

Abstract

We introduce a new non-perturbative method to tune the parameters of the Columbia formulation of an anisotropic, clover-improved relativistic heavy-quark (RHQ) action. By making use of suitable observables which can be computed at a sequence of heavy-quark mass values, employing an $O(a)$ -improved discretized action with domain-wall chiral fermion, and safely interpolated between the accessible heavy-quark mass region and the static point predicted by heavy-quark effective theory, we are able to precisely determine the unknown coefficients of the RHQ action. In this proof-of-principle study we benefit from the RBC/UKQCD Iwasaki gauge configurations with 2 + 1 flavors of dynamical quarks, at three values of the lattice spacing varying from 0.11 to 0.063 fm. Preliminary results and applications to bottom spectroscopy are also presented.

Advantages of the new method

- Do not need to sacrifice lattice predictivity using experimental inputs for mass splittings
- Can predict tuned RHQ parameters for any heavy-meson mass
- Preliminary results show that an extension to non-perturbative renormalization and operator improvement in a position-space scheme is possible

Simulation details

- Domain-wall fermions for the light quarks (u, d, s)
- Iwasaki gauge action
- Configurations generated by RBC and UKQCD Collaborations

ensemble	$(L/a)^3 \times (T/a)$	$\approx a$ (fm)	am_l	am_s	M_π (MeV)
24l	$24^3 \times 64$	0.11	0.005	0.04	340
24lh	$24^3 \times 64$	0.11	0.01	0.04	426
32l	$32^3 \times 64$	0.083	0.004	0.03	302
32lfine	$32^3 \times 64$	0.063	0.0047	0.0186	371

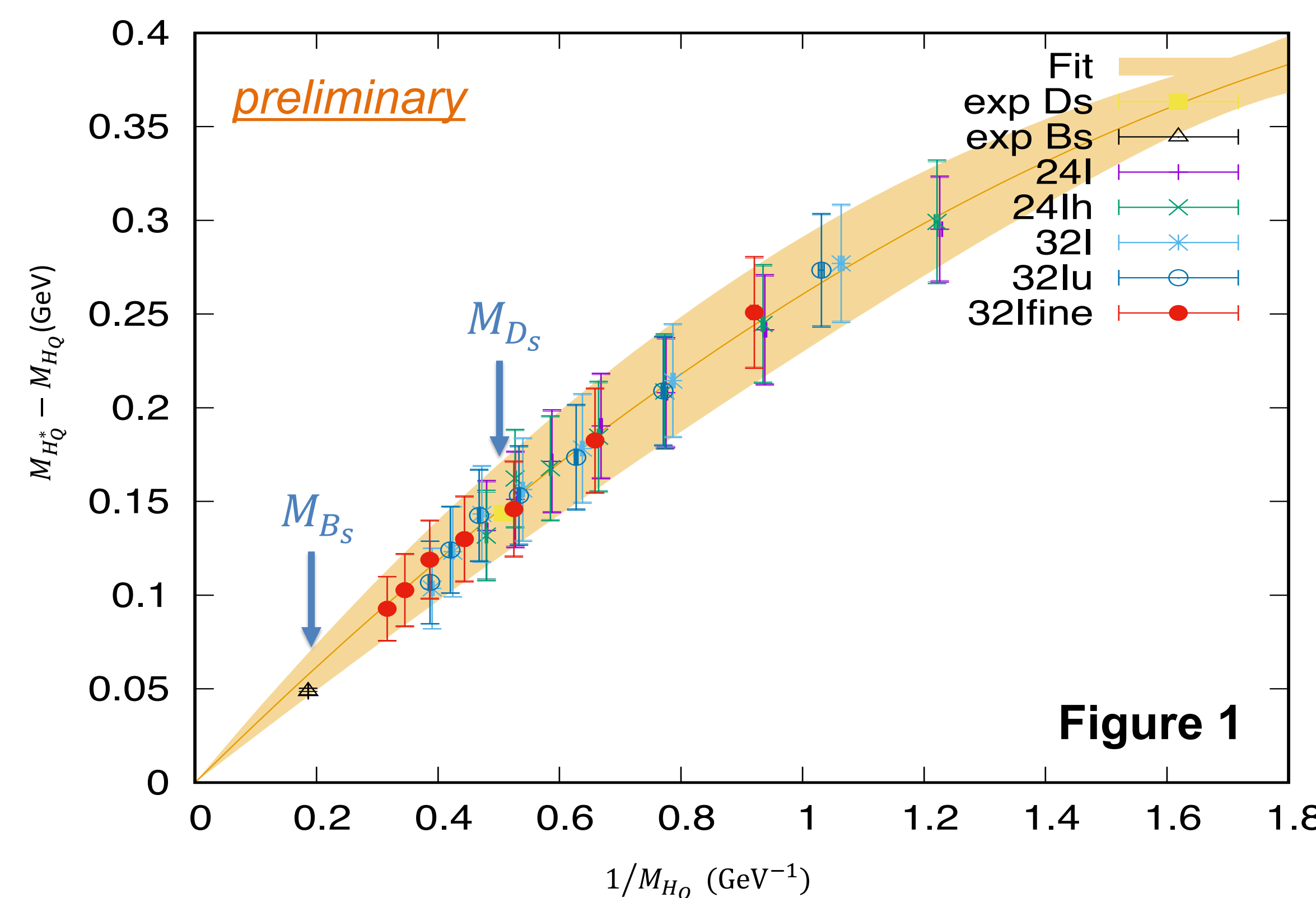
RHQ action for the b -quark

- Relativistic Heavy Quark action developed by Christ, Li and Lin (2007)
- Builds upon Fermilab approach by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation
- Improved action describes energies and amplitudes of on-shell states

- Heavy-quark mass is treated to all orders in $(m_Q a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
 - resulting errors will be of $O(\vec{p}^2 a^2)$
 - allows computation of heavy-light quantities with discretization errors of the same size as in light-light systems
- Depends on three parameters
 - bare quark mass m_0
 - anisotropy parameter $\zeta(m_0 a)$
 - coefficient c_P ($m_0 a$) of an isotropic Sheikholeslami and Wohlert term
- Applies for all values of the heavy-quark mass, for both heavy-heavy and heavy-light systems
- Has a smooth continuum limit

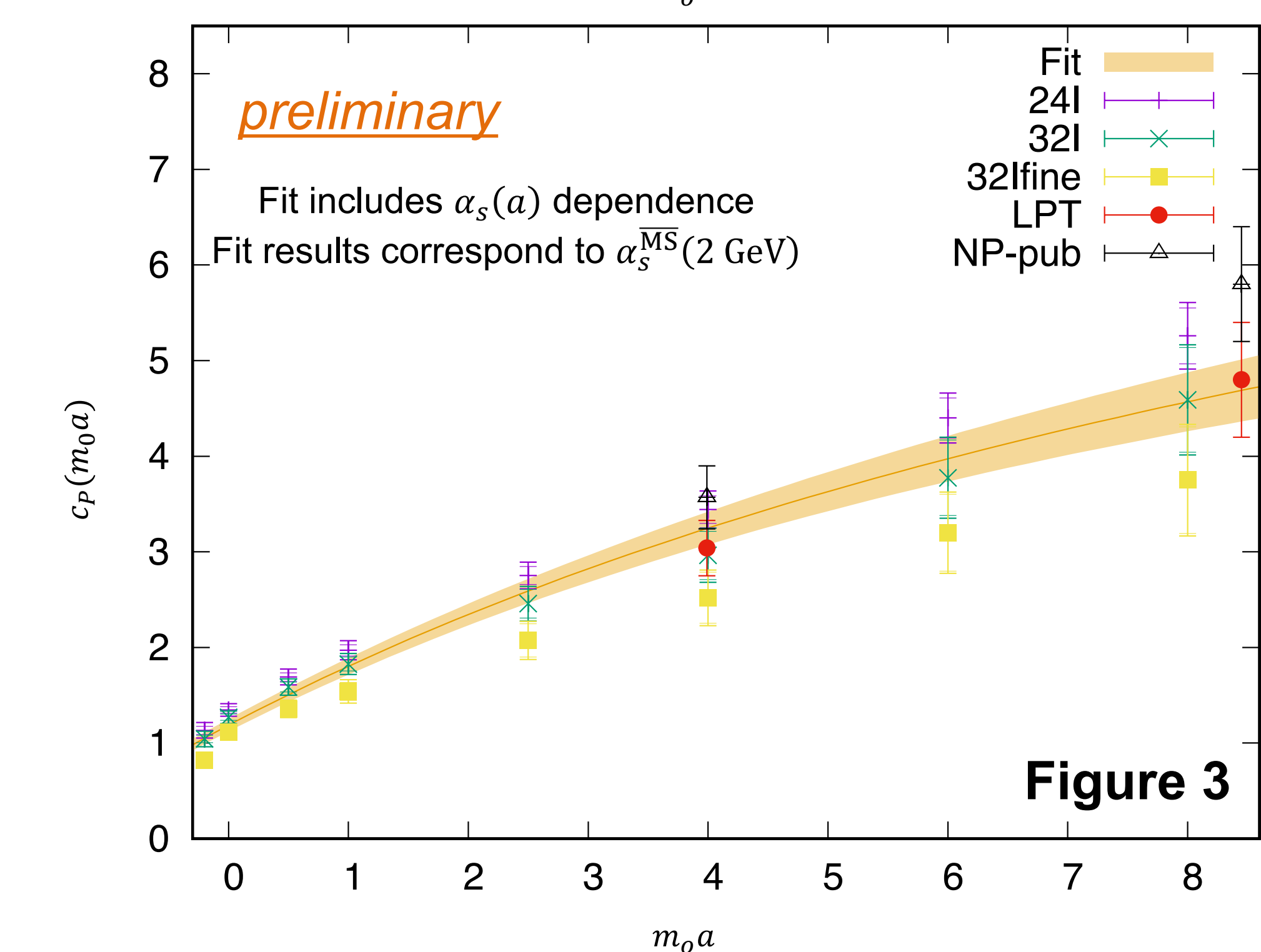
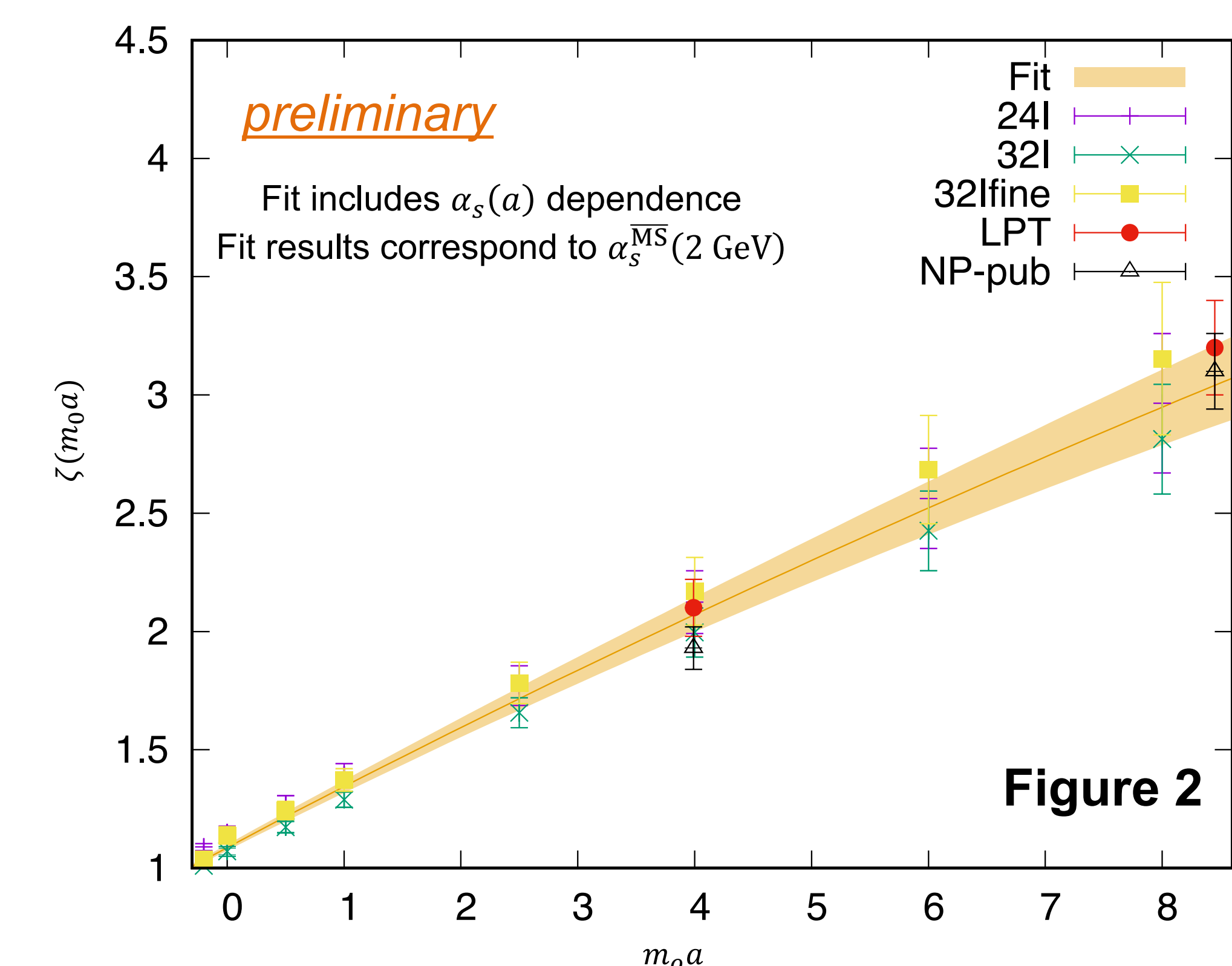
Non-perturbative tuning of the RHQ action parameters

- Compute the hyperfine splitting $\Delta_{HQ} = M_{H_Q^*} - M_{H_Q}$ of heavy-light mesons at a sequence of heavy-quark mass values (up to $m_Q \approx 2m_c$) using the DW action
- Interpolate between the accessible heavy-quark mass region and the known static point predicted by HQET (Isgur and Wise 1989, Neubert 1992), see Figure 1



- Compute Δ_{HQ} at a sequence of $m_0 a$ values using the RHQ action. Start from educated guesses for the parameters c_P and ζ

- Measure the ratio $M_{HQ}^{rest} / M_{HQ}^{kinetic}$
- Probe parameter space at five points for each $m_0 a$ value, by varying one of the two parameters $\{c_P, \zeta\}$ by a chosen uncertainty $\pm \sigma_{\{c_P, \zeta\}}$
- Assume linearity to relate parameters and observables
- Obtain tuned parameters (see Figures 2 and 3) by requiring that Δ_{HQ} agrees with the DW fit result and that $M_{HQ}^{rest} = M_{HQ}^{kinetic}$



Conclusions

- In Figures 2 and 3 we compare our non-perturbative determinations with lattice perturbation theory predictions (LPT) and previous results (NP-pub), see Aoki *et al.* & Lehner 2012. Good agreement is found.
- Preliminary results:
 - $\Delta_{B_s} = 54(11)_{stat}(3)_{syst}$ MeV [$\Delta_{B_s}^{exp} = 49(2)$ MeV]
 - $\Delta_{B_c} = 50(9)_{stat}(2)_{syst}$ MeV