Semileptonic $b \to u$ decays and $|V_{ub}|$

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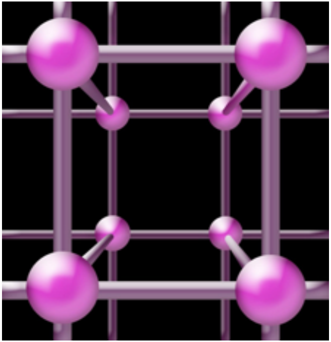
Motivation / Background

The Cabibbo–Kobayashi–Maskawa (CKM) matrix element $|V_{ub}|$...

- describes the coupling between u and b quarks in the weak interaction
- is one of the fundamental parameters of the Standard Model
- is the focus of a longstanding puzzle: a tension between inclusive & exclusive determination

Both leptonic and semileptonic decays can be used to extract $|V_{ub}|$.



In this talk we will focus on the semileptonic decays and the status of Lattice QCD calculations of the form factors.

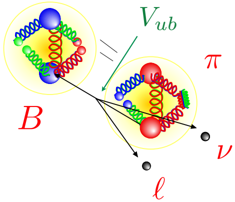
Semileptonic decays

CKM matrix element V_{ub} relates to the differential decay rate

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |k_\pi|^3 |f_+(q^2)|^2$$

- Experimental (partial) branching fraction
- CKM matrix element
- Form factor (from theory, usually Lattice QCD)

k_π = pion momentum
q = momentum transfer



Several decays can be used:

- $B \to \pi \ell \nu$
  Good experimental data
- $B_s \to K \ell \nu$
  Easier on lattice than $B \to \pi$
  Measured by LHCb, 2 wide bins
- $B_c \to D \ell \nu$
  Measurement by LHCb forthcoming?

Form factors and matrix elemets

For pseudoscalar to pseudoscalar decays

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[(p_B+k_\pi)^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2}q^\mu.$$

A useful parametrisation is

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{M_B}\left[f_\parallel(E_\pi)v^\mu + k^\mu_{\pi,\perp} f_\perp(E_\pi)\right].$$

- $p_B = B$ meson four-momentum
- k_π = pion four-momentum
- $q^\mu = p_B^\mu - k_\pi^\mu$ = momentum transfer
- $v^\mu = p_B/M_B$ = heavy quark velocity
- $E_\pi = v \cdot k_\pi = (M_B^2 + M_\pi^2 - q^2)/(2M_B)$ = pion energy
- $k^\mu_{\pi,\perp} = k_\pi^\mu - (v \cdot k_\pi)v^\mu$ = projection of the pion momentum in the direction perpendicular to v^μ

Form factors and vector-current matrix elements

The two form factor definitions are related by

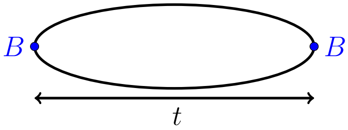
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}} \left[f_\parallel(E_\pi) + (M_B - E_\pi) f_\perp(E_\pi) \right]$$

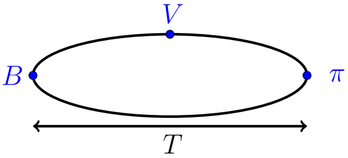
$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 - M_\pi^2} \left[(M_B - E_\pi) f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2) f_\perp(E_\pi) \right]$$

The B meson is often kept at rest, and the vector-current matrix elements calculated on the lattice can be directly related to the form factors:

$$f_\parallel(E_\pi) = \frac{\langle \pi(k_\pi) | V^0 | B \rangle}{\sqrt{2M_B}}$$

$$f_\perp(E_\pi) = \frac{\langle \pi(k_\pi) | V^i | B \rangle}{\sqrt{2M_B}} \frac{1}{k_\pi^i}.$$



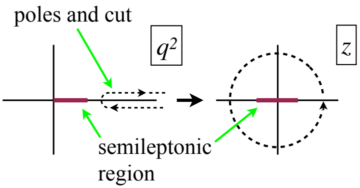


Parametrising q^2 dependence: z-expansion

$$z = z(q^2, t_0) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (M_B \pm M_\pi)^2$$

t_0 is a free parameter. The choice $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$ gives $|z| < 0.3$.



BGL (Boyd, Grinstein, Lebed) parametrisation:

$$f_i = \frac{1}{\mathcal{P}_i(q^2)\phi_i(q^2, t_0)} \sum_{n=0}^{N_z} a_n^i z^n, \text{ where } i \in \{+, 0\}, \mathcal{P}_+ = z(q^2, M_{B^*}^2), \mathcal{P}_0 = 1.$$

$\phi_i(q^2, t_0)$ are analytic functions (see PRD **56** (1997) 6895). Often $\phi_0(q^2, t_0) = 1$.

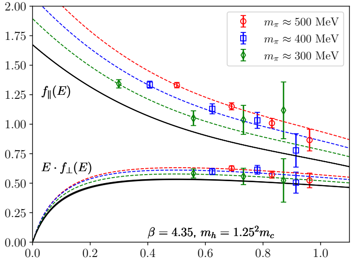
BCL (Bourrely, Caprini, Lellouch) parametrisation (see PRD **79** (2009) 013008):

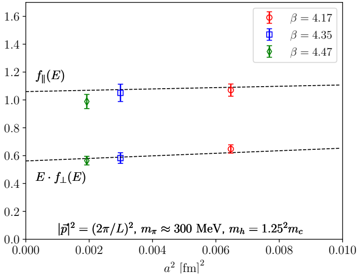
$$f_+ = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{n=0}^{N_z - 1} b_n^+ \left[z^n - (-1)^{n - N_z} \frac{n}{N_z} z^{N_z} \right]; \quad f_0 = \sum_{n=0}^{N_z} b_n^0 z^n$$

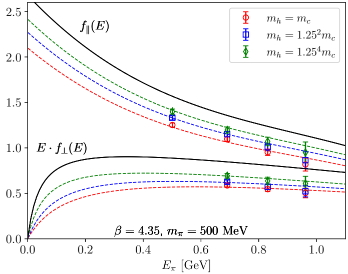
Both of these parametrisations are widely used.

JLQCD: $B \to \pi \ell \nu$

- 2+1 flavor ensembles generated by JLQCD Collaboration
- all sea and valence quarks use Möbius Domain Wall Fermion action
- heavy quark fully relativistic, extrapolated to physical b mass

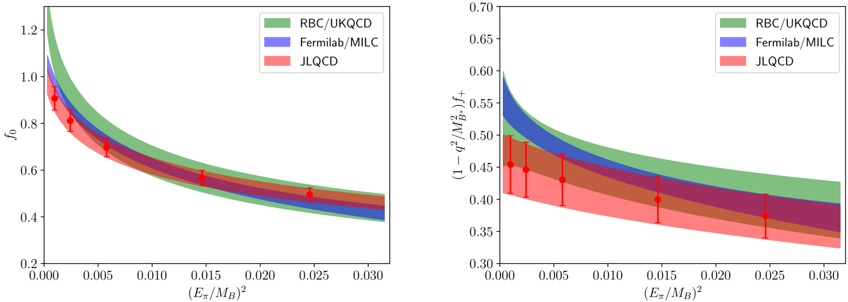






B. Colquhoun, S. Hashimoto, T. Kaneko, J. K., see e.g. arXiv:1912.02409.

JLQCD: $B \to \pi\ell\nu$



Comparison of the JLQCD form factors at physical parameters with RBC/UKQCD (arXiv:1501.05373) and Fermilab/MILC (arXiv:1503.07839) shows that the shapes are slightly different especially at small E_π. This may hint to some systematics, though the statistical significance is limited.

RBC/UKQCD: $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$

- RBC/UKQCD's 2+1 flavor domain-wall fermion + Iwasaki gauge action ensembles
- Domain-wall fermions for up/down, strange, and charm quarks
- Relativistic heavy quark (RHQ) action for b quarks
  - tuned to physical b quark mass, no extrapolation
  - different action for b than for lighter quarks
- Update of the 2015 calculation: add finer lattice spacing, improved analysis and systematics

Two parallel talks at this conference:

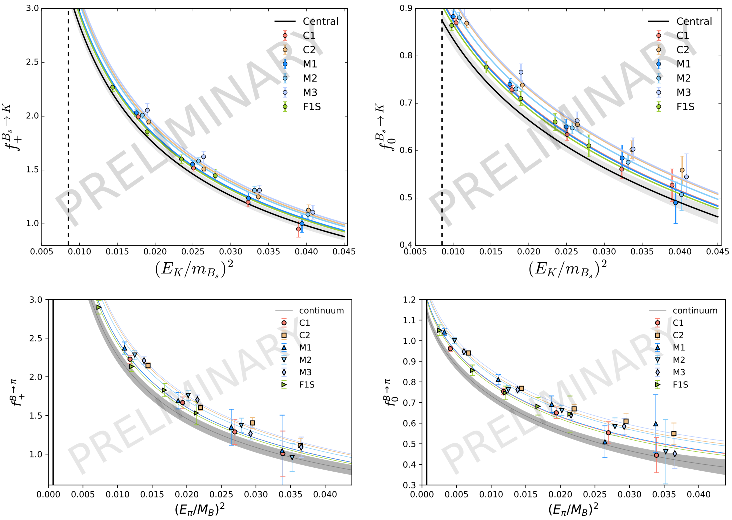
✓ **Thursday 13:45 (US/Eastern) Ryan Hill**
Semileptonic form factors for $B \to \pi \ell \nu$ decays

✓ **Thursday 14:00 (US/Eastern) Jonathan Flynn**
Form factors for semileptonic $B_s \to K \ell \nu$ and $B_s \to D_s \ell \nu$ decays

See arXiv:2012.04323 and arXiv:1912.09946 for previous reports.

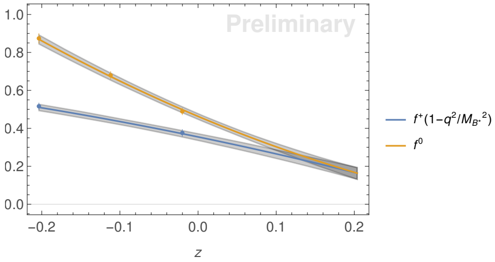
J. Flynn, R. Hill, A. Jüttner, A. Soni, J.T. Tsang, O. Witzel

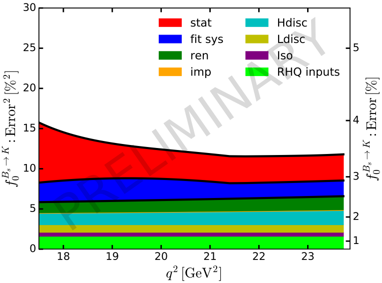
RBC/UKQCD: $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$

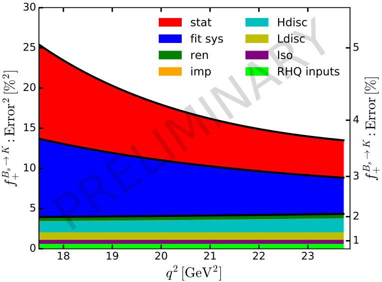


Figures courtesy of Oliver Witzel.

RBC/UKQCD: $B_s \to K\ell\nu$ z-expansion and error budgets







- Parametrisation of the physical form factors using z-expansion with BCL Ansatz.
- Full error budgets include statistical and all systematic uncertainties.

Figures courtesy of Oliver Witzel.

Fermilab/MILC: $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$

- 2+1+1 flavor ensembles generated by the MILC Collaboration
- sea and valence quarks use Highly Improved Staggered Quark (HISQ) formalism
- heavy HISQ method used for b quark, extrapolation to physical B meson mass

Parallel talk:

✓ **Thursday 21:00 (US/Eastern) Andrew Lytle**
B-meson semileptonic decays with highly improved staggered quarks

See arXiv:1912.13358 for previous report by Fermilab/MILC.

C. DeTar, A. El-Khadra, E. Gamiz, S. Gottlieb, W. Jay,
A. Kronfeld, A. Lytle, J. Simone

HPQCD: $B_c \to D\ell\nu$

- 2+1+1 flavor ensembles generated by the MILC Collaboration
- sea and valence quarks use Highly Improved Staggered Quark (HISQ) formalism
- heavy HISQ method (first used in arXiv:1004.4285) for b quark, extrapolation to physical B meson mass

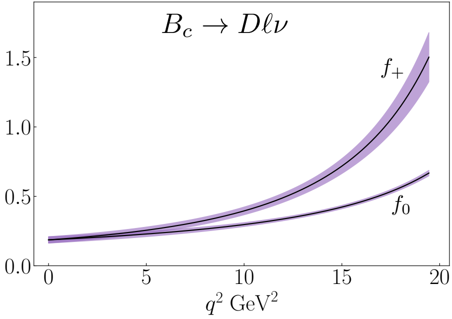


Figure courtesy of Laurence Cooper.

Parallel talk:

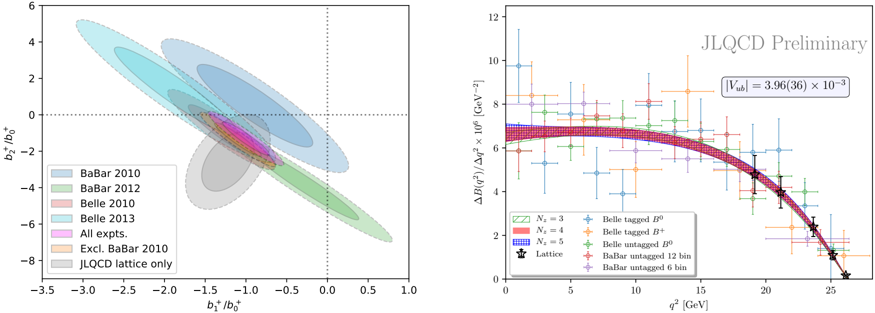
✓ **Thursday 6:45 (US/Eastern) Laurence Cooper**
Form factors for $B_c^+ \to D^0\ell^+\nu$ and rare $B_c^+ \to D_s^+\ell^+\ell^-$ with HISQ

L. Cooper, C. Davies, M. Wingate

z-expansion and extracting $|V_{ub}|$

- To compare the shape of the form factors from experiment and lattice, we fit each data set on its own and compare b_1^+/b_0^+, b_2^+/b_0^+
- To extract $|V_{ub}|$, we fit form factors together with experimental results, integrating over each q^2 bin i:

$$\Delta\Gamma_i(B \to \pi \ell \nu) = \int_{q^2_{i,\mathrm{min}}}^{q^2_{i,\mathrm{max}}} \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |k_\pi|^3 |f_+(q^2)|^2 \mathrm{d}q^2$$



Figures courtesy of Brian Colquhoun.

Dispersive Matrix Method

Unitarity description of the FFs for V_{CKM} extraction

The dispersive matrix (DM) method determines the Form Factors (FFs) in the whole kinematical range of values of q^2 by using **two inputs**:

- computations of the FFs on the lattice in the high-q^2 regime
- non-perturbative values of the susceptibilities computing on the lattice the two-point correlation functions of the corresponding quark currents

See Martinelli et al.: arXiv:2105.07851 for the b → c case

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

Original idea from L. Lellouch: NPB, 479 (1996)
New developments in Di Carlo et al.: arXiv:2105.02497

The resulting non-perturbative bands of the FFs will be

- **entirely based on first principles (LQCD, no perturbative evaluation of the susceptibilities, no series expansion)**
- **independent of any assumption on the functional dependence on the momentum transfer**
- **independent of the experimental determinations of the differential decay widths**

See M.Naviglio's talk on July 29th for further details!

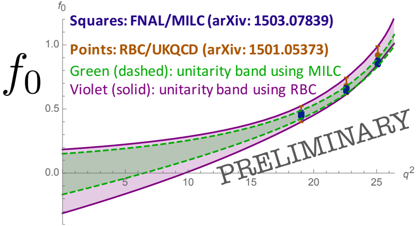
→ No HQET, no truncation, no perturbative bounds with respect to the well-known CLN, BCL and BGL parametrizations

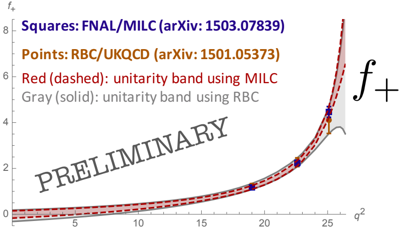
Slide courtesy of Ludovico Vittorio.

Dispersive Matrix Method

Unitarity description of the FFs of the semileptonic B → π decays

In this way we obtain unitarity bands of the FFs as functions of the momentum transfer in the whole kinematical regime.
By using ***only*** **FNAL/MILC** or **RBC/UKQCD** data as inputs ***without experimental data***:





Our extrapolation at zero momentum transfer reads

$$f^{\pi}(q^2=0)|_{\text{RBC/UKQCD}} = -0.06 \pm 0.25$$
$$f^{\pi}(q^2=0)|_{\text{FNAL/MILC}} = -0.01 \pm 0.16$$

Agreement with the BCL extrapolations by:

$$f(0) = 0.01(24)$$ **RBC/UKQCD (arXiv: 1501.05373)**

$$f(0) = 0.20(14)$$ **FNAL/MILC (arXiv: 1503.07839)**

Slight discrepancy with Light Cone Sum Rules: $f(0) = 0.28(3)$ **LCSR (arXiv:2102.07233)**

Slide courtesy of Ludovico Vittorio.

Dispersive Matrix Method

Exclusive Vub determination through unitarity

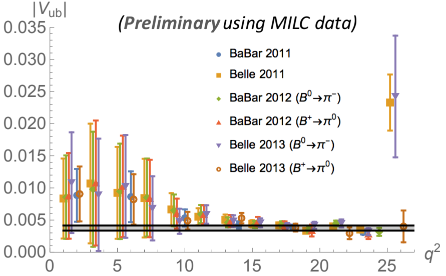
Starting from the FFs bands, we use the experimental data ***only*** to compute bin-per-bin

Multiplicity factor — Exper. data

$$|V_{ub}| = \sqrt{\frac{C_v}{\tau_{B^\kappa}} \times \frac{\Delta\mathcal{B}|_{exp}}{\Delta\zeta}}$$

B0/B+ meson lifetime — Theor. decay width

See L.Vittorio's talk on July 29th
for an analogous study of BD(*) decays



The final weighted mean reads with the MILC inputs ***(black band)***

Agreement with the *inclusive value*!

$$|V_{ub}|^{B\pi}_{\mathrm{FNAL/MILC}} \times 10^3 = 3.76(40) \quad \Longrightarrow \quad |V_{ub}|_{incl} \times 10^3 = 4.10 \pm 0.09 \pm 0.22 \pm 0.15$$

arXiv:2102.00020

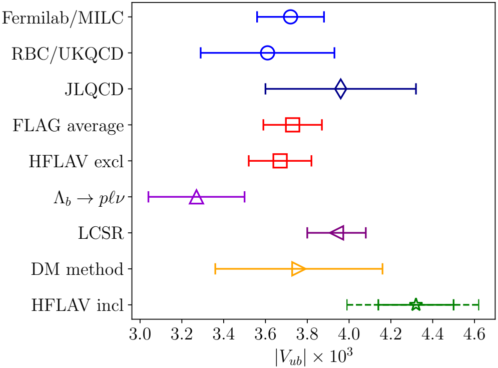
This result comes from a fully non-perturbative and model-independent description of the FFs!!

Improvements in precision are expected by considering together the LQCD and experimental data in the DM method:

the complete analysis will be presented in a forthcoming paper!

Slide courtesy of Ludovico Vittorio.

Summary: $|V_{ub}|$



Collaboration	$|V_{ub}| \times 10^3$
Fermilab/MILC[1]	3.72(16)
RBC/UKQCD[2]	3.61(32)
JLQCD[3]	3.96(36)
FLAG average[4]	3.73(14)
HFLAV exclusive[5]	3.67(15)
$\Lambda_b \to p\ell\nu$ FF[6]	3.27(23)
Light Cone SR[7]	3.94(14)
DM method[8]	3.76(40)
HFLAV incl. GGOU[9]	4.32(18)

[1] arXiv:1503.07839; [2] arXiv:1501.05373; [3] Work in progress;
[4] arXiv:1902.08191; [5] https://hflav.web.cern.ch; [6] arXiv:1503.01421;
[7] arXiv:2103.01809; [8] This conference (using Fermilab/MILC form factors);
[9] https://hflav.web.cern.ch, the data point is from GGOU analysis, and the second (dashed) error bar shows the spread of values from using different frameworks

Parallel talks at this conference

- ✓ **Thursday 6:45 (US/Eastern) Laurence Cooper (HPQCD)**
  Form factors for $B_c^+ \to D^0 \ell^+ \nu$ *and rare* $B_c^+ \to D_s^+ \ell^+ \ell^-$ *with HISQ*
- ✓ **Thursday 13:45 (US/Eastern) Ryan Hill (RBC/UKQCD)**
  Semileptonic form factors for $B \to \pi \ell \nu$ *decays*
- ✓ **Thursday 14:00 (US/Eastern) Jonathan Flynn (RBC/UKQCD)**
  Form factors for semileptonic $B_s \to K \ell \nu$ *and* $B_s \to D_s \ell \nu$ *decays*
- ✓ **Thursday 14:45 (US/Eastern) Manuel Naviglio**
  Non-Perturbative Bounds for Semileptonic Decays in Lattice QCD
- ✓ **Thursday 21:00 (US/Eastern) Andrew Lytle (Fermilab/MILC)**
  B*-meson semileptonic decays with highly improved staggered quarks*

Work is also being done on the **inclusive** semileptonic decays.
See parallel talk

- ✓ **Wednesday 21:45 (US/Eastern) Shoji Hashimoto**
  Composition of the inclusive semileptonic decay of B *meson*

G. Bailas, P. Gambino, T. Kaneko, S. Machler. See arXiv:2005.13730.

Thank you!