# Charmonium-like resonances in coupled $D\bar{D} - D_s\overline{D_s}$ scattering

$\bar{c}\,c$ $\bar{c}c\bar{q}q$ $\bar{c}c\bar{s}s$


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Lattice 2021, MIT

July 28th 2021

SP, S. Collins, D. Mohler, M. Padmanath, S. Piemonte


2011.02541, *JHEP* 06 (2021) 035 (resonances with spins $J^{PC}=0^{++}$, $2^{++}$ )

# Motivation to study charmonium resonances

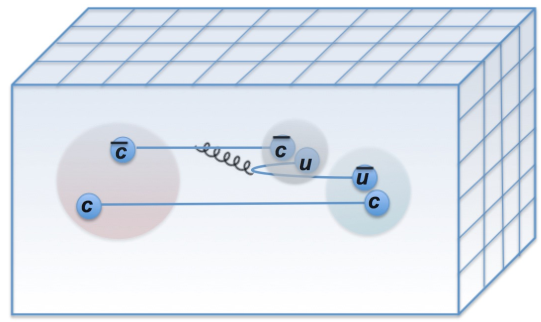
Experimentally discovered exotic hadrons

- Most of them contain cc
- All of them are resonances (decay strongly)

$$Z_c : \bar{c}c\bar{d}u$$

$$P_c : \bar{c}cuud$$

$$X(3872) : \bar{c}c\bar{q}q$$



Current study:

Charmonium(like) resonances with **isospin=0** and J$^{PC}$ **=0$^{++}$ , 2$^{++}$**

q=u,d

$\bar{c}\ c$ $\quad$ $\bar{c}q\bar{q}c$ $\quad$ $\bar{c}s\bar{s}c$

$\bar{D}D$ $\quad$ $\bar{D}_sD_s$

coupled-channel scattering $\quad D\bar{D} - D_s\bar{D}_s$

conventional + exotic ?

This is the first extraction of the scattering matrix for coupled channels in the charmonium sector

The only previous lattice studies that into account decaying nature of charmonium resonances

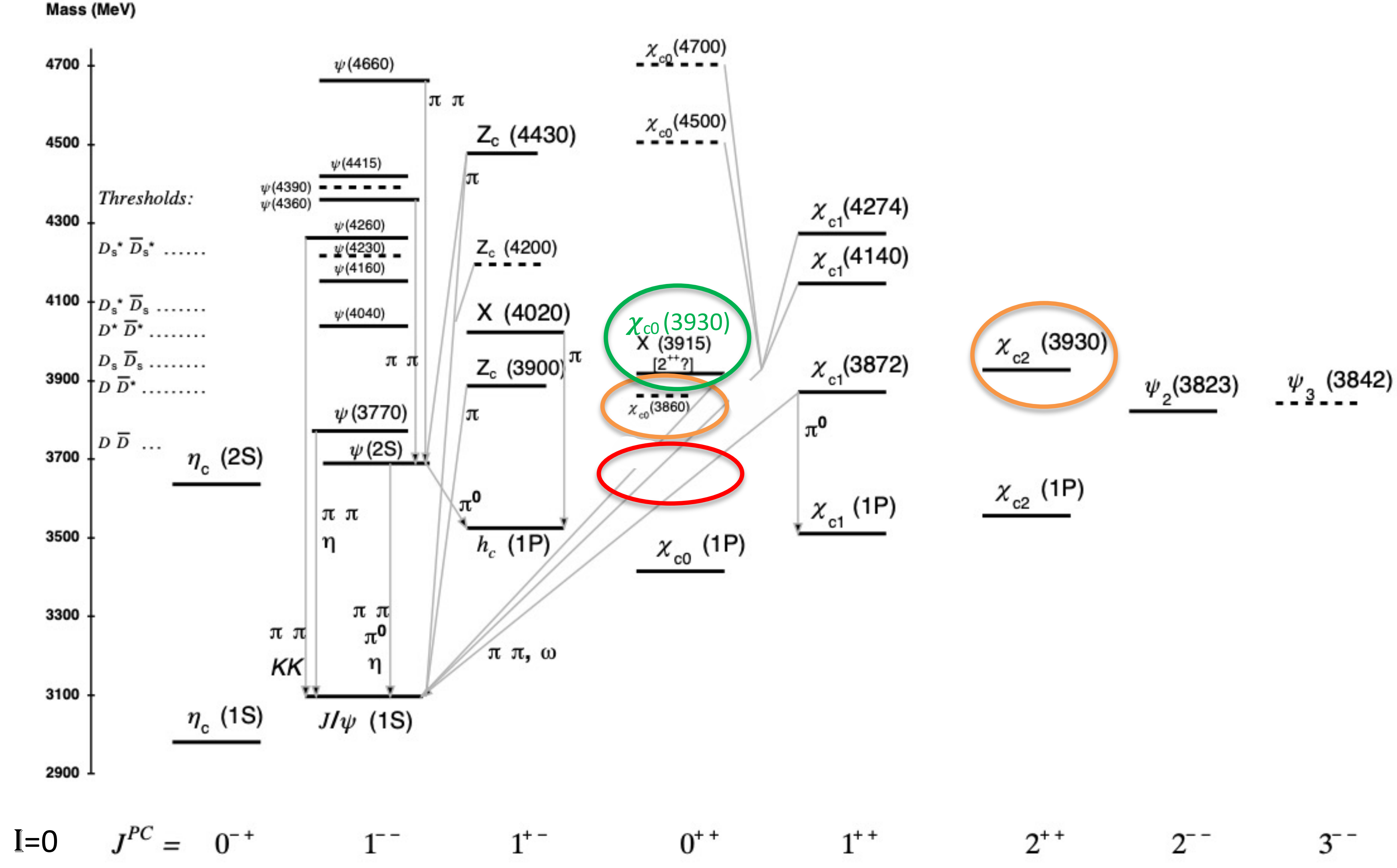
J=0,1 : Lang, Leskovec, Mohler, SP, 1503.05363, JHEP 2015 , conclusion concerning J=0 remains puzzling

J=1,3: Piemonte, Collins, Padmanath, Mohler, SP, 1905.03506, PRD 2019

J=2 : /

PDG 2019

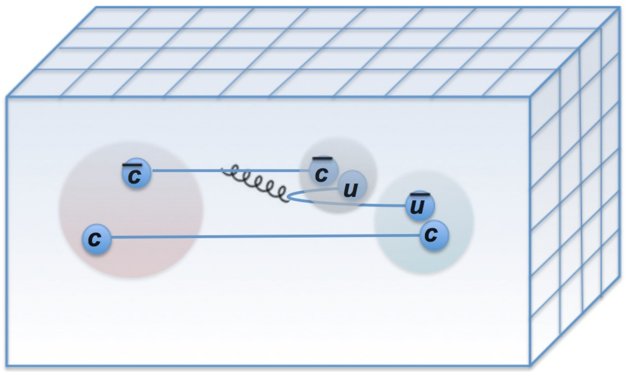
# Charmonium system: experimental status (PDG) and summary of lattice results




this lattice study: postdicted | predicted, exp discovered | predicted, exp not (yet) discovered

likely nature : $\bar{c}\ c$ | $\bar{D}_s D_s$ | $\bar{D} D$

# Lattice details




CLS ensembles with u/d, s dynamical quarks

a≃0.086 fm

$N_L$=24, 32

lat exp

$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_u+m_d+m_s = m_u^{exp} + m_d^{exp} + m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

| m [MeV] | lat | exp |
|---|---|---|
| $m_\pi$ | 280(3) | 137 |
| $m_D$ | 1927(2) | 1867 |
| $m_{Ds}$ | 1981(1) | 1968 |
| $M_{av}$ | 3103(3) | 3068 |

$$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$

separation between $D\bar{D}$ and $D_s\bar{D}_s$ thresholds smaller than in exp

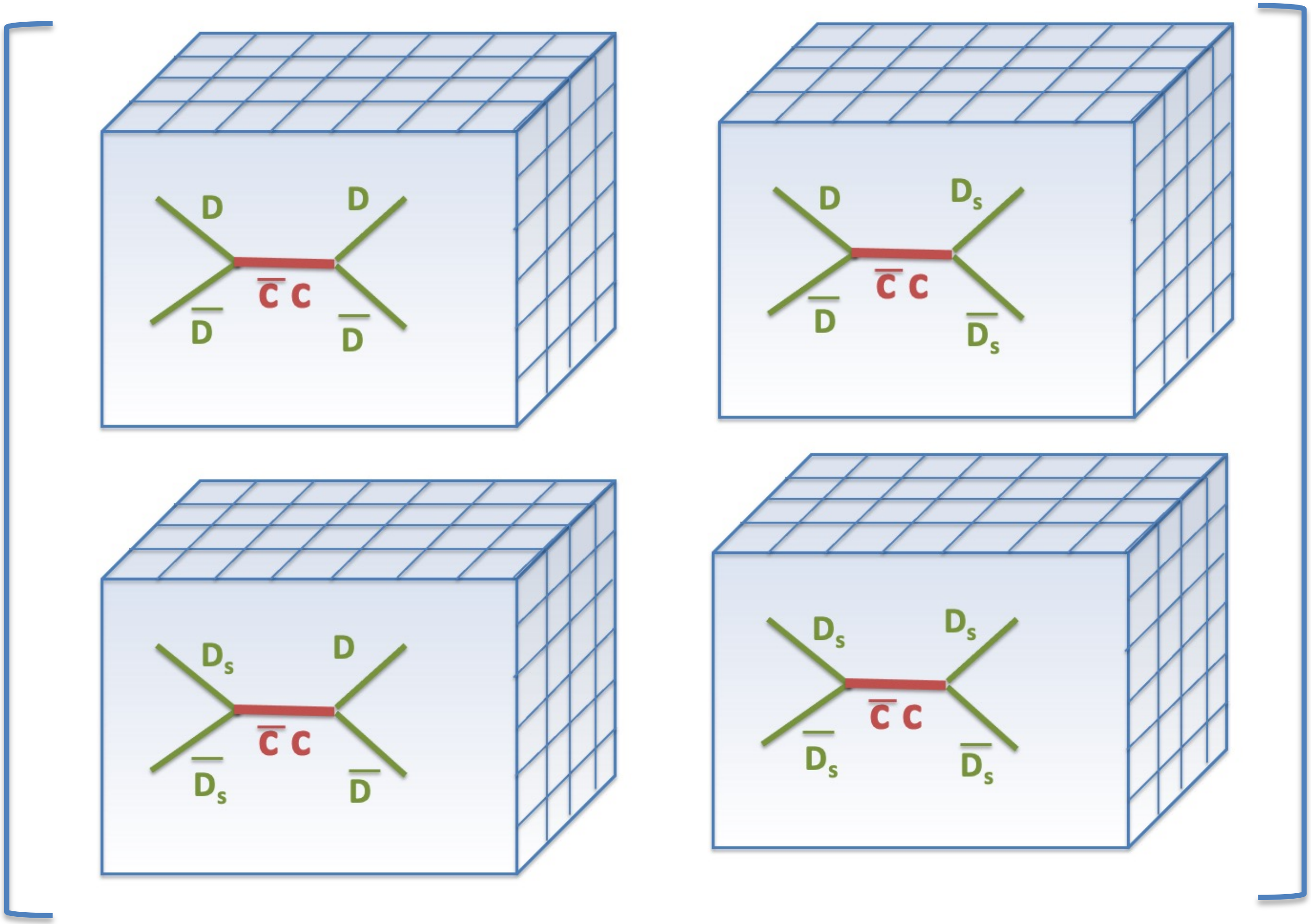
Wick contractions evaluated with distillation or stochastic distillation method.

# Charmonium resonances in coupled $D\bar{D} - D_s \bar{D}_s$ scattering

aim: extract scattering matrix $t_{ij}(E)$ illustrated below using Luscher's finite volume method

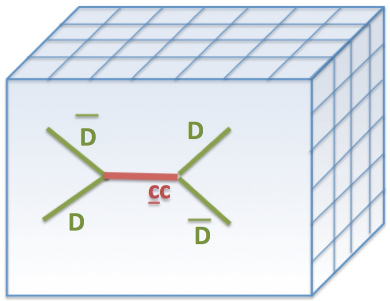
$t_l(E) =$

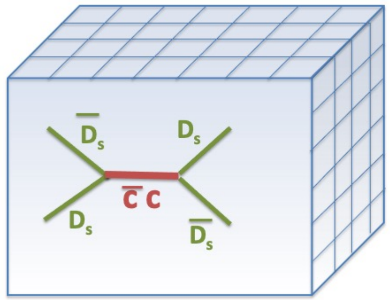
$l=0, 2$



# Towards $E_n$ for coupled-channel $D\bar{D} - D_s \bar{D}_s$ scattering

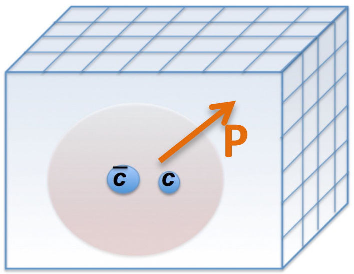
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$




## Implemented operators

$$O^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$



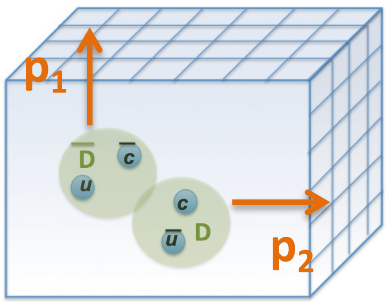

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

P: 0
(0,0,1) $2\pi/N_L$
(1,1,0) $2\pi/N_L$

$N_L$=24, 32

$$O^{\bar{D}D} = (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2} = \bar{D}(\vec{p}_1)\ \ D(\vec{p}_2)$$

$$O^{\bar{D}_s D_s} = (\bar{c}\Gamma_1 s)_{\vec{p}_1} (\bar{s}\Gamma_2 c)_{\vec{p}_2} = \bar{D}_s(\vec{p}_1)\ \ D_s(\vec{p}_2)$$




irreps studied

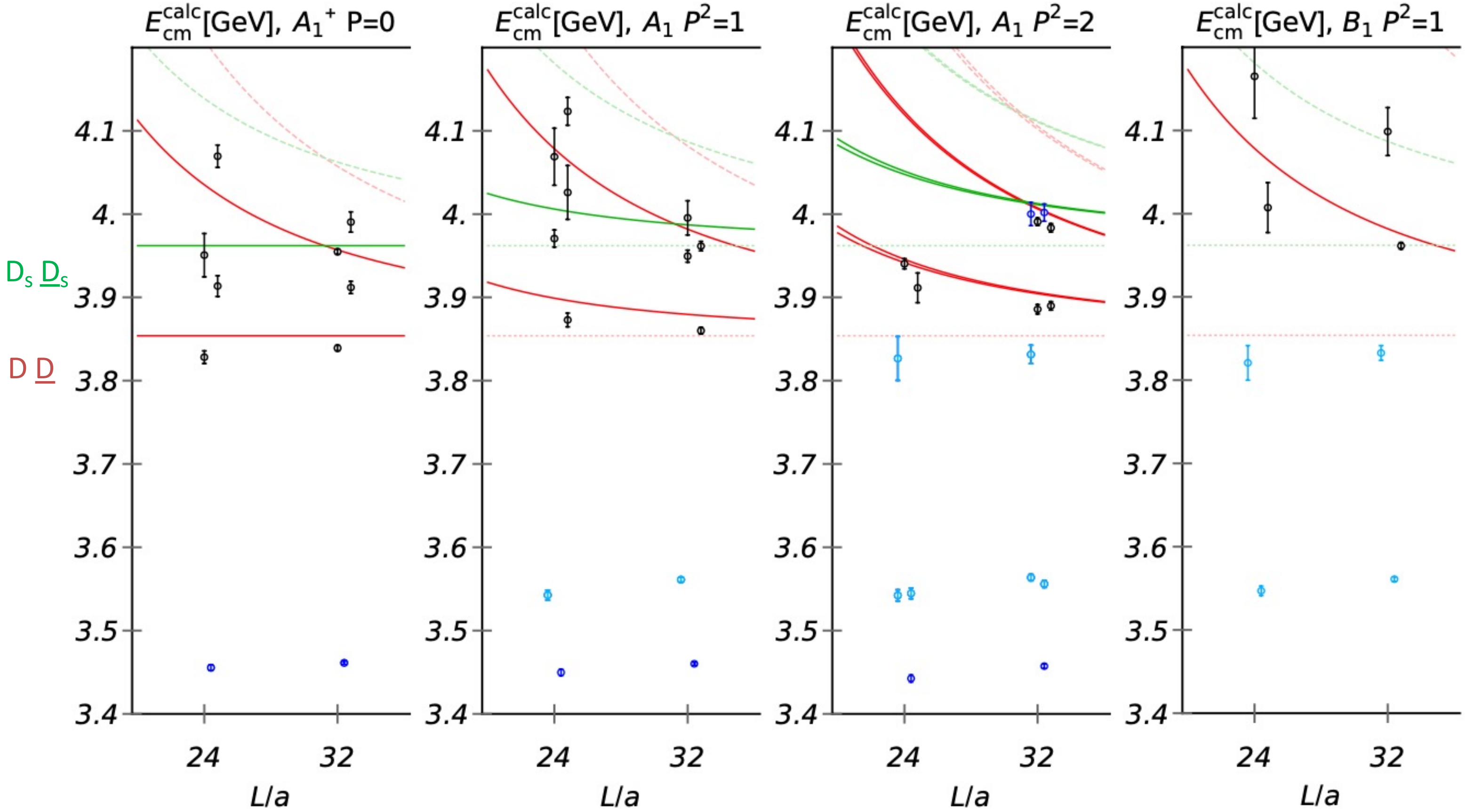
| | | |
|---|---|---|
| i) $\Lambda^P = A_1^+$ , | $\lvert\vec{P}\rvert^2 = 0$ , | $J^P[\lambda] = 0^+[0]$ , |
| ii) $\Lambda = A_1$ , | $\lvert\vec{P}\rvert^2 = 1$ , | $J^P[\lambda] = 0^+[0],\ 2^+[0]$ , |
| iii) $\Lambda = A_1$ , | $\lvert\vec{P}\rvert^2 = 2$ , | $J^P[\lambda] = 0^+[0],\ 2^+[0],\ 2^{\pm}[2]$ |
| iv) $\Lambda = B_1$ , | $\lvert\vec{P}\rvert^2 = 1$ , | $J^P[\lambda] = 2^{\pm}[2]$ . |

$$O^{J/\psi\ \omega} = J/\psi(\vec{p}_1)\ \ \omega(\vec{p}_2)$$

$$O^{\bar{D}^* D^*} = \bar{D}^*(\vec{p}_1)\ \ D^*(\vec{p}_2)$$

# Energies of eigen-states $E_n$ in irreps that contain $J^{PC}=0^{++}, 2^{++}$

for $m_D$=1927 MeV




non-interacting energies denoted

D $\underline{D}$

$D_s$ $\underline{D}_s$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{-+}$

black circles used for

scattering analysis

Extraction of matrix t(E) : i,j=1,2 1: $D\underline{D}$, 2: $D_s\underline{D}_s$, l=0,2

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_j^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij}$$

$$\det[\tilde{K}^{-1}_{l;ij}(E_{cm})\ \delta_{ll'} - B^{\vec{P},\Lambda}_{ll';i}(E_{cm})\ \delta_{ij}] = 0$$

known matrix (we take into account that it is not diagonal in l=0,2)

package TwoHadronsInBox by C. Morningstar et al employed [1707.05817]

$$\rho_i \equiv 2p_i/E_{cm}$$

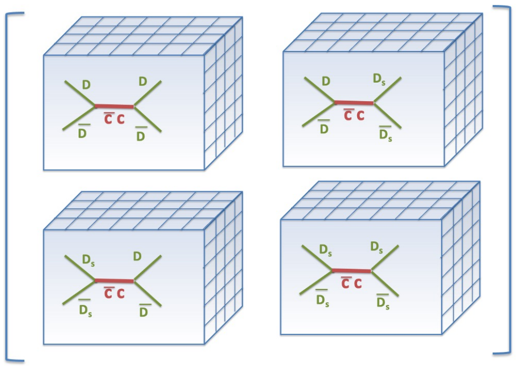
Parametrization for K(s) matrix in each of two energy regions

$$\frac{\tilde{K}^{-1}_{ij}(s)}{\sqrt{s}} = a_{ij} + b_{ij}s$$

$$s = E^2_{cm}$$

we verified aposteriory that both regions can be smoothly connected

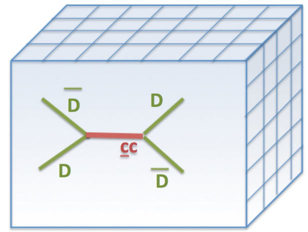
$t_1(E)$ =



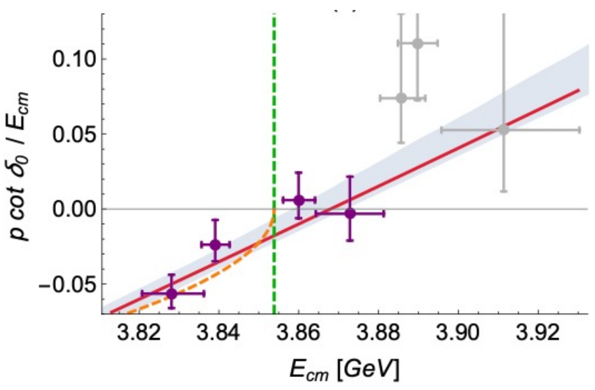
# $J^{PC}=0^{++}$ (l=0) for low energy region :

unexpected shallow bound state slightly below D$\bar{D}$ threshold

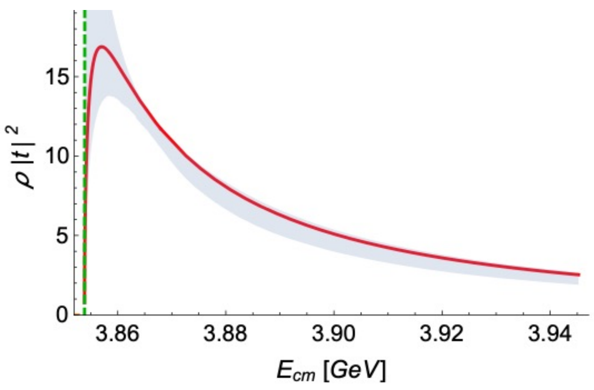
$\bar{D}D$




D $\bar{D}$




D $\bar{D}$




$$m - 2\, m_D = -4.0\ ^{+3.7}_{-5.0}\ \mathrm{MeV}$$

$$D\bar{D} \rightarrow D\bar{D}$$

one-channel D$\bar{D}$ and two-channel analysis give consistent results

exp: not claimed yet

possible hint from Belle [0708.3812] and BaBar [1002.0281 ] ?

look for peak above D$\bar{D}$ threshold

see strategies [Oset et al 1512.04048, 2004.05204, 2010.15431, 1211.1862]

lattice : a virtual bound state pole is present but not mentioned in

[Lang, Leskovec, Mohler, SP, 1503.05363 ]

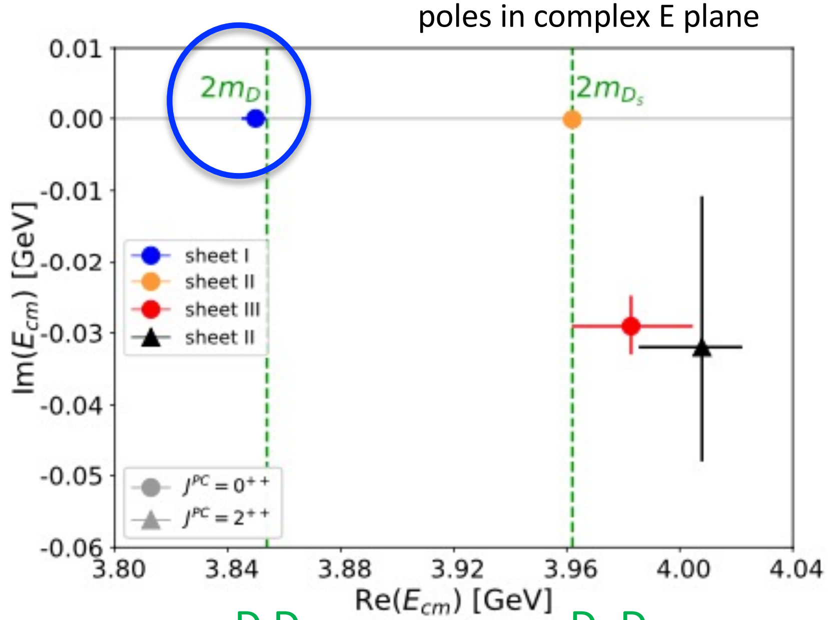
pheno: predicted by effective models with vector meson exchange

[Oset et al, many works, eg. state at ~1720 MeV in Table 4 of 0612179]

predicted as spin partner of X(3872)

[Hildago Duque et al 1305.4487, Baru et al 1605.09649]

poles in complex E plane




D $\bar{D}$ $D_s$ $\bar{D}_s$

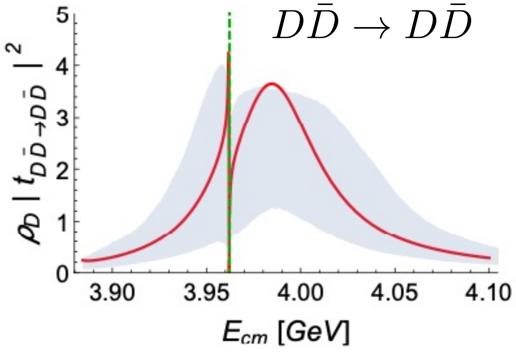
# $J^{PC}=0^{++}$ : higher energy region around $D_s\bar{D}_s$ threshold

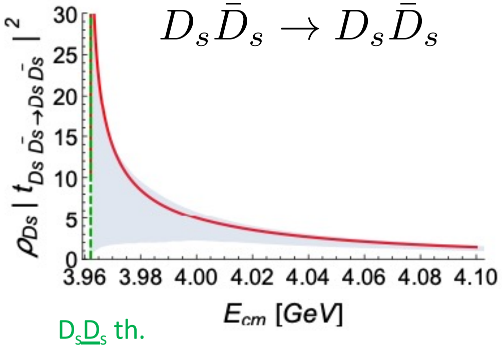
$$D\bar{D} - D_s\bar{D}_s$$

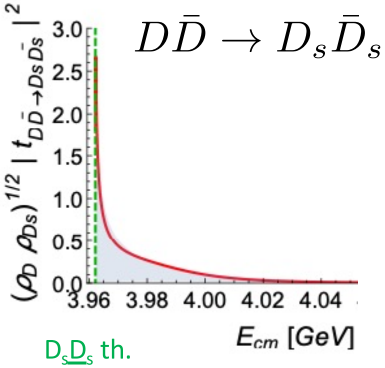
near pole

$$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$



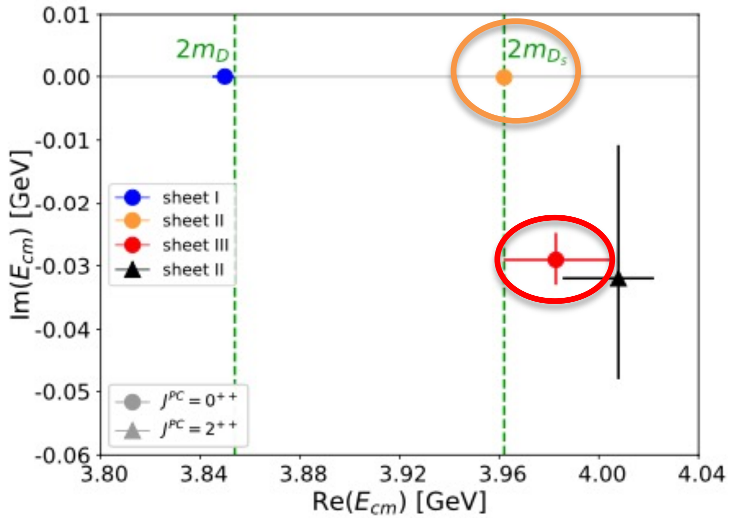




- conventional broad resonance coupling mostly to $D\bar{D}$ — $\bar{c}\ c$

lat $\quad m - M_{av} = 880^{+28}_{-20}\,\text{MeV}\,, \quad g = 1.35\ ^{+0.04}_{-0.08}\,\text{GeV}$

exp $\chi_{c0}(3860) : m - M_{av} = 793\ ^{+48}_{-35}\,\text{MeV}\,, \quad g = 2.5^{+1.2}_{-0.9}\,\text{GeV}$

Belle 2017 $\quad M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$




- state near $D_s\bar{D}_s$ threshold coupling mostly to $D_s\bar{D}_s$ — $\bar{D}_s D_s$ — $\bar{c}s\bar{s}c$

lat $\quad m - 2m_{D_s} = -0.2\ ^{+0.16}_{-4.9}\,\text{MeV}\,, \quad g = 0.10\ ^{+0.21}_{-0.03}\,\text{GeV}$

LHCb 2009.00026

exp $\chi_{c0}(3930) : \ m - 2m_{D_s} = -12.9 \pm 1.6\,\text{MeV}\,, \ \Gamma = 17 \pm 5\,\text{MeV}\,, \ g = 0.67 \pm 0.10\,\text{GeV}$

exp $X(3915) : \ m - 2m_{D_s} = -18.3 \pm 1.9\,\text{MeV}\,, \ \Gamma = 20 \pm 5\,\text{MeV}\,, \ g = 0.72 \pm 0.10\,\text{GeV}$

Babar (those two are likely the same exp state: to be verified by exp)

$D_s\bar{D}_s$ nature explains why width to $D\bar{D}$ is so small

Pheno predictions of $c\bar{s}s\bar{c}$ state:

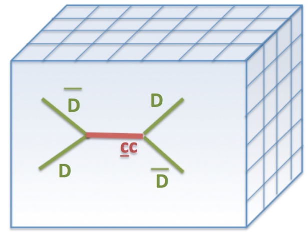
Lebed and Polosa: 1602.08421

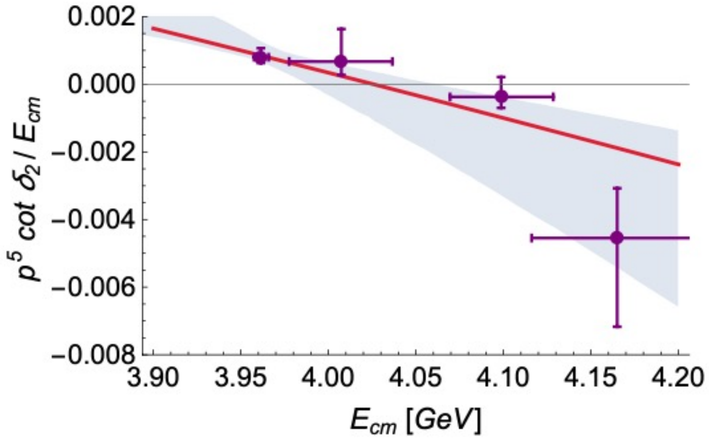
2103.12425

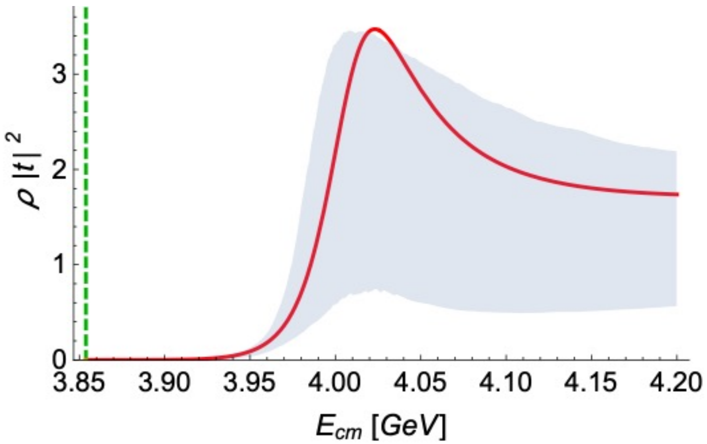
Guo et al, virtual state 2101.01021

# $J^{PC}=2^{++}$ (l=2) : conventional resonance

$\bar{c}\ c$




$$D\bar{D} \to D\bar{D}$$

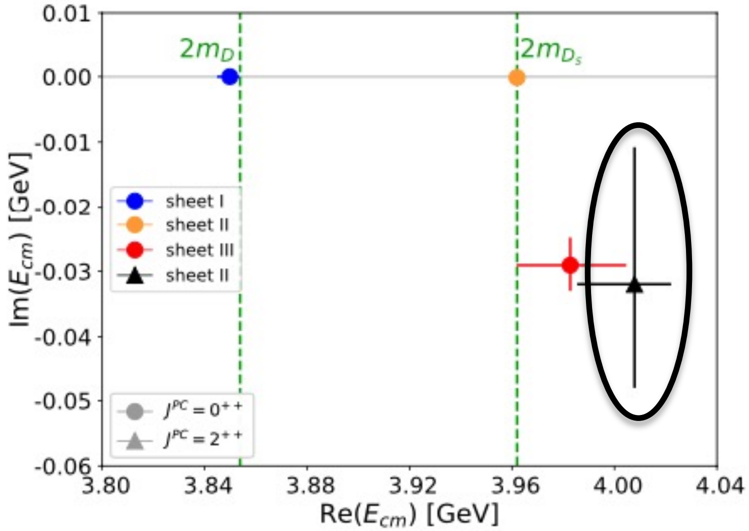
- 2++ resonance $\qquad \Gamma \equiv g^2 p_D^{2l+1}/m^2$

lat $\quad \chi_{c2}(3930): \ m - M_{av} = 904\ ^{+14}_{-22}\,\mathrm{MeV}, \quad g = 4.5^{+0.7}_{-1.5}\,\mathrm{GeV}^{-1}$

exp $\chi_{c2}(3930): \ m - M_{av} = 854 \pm 1\,\mathrm{MeV}, \quad g = 2.65 \pm 0.12\,\mathrm{GeV}^{-1}$

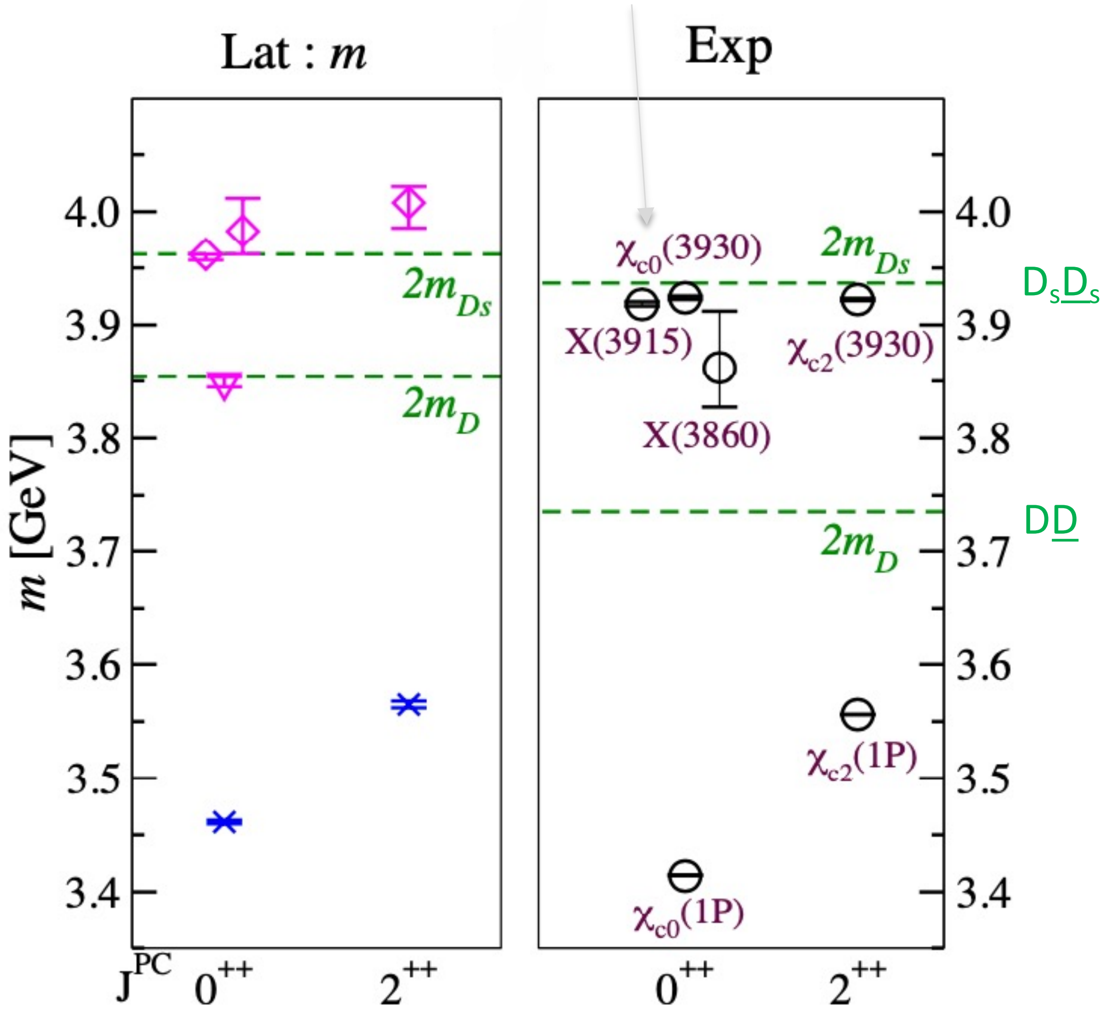
PDG

$$M_{av} = \tfrac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$



# $J^{PC}=0^{++}, 2^{++}$: summary of masses for charmonium-like states

those two are likely the same state




| m [MeV] | lat | exp |
|---|---|---|
| $m_\pi$ | 280(3) | 137 |
| $m_D$ | 1927(2) | 1867 |
| $m_{Ds}$ | 1981(1) | 1968 |
| $M_{av}$ | 3103(3) | 3068 |

$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

# $J^{PC}=0^{++}, 2^{++}$ : summary of masses for charmonium-like states

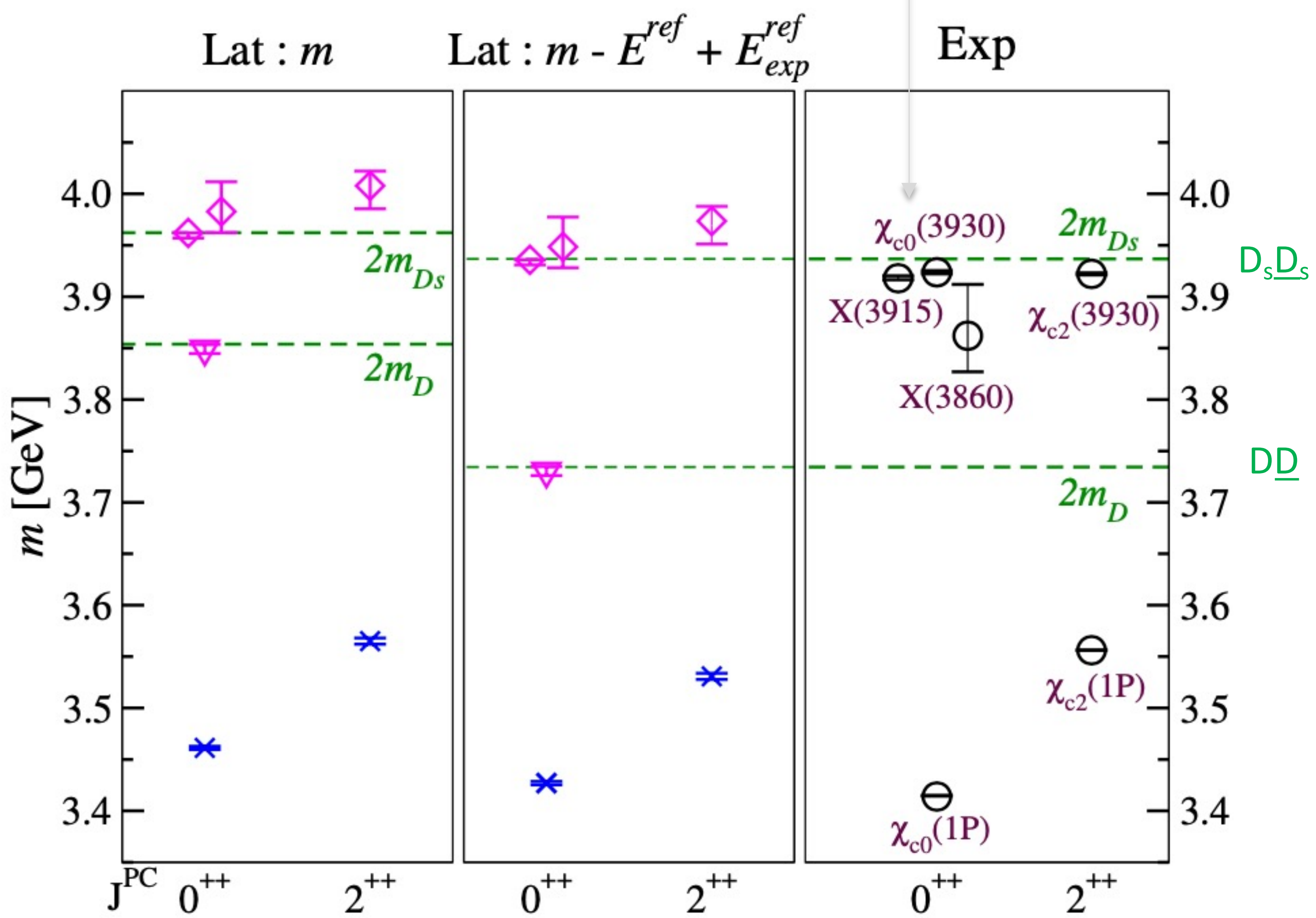
$E^{ref}$

$2m_{D(s)}$: for state closest to $D_s\bar{D}_s$ th.

$M_{av}$ : for other four states

$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

those two are likely the same state



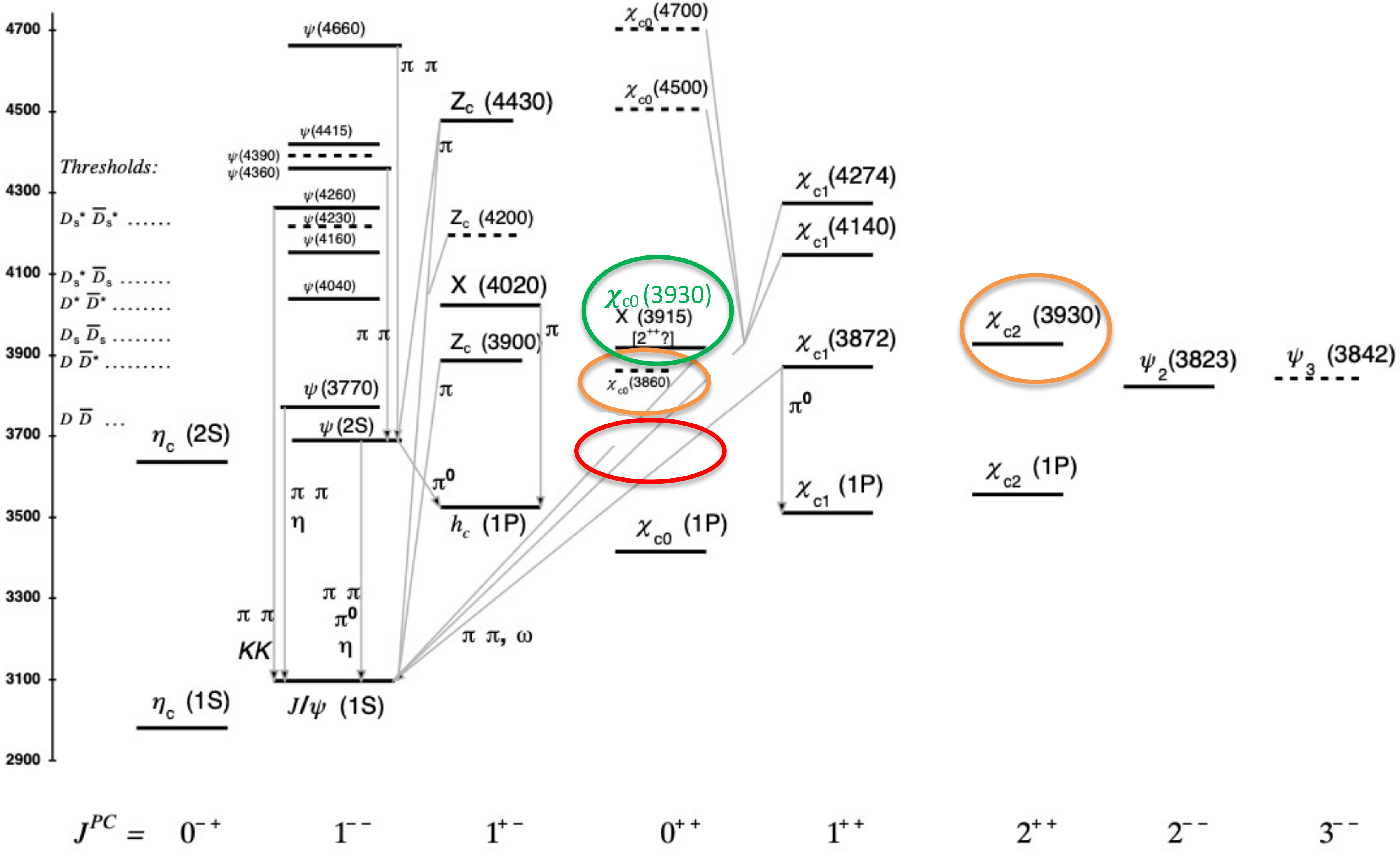
# Challenges

- accurate determination of highly excited $E_n$
- several $J^P$ contribute to each irrep
- extraction of scattering matrices for coupled-channel scattering

# Simplifications / assumptions

- $\eta_c\ \eta$ omitting from operator basis, assuming to be decoupled from $D\underline{D}$, $D_s\underline{D}_s$
- $J/\psi\ \omega$ taking into operator basis, assuming to be decoupled from $D\underline{D}$, $D_s\underline{D}_s$
- analysis of $D\underline{D}$ with $l=2$: assuming negligible coupling to $D_s\underline{D}_s$
- further assumptions: see section 5 of 2011.02541, JHEP

## **Conclusions** ... on charmonium-like states with $J^{PC} = 0^{++}, 2^{++}$

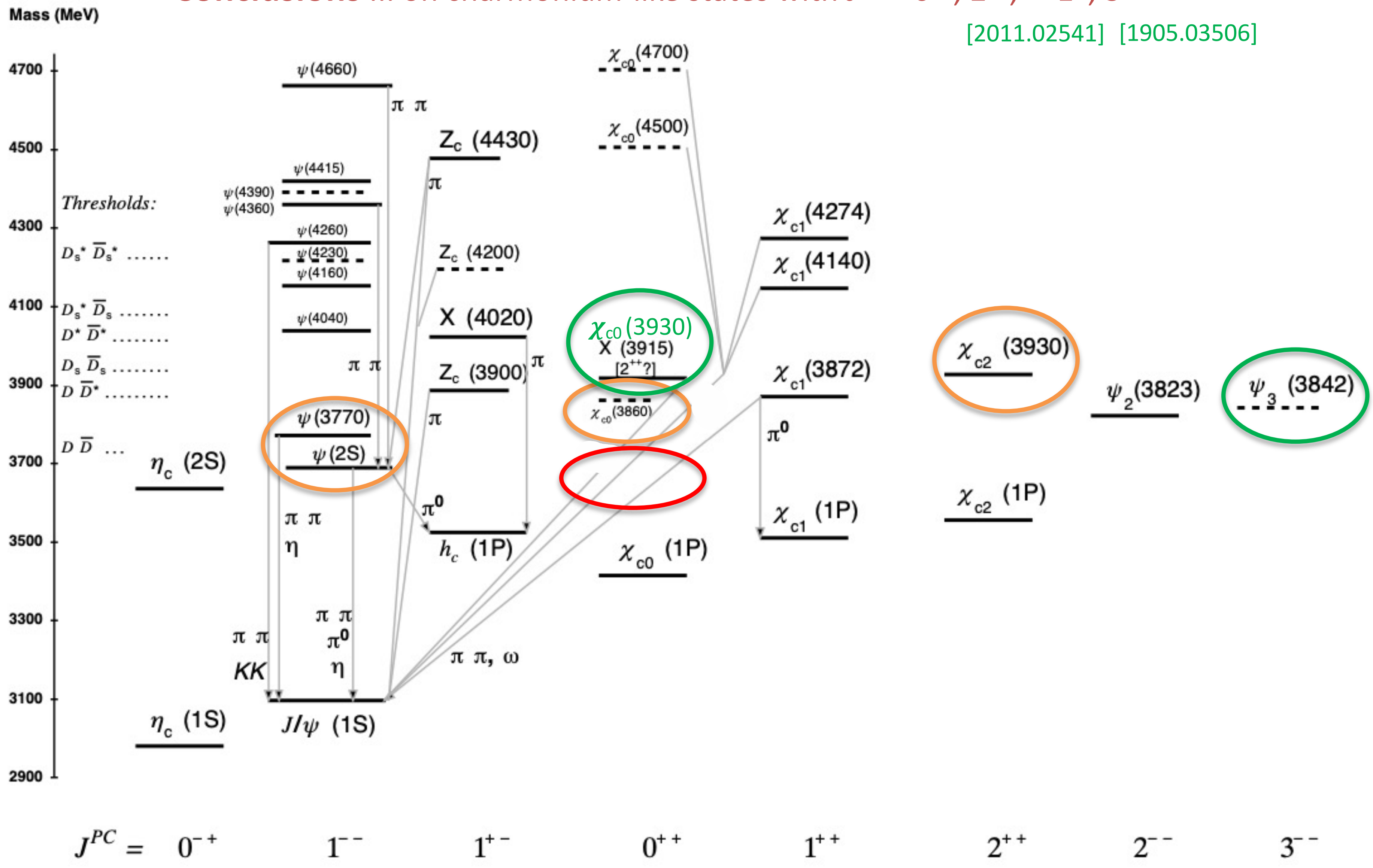



| | | | |
|---|---|---|---|
| this lattice study: | postdicted | predicted, exp discovered | predicted, exp not (yet) discovered |
| likely nature : | $\bar{c}\ c$ | $\bar{D}_s D_s$ | $\bar{D} D$ |

PDG 2019

# **Conclusions** ... on charmonium-like states with $J^{PC}$= $0^{++}$, $2^{++}$, $1^{--}$, $3^{--}$

[2011.02541] [1905.03506]

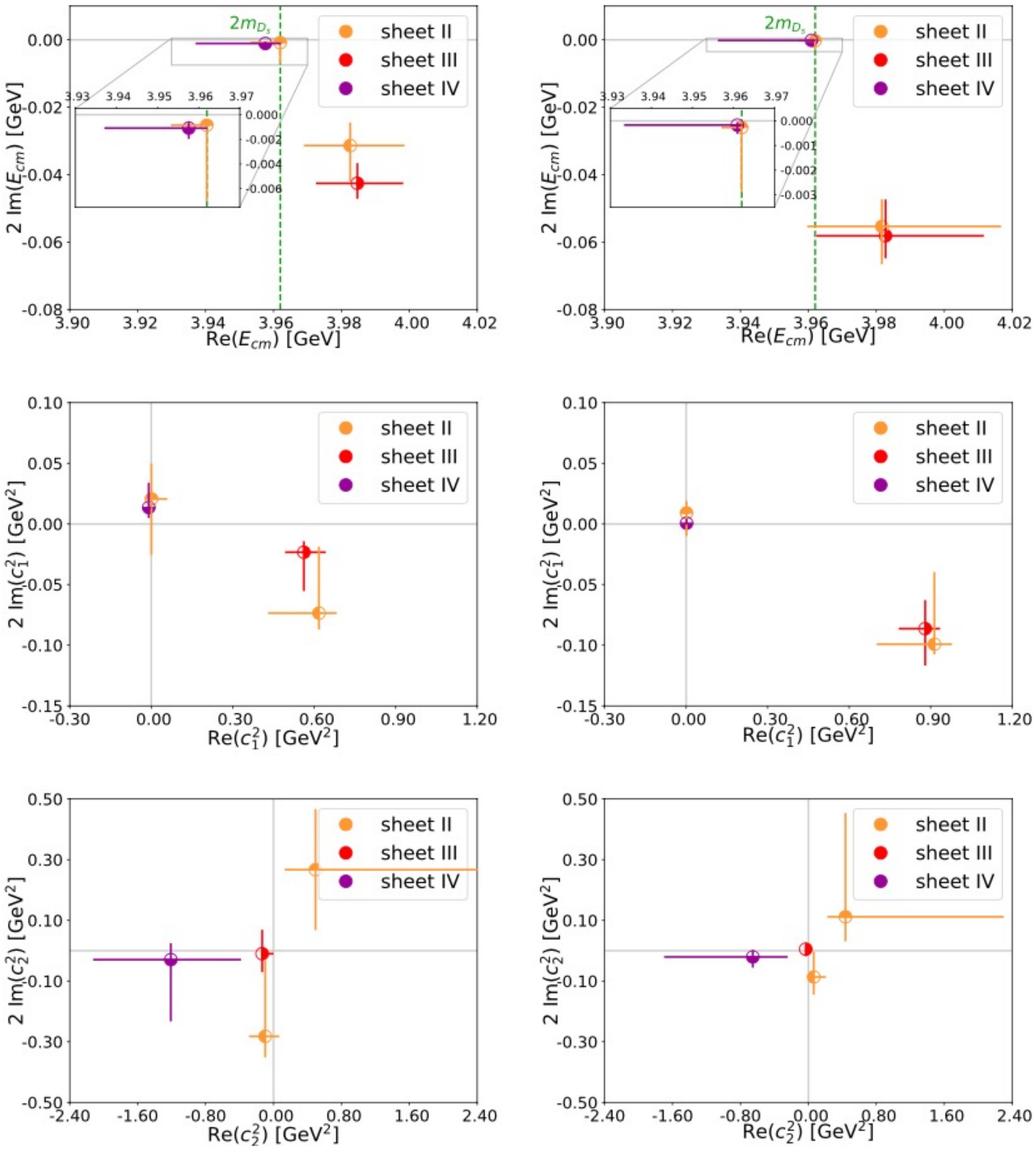



this lattice study: postdicted | predicted, exp discovered | predicted, exp not (yet) discovered

# Backup

# T(E) near poles

$$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}} \quad \text{for} \quad E_{cm} \simeq E^p_{cm}\ , \qquad i = \ 1\ (D\bar{D}),\ 2\ (D_s\bar{D}_s)$$



# Analysis combining lower and higher energy regions

confirms conclusions based on analysis of seperate energy regions

$$\Omega(E_{\rm cm}) = \frac{\det(A)}{\det((\mu^2 + AA^\dagger)^{1/2})}\,, \quad A(E_{\rm cm}) = \tilde{K}^{-1}(E_{\rm cm}) - B(E_{\rm cm})$$

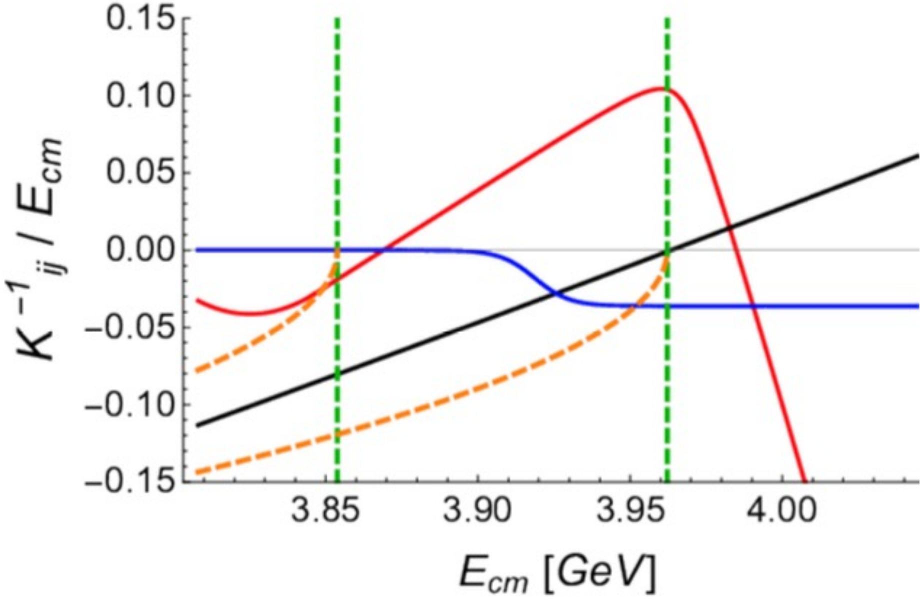



**Figure 15**. The modeled $\tilde{K}^{-1}/\sqrt{s}$ -matrix elements for the coupled $D\bar{D} - D_s\bar{D}_s$ scattering: red for $D\bar{D} \to D\bar{D}$, black for $D_s\bar{D}_s \to D_s\bar{D}_s$ and blue for $D_s\bar{D}_s \to D\bar{D}$. The green dashed lines indicate the $D\bar{D}$ and the $D_s\bar{D}_s$ thresholds. The left orange dashed line gives the bound state constraint for $D\bar{D}$ channel. The right orange line gives the bound state constrain for the $D_s\bar{D}_s$ channel in the limit that the two channels are decoupled.

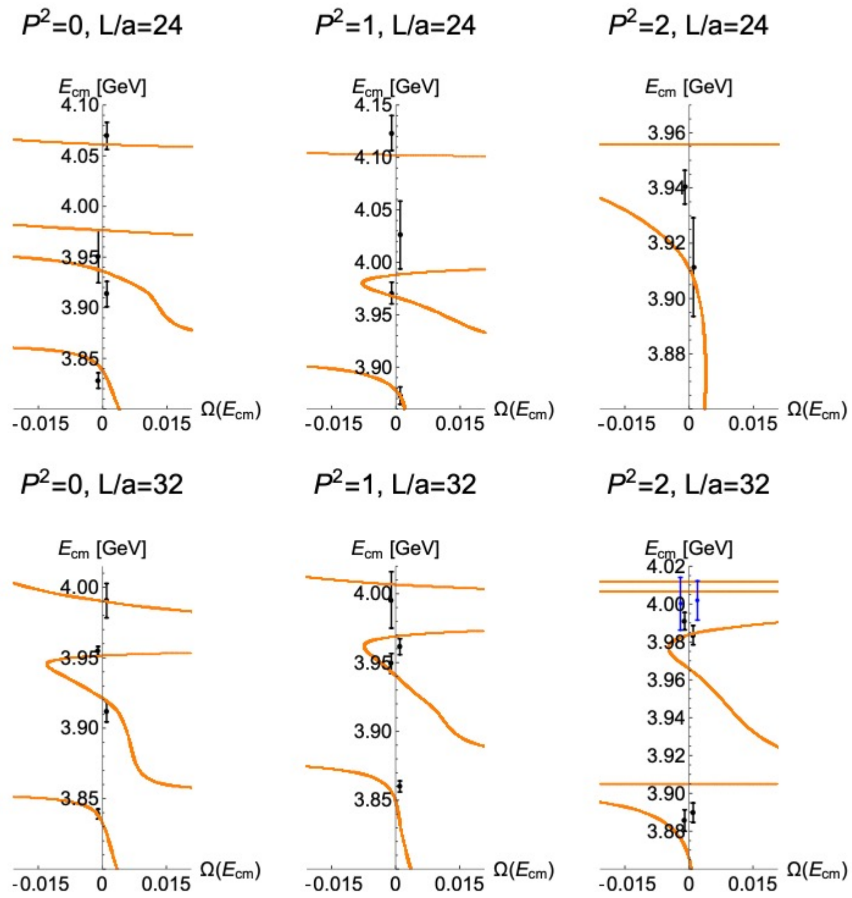
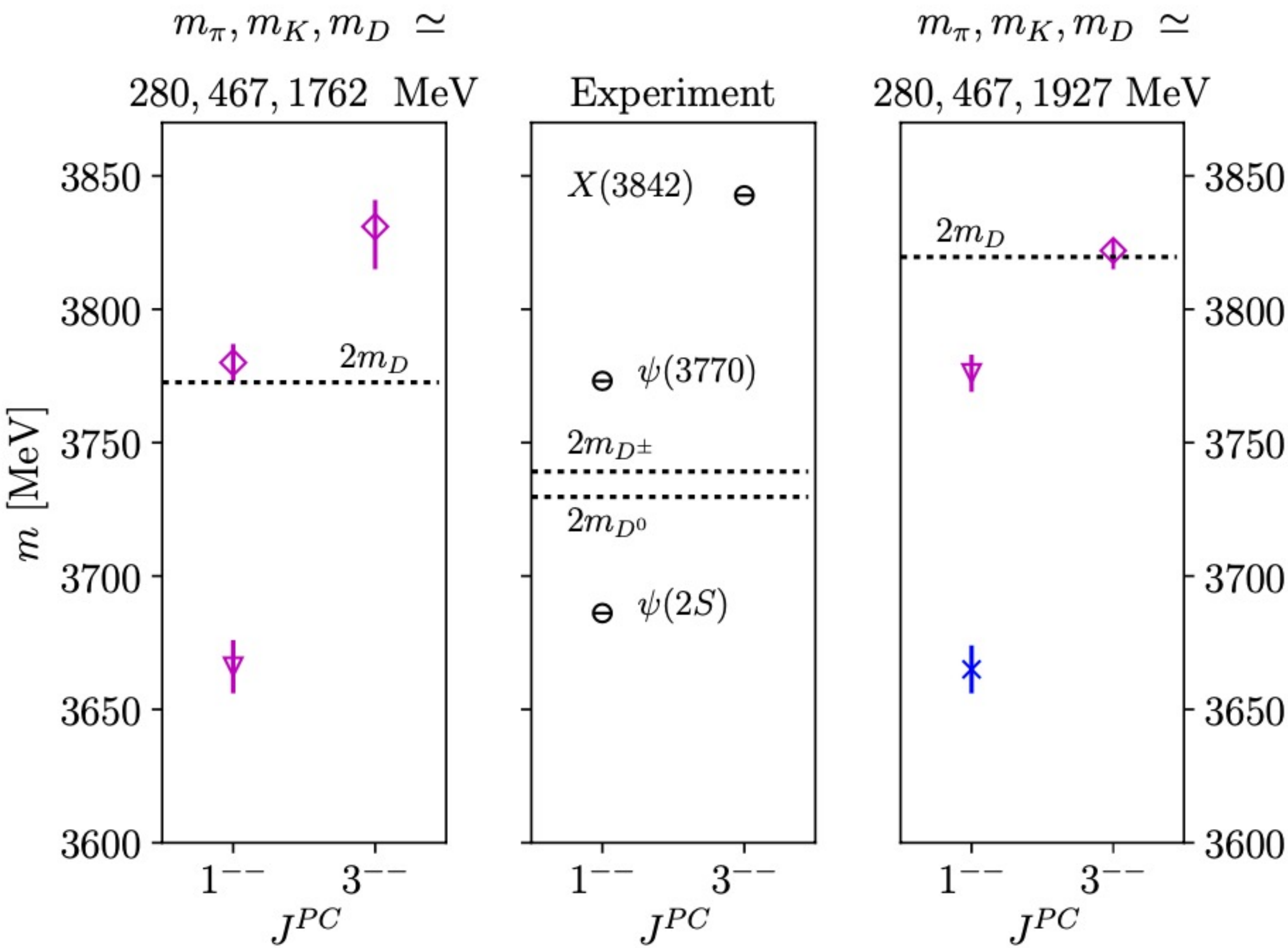



**Figure 16**. The $\Omega(E_{\rm cm})$ function (defined in eq. (C.1)) for the scattering matrix of the coupled channels $D\bar{D} - D_s\bar{D}_s$ in the wider energy region ($E_{\rm cm} \simeq 2m_D - 4.1$ GeV) is given by the orange line. The observed eigen-energies are given by circles and the coloring is the same as in figure 2.

# Charmonium-like states with $J^{PC}=1^{--},3^{--}$

[1905.03506, PRD 2019]



# Results based on cc operators and the challenge to determine $J^P$

[1811.04116, PRD 2019]

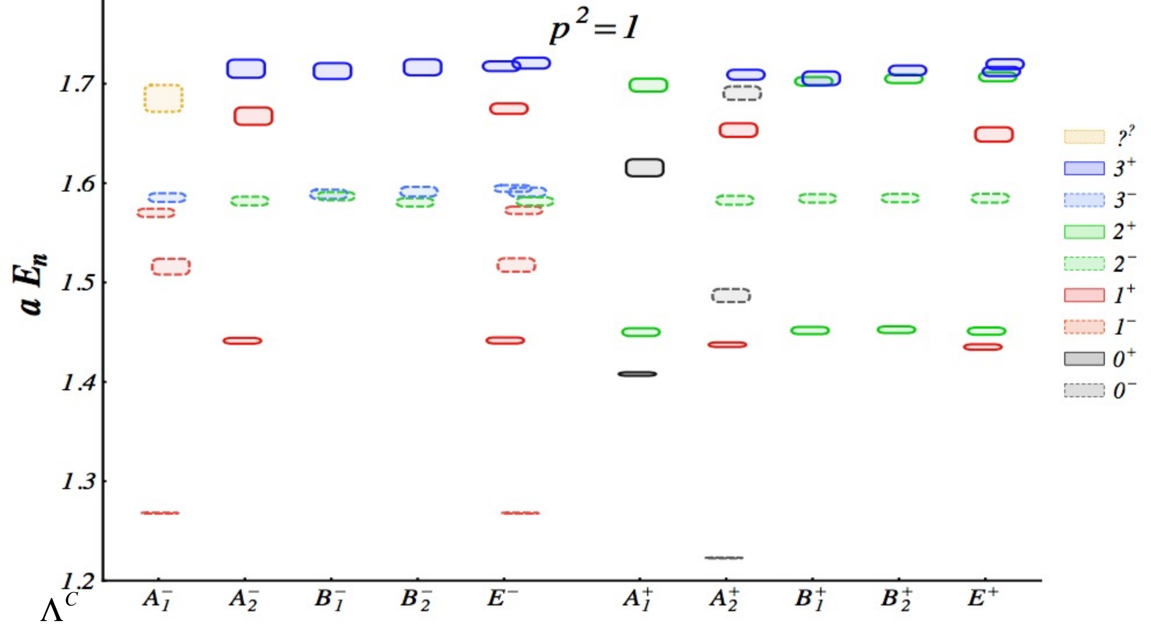
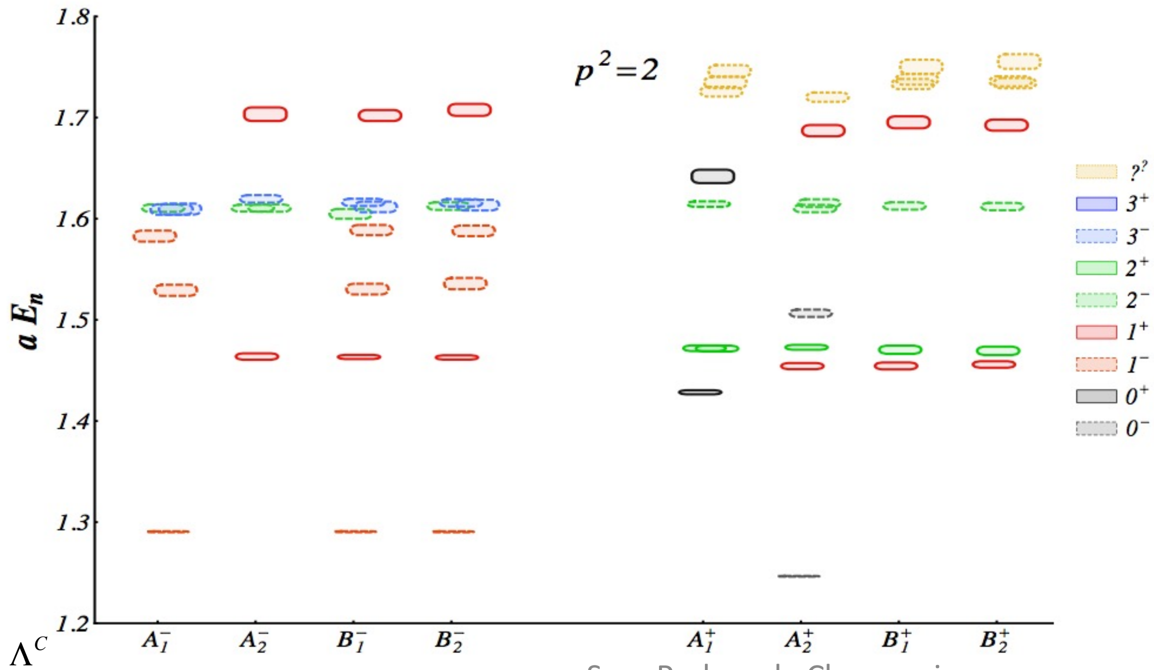



FIG. 11. $J^P$-identified charmonium spectrum in the moving frame with $\mathbf{p}=(0,0,1)$. Irreps $\Lambda^C$ of group $Dic_4$ are presented. The colors indicate $J^P$ of states according to the color-coding (21).

| $\mathbf{p}=(0,0,1)$, $Dic_4$ | | |
|---|---|---|
| $\Lambda$ (*dim*) | $\lvert\lambda\rvert^{\tilde\eta}$ | $J^P$ (at rest) |
| $A_1$ (1) | $0^+$ | $0^+,\ 1^-,\ 2^+,\ 3^-$ |
| $A_2$ (1) | $0^-$ | $0^-,\ 1^+,\ 2^-,\ 3^+$ |
| $E$ (2) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |
| $B_1$ (1) | 2 | $2^\pm,\ 3^\pm$ |
| $B_2$ (1) | 2 | $2^\pm,\ 3^\pm$ |




| $\mathbf{p}=(1,1,0)$, $Dic_2$ | | |
|---|---|---|
| $\Lambda$ (*dim*) | $\lvert\lambda\rvert^{\tilde\eta}$ | $J^P$ (at rest) |
| $A_1$ (1) | $0^+$ | $0^+,\ 1^-,\ 2^+,\ 3^-$ |
| | 2 | $2^\pm,\ 3^\pm$ |
| $A_2$ (1) | $0^-$ | $0^-,\ 1^+,\ 2^-,\ 3^+$ |
| | 2 | $2^\pm,\ 3^\pm$ |
| $B_1$ (1) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |
| $B_2$ (1) | 1 | $1^\pm,\ 2^\pm,\ 3^\pm$ |
| | 3 | $3^\pm$ |

lower $m_c$ : mD=1762(2) MeV