Impact of Lattice QCD on Transverse Momentum Parton Phenomenology

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Lattice 2021
Zoom/Gather@MIT
July 29, 2021

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U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Collaboration

Outline

- Transverse Momentum Dependent (TMD) distributions:
  Introduction

TMD Cross Sections

- Factorization & Evolution
- Perturbative & Nonperturbative regimes
- Global Fits and Phenomenology

TMDs, Quasi-TMDs, and Lattice Results

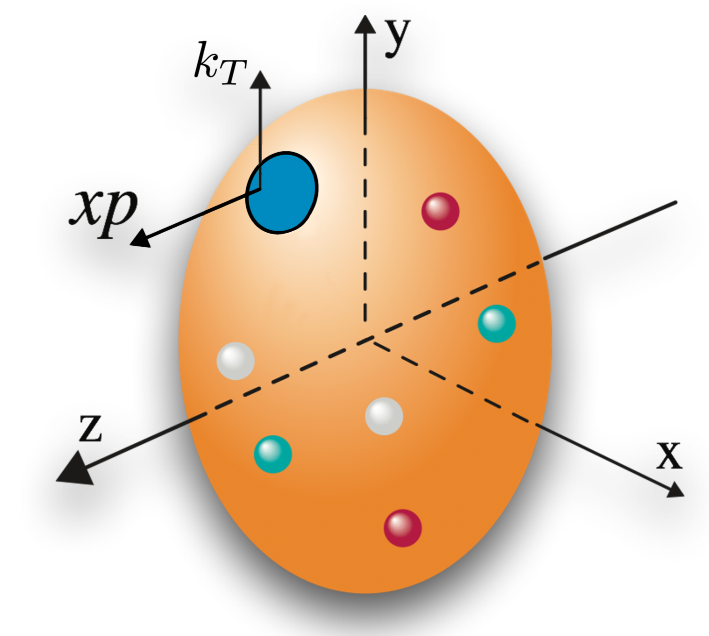
- Definitions and Relations
- Non-perturbative anomalous dimension (CS Kernel)
- Non-perturbative Soft Function (TMDPDF)
- Future prospects and directions

PDFs

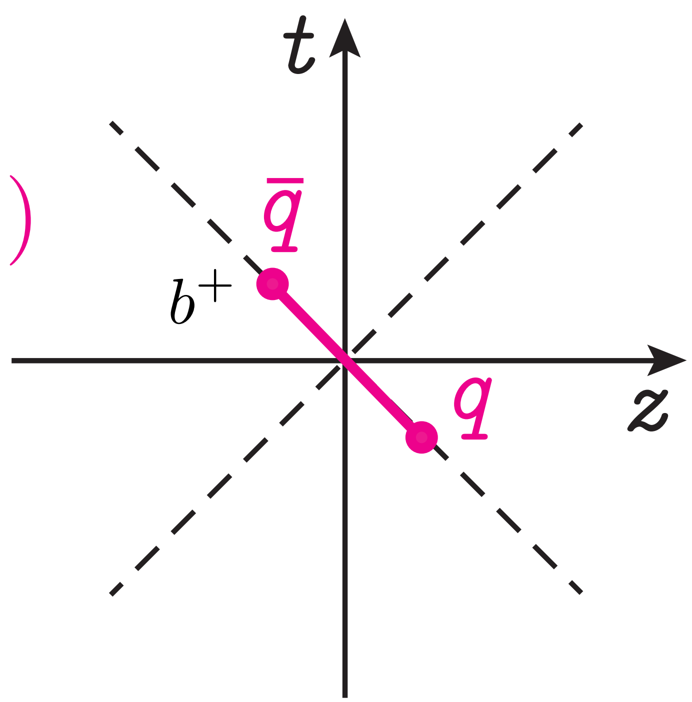
$$f_{q/P}(x,\mu)$$

longitudinal

$$f_{q/P}(x) = \int \frac{db^+}{4\pi}\, e^{-\frac{i}{2}b^+ x P^-} \langle P|O(b^+)|P\rangle$$



- **ubiquitous in description of collider physics processes**



light-cone sensitive operators

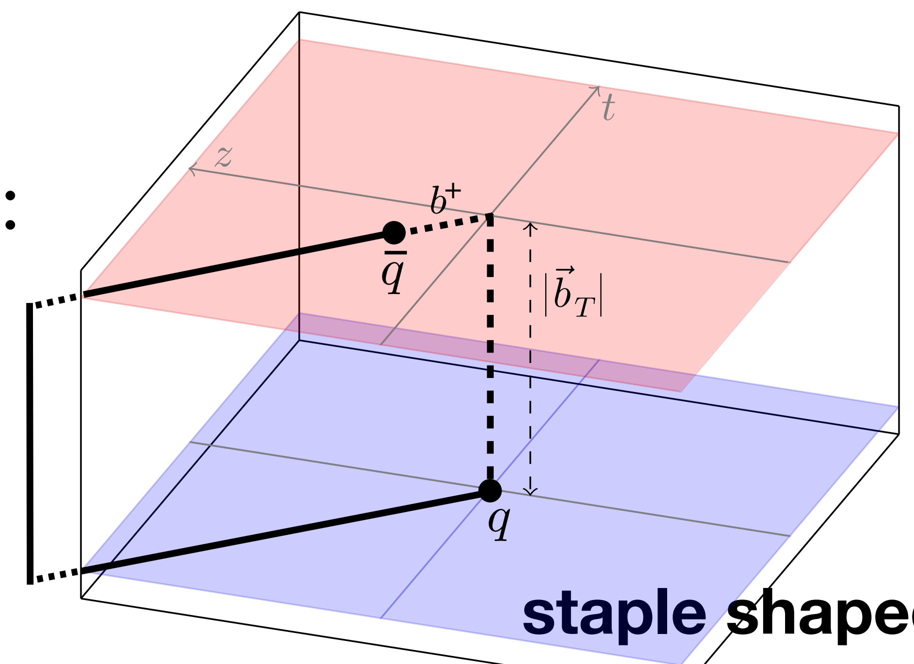
TMDs

$$f_{q/P}(x, k_T, \mu, \zeta)$$

longitudinal & Transverse

- **key information about the structure of hadrons**

$O(b^+, b_T)$:



staple shaped Wilson lines

$$f_{q/P}(x, k_T) \sim \int \frac{db^+ d^2b_T}{2(2\pi)^3}\, e^{-\frac{i}{2}b^+ x P^- + i\vec{b}_T\cdot\vec{k}_T} \langle P|O(b^+, b_T)|P\rangle$$

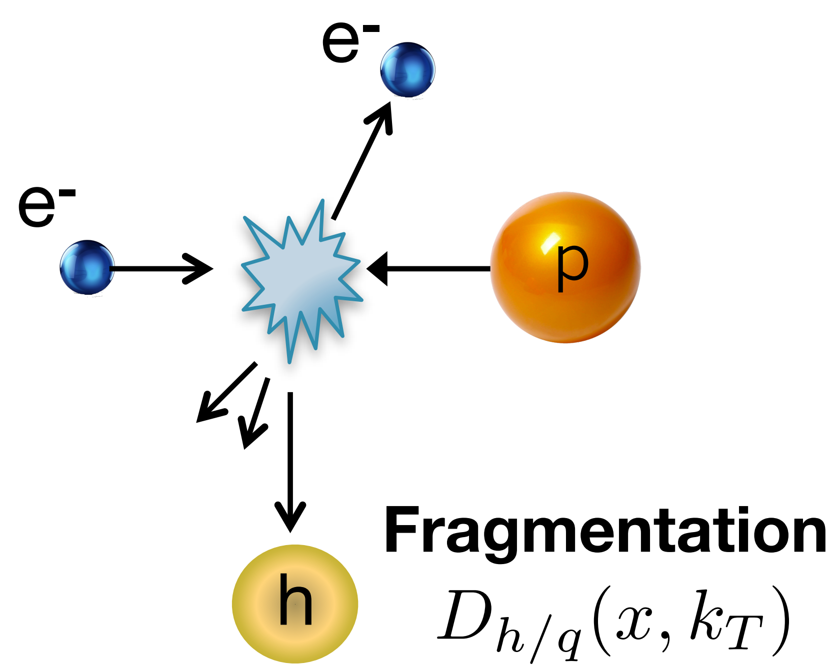
+ complications

TMD Factorization

(schematic)

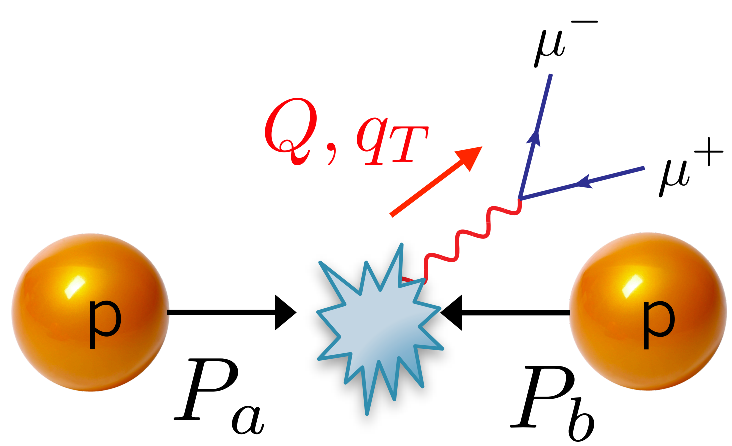
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



Drell-Yan

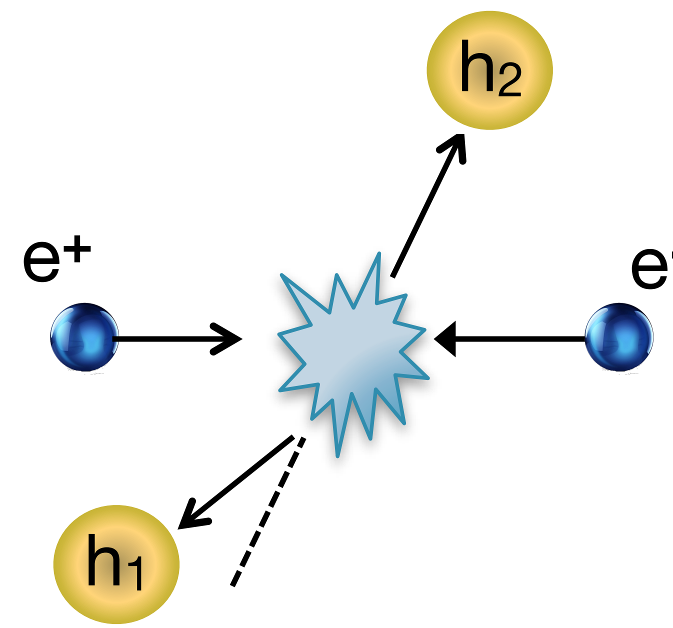
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



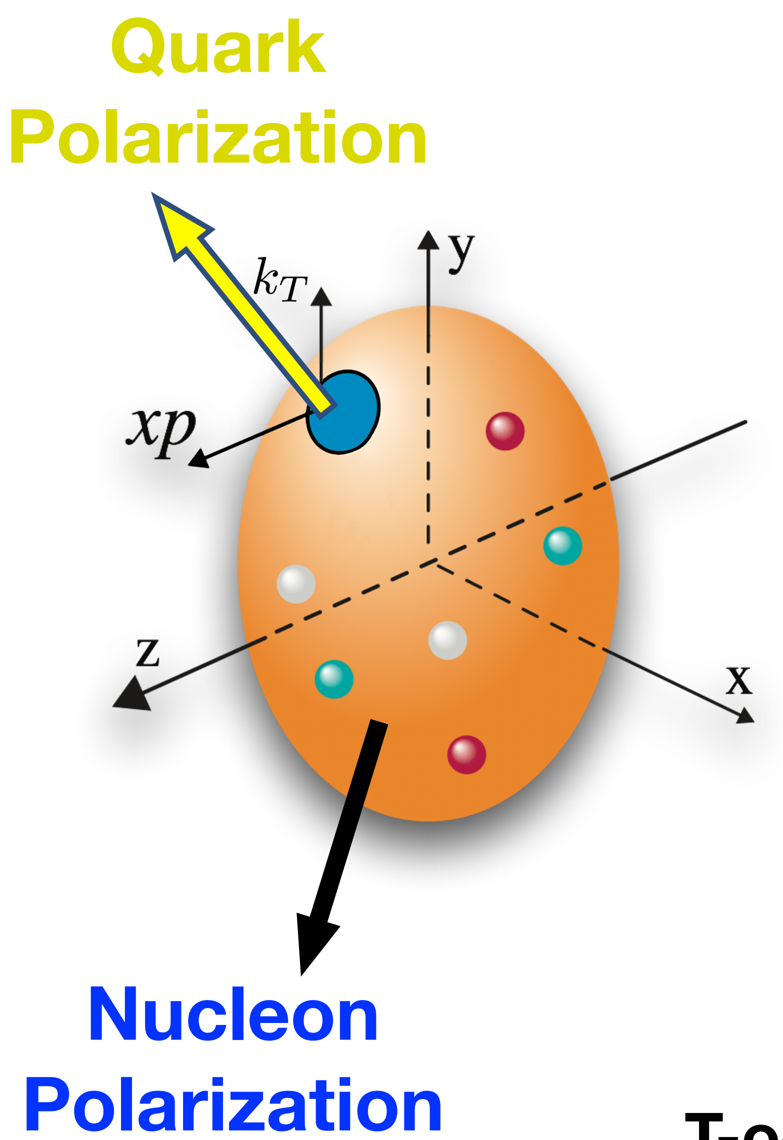
$$q_T \ll Q$$

Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



TMDs with Polarization



Nucleon Polarization	Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$
T	$f_{1T}^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$	$h_1(x,k_T^2)$ Transversity $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

T-odd sign flip: $f_{1T}^{\perp\,\mathrm{SIDIS}} = -f_{1T}^{\perp\,\mathrm{DY}}$, $h_1^{\perp\,\mathrm{SIDIS}} = -h_1^{\perp\,\mathrm{DY}}$

Have flavor indices, like longitudinal PDFs.

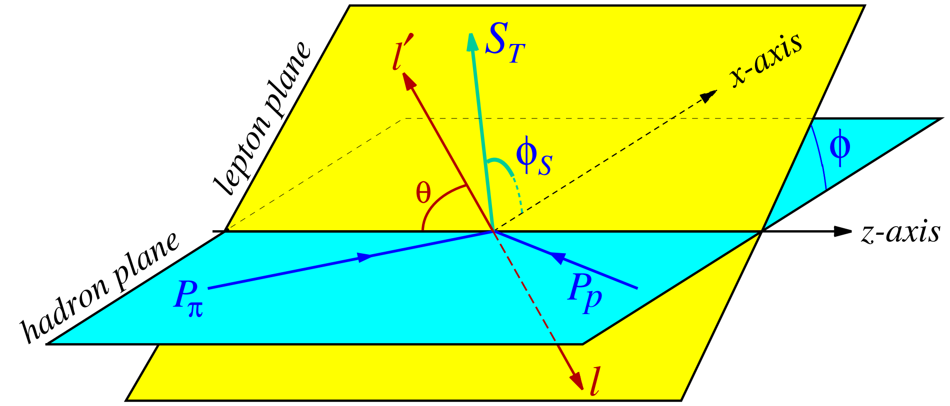
Analogous tables for Frag. functions, gluons, etc.

Can explore the 3D Structure of Hadrons using all these distributions!

Observables

Drell-Yan with pol. proton:

$$\pi p \xrightarrow{\gamma^*} \ell^+ \ell^- X$$



$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,Q^2}\Big\{\Big[(1+\cos^2\theta)F_{UU}^1 + \sin^2\theta\cos(2\phi)F_{UU}^{\cos 2\phi}\Big] + S_L\sin^2\theta\sin(2\phi)F_{UL}^{\sin 2\phi} + S_T(1-\cos^2\theta)\sin\phi_S F_{UT}^{\sin\phi_S} + S_T\sin^2\theta\Big[\sin(2\phi+\phi_S)F_{UT}^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big]\Big\}$$

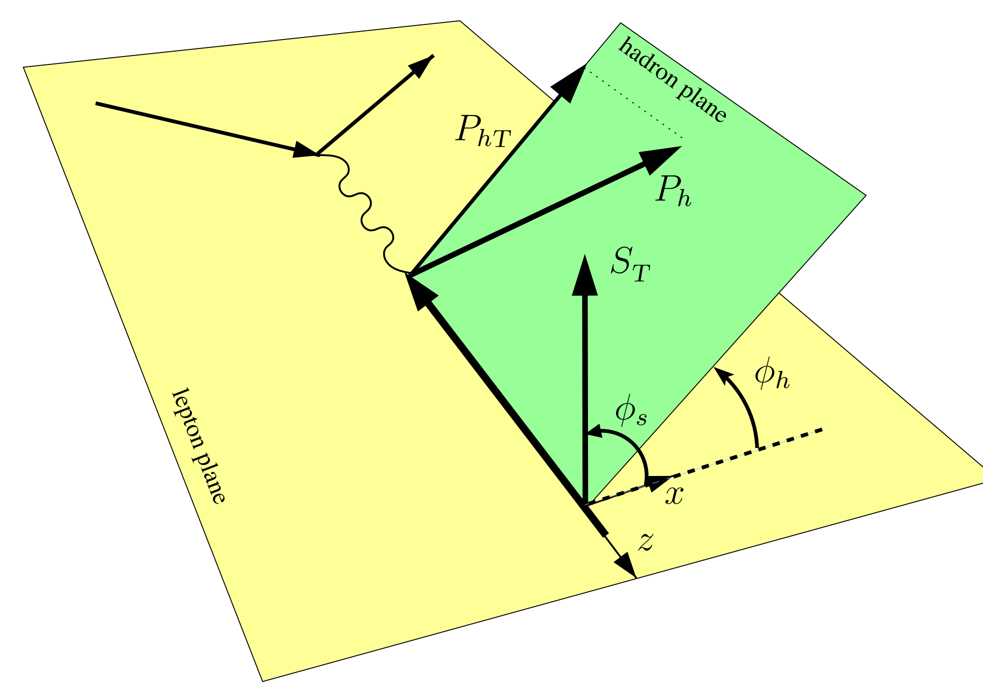
products of TMDPDFs:

$$f_{1,\pi}f_{1,p},\ \ h_{1,\pi}^{\perp}h_{1,p}^{\perp},\ \ \ldots$$

SIDIS with polarized electron & proton:

$$e^- p \xrightarrow{\gamma^*} e^- h X$$

$$\frac{d^6\sigma}{dx_B\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x_B\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda p_2\,F_{LL} + S_T\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} + S_T\sin(\phi_h+\phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)} + S_T\sin(3\phi_h-\phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$



TMDPDF * TMDFF

$$f_{1,p}D_{1,h},\ \ h_{1,p}^{\perp}H_{1,h}^{\perp},\ \ \ldots$$

TMD Factorization

CSS (Collins, Soper, Sterman)
SCET (Soft Collinear Effective Theory)

- rigorous QFT based derivation of cross sections
- based on analysis of momentum regions

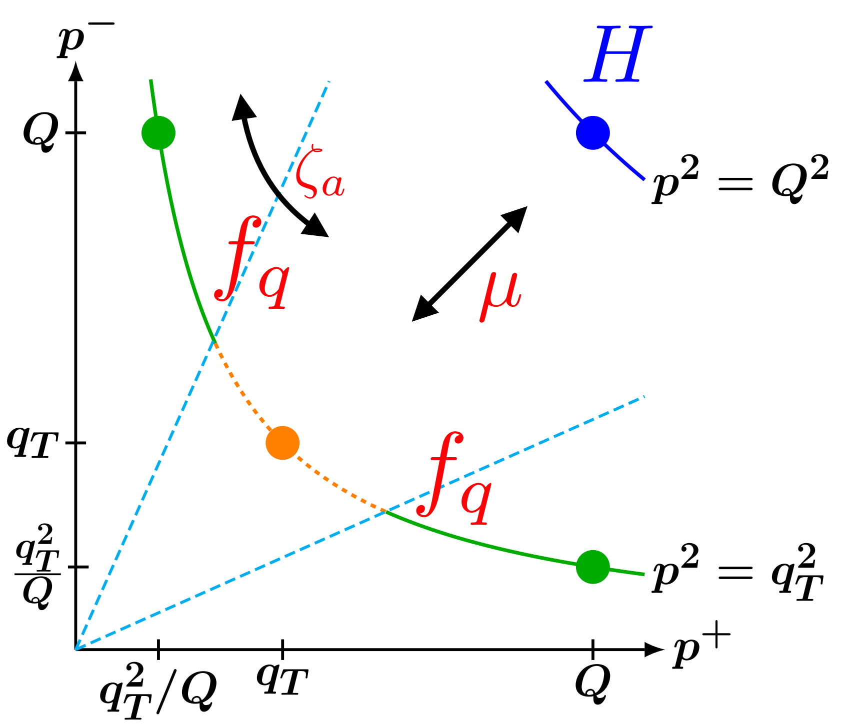
eg. Drell-Yan

$$\sigma(q_T, Q) = H(Q,\mu) \int d^2\vec{b}_T \, e^{i\vec{q}_T\cdot\vec{b}_T} \, f_q(x_a, \vec{b}_T, \mu, \zeta_a) \, f_q(x_b, \vec{b}_T, \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Hard virtual corrections

FT

$$f_q(x_a, \vec{k}_T, \mu, \zeta_a)$$



μ = renormalization scale

ζ = Collins-Soper parameter

$$\zeta_a = (x_a P_a^-)^2 = (2x_a P^z)^2$$

$$\zeta_a \zeta_b = Q^4$$

think: $\zeta \sim Q^2$

TMD Factorization

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SCET (Soft Collinear Effective Theory)

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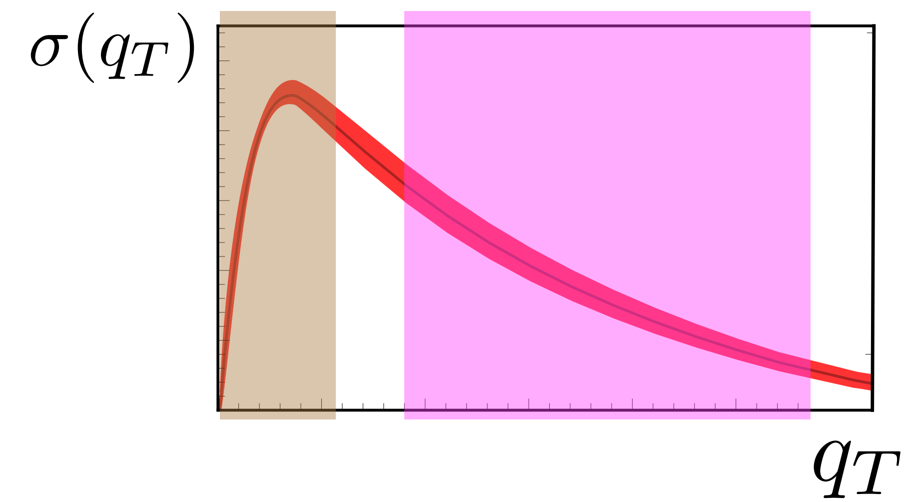
eg. Drell-Yan

$$\sigma(q_T, Q) = H(Q,\mu) \int d^2\vec{b}_T \, e^{i\vec{q}_T\cdot\vec{b}_T} \, f_q(x_a, \vec{b}_T, \mu, \zeta_a) \, f_q(x_b, \vec{b}_T, \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$



- **nonperturbative** $k_T \sim b_T^{-1} \sim \Lambda_{\rm QCD}$ $\quad f_q(x, \vec{k}_T, \mu, \zeta)$

- **perturbative** $k_T \sim b_T^{-1} \gg \Lambda_{\rm QCD}$

$$f_q(x, \vec{k}_T, \mu, \zeta) = \sum_i \int \frac{dy}{y} \, C_{qi}\left(\frac{x}{y}, \vec{k}_T, \mu, \zeta\right) f_i(y, \mu)$$

perturbative PDF

now known to $\mathcal{O}(\alpha_s^3)$ for unpolarized case

Ebert, Mistlberger, Vita '20
Luo, Yang, Zhu, Zhu '20

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

Solution:

$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\right] \times f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

Useful: Connect Lattice calculation or model with $\mu_0 \sim \sqrt{\zeta_0} \sim \text{few GeV}$

to scales needed in factorization theorem: $\mu \sim \sqrt{\zeta} \sim Q$

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

$$\mu \frac{d}{d\mu} \gamma_\zeta^q(\mu, b_T) = 2\zeta \frac{d}{d\zeta} \gamma_\mu^q(\mu, \zeta) = -2\Gamma_{\rm cusp}^q[\alpha_s(\mu)]$$

path independent

All Orders form:

$$\gamma_\mu^q(\mu, \zeta) = \Gamma_{\rm cusp}^q[\alpha_s(\mu)] \ln \frac{\mu^2}{\zeta} + \gamma_\mu^q[\alpha_s(\mu)]$$

$$\gamma_\zeta^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^q[\alpha_s(\mu')] + \gamma_\zeta^q[\alpha_s(1/b_T)]$$

- **Perturbative at short distance** $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$

$$\gamma_\zeta^q[\alpha_s] = \alpha_s\, \gamma_\zeta^{q(1)} + \alpha_s^2\, \gamma_\zeta^{q(2)} + \alpha_s^3\, \gamma_\zeta^{q(3)} + \ldots$$

3-loop result: Li, Zhu 2016

→ **LL, NLL, NNLL, N3LL, … results**

- **For** $b_T^{-1} \sim \Lambda_{\rm QCD}$ **the CS kernel** $\gamma_\zeta^q(\mu, b_T)$ **becomes nonperturbative**

(even if the evolution variables μ, ζ are perturbative)

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

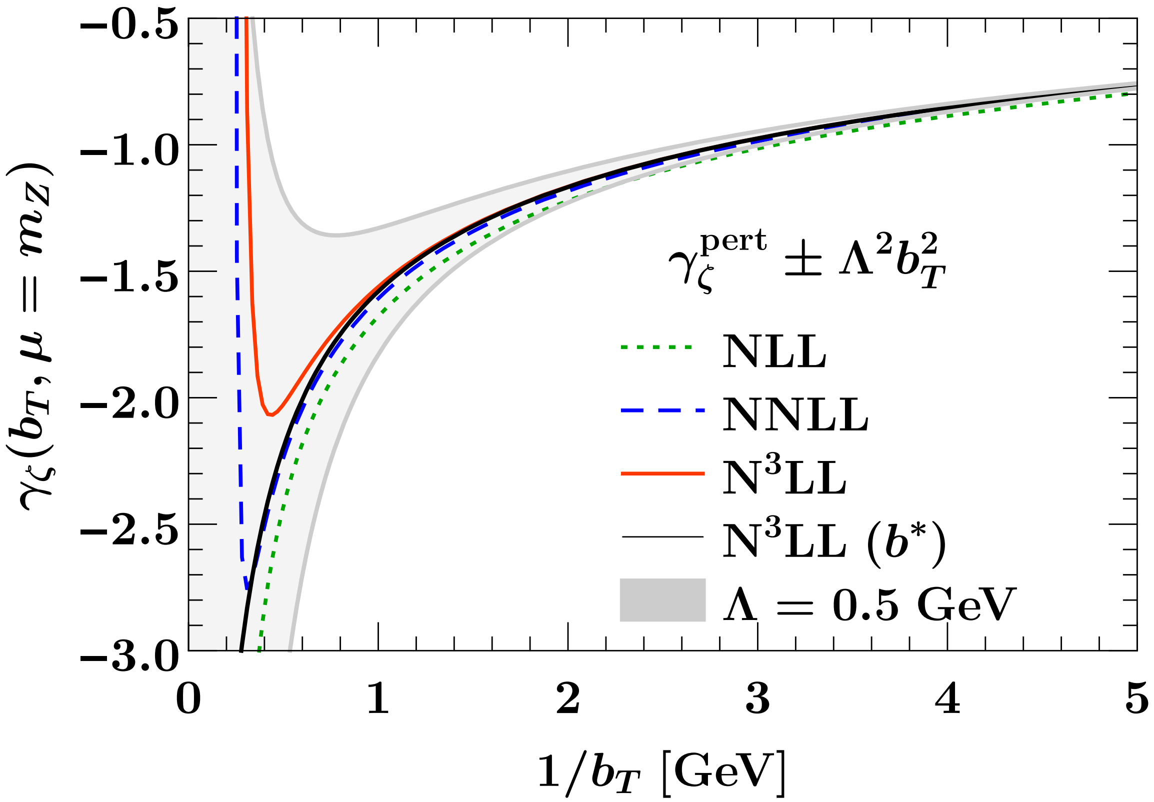
Solution:

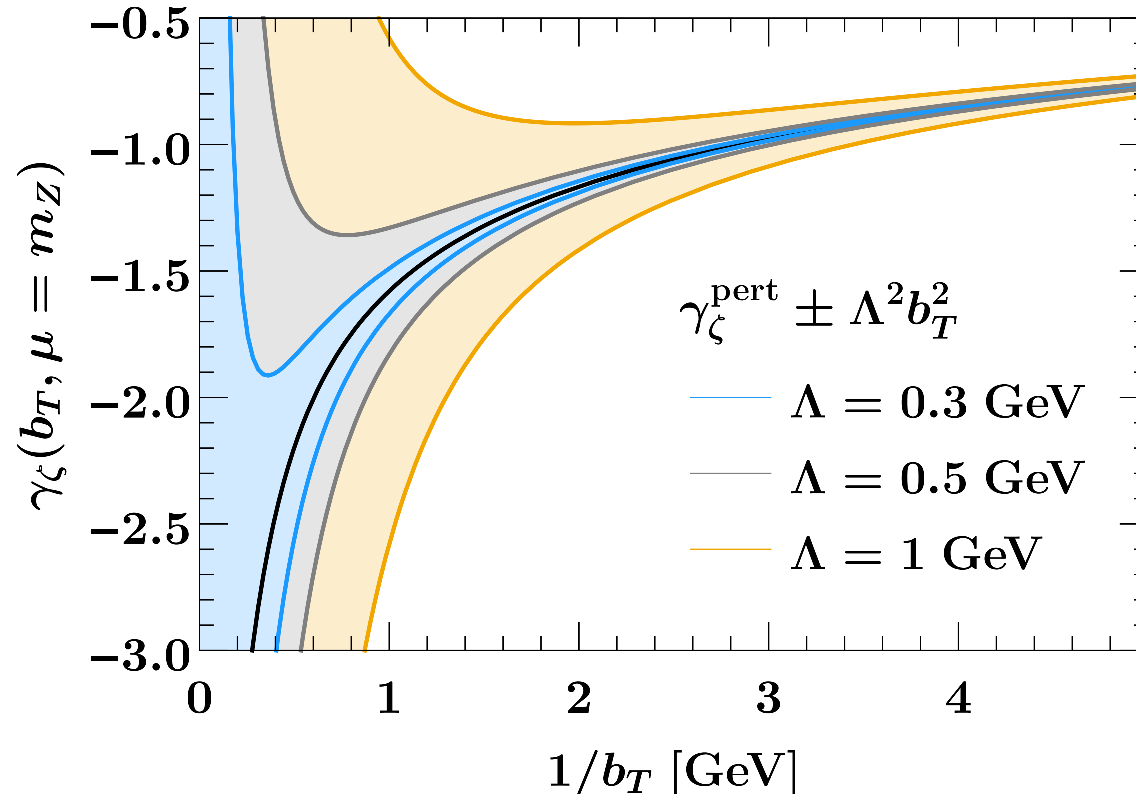
$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\Big] \exp\Big[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\Big] \times f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

Much more complicated than longitudinal PDFs.

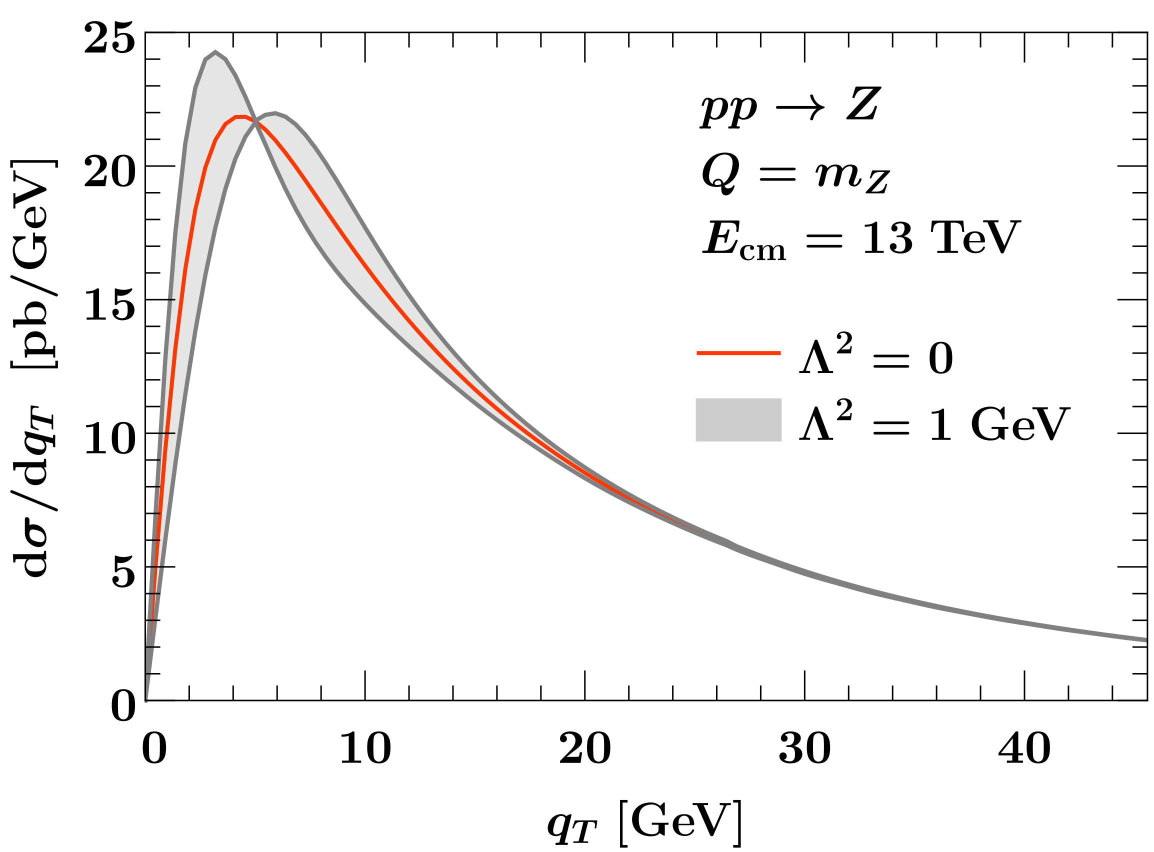
Have non-perturbative contributions for CS kernel γ_ζ^q and boundary condition $f_q(x, \vec{b}_T, \mu_0, \zeta_0)$

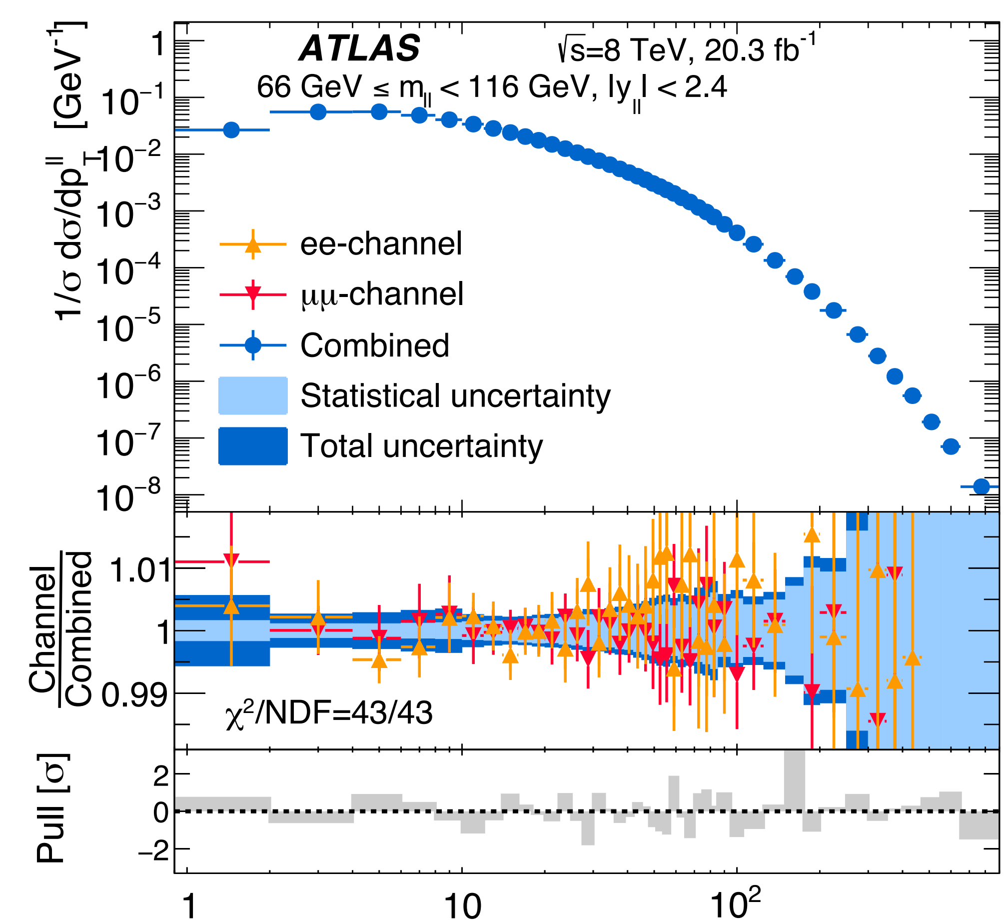
$\gamma_\zeta^q(\mu, b_T)$ Nonperturbative Contributions to Rapidity Anom. Dim.





Drell-Yan Cross Section:





[ATLAS '15]

Global Fits

SV19 = Scimemi, Vladimirov (1912.06532)

Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

Common features:

- **Unpolarized Data with constraint:** $q_T/Q < 0.2 - 0.25$ **(4-6% power corrections)**
- **Perturbative accuracy: N3LL resummation + NNLO matching to PDF**
- **Neglect small contributions from Boer-Mulders terms (higher twist for pert. b_T)**
- **Common b_T dependence for all flavors:**

$$f_{1,i/h}(x, b_T, \mu, \zeta) = f^{\rm pert}_{1,i/h}(x, b_T, \mu, \zeta)\, f_1^{\rm NP}(x, b_T)$$

(similar for TMDFF D_1)

$$f^{\rm pert}_{1,i/h}(x, b_T, \mu, \zeta) = \sum_j \int \frac{dy}{y} C_{ij}(x/y, b_T, \mu, \zeta) f_j(y, \mu)$$

- **Longitudinal PDFs input from PDF sets (MMHT, NNPDF, etc)**

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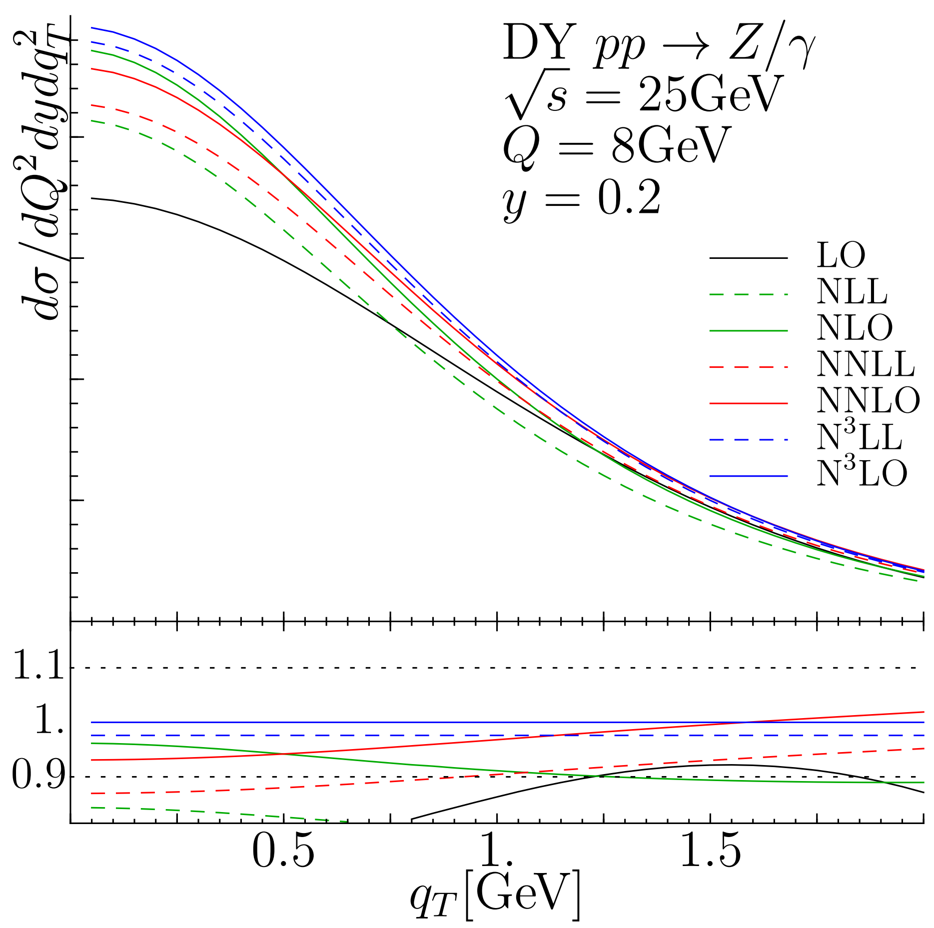
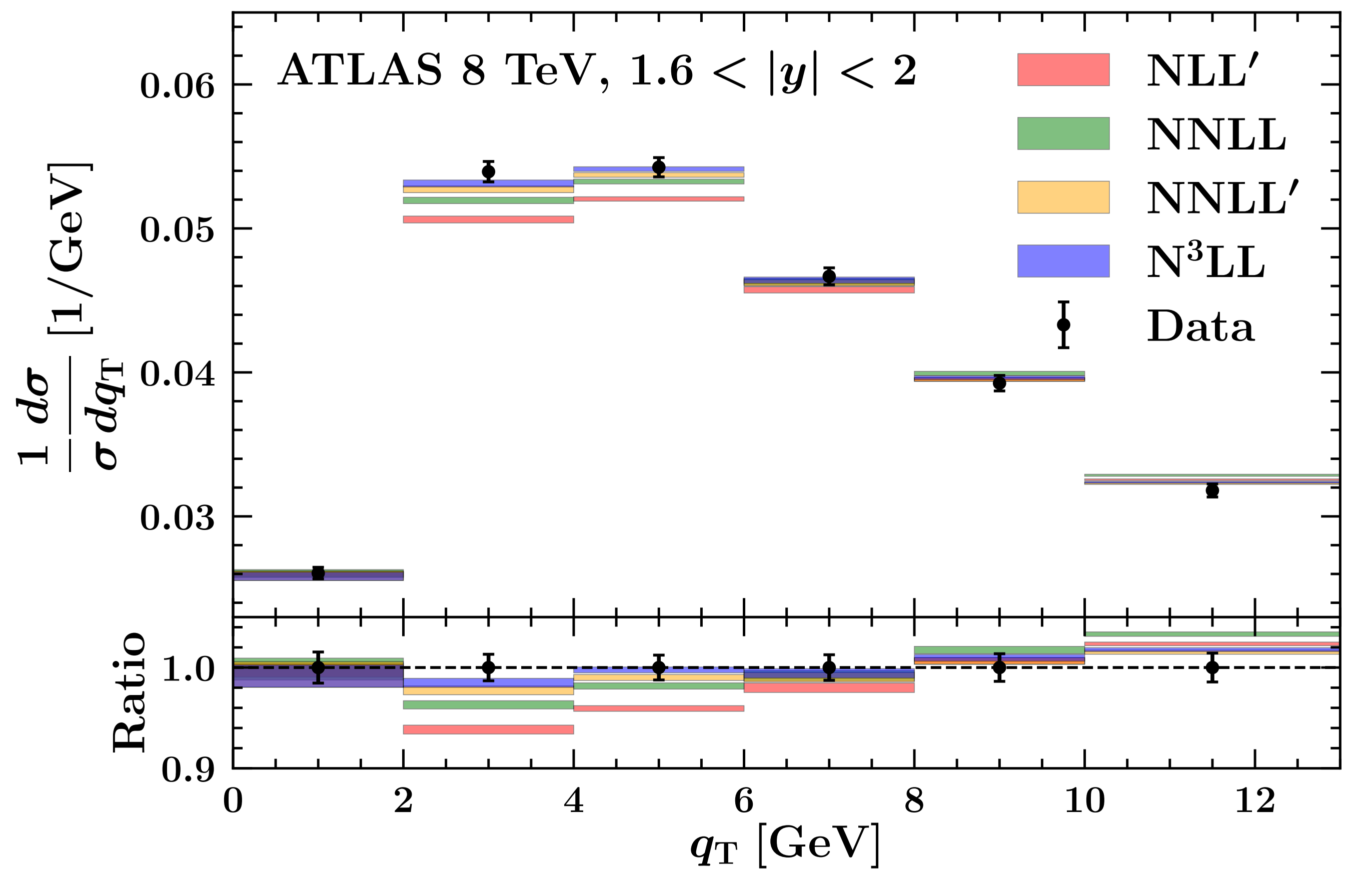
Common features:

Good Perturbative convergence:

SV19



Pavia19



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Differences:

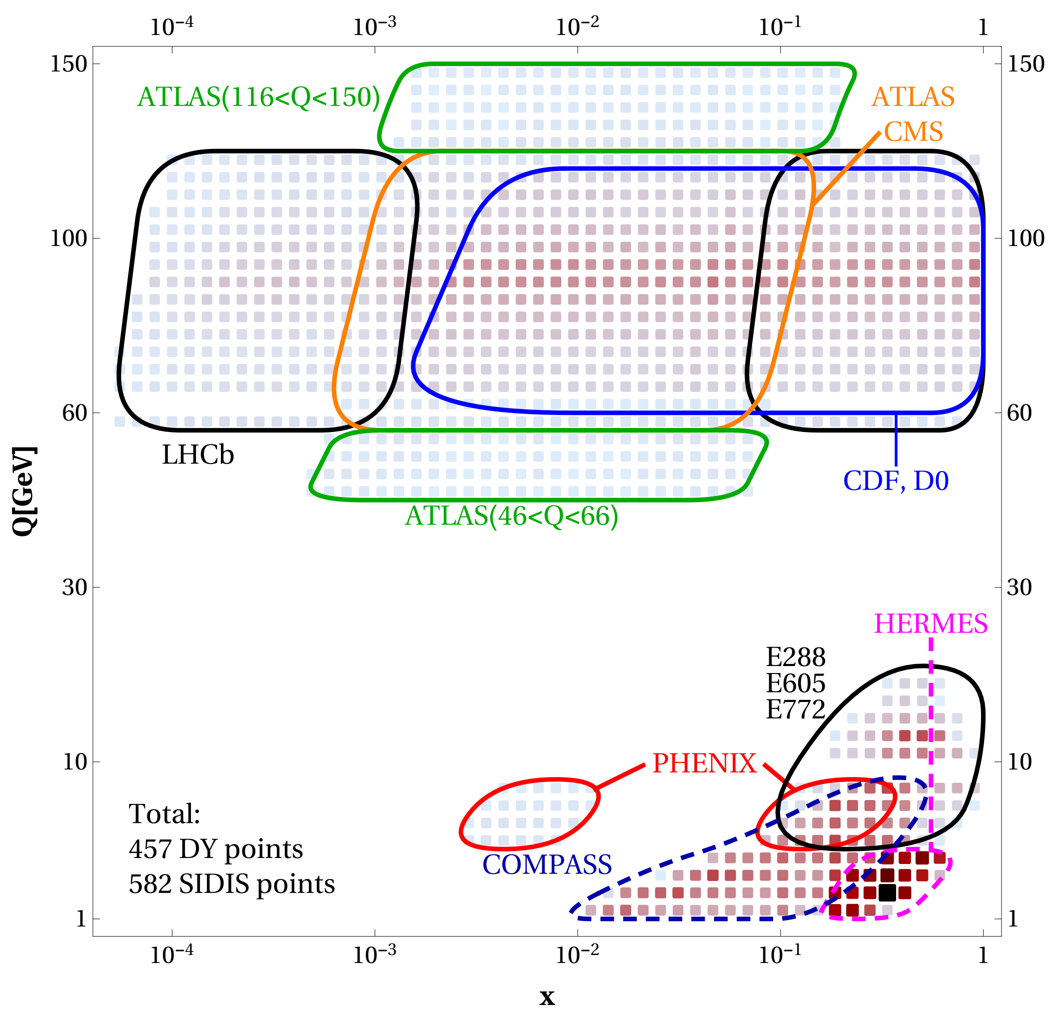
Some differences in solution of evolution equations (not discussed here)

Datasets used

SV19

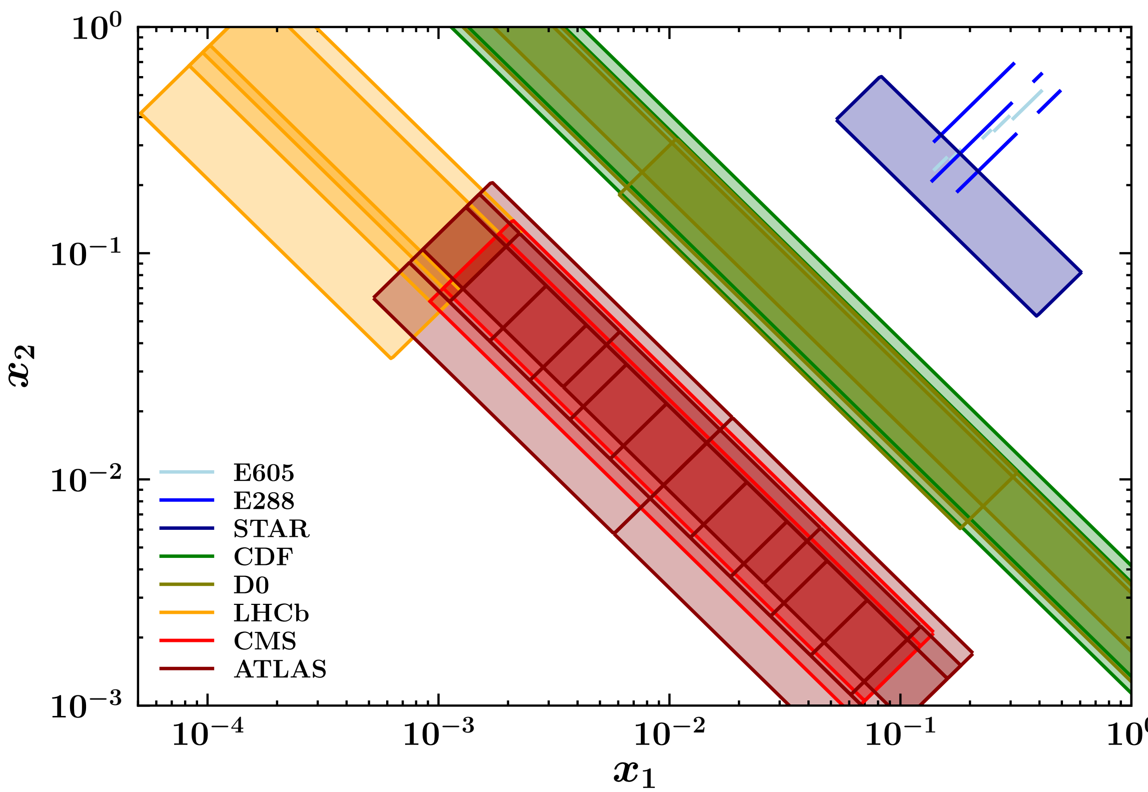
Drell-Yan (457 bins)

SIDIS (582 bins)



Pavia19

Drell-Yan (353 bins)



$$x_1 = Qe^{y}/\sqrt{s}, \quad x_2 = Qe^{-y}/\sqrt{s}$$

Global Fits

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Differences:

Non-perturbative Models

SV19

TMDPDF: 5
TMDFF: 4
CS kernel: 2

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z+\eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right)$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b^*) - \frac{1}{2}c_0 b b^*$$

$$b^*(b) = \frac{b}{\sqrt{1+\boldsymbol{b}^2/B_{\mathrm{NP}}^2}}$$

Pavia19

TMDPDF: 7
CS kernel: 2

$$f_{\mathrm{NP}}(x,b_T) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda\exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b_*) - \frac{1}{2}(g_2 b_T^2 + g_{2B} b_T^4)$$

$$b_*(b_T) = b_{\max}\left(\frac{1-\exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

Note: model form for b* used to split perturbative & non-perturbative parts

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Fit Results:

SV19

$$\chi^2/N_{pt} = 1.06$$

NP-parameters			
RAD	$B_{\rm NP} = 1.93 \pm 0.22$	$c_0 = (4.27 \pm 1.05) \times 10^{-2}$	
TMDPDF	$\lambda_1 = 0.224 \pm 0.029$	$\lambda_2 = 9.24 \pm 0.46$	$\lambda_3 = 375. \pm 89.$
	$\lambda_4 = 2.15 \pm 0.19$	$\lambda_5 = -4.97 \pm 1.37$	
TMDFF	$\eta_1 = 0.233 \pm 0.018$	$\eta_2 = 0.479 \pm 0.025$	
	$\eta_3 = 0.472 \pm 0.041$	$\eta_4 = 0.511 \pm 0.040$	

Pavia19

$$\chi^2/N_{pt} = 1.02$$

Parameter	Value
g_2	0.036 ± 0.009
N_1	0.625 ± 0.282
α	0.205 ± 0.010
σ	0.370 ± 0.063
λ	0.580 ± 0.092
N_{1B}	0.044 ± 0.012
α_B	0.069 ± 0.009
σ_B	0.356 ± 0.075
g_{2B}	0.012 ± 0.003

Low and High energy data are well described

RAD parameters are less sensitive to input PDF set

Universality of RAD satisfied by DY vs. SIDIS data

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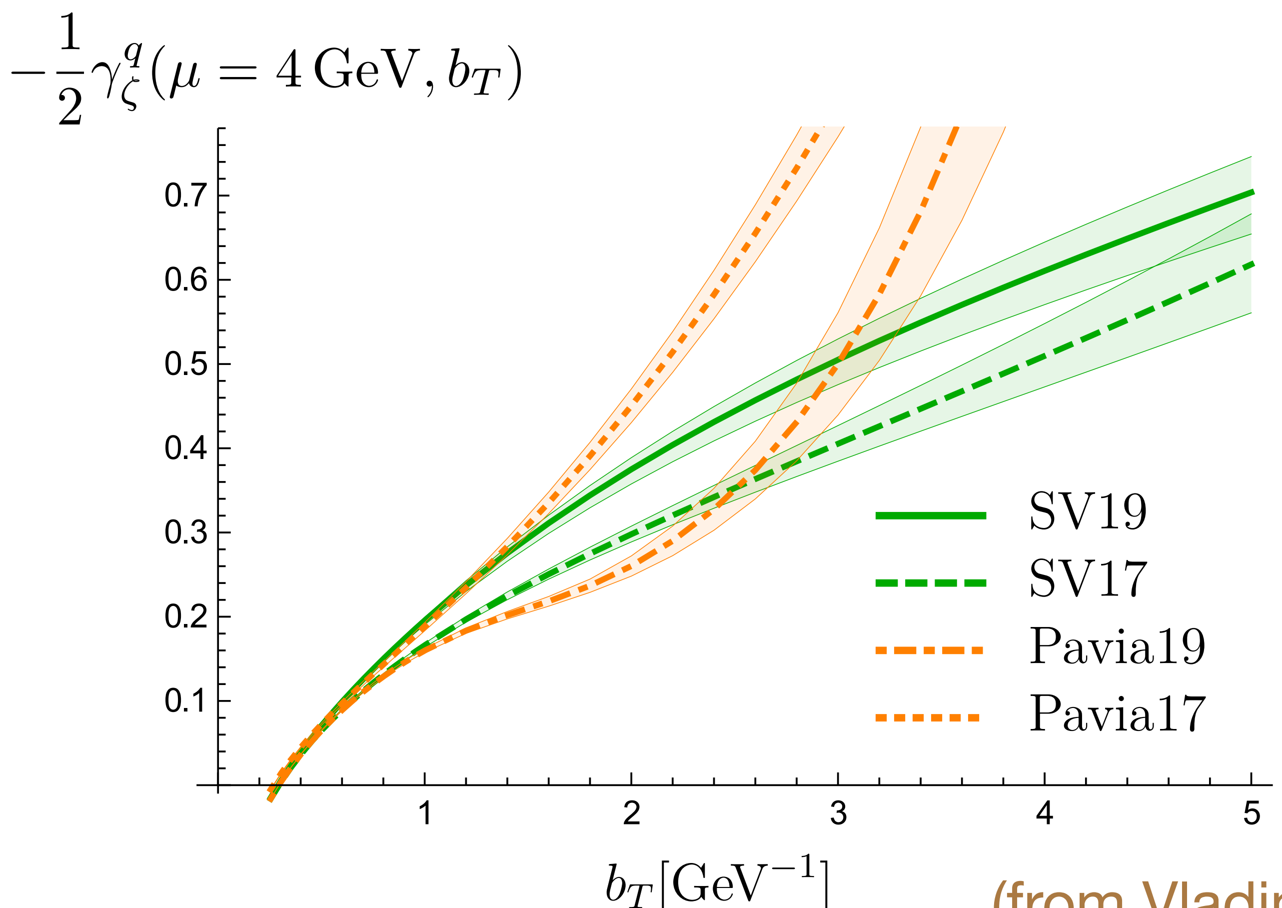
Fit Results:

Comparison of results for CS Kernel in non-perturbative regime:



(from Vladimirov, 2003.02288)

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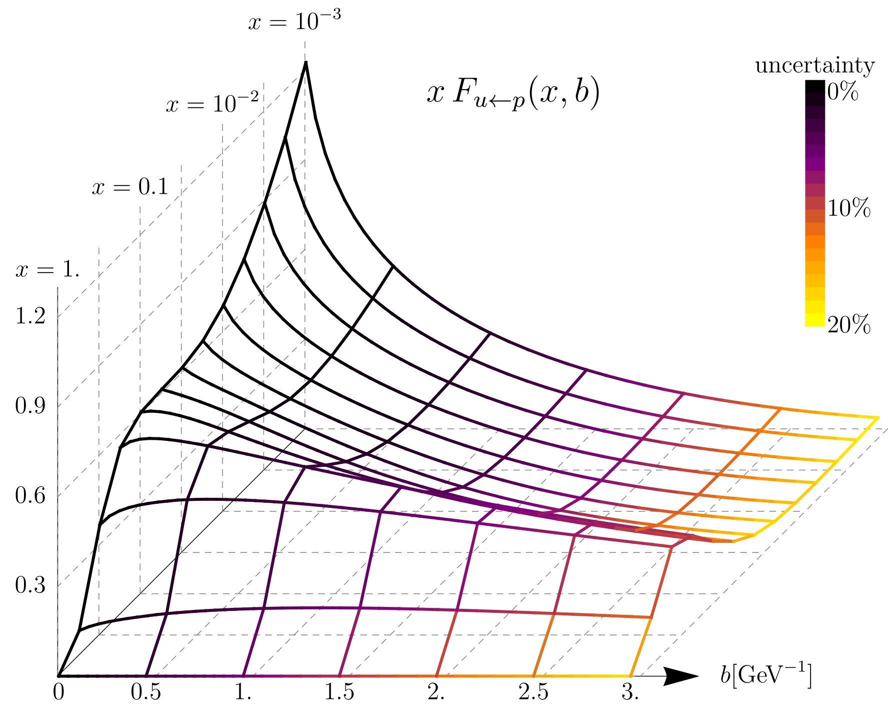
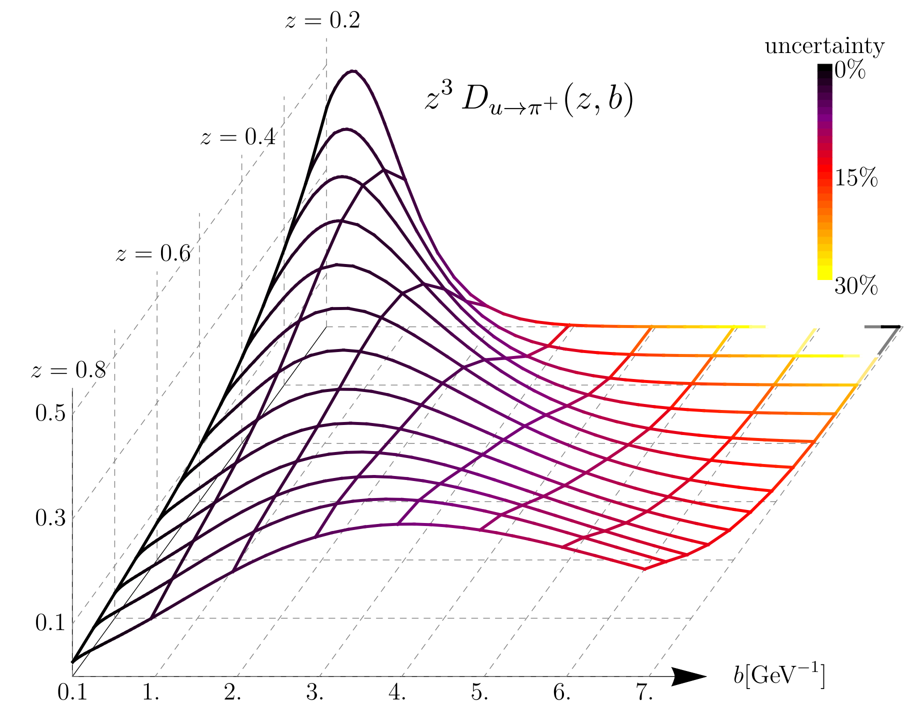
Fit Results:

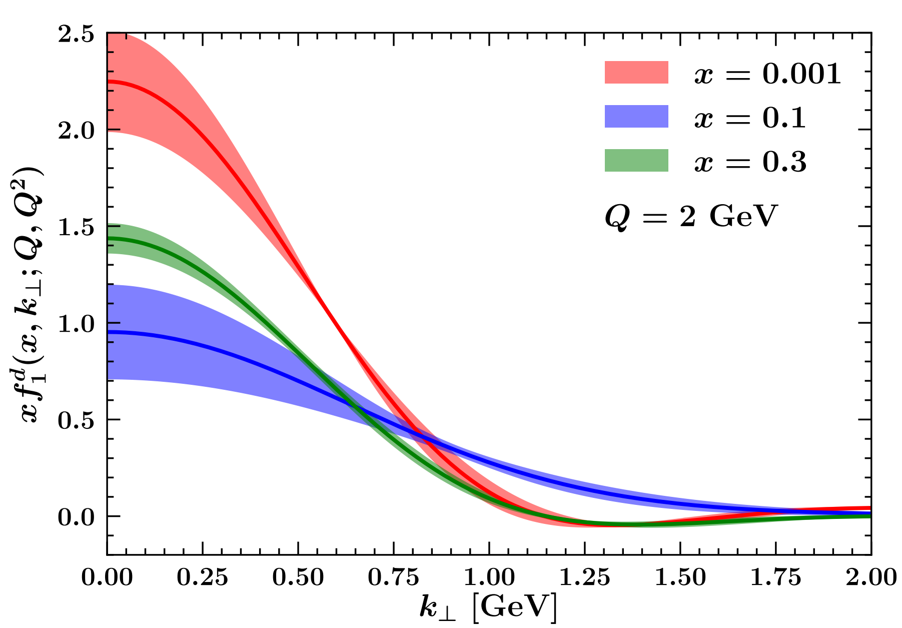
Results for intrinsic TMDPDF (& TMDFF)

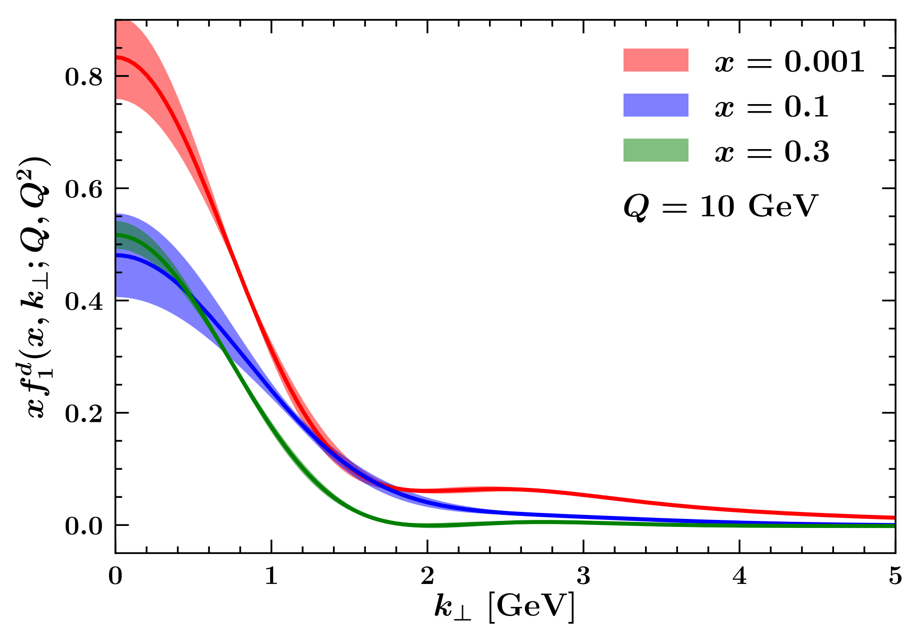
SV19





Pavia19





Quite precise determinations if we assume a given fit form.

Global Fits

Bury, Prokudin, Vladimirov (2012.05135)

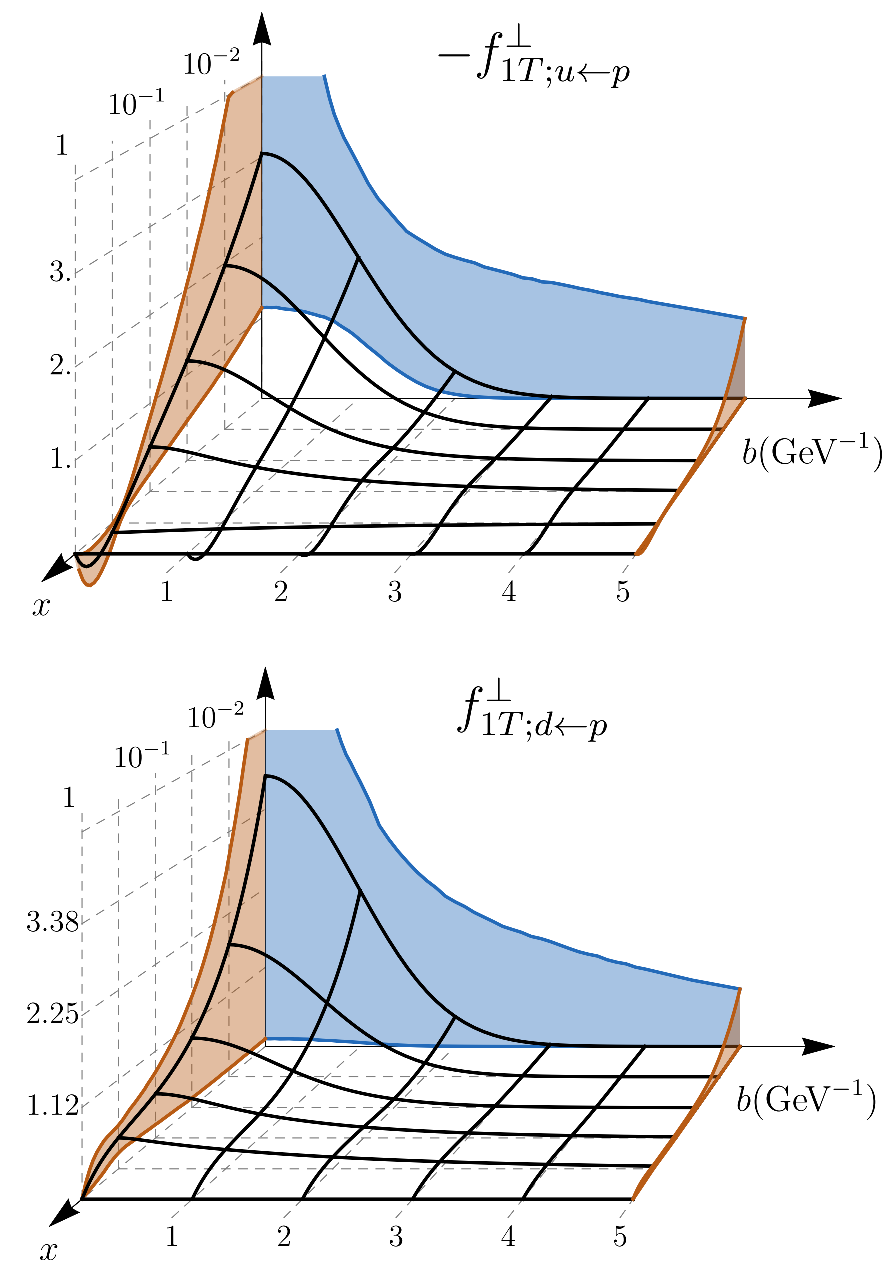
Extraction of Sivers function from global fit to SIDIS, DY, and W/Z data [76 bins: HERMES, COMPASS, Jlab (SIDIS); STAR(W/Z); COMPASS (DY)]

$$f_{1T}^{\perp\,\mathrm{SIDIS}} = -f_{1T}^{\perp\,\mathrm{DY}}$$

N3LL analysis following SV19

Flavor dependent parametrization (no matching)

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0+xr_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$



<u>Results:</u>

Good global fit: $\chi^2/N_{pt} = 0.88$

Opposite signs for up and down Sivers functions

Data not precise enough to confirm sign flip

$f_{1T}^{\perp\,\mathrm{SIDIS}} = +f_{1T}^{\perp\,\mathrm{DY}}$ **gives** $\chi^2/N_{pt} = 1.0$

Targets for Lattice QCD:

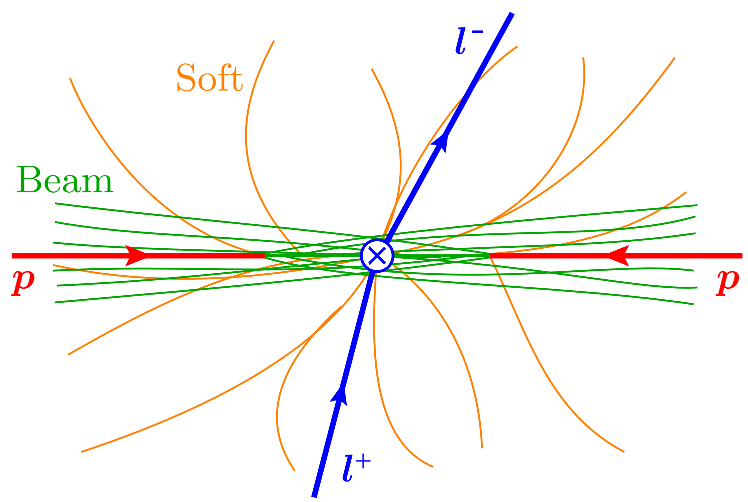
- Non-perturbative CS Kernel
- Info on Spin-dependent TMDPDFs (in ratios)
- Info about 3D structure, x and b_T (in ratios)
- proton vs. pion TMDPDFs (in ratios)
- flavor dependence of TMDPDFs (in ratios)
- TMDPDF with x and b_T (normalization)
- Gluon TMDPDFs [repeat items above]

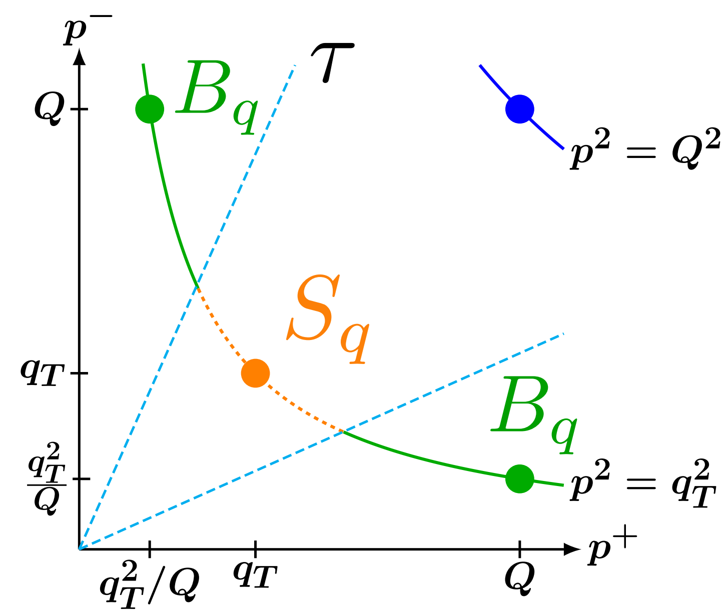
TMD Definitions

Beam Function

Soft factor

$$f_q(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to 0,\tau\to 0} Z_{\rm uv}(\epsilon,\mu,\zeta) B_q(x,\vec{b}_T,\epsilon,\tau,\zeta)\ \Delta_q(b_T,\epsilon,\tau)$$





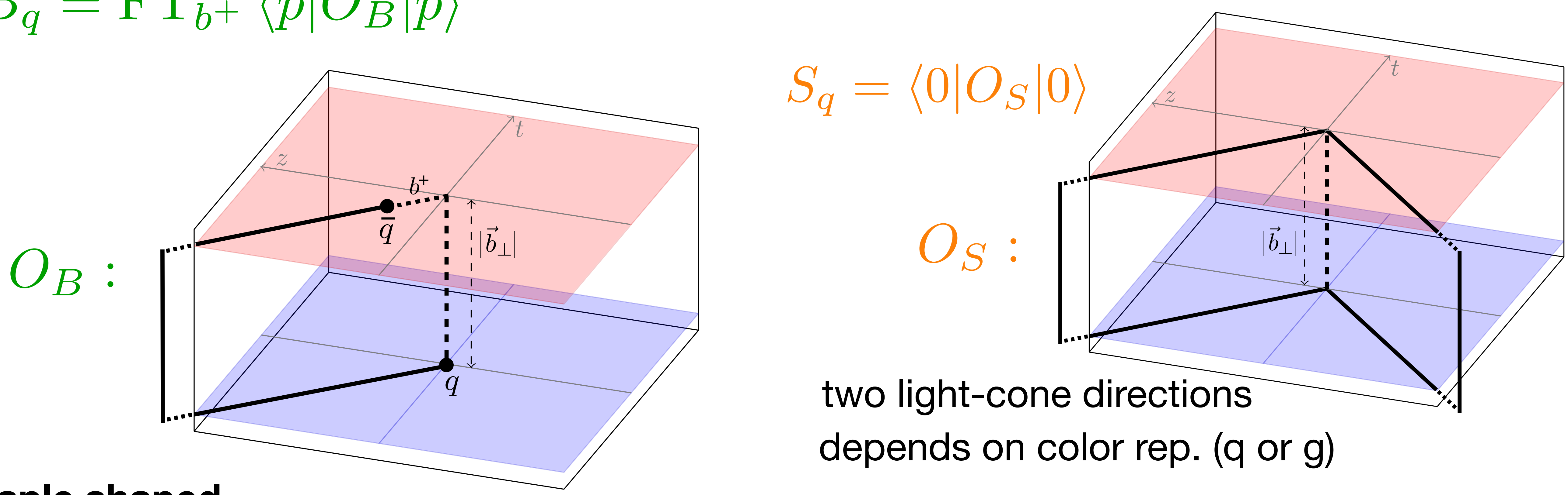
contains

S_q & subtractions

$\Delta_q = 1/\sqrt{S_q}$

$$B_q = \mathrm{FT}_{b^+}\,\langle p|O_B|p\rangle$$

$$S_q = \langle 0|O_S|0\rangle$$



two light-cone directions

depends on color rep. (q or g)

staple shaped Wilson lines

Lattice calculations must overcome light-cone nature of objects.

Quasi-PDFs

(Xiangdong Ji 2013)

- **Consider a purely spatial operator**

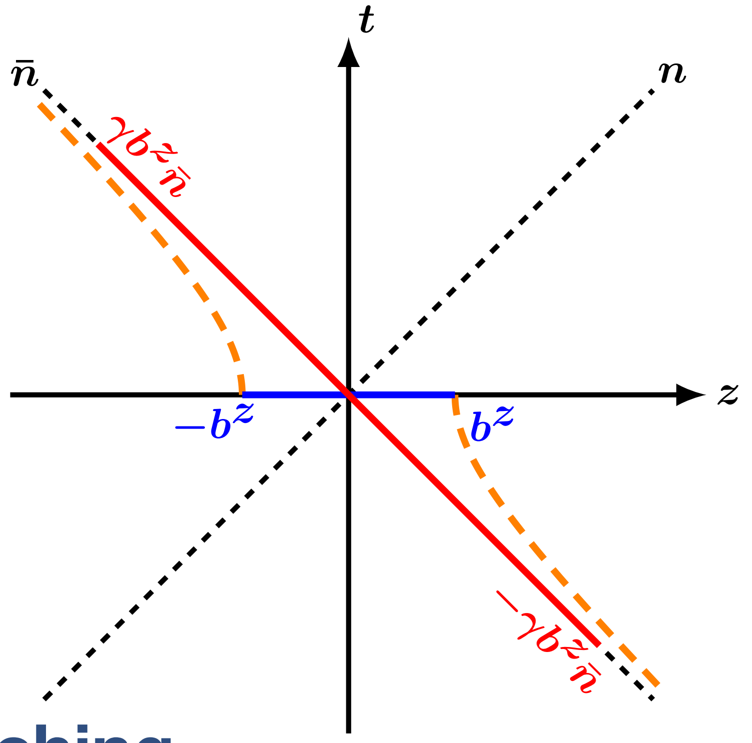
$$\tilde{f}_q(x, P^z, \epsilon) = \int \frac{db^z}{4\pi} e^{ib^z x P^z} \langle p(P) | \bar{q}(b^z) W_z(b^z, 0) \gamma^0 q(0) | p(P) \rangle$$

quasi-PDF

- **Relate to light-cone operator for PDF by a boost**

$$\text{boost to } \mathcal{O} \Leftrightarrow \text{boost to proton state}$$

take $\Lambda_{\rm QCD} \ll P^z$ (finite large P^z) **"LaMET"**



- **quasi-PDF and PDF must have same IR physics**

- **Differences in UV accounted for by perturbative matching**

$$\tilde{f}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|} C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right) \boxed{f_j(y, \mu)} + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

quasi-PDF computable with Lattice QCD

Perturbative matching coefficient

PDF

Power corrections

Quasi-TMDPDFs

UV renormalization & scheme change

$$\tilde{f}_q(x,\vec{b}_T,\mu,P^z) = \int \frac{db^z}{2\pi}\, e^{ib^z(xP^z)} \lim_{\substack{a\to 0\\ L\to\infty}} \tilde{Z}'_q(b^z,\mu,\tilde{\mu})\tilde{Z}^q_{\rm uv}(b^z,\tilde{\mu},a) \times \tilde{B}_q(b^z,\vec{b}_T,a,L,P^z)\tilde{\Delta}^q_S(b_T,a,L)$$

quasi-Beam function

quasi-soft factor

a = lattice spacing (UV regulator)

- needs to be computable with Lattice QCD
- must have same IR physics as TMDPDF

  (including $b_T \sim \Lambda^{-1}_{\rm QCD}$ dependence)

(isovector quark operators u-d, from here on)

Quasi-Beam Functions

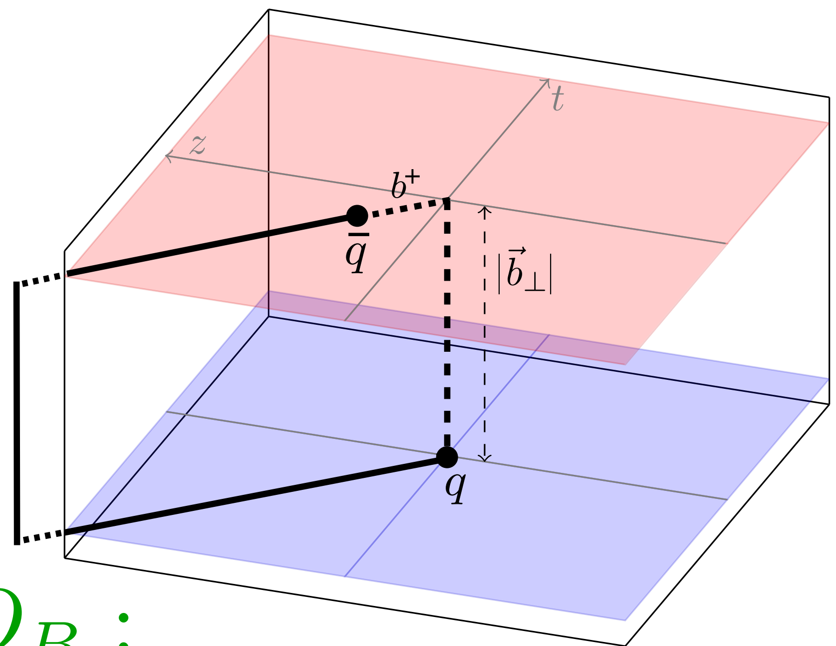
$\tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$

Natural Quasi-Beam Function

$\tilde{B}_q = \langle p|\tilde{O}_B|p\rangle$

Beam Function

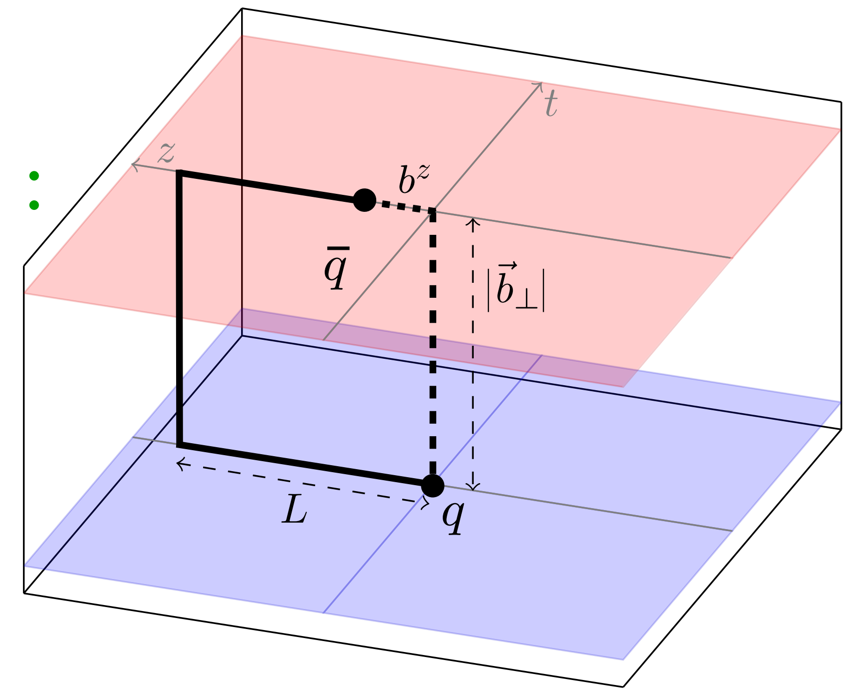
$B_q = \langle p|O_B|p\rangle$



O_B :

Connected by boost (for bare operators)

$\tilde{O}_B$:



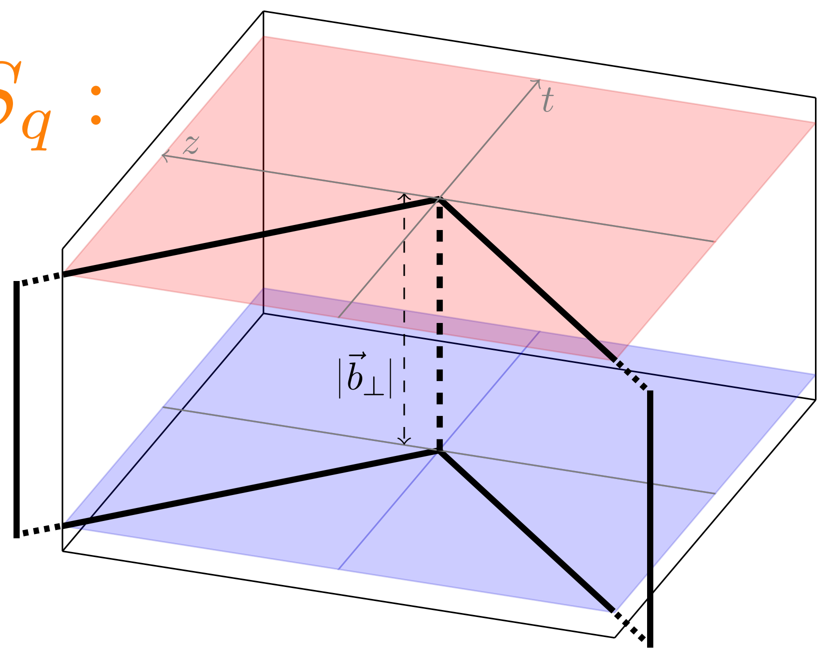
$$\frac{1}{P^z} \ll b_T \ll L$$

- **Finite length L for Wilson lines, regulates rapidity divergences** $\frac{1-e^{-ik^zL}}{k^z}$
- **Spatial lines, so have power law UV divergence** $\propto \text{length} = 2L + b_T - b^z$

Quasi-Soft Function

$\tilde{\Delta}_S^q = 1/\sqrt{\tilde{S}_q}$ $\tilde{S}_q = \langle 0|\tilde{O}_S|0\rangle$

S_q :



? $\leftarrow$ $\tilde{S}_q$

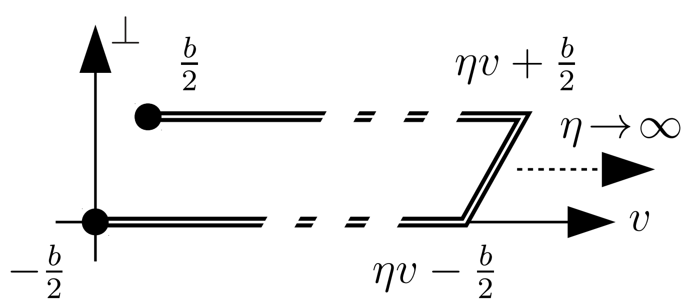
No connection via a boost.

One way around this is to study ratios where $\tilde{S}_q$ cancels.

Musch, Hagler, Engelhardt, Negele, Schafer ’10,’11,’15
Yoon et.al.’17

Use Lorentz Invariance to relate space-like and equal-time paths

$$\widetilde{\Phi}_i^{[\Gamma]}(b,P',P,S,v,\eta,a) = \frac{1}{2}\left\langle P',S\middle|\bar{\psi}_i^0(b^\mu/2)\Gamma W^v_{\sqsubset\eta}(b^\mu/2,-b^\mu/2)\psi_i^0(-b^\mu/2)\middle|P,S\right\rangle$$



connection to bare (Collins) TMDPDF requires $\eta \to \infty$, $\hat{\zeta} \to \infty$

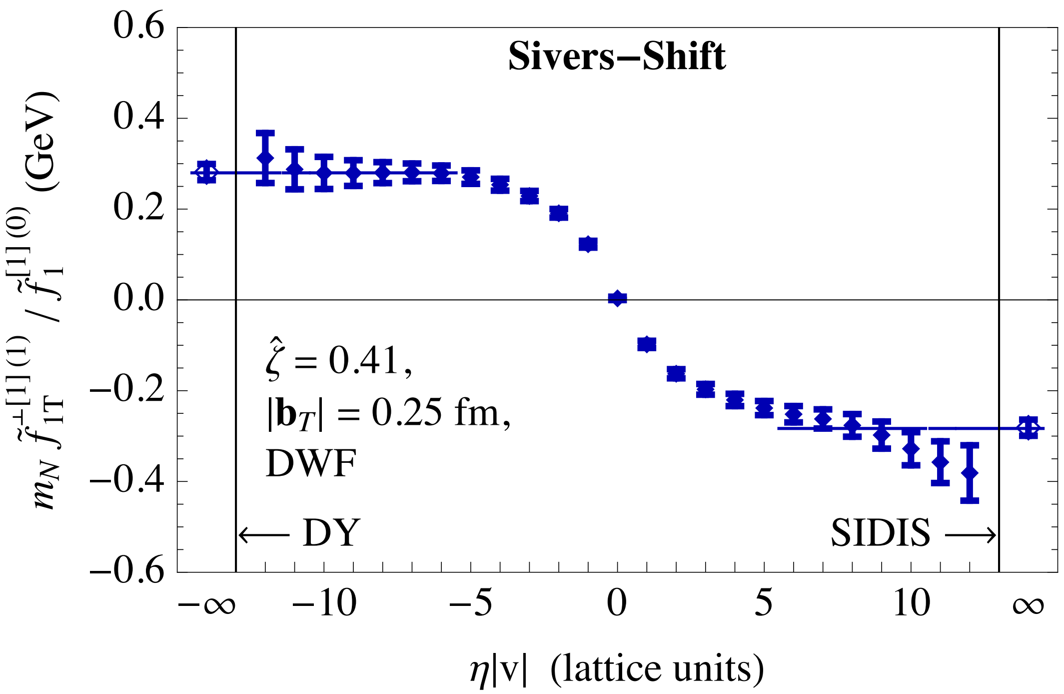
Lorentz Invariant
$P \cdot b$
b^2
$\hat{\zeta} = \dfrac{v \cdot P}{m_p\sqrt{-v^2}}$
$\dfrac{v \cdot b}{\sqrt{-v^2}}$
$\eta^2 v^2$

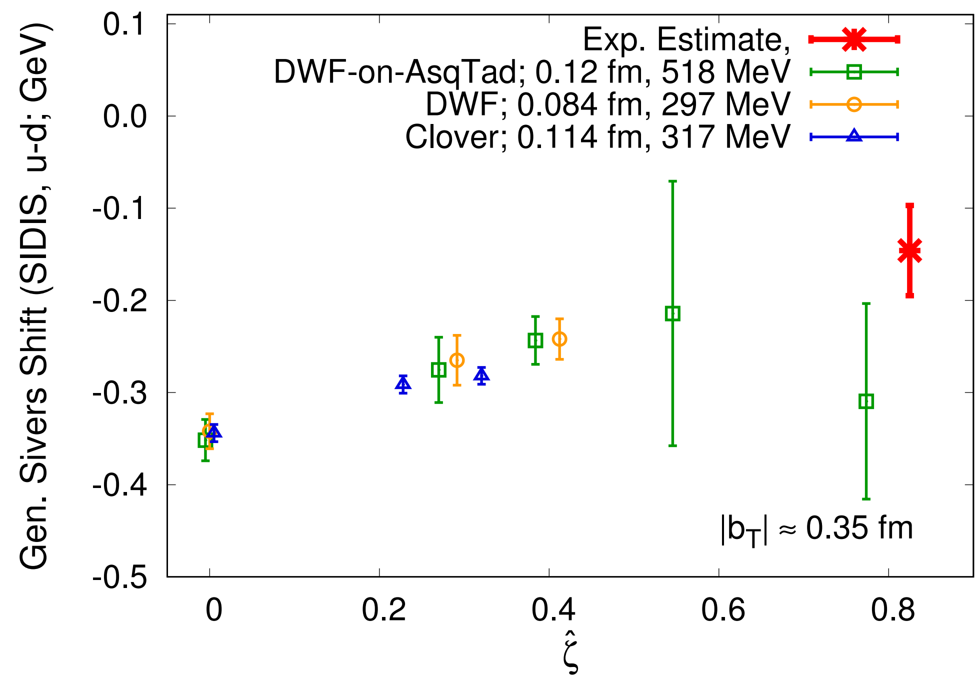
Lorentz Invariant Approach

Yoon, Engelhardt, Gupta, Bhattacharya, Green, Musch, Negele, Pochinsky, Schafer, Syritsyn (1706.03406)

Eg. ratio constraining Sivers function (u-d flavor)

$$\frac{\int dx\, f_{1T}^{\perp}(x, b_T, \ldots)}{\int dx\, f_1(x, b_T, \ldots)} = \lim_{\eta\to\infty, \hat{\zeta}\to\infty} \frac{\tilde{\Phi}^{[f_{1T}^{\perp}]}(b^z = 0, b_T, a, \ldots)}{\tilde{\Phi}^{[f_1]}(b^z = 0, b_T, a, \ldots)} + \ldots$$





Observe sign flip in gen. Sivers shift

Correct trend towards experimental result

Quasi-Soft Function

$\tilde{\Delta}_S^q = 1/\sqrt{\tilde{S}_q}$ $\tilde{S}_q = \langle 0|\tilde{O}_S|0\rangle$

- **Cancel power law dependence on L, length =** $2(2L+b_T)$
- **Needed to reproduce infrared structure.**

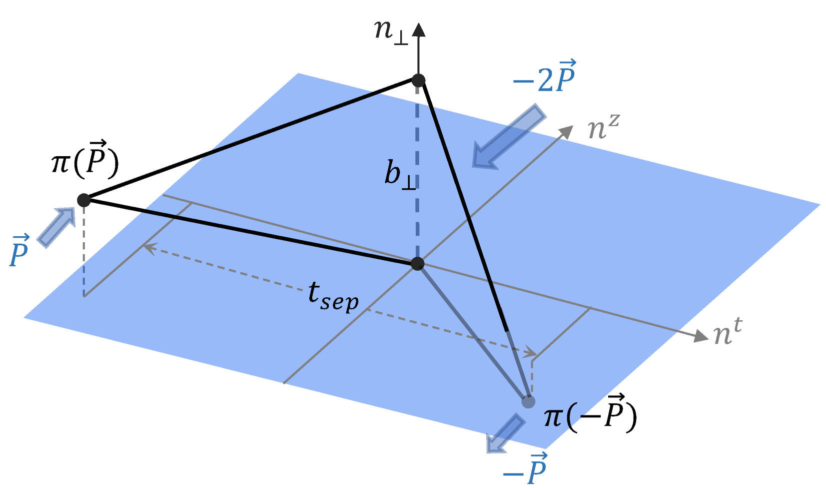
Can be extracted from TMD factorization theorem for light meson form factor F
& quasi-TMD light meson wavefunction $\tilde{\phi}$

[Ji, Liu, Liu 1910.11415]

(see parallel talk by Yizhuang Liu)

$$\tilde{S}_q = \frac{F(b_\perp, P\cdot P')}{\int dx dx' H(x,x',P,P')\widetilde{\phi}(x',b_\perp,P')\widetilde{\phi}^\dagger(x,b_\perp,P)}$$

$$F(b_\perp, P^z) = \langle \pi(-\vec{P})|(\overline{q}_1\Gamma q_1)(\vec{b})(\overline{q}_2\Gamma q_2)(0)|\pi(\vec{P})\rangle_c$$



$$\tilde{\phi}(x,b_\perp,P) = \lim_{L\to\infty} P^z \int \frac{dz}{4\pi} \frac{\langle P|\bar{\psi}(z\hat{z}/2+\vec{b}_\perp)\tilde{\Gamma}W_z\psi(-z\hat{z}/2)|0\rangle}{\sqrt{Z_E(2L,b_\perp,Y=0)}}$$

Quasi-TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0 \\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a) \times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

- **linear divergences in L cancel**
- $\tilde{Z}^q_{\rm uv}$ **multiplicative renormalization (matrix with operator mixing on lattice)**
- $\tilde{Z}'_q$ **converts lattice friendly scheme ($\tilde{\mu}$) to $\overline{\rm MS}$ (μ)**

Relation between Quasi-TMDPDF & TMDPDF

[Ebert, IS, Zhao '18]
[Ji, Liu, Liu '19]

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

nonperturbative quasi-TMDPDF — **perturbative kernel** — **nonperturbative CS kernel** — **nonperturbative TMDPDF**

$$+\mathcal{O}\Big(\frac{1}{(b_T x P^z)^2}, \frac{\Lambda_{\rm QCD}}{xP^z}\Big)$$

(Note: no convolution in x)

- $C^{\rm TMD}(\mu, xP^z)$ **is spin independent**

$$\frac{g_{1L}(x,b_T,\mu,\zeta)}{f_1(x,b_T,\mu,\zeta)} = \frac{\tilde{g}_{1L}(x,b_T,\mu,P^z)}{\tilde{f}_1(x,b_T,\mu,P^z)}\,, \quad \frac{h_1(x,b_T,\mu,\zeta)}{f_1(x,b_T,\mu,\zeta)} = \frac{\tilde{h}_1(x,b_T,\mu,P^z)}{\tilde{f}_1(x,b_T,\mu,P^z)}\,, \quad \frac{h_{1T}^\perp(x,b_T,\mu,\zeta)}{f_1(x,b_T,\mu,\zeta)} = \frac{\tilde{h}_{1T}^\perp(x,b_T,\mu,P^z)}{\tilde{f}_1(x,b_T,\mu,P^z)}$$

Ebert, Schindler, IS, Zhao '20; Vladimirov, Schafer '20

$$\frac{f_1^\perp(x,b_T,\mu,\zeta)}{f_1(x,b_T,\mu,\zeta)} = \frac{\tilde{f}_1^\perp(x,b_T,\mu,P^z)}{\tilde{f}_1(x,b_T,\mu,P^z)}$$

Ji, Liu, Schaefer, Yuan '20

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

quasi-Beam fns.

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z) \int db^z e^{ib^z xP_1^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z) \int db^z e^{ib^z xP_2^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_2^z)}$$

- **does not require $\tilde{\Delta}_S^q$**
- **LHS independent of $P_1^z, P_2^z, x,$ hadron state, spin**
- **can setup $\tilde{Z}^q_{\rm uv}$ to remove power law divergences in num/den**

$C^{\text{TMD}}(\mu, xP^z)$ in $\overline{\text{MS}}$ at 1-loop

[Ji, Jin, Yuan, Zhang, Zhao '18]
[Ebert, IS, Zhao '18]

$$C^{\text{TMD}}(\mu, xP^z) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{(2xP^z)^2}{\mu^2} + 2\ln\frac{(2xP^z)^2}{\mu^2} - 4 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

$\tilde{Z}^q_{\text{uv}}, \tilde{Z}'_q$ **One-loop renormalization with lattice regularization, RI/MOM**

Constantinou, Panagopoulos, Spanoudes, 1901.03862

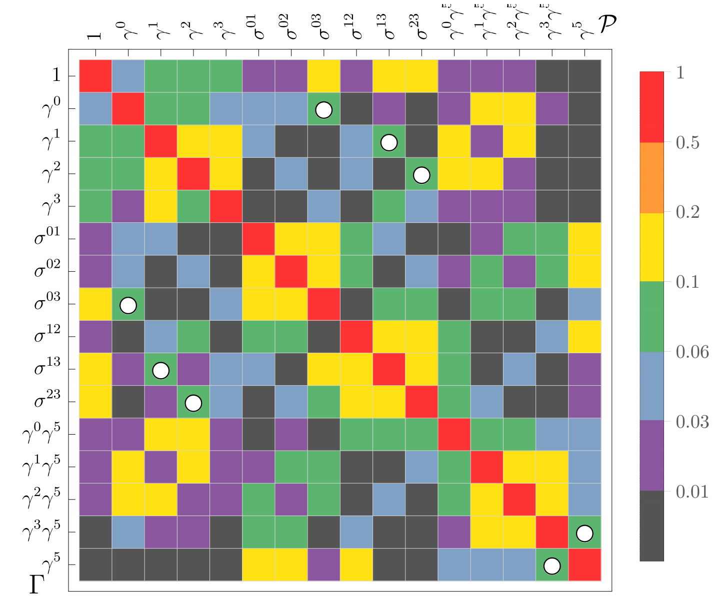
$\tilde{Z}'_q$ **conversion factor between RI/MOM scheme and $\overline{\text{MS}}$ at 1-loop**

Ebert, IS, Zhao, 1910.08569

$\tilde{Z}^q_{\text{uv}}$ **Nonperturbatively on Lattice in an RI/MOM scheme**

P. Shanahan, M. Wagman, Y. Zhao, 1911.00800

full 16x16 mixing matrix



First Lattice Result for Rapidity Anomalous Dimension

P. Shanahan, M. Wagman, Y. Zhao arXiv:2003.06063

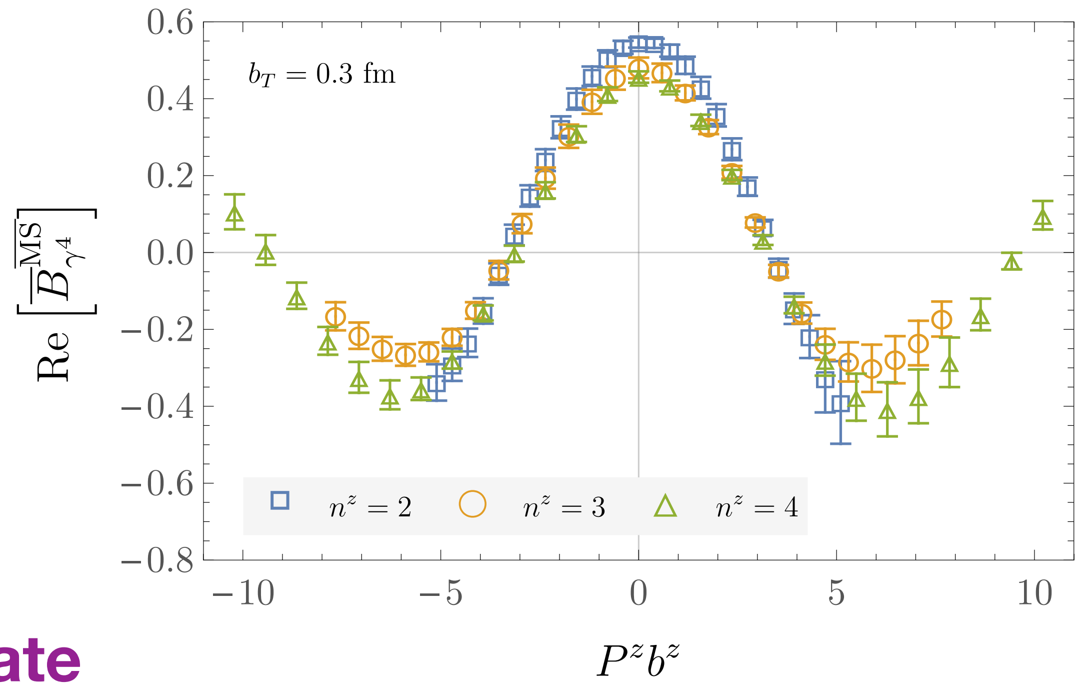
nf=0 (quenched) simulation

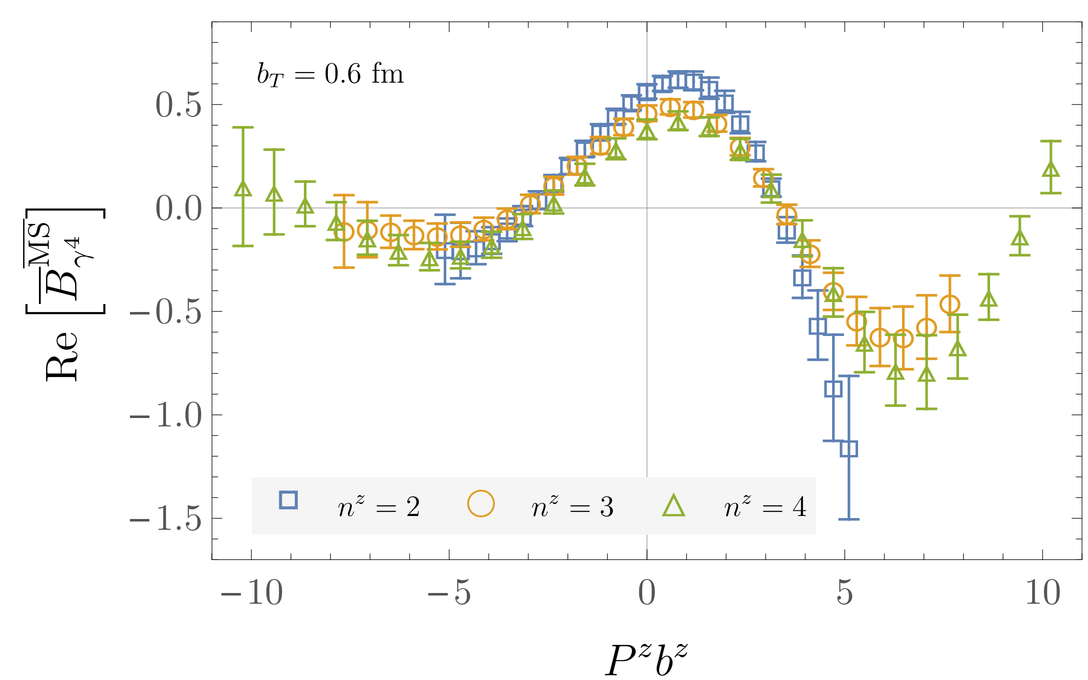
Exploits universality: uses 1.2 GeV pseudoscalar meson

Includes nonperturbative renormalization

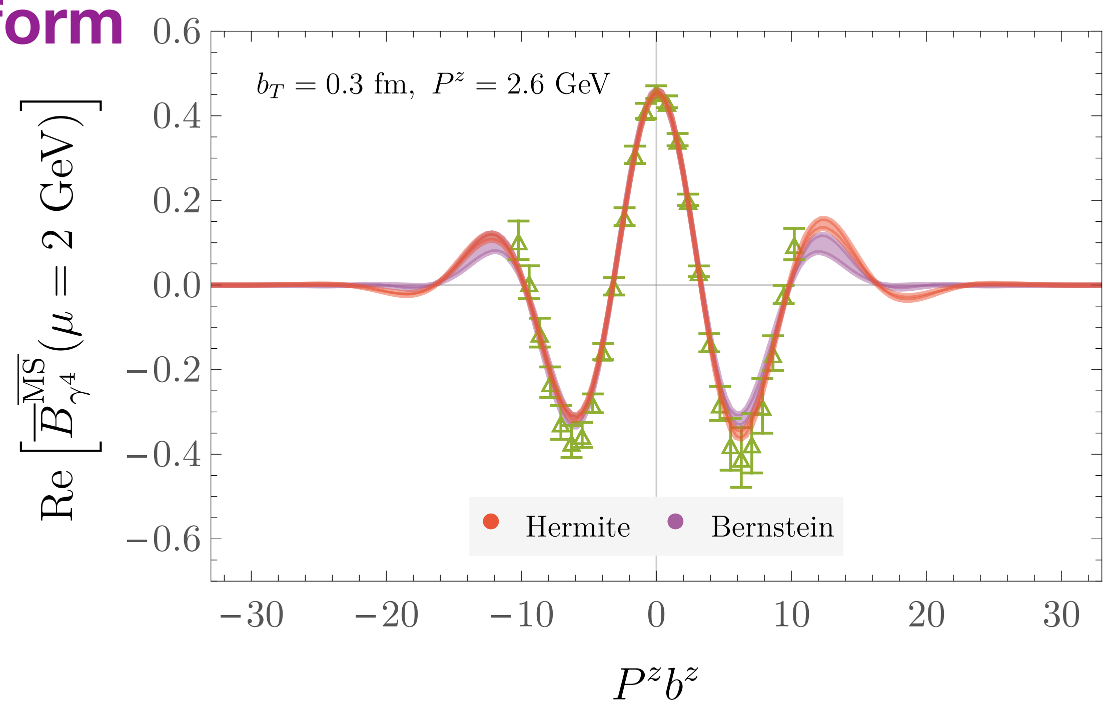
$P^z \in \{1.29, 1.94, 2.58\}$ GeV

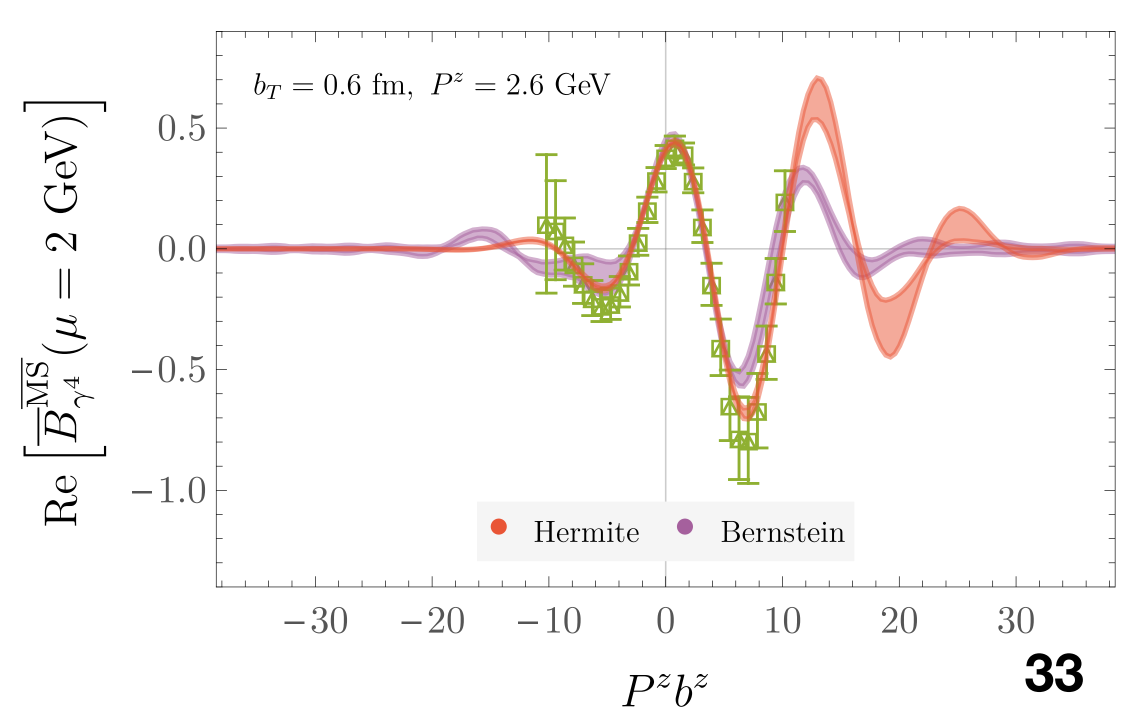
Renormalized quasi-Beam Fn.





Fits to facilitate Fourier transform





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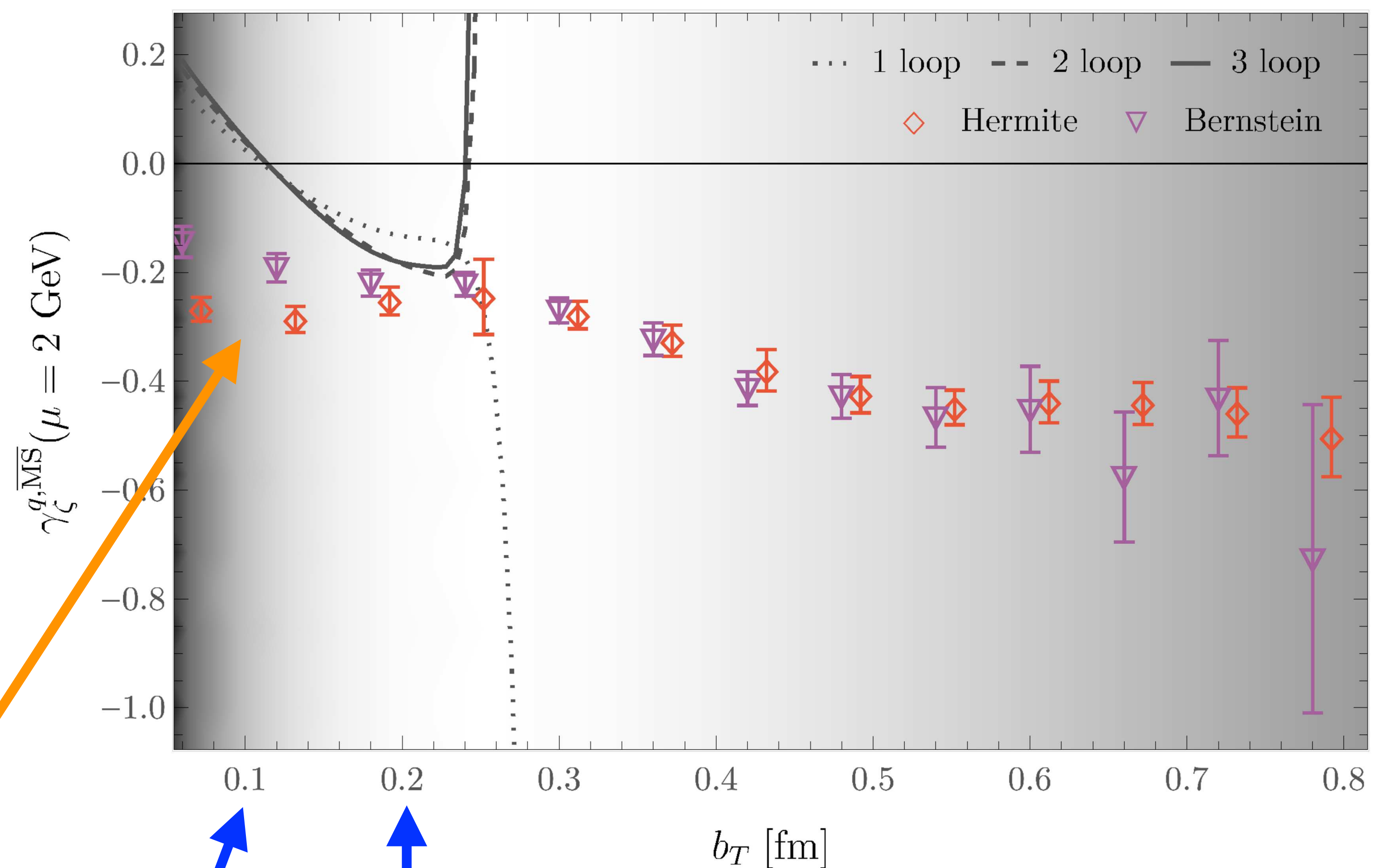
Includes nonperturbative renormalization, tree level matching

$P^z \in \{1.29, 1.94, 2.58\}$ GeV

Result for Nonperturbative TMD Rapidity Anomalous Dimension (nf=0)



Larger $1/(b_T P^z)$ power corrections (not included in error bars)

$(2\text{ GeV})^{-1}$ $(1\text{ GeV})^{-1}$

TMD Soft Function Calculation

Zhang, Hua, Hua, Ji, Liu^2, Schlemmer Schafer, Sun, Wang, Yang (LPC) 2005.14572

nf=2+1 simulation, $m_\pi = 547\,\mathrm{MeV}$

No renormalization, tree level matching

(see parallel talk by Qi-An Zhang)

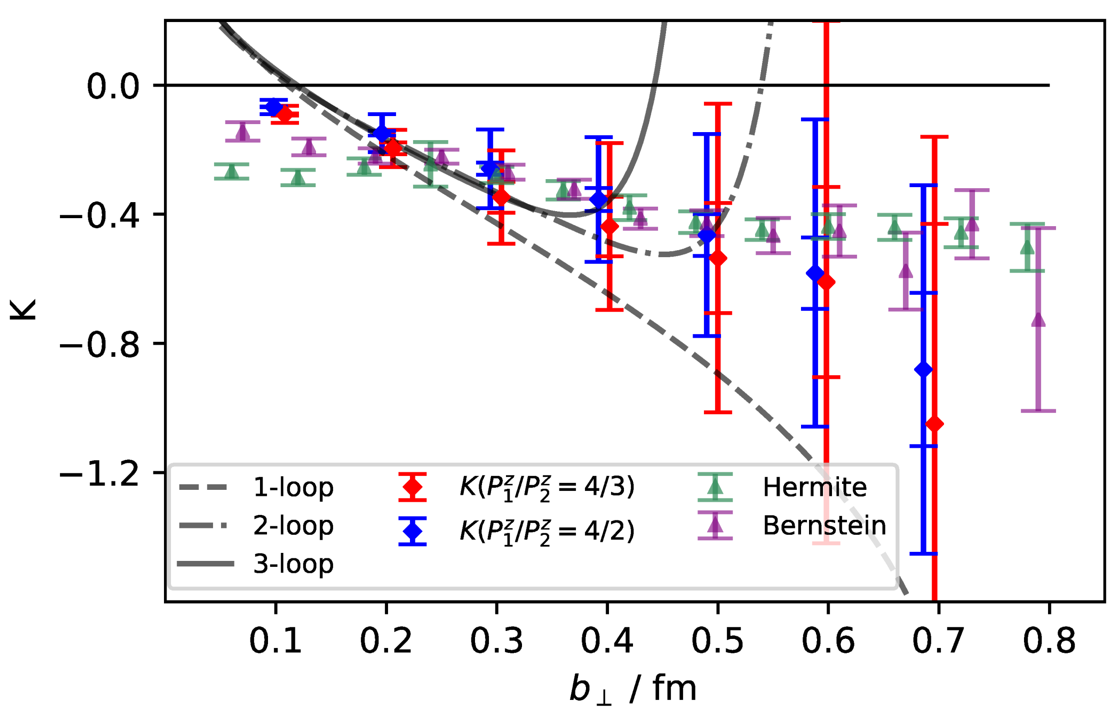
(also parallel talk by Schicheng Xia)
(also parallel talk by Min-Hua Chu)

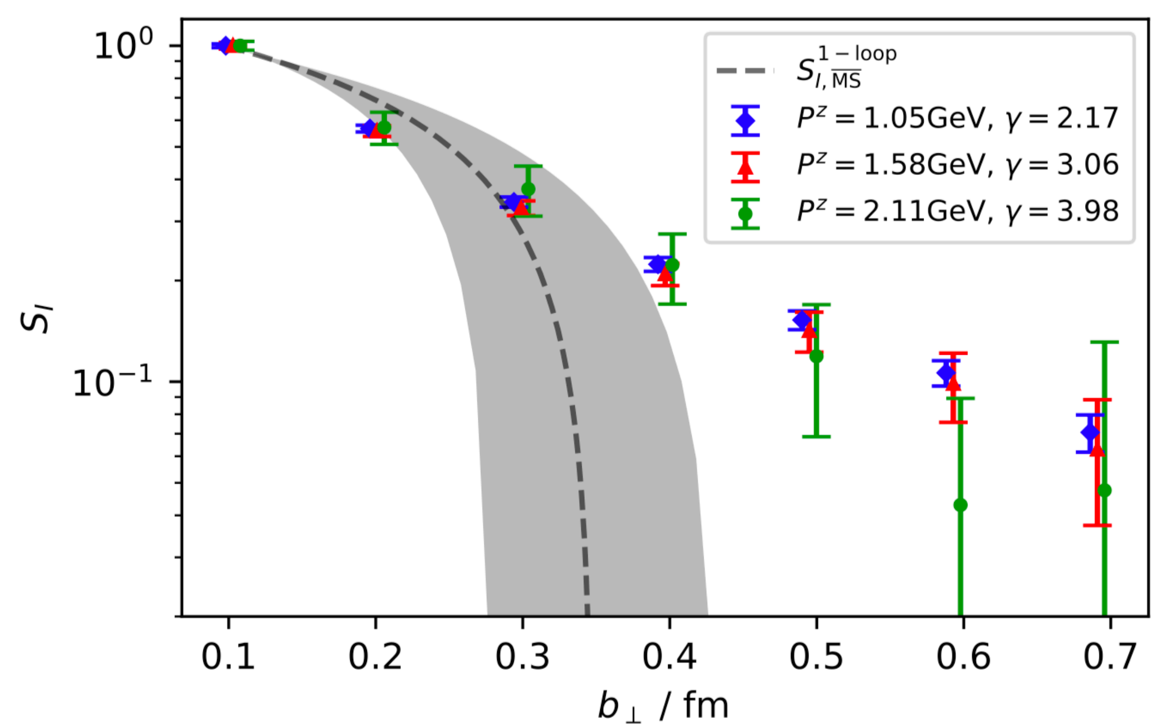
$$\tilde{S}_q = \frac{F(b_\perp, P\cdot P')}{\int dx dx' H(x,x',P,P')\widetilde{\phi}(x',b_\perp,P')\widetilde{\phi}^\dagger(x,b_\perp,P)}$$

$$\tilde{S}_q(b_\perp,\mu,Y) = e^{Y\gamma_\zeta(\mu,b_\perp)} S_I^{-1}(b_\perp,\mu)$$

Rapidity Anom. Dim.

Intrinsic nonperturbative soft function





CS Kernel Calculation

Schlemmer, Vladimirov, Zimmermann, Engelhardt
Schafer (Regensburg/NMSU) 2103.16991

(see parallel talk by Maximilian Schlemmer)

nf=2+1 simulation, $m_\pi = 422\,\mathrm{MeV}$

No renormalization & 1-loop matching

$P^z = 1.25, 1.73, 2.27\mathrm{GeV}$

Assumes certain NP factor is constant (supported by pheno-extractions), no need for Fourier Trnsfm.

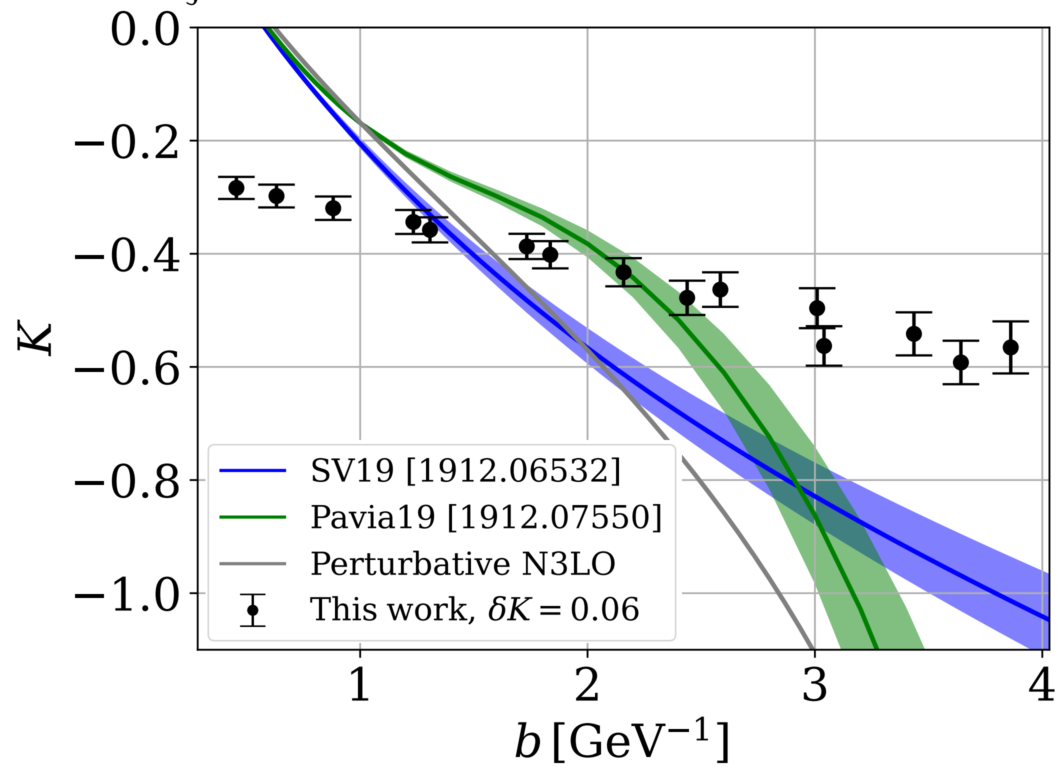
Vladimirov, Schafer 2002.07527

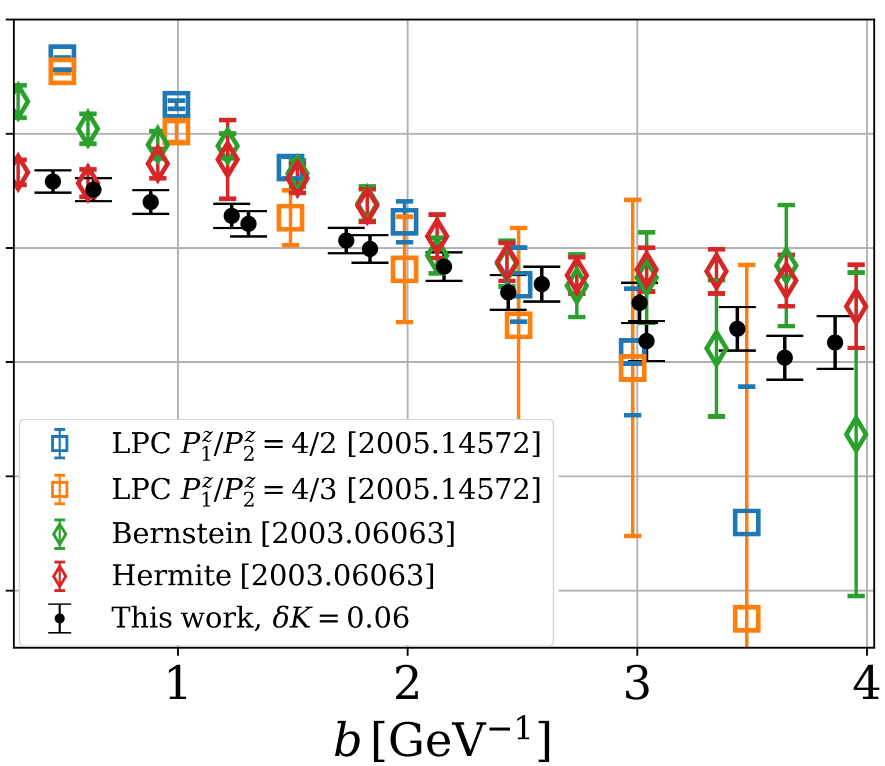
$$\frac{\int dx_1 e^{ix_1 \ell P_1^+} |C_H(|x_1|P_1^+/\mu)|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x_1, b; \mu, \zeta_1)}{\int dx_2 e^{ix_2 \ell P_2^+} |C_H(|x_2|P_2^+/\mu)|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x_2, b; \mu, \zeta_2)} = \left(\frac{P_1^+}{P_2^+}\right)^{K(b,\mu)} \mathbf{r}^{[\Gamma]}_{f\leftarrow h}(b, \mu; P_1, P_2) + \ldots$$

assume NP factor is constant

$$\mathbf{r}^{[\Gamma]}_{f\leftarrow h}(b, \mu; P_1, P_2) = 1 + 4C_F \frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{P_1^+}{P_2^+}\right) \left[1 - \ln\left(\frac{4P_1^+ P_2^+ |v^-|^2}{\mu^2}\right) - 2\mathbf{M}^{[\Gamma]}_{f\leftarrow h}(b, \mu)\right]$$

$K = \gamma_\zeta^q$





TMD Soft function & CS Kernel Calculation

nf=2+1 simulation, $m_\pi = 350\,\mathrm{MeV}$

Li, Xia, Alexandrou, Chichy, Constantinou, Feng, Hadjiyannakou, Jansen, Liu, Scapellato, Steffens, Tarello (ETMC/PKU) 2106.13027

Ratio scheme renormalization & tree level matching

(cf. plenary talk by Chichy)

Study of spin projections & mixing free combinations

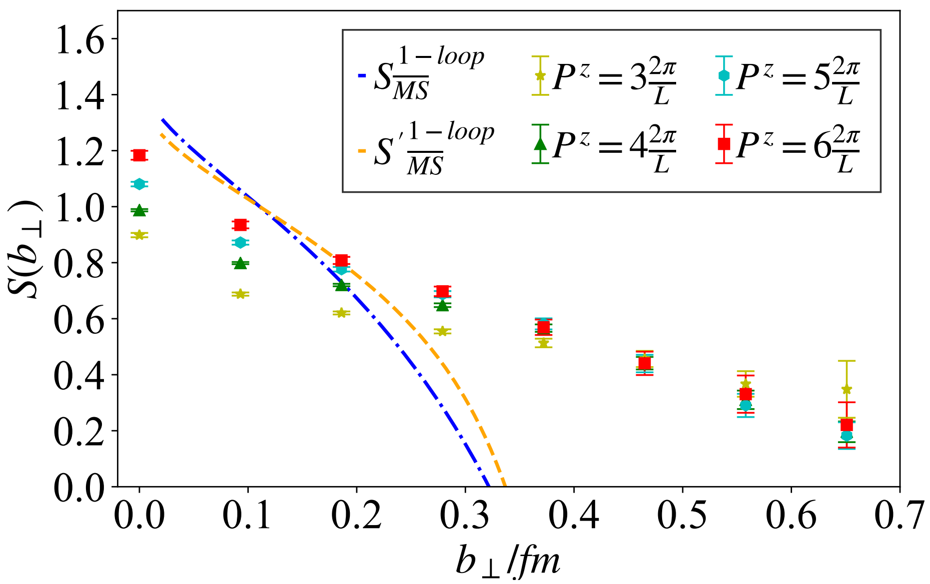
$P^z = (1.7 - 3.3)\,\mathrm{GeV}$

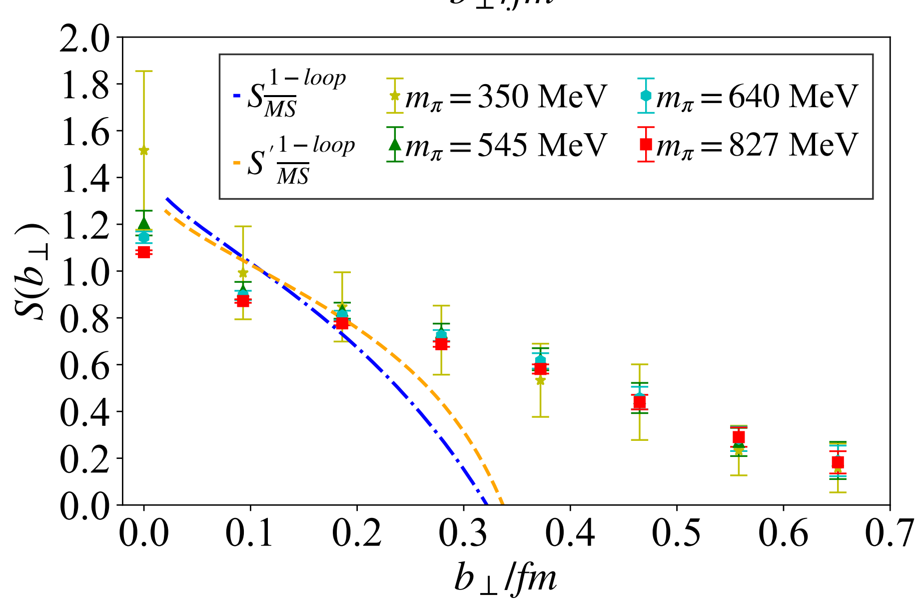
$$\tilde{S}_q = \frac{F(b_\perp, P\cdot P')}{\int dx dx' H(x,x',P,P')\widetilde{\phi}(x',b_\perp,P')\widetilde{\phi}^\dagger(x,b_\perp,P)}$$

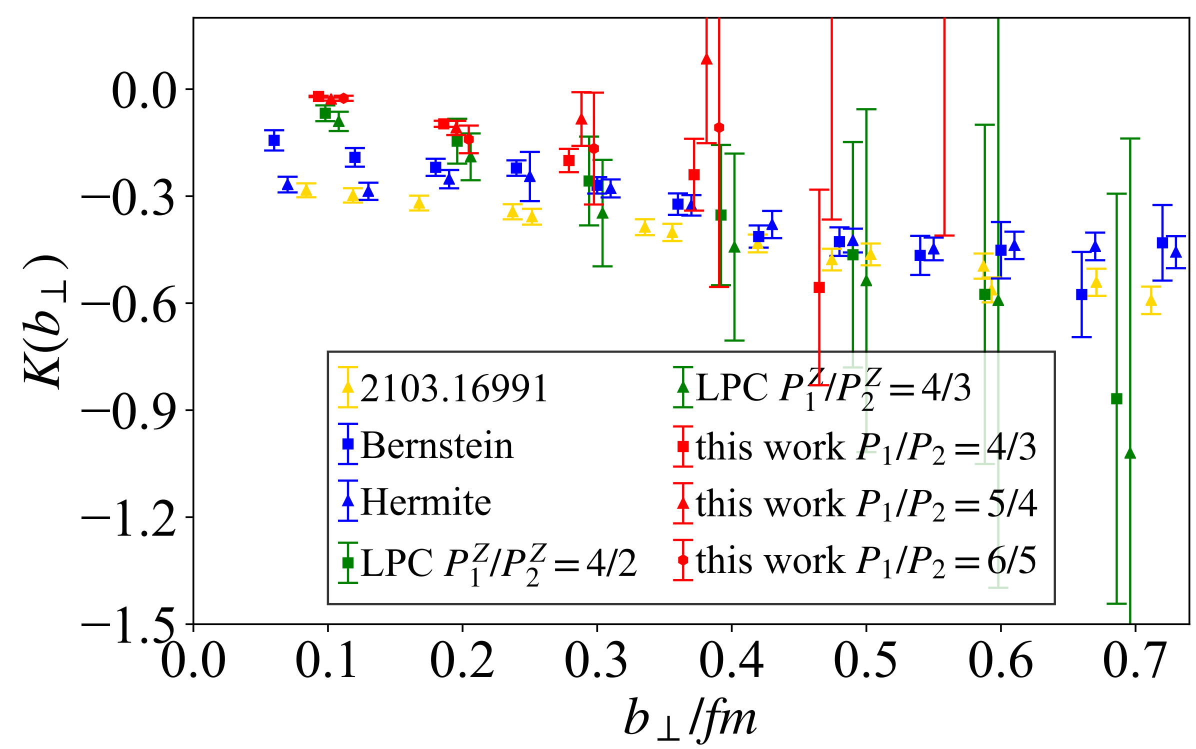
$$K = \gamma_\zeta^q$$

$$K(b_\perp,\mu) = \lim_{l\to\infty} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left|\frac{\phi(b_\perp,l,P_1^z)/E_1}{\phi(b_\perp,l,P_2^z)/E_2}\right|$$

$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln\left|\frac{C^{wf}_{\Gamma_\phi}(b_\perp,P_1^z)}{C^{wf}_{\Gamma_\phi}(b_\perp,P_2^z)}\frac{C^{wf}_{\Gamma_\phi}(0,P_2^z)}{C^{wf}_{\Gamma_\phi}(0,P_1^z)}\right|$$







CS Kernel Calculation

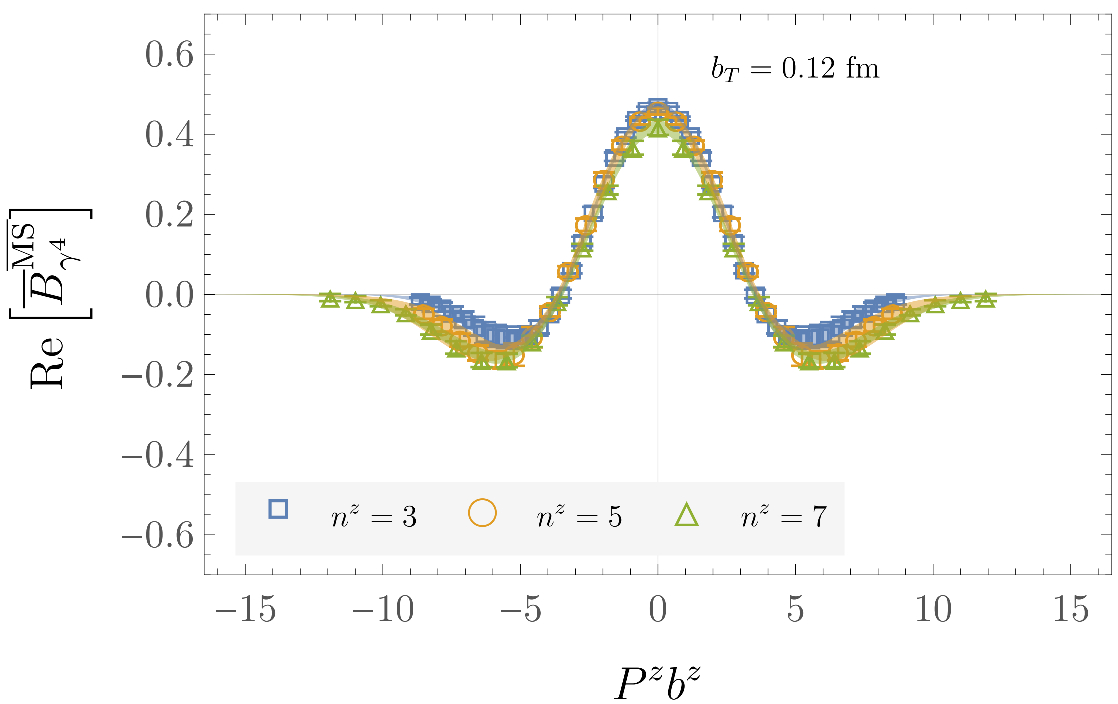
Shanahan, Wagman, Zhao (SWZ) 2107.11930

nf=2+1+1 simulation, MILC configs with staggered quarks, m_π^{phys}

RIMOM matrix renormalization, conversion to $\overline{\text{MS}}$, and one loop matching

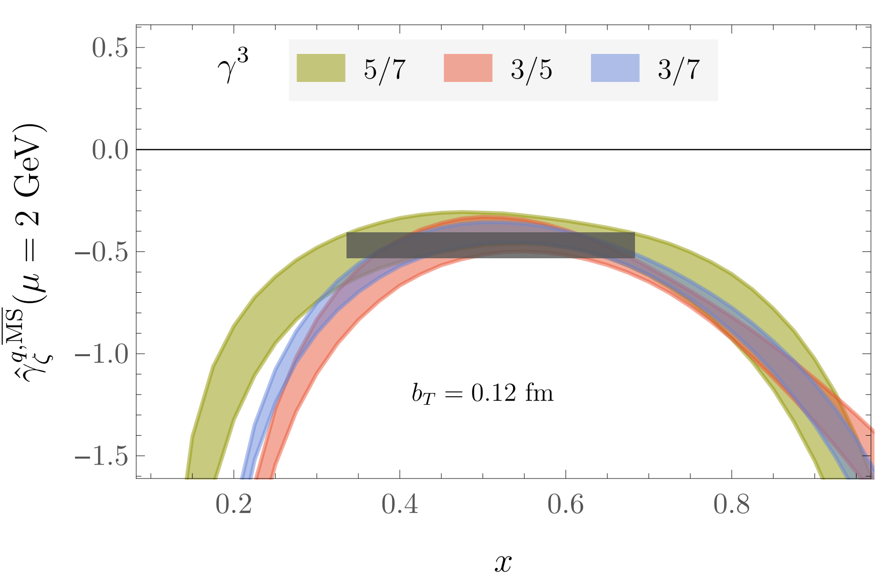
matrix elements with pion state, $m_\pi = 538\,\text{MeV}$, $\quad P^z = 0.65, 1.1, 1.5\,\text{GeV}$

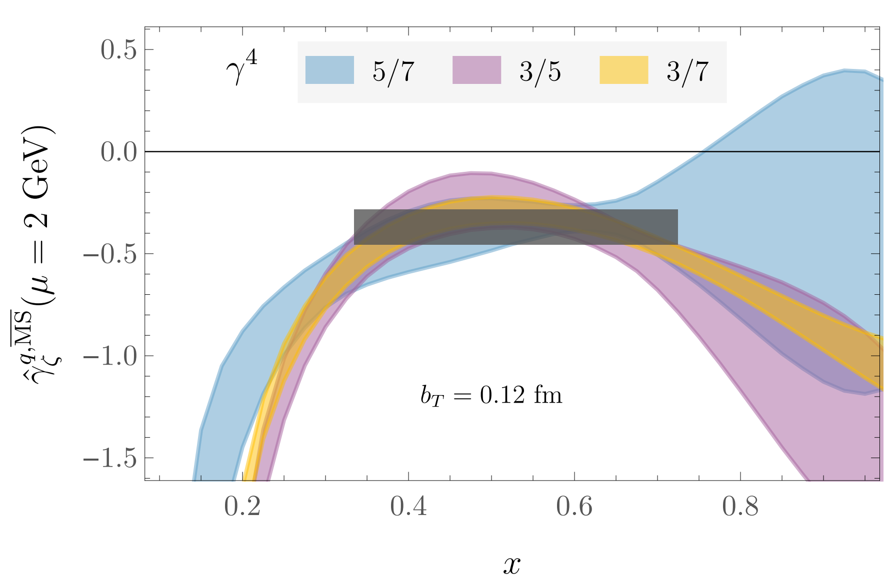
$$\hat{\gamma}_\zeta^q(\mu, b_T; P_1^z, P_2^z, x) \equiv \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C_{\text{ns}}^{\text{TMD}}(\mu, xP_2^z)}{C_{\text{ns}}^{\text{TMD}}(\mu, xP_1^z)} \frac{\int \text{d}b^z e^{-ib^z x P_1^z} P_1^z \hat{B}_{\gamma^4}^{\overline{\text{MS}}}(\mu, b^z, b_T, P_1^z)}{\int \text{d}b^z e^{-ib^z x P_2^z} P_2^z \hat{B}_{\gamma^4}^{\overline{\text{MS}}}(\mu, b^z, b_T, P_2^z)}\right]$$



Fits to enable Fourier transform

$$f_{\text{Re}}(\sigma, \{r_n\}; b^z) = \exp[-(b^z)^2/(2\sigma^2)] \sum_{n=0}^{n_{\max}} r_n (b^z)^{2n}$$





CS Kernel Calculation

Shanahan, Wagman, Zhao (SWZ) 2107.11930

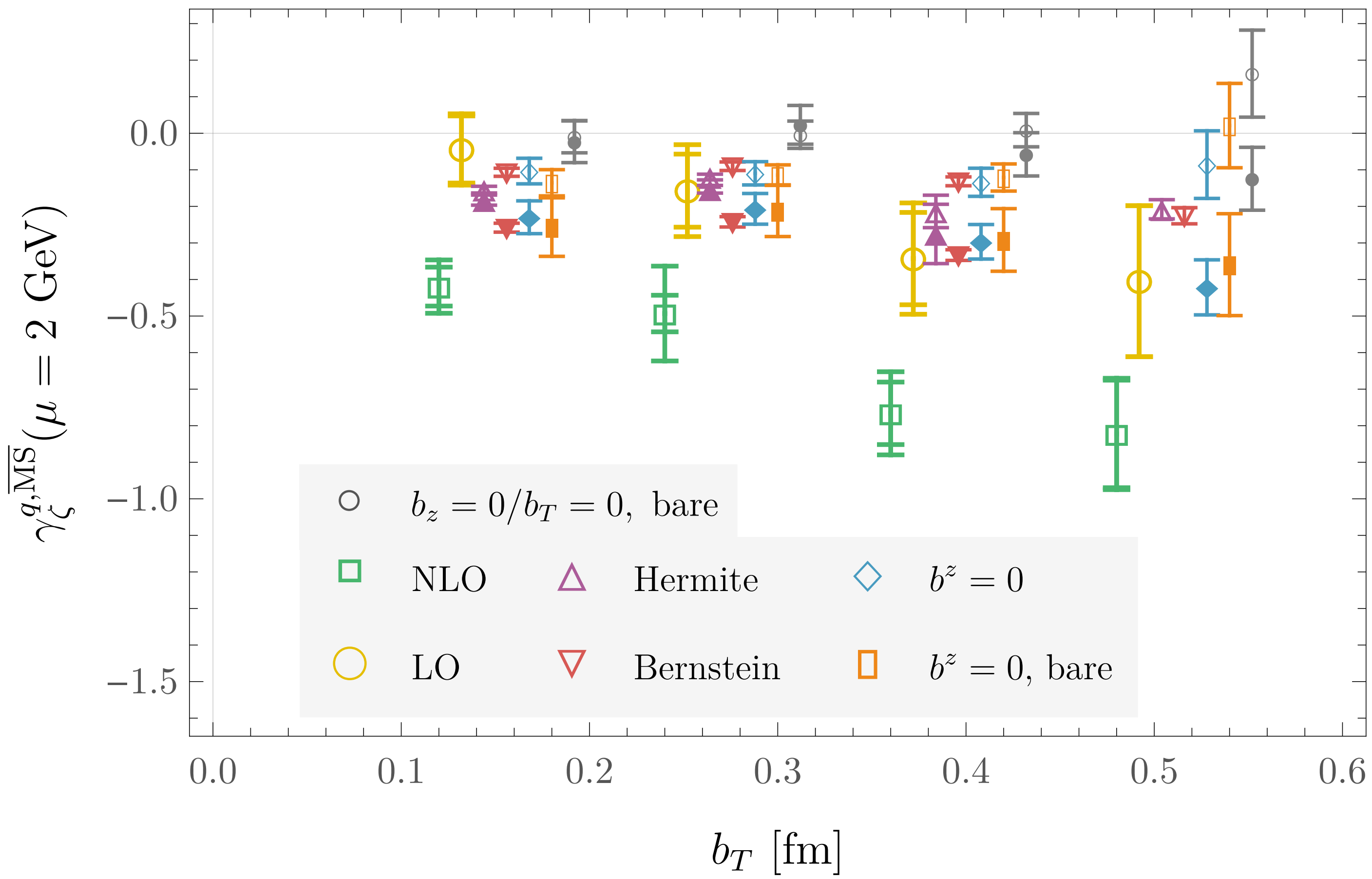
nf=2+1+1 simulation, MILC configs with staggered quarks, $m_\pi^{\rm phys}$

RIMOM matrix renormalization, conversion to $\overline{\rm MS}$, and one loop matching

matrix elements with pion state, $m_\pi = 538\,{\rm MeV}, \quad P^z = 0.65, 1.1, 1.5\,{\rm GeV}$

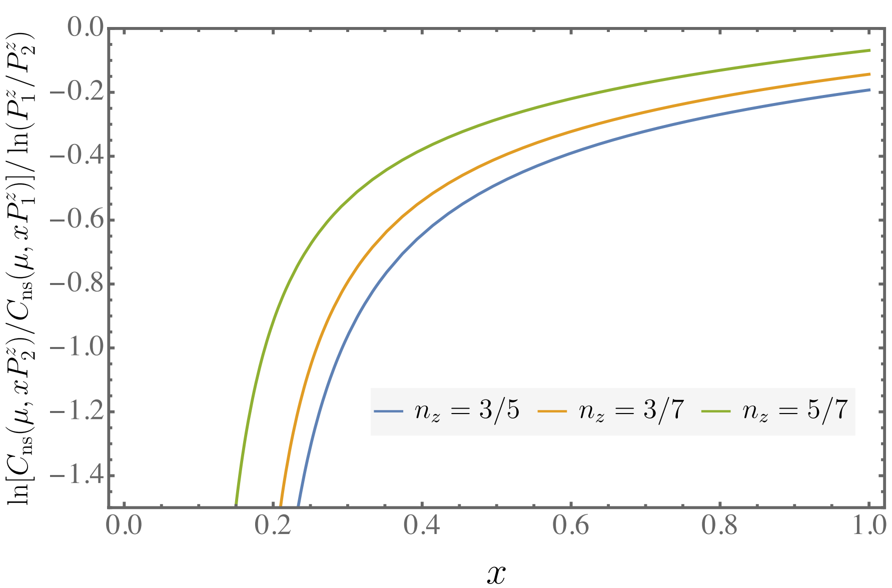
$$\hat{\gamma}^q_\zeta(\mu, b_T; P_1^z, P_2^z, x) \equiv \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{C^{\rm TMD}_{\rm ns}(\mu, xP_2^z)}{C^{\rm TMD}_{\rm ns}(\mu, xP_1^z)} \frac{\int {\rm d}b^z e^{-ib^z xP_1^z} P_1^z \hat{B}^{\overline{\rm MS}}_{\gamma^4}(\mu, b^z, b_T, P_1^z)}{\int {\rm d}b^z e^{-ib^z xP_2^z} P_2^z \hat{B}^{\overline{\rm MS}}_{\gamma^4}(\mu, b^z, b_T, P_2^z)}\right]$$

Results with/without 1-loop matching, and various renormalization assumptions



Significant correction from NLO matching

perturbative contribution:



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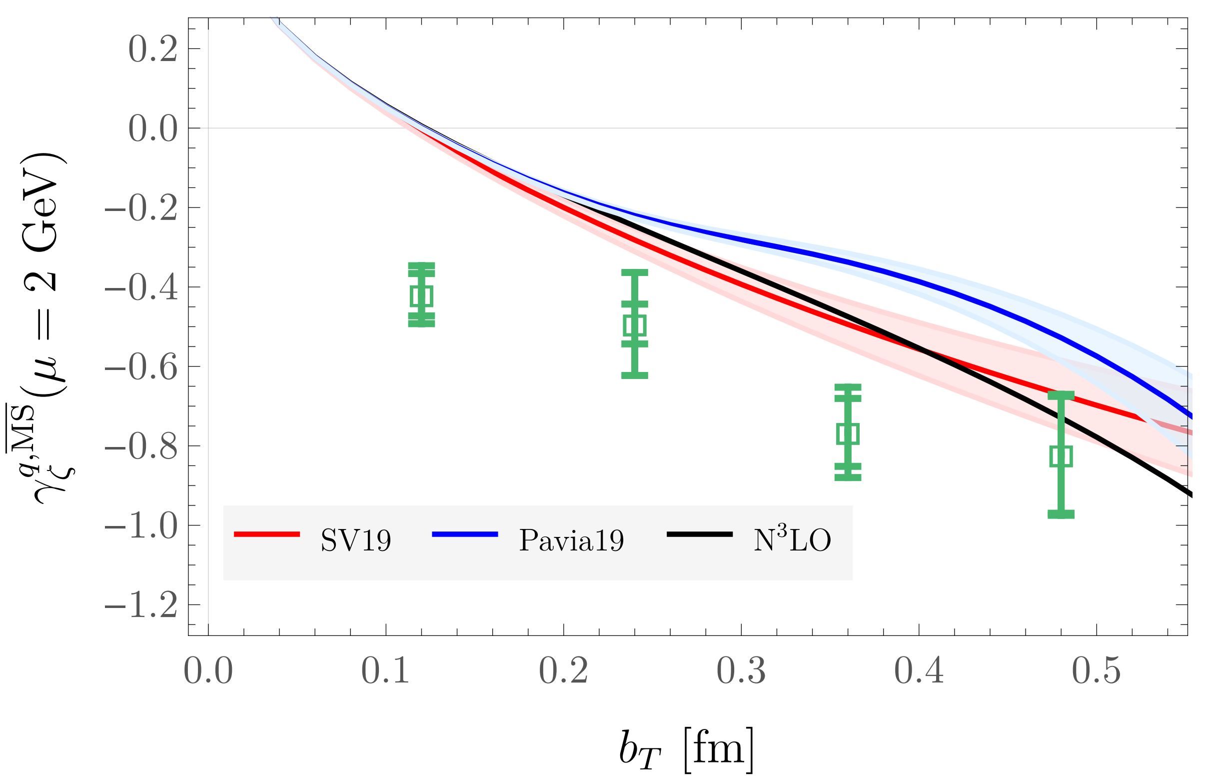
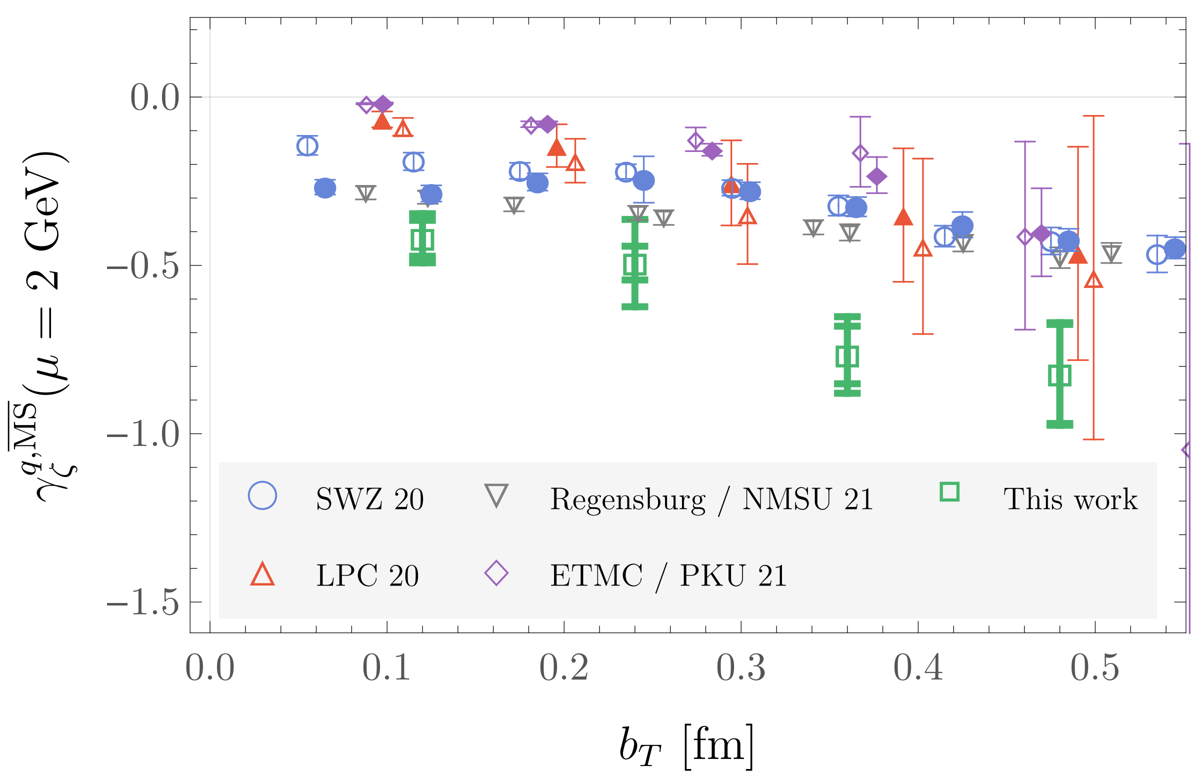
Comparison with Global fits & earlier literature:





Differences can be explained by combination of $\mathcal{O}(1/(b_T x P^z)^2)$ power corrections and LO vs. NLO matching

Summary

- Major advances in phenomenology & theory for TMDPDFs
- Lattice determination of TMDPDFs is rapidly advancing field, hard work, but shows significant promise

Targets:

- Non-perturbative CS Kernel 👍
- Info on Spin-dependent TMDPDFs (in ratios) 👍
- Info about 3D structure, x and b_T (in ratios)
- proton vs. pion TMDPDFs (in ratios)
- flavor dependence of TMDPDFs (in ratios)
- TMDPDF with x and b_T (normalization) 👍
- Gluon TMDPDFs ?
- Fragmentation ???