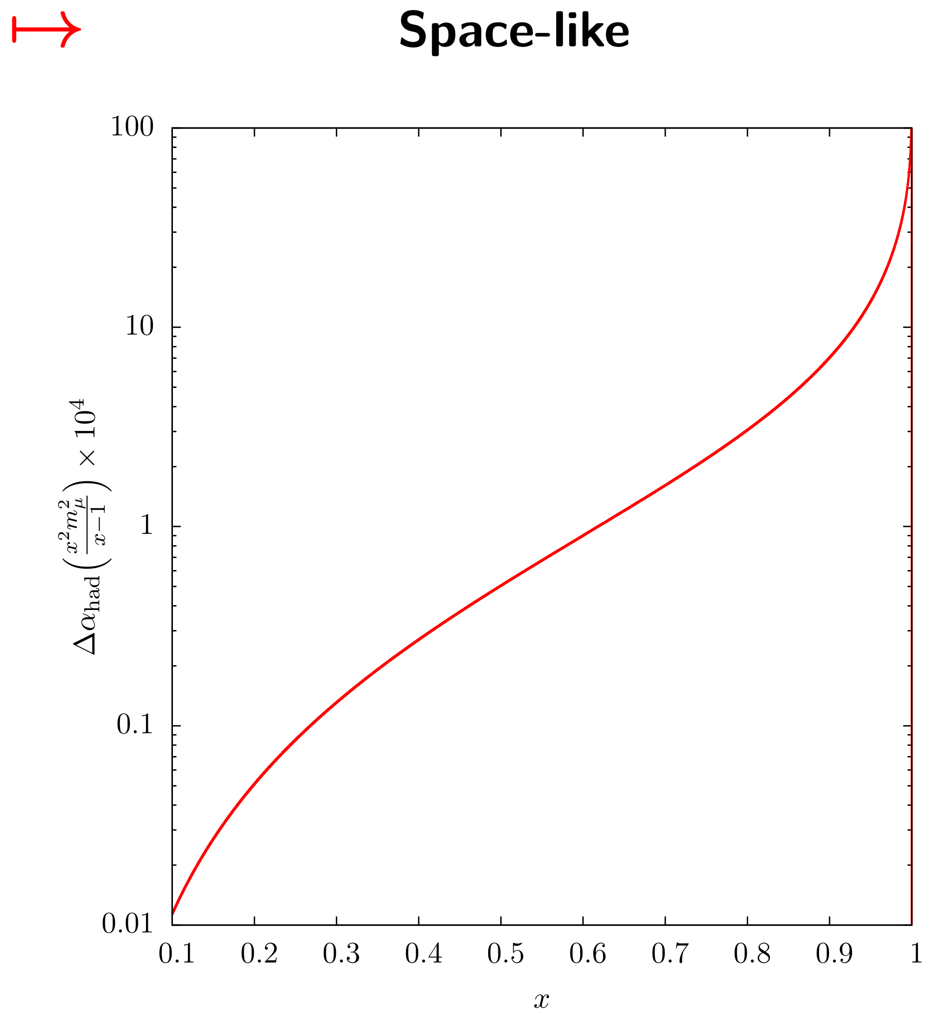
For intermediate window:
~30% from $\sigma(\pi\pi) \lesssim 1\,\mathrm{GeV}$

Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

LOI June 2019 [P. Banerjeei et al, arXiv:2004.13663, Eur.Phys.J.C 80 (2020)]





Smooth function

CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- pilot run in 2021
- full apparatus in 2023-2024

μONe