The semiclassical ensembles of instanton-dyons describe deconfinement and chiral phase transitions in the usual and deformed QCD

Edward Shuryak
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Lattice 2021, July 27, virtual at MIT

collaborators
Dallas DeMartini
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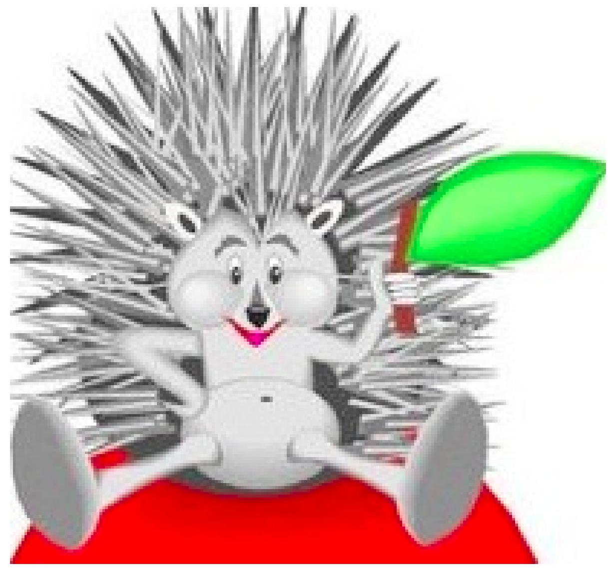
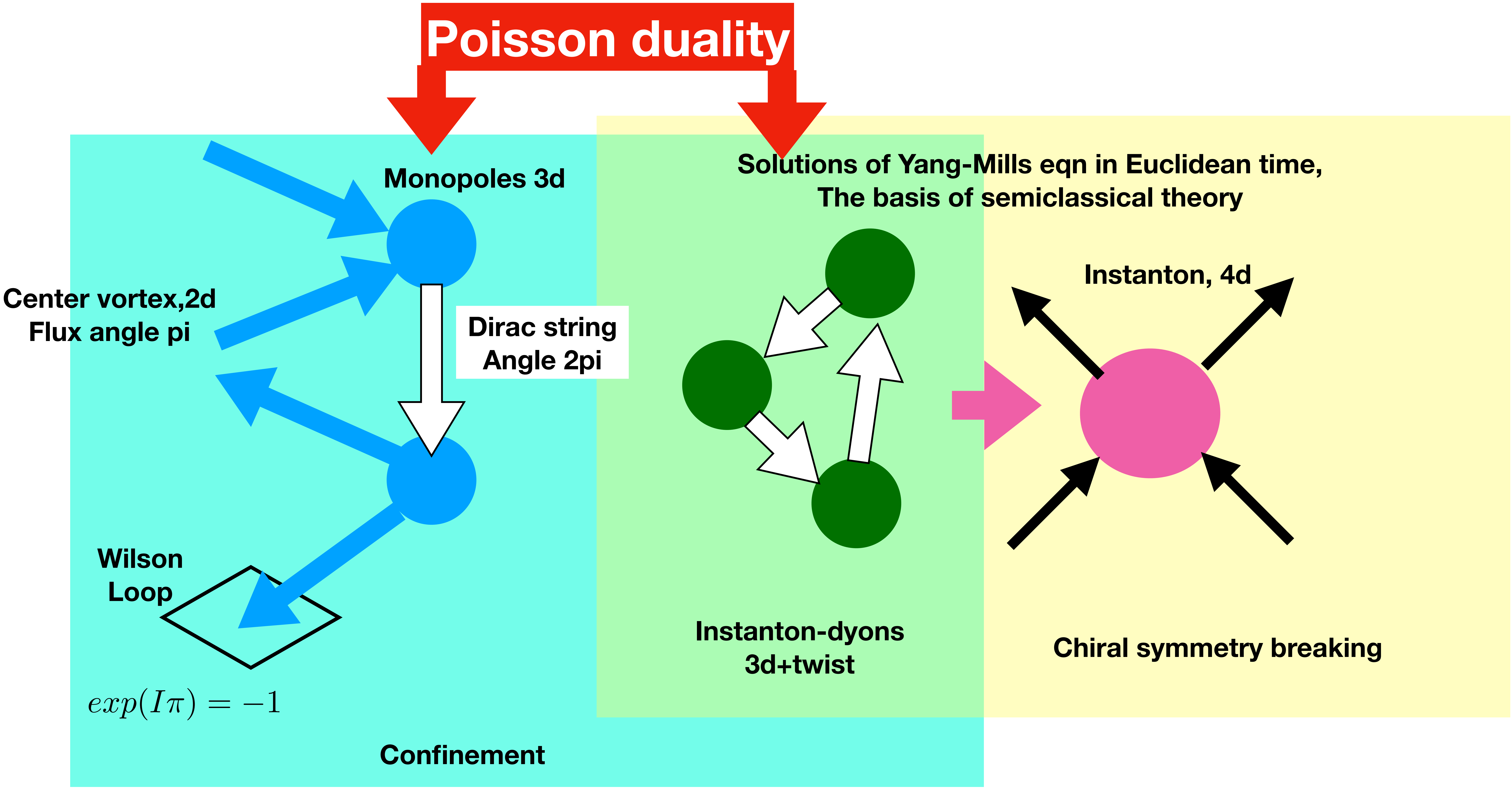
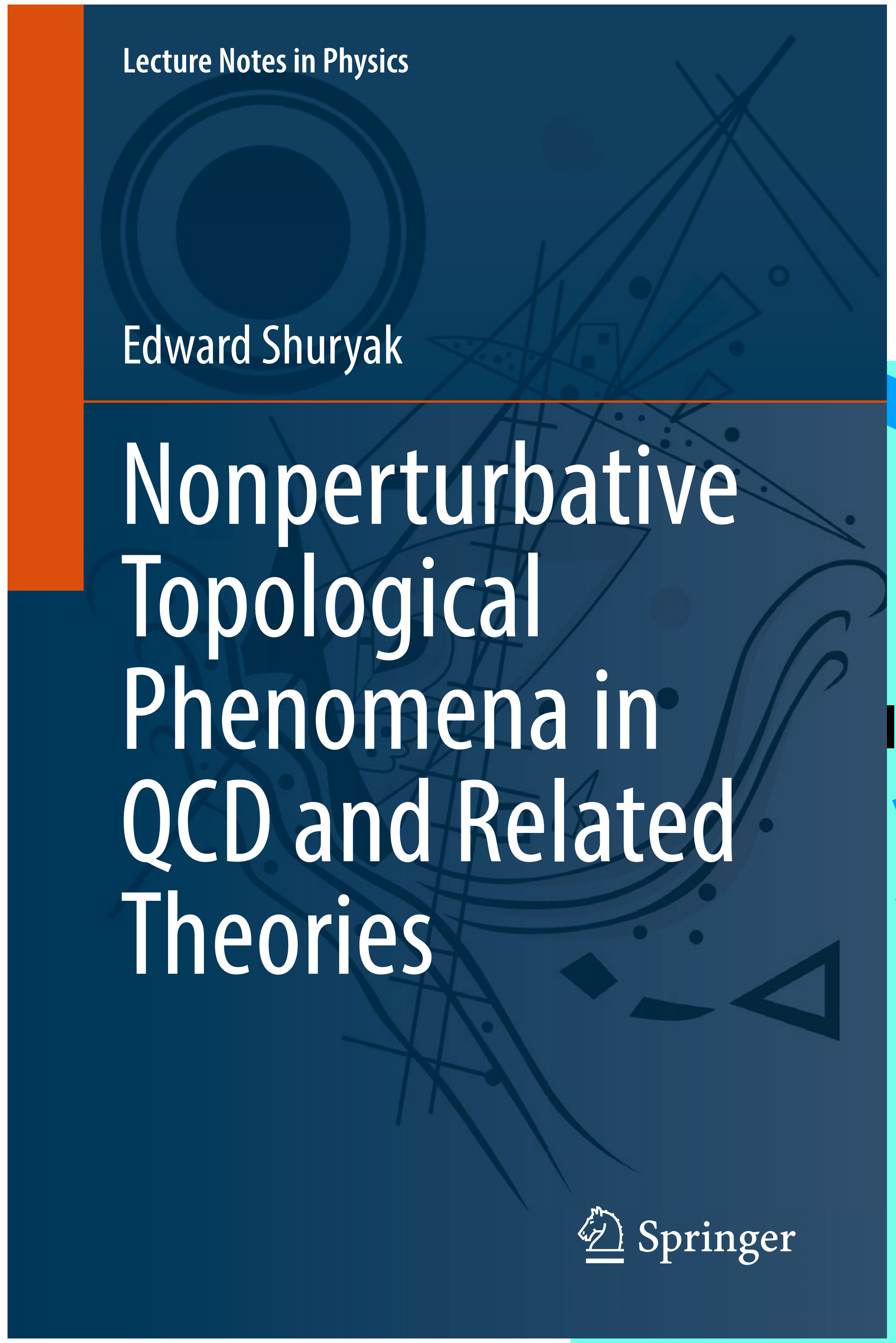
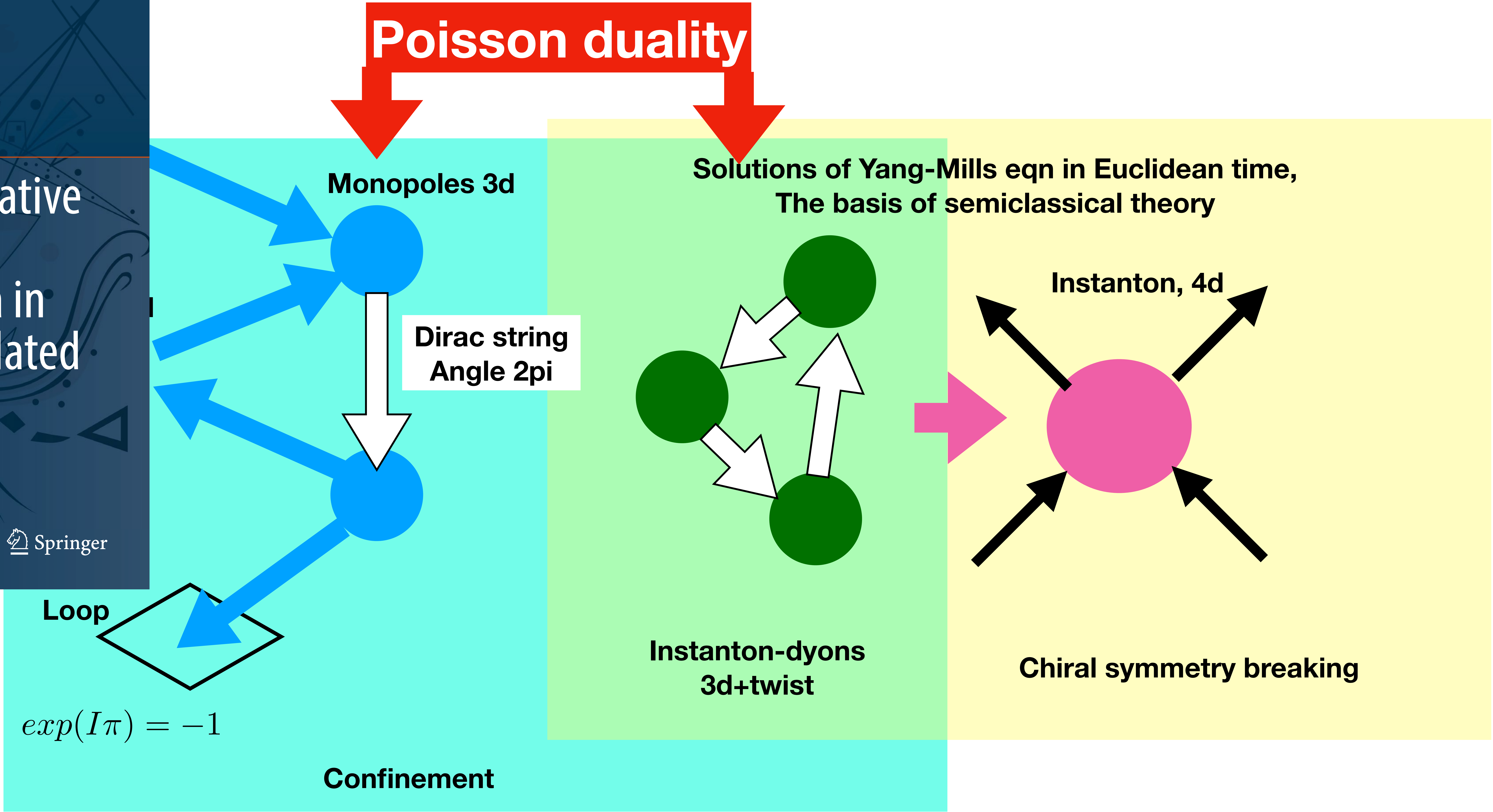


outline

- map of gauge topology
- VEV of Polyakov line and the instanton-dyons
- Dirac zero and quasizero fermionic states on the lattice
- Deconfinement transition,
- QCD deformation via Polyakov line operators
- Chiral symmetry breaking
- QCD deformation via quark periodicity phases
- Poisson duality Instanton-dyons <=> Monopoles


Poisson duality
Monopoles 3d
Solutions of Yang-Mills eqn in Euclidean time,
The basis of semiclassical theory
Center vortex,2d
Flux angle pi
Dirac string
Angle 2pi
Instanton, 4d
Wilson
Loop
$exp(I\pi) = -1$
Instanton-dyons
3d+twist
Chiral symmetry breaking
Confinement


Lecture Notes in Physics
Edward Shuryak
Nonperturbative Topological Phenomena in QCD and Related Theories
Springer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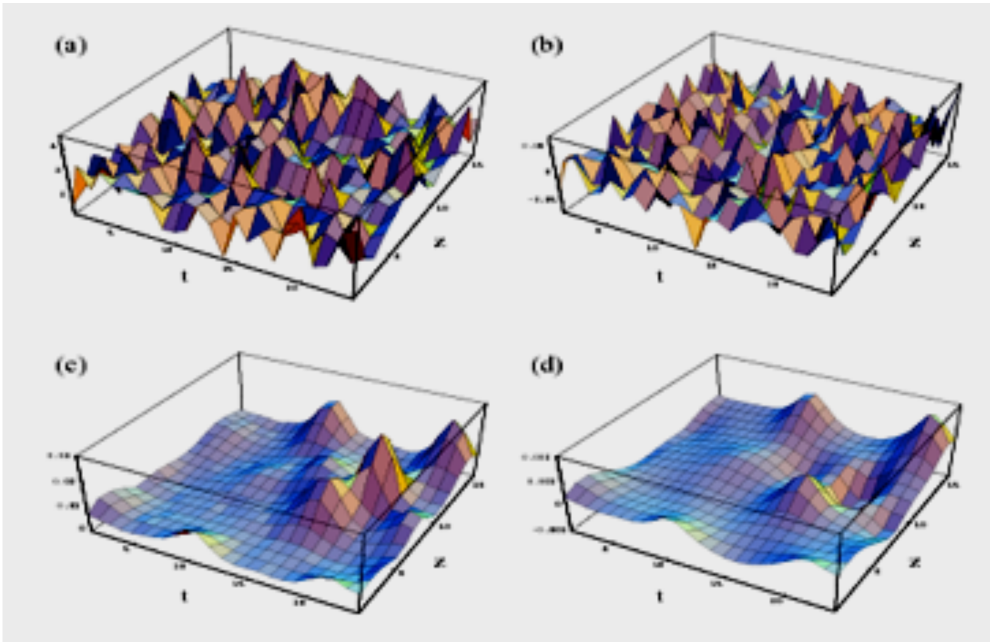
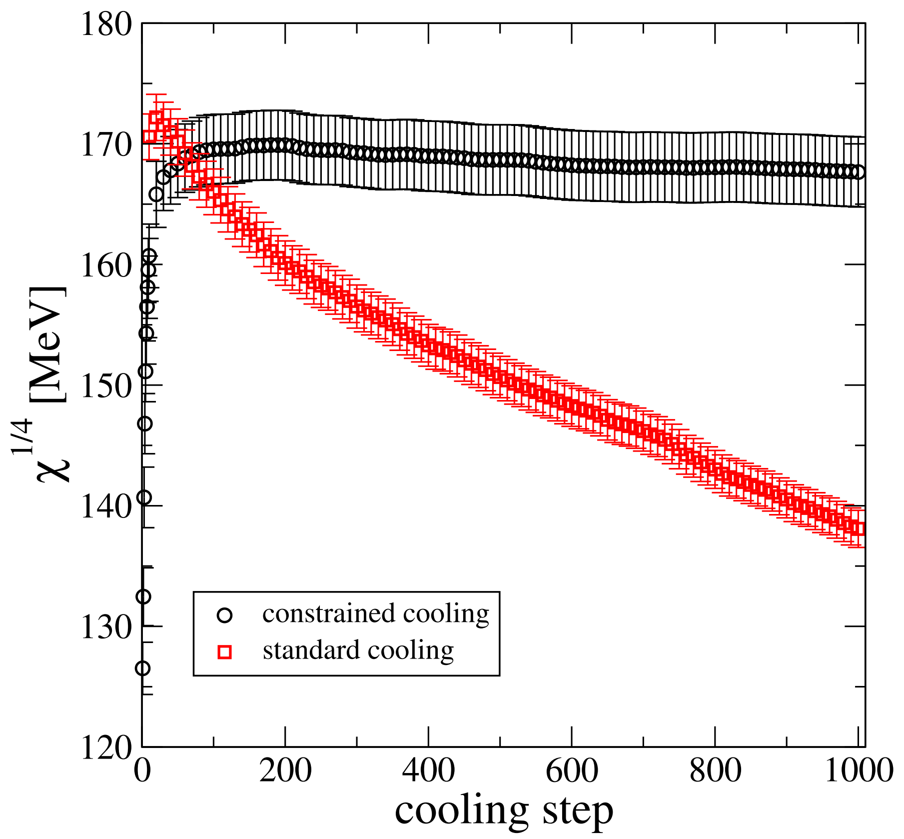
“action cooling” is known to eliminate gluons and lead to instantons

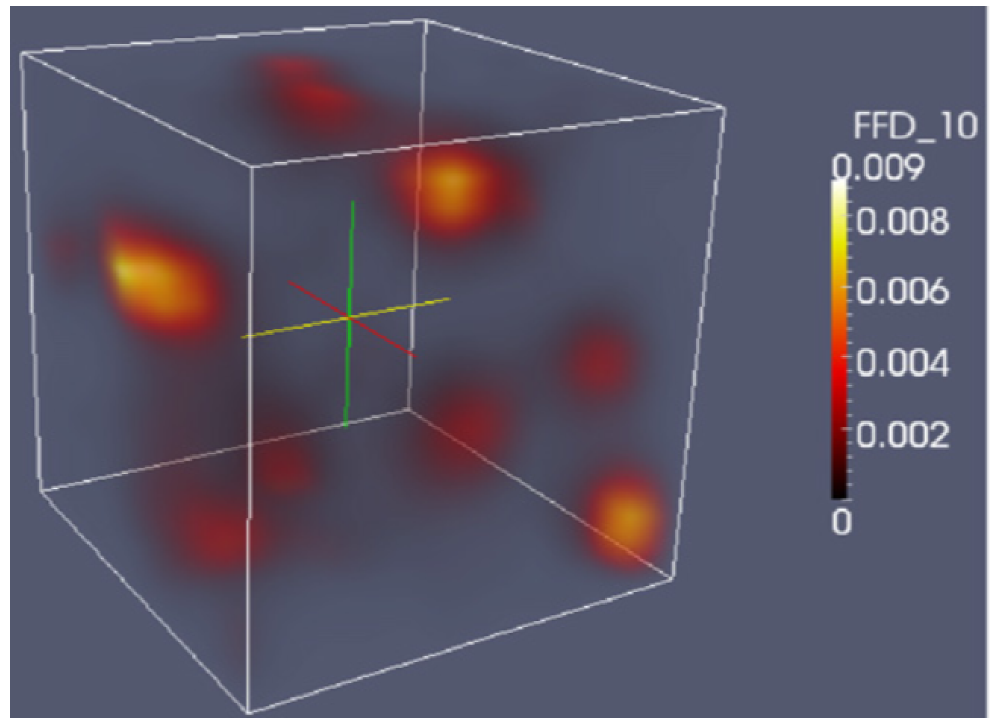


Negele et al, 97

perhaps dyons were first observed in “constrained cooling” preserving local L



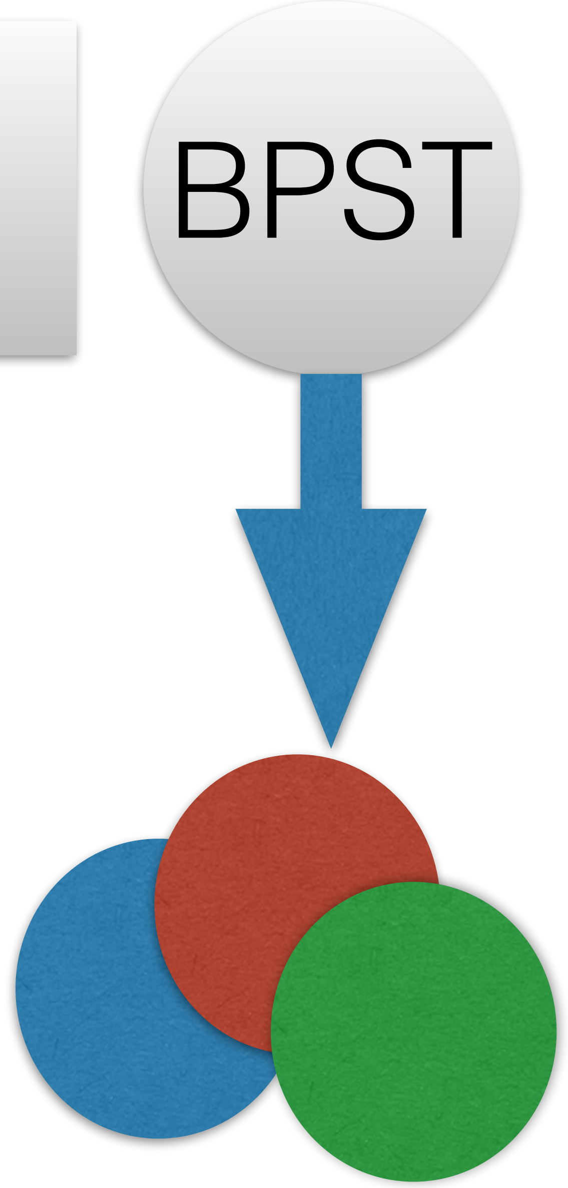
Langfeld and Ilgenfritz, 2011



while the total top.charge of the box is always integer, local bumps are not! They are all (anti)selfdual But top charge and actions Were not integers!

a lot of work on finding instanton-dyons was done by C.Gatringer et al, Ilgenfritz et al

Non-zero Polyakov line splits instantons into Nc instanton-dyons (Kraan,van Baal, Lee,Lu 1998)



Explained mismatch of quark condensate in SUSY QCD

V.Khoze (jr) et al 2001

Explained confinement by back reaction to free energy

D.Diakonov 2012, Larsen+ES,Liu,Zahed+ES 2016

Explain chiral symmetry breaking in QCD and in setting with modified fermion periodicities

R.Larsen+ES 2017, Unsal et al 2017

Pierre van Baal

QCD with near-real quark masses, at T slightly above Tc

the cleanness case: domain wall fermions Q=1 configurations Nt=8,Nx=32, T/Tc=1,1.08

excellent agreement of the shape with analytic formulae

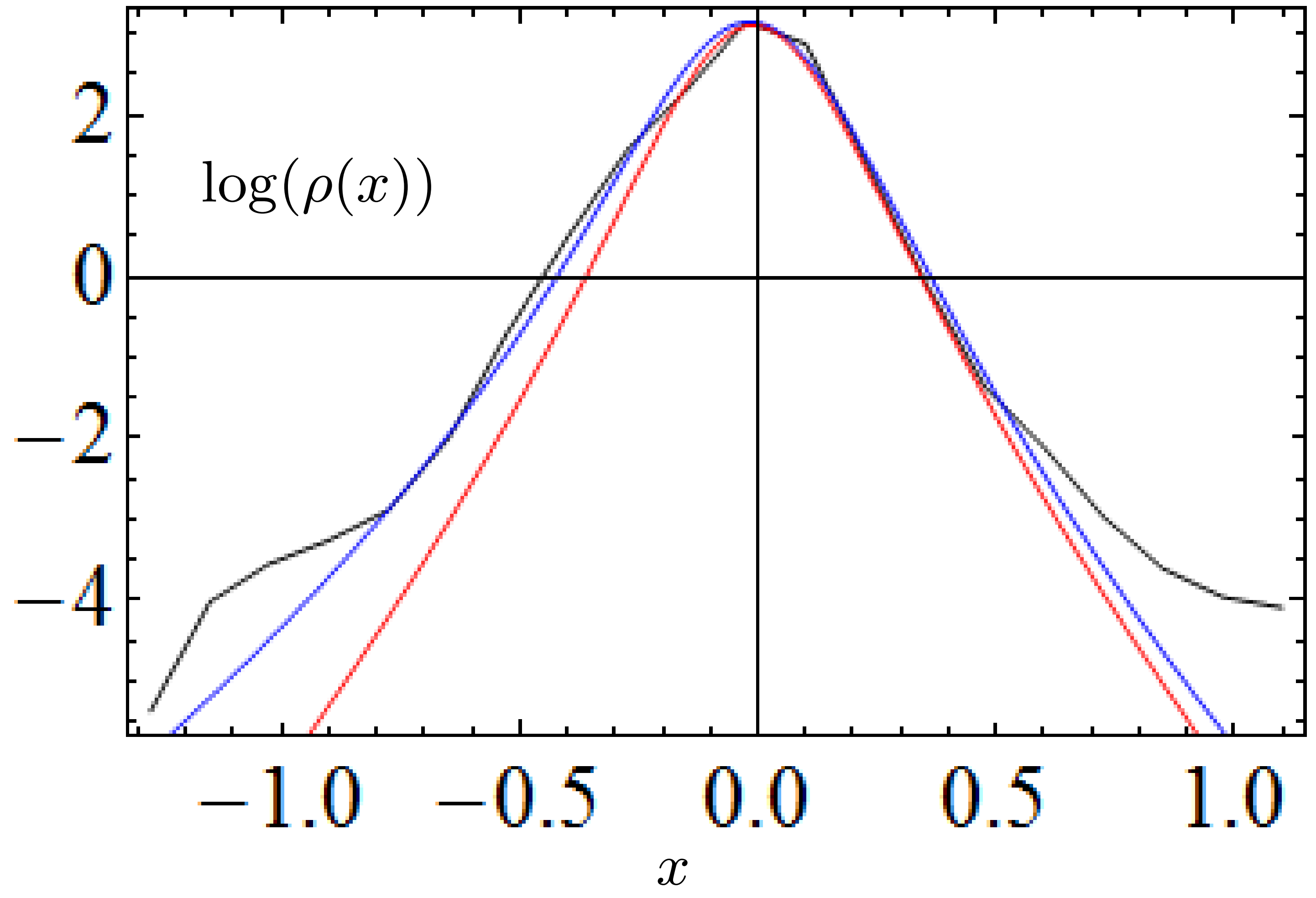


FIG. 8: $\log(\rho(x))$ of the zero mode of conf. 2960 at $\phi = \pi$ (black) and the log of the analytic formula for $P = 0.4$ and $P = 1$ though the maximum. $T = 1.08T_c$. Red peak only has been scaled to fit in height, while blue peak uses the found normalization.

- *Phys.Lett.B* 794 (2019) 14-18 • e-Print: 1811.07914 [hep-lat]
- *Phys.Rev.D* 102 (2020) 3, 034501 • e-Print: 1912.09141

* **correlations with local Polyakov loop, in progress**

- Rasmus N. Larsen, Sayantan Sharma, Edward Shuryak

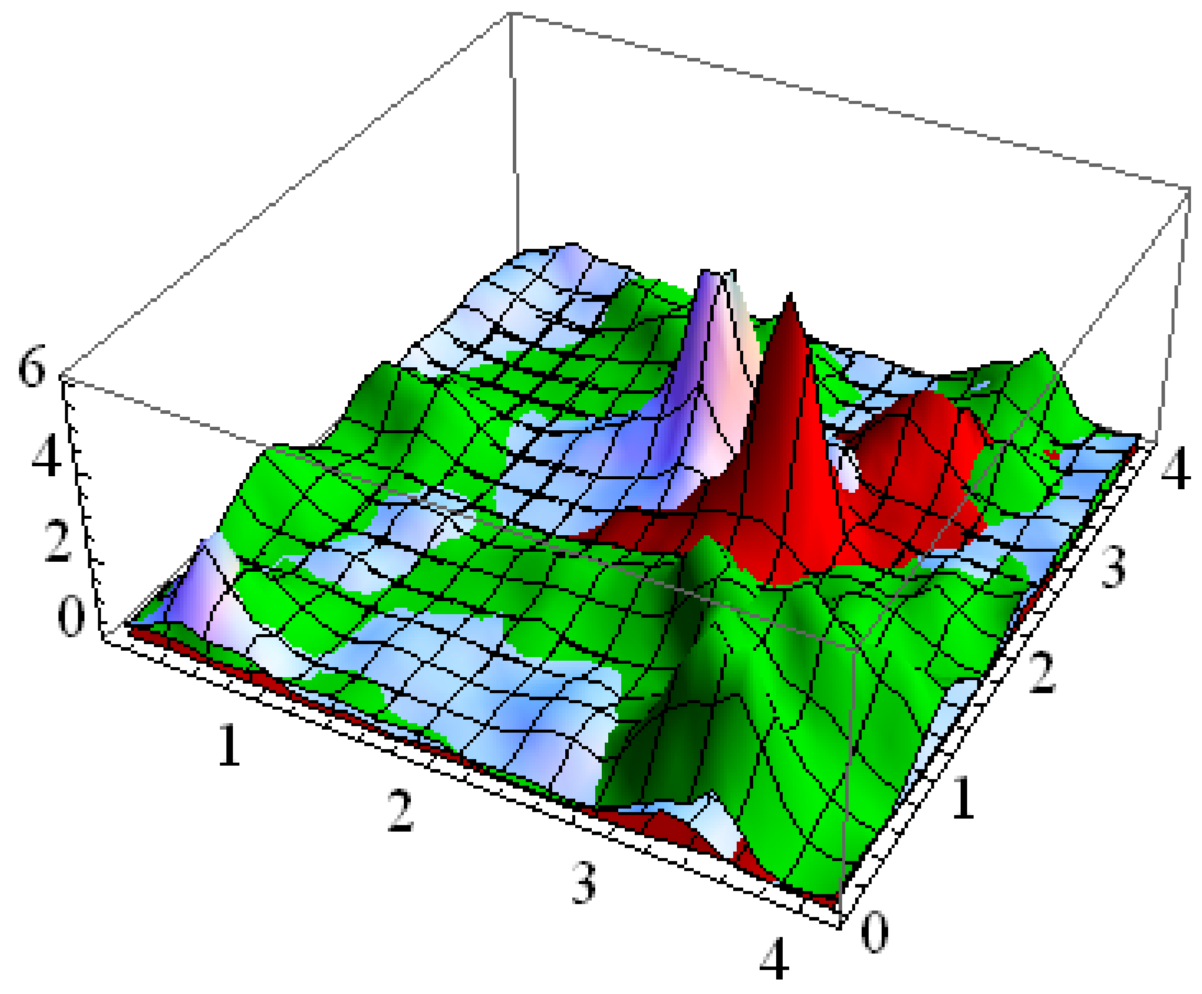


FIG. 17: $\rho(x, y)$ of the zero mode of conf. 2660 at $T = T_c$. $\phi = \pi$(red), $\phi = \pi/3$(blue), $\phi = -\pi/3$(green). Peak height has been scaled to be similar to that of $\phi = \pi$.

extracting the shape of the fermonic zero mode and modyfying the phase one can find all 3 dyons

We found that their fields interfere with each other the interaction between them Is in excellent agreement with van Baal analytic formulae

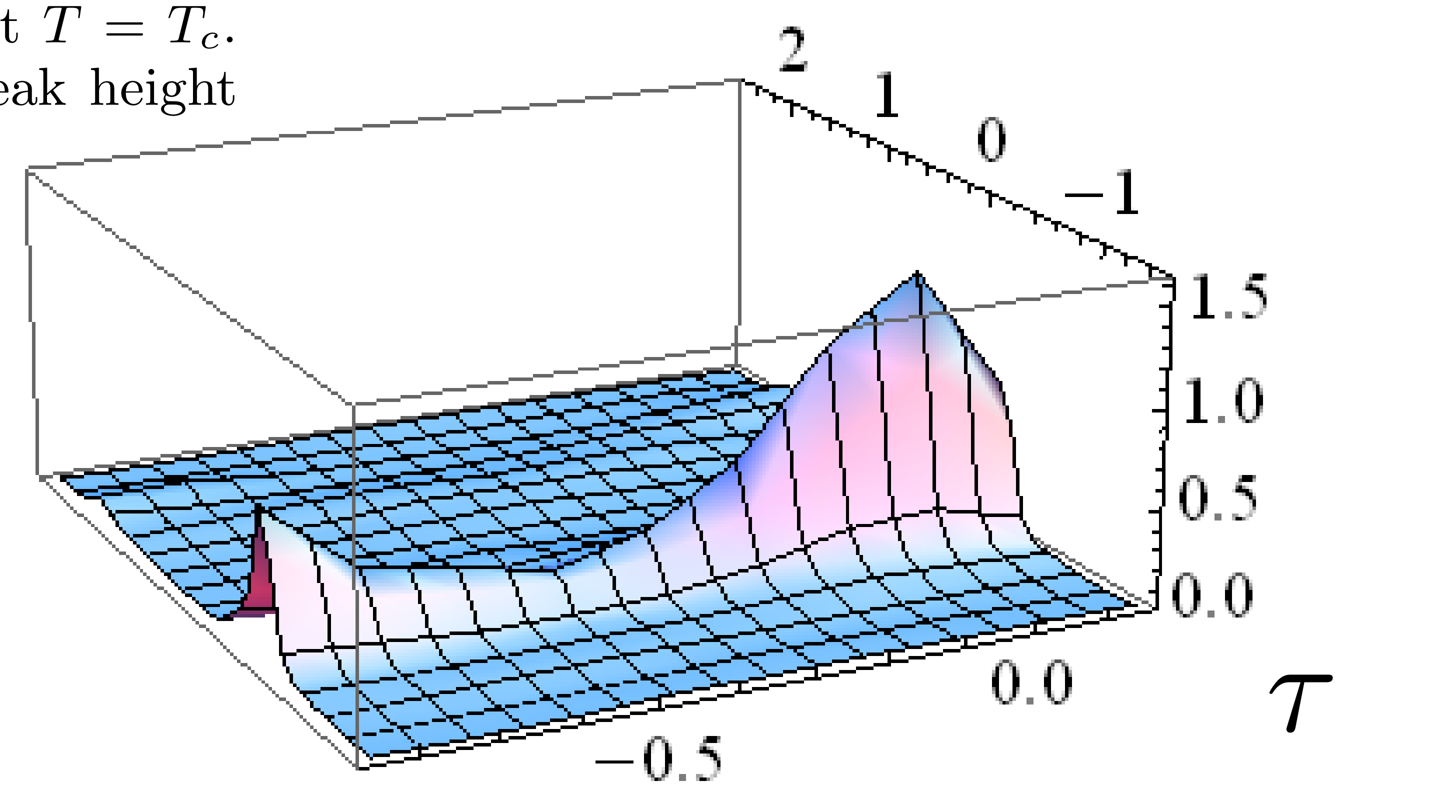


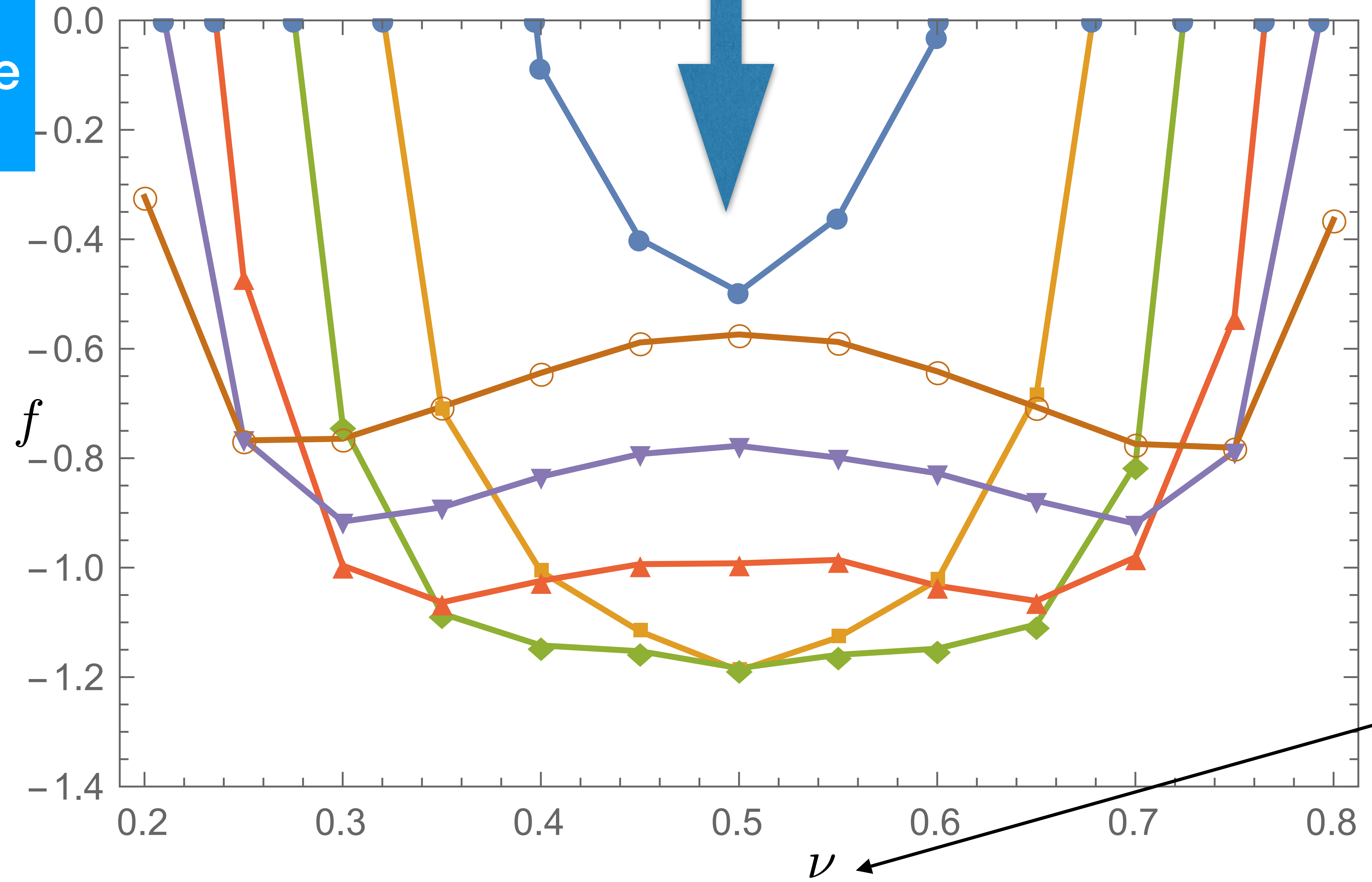
FIG. 3: $\rho(x, t)$ of the zero mode of conf. 2000 at $\phi = \pi/3$. $T = T_c$.

SU(2) pure gauge deconfinement transition in the dyon ensemble is second order

(with Rasmus Larsen)

confined

free energy vs holonomy



$$\langle A_4^3 \rangle = v\frac{\tau^3}{2} = 2\pi T\nu\frac{\tau^3}{2}$$

$$< P >= cos(\pi\nu) \to 0$$

$$if \ \nu = 1/2$$

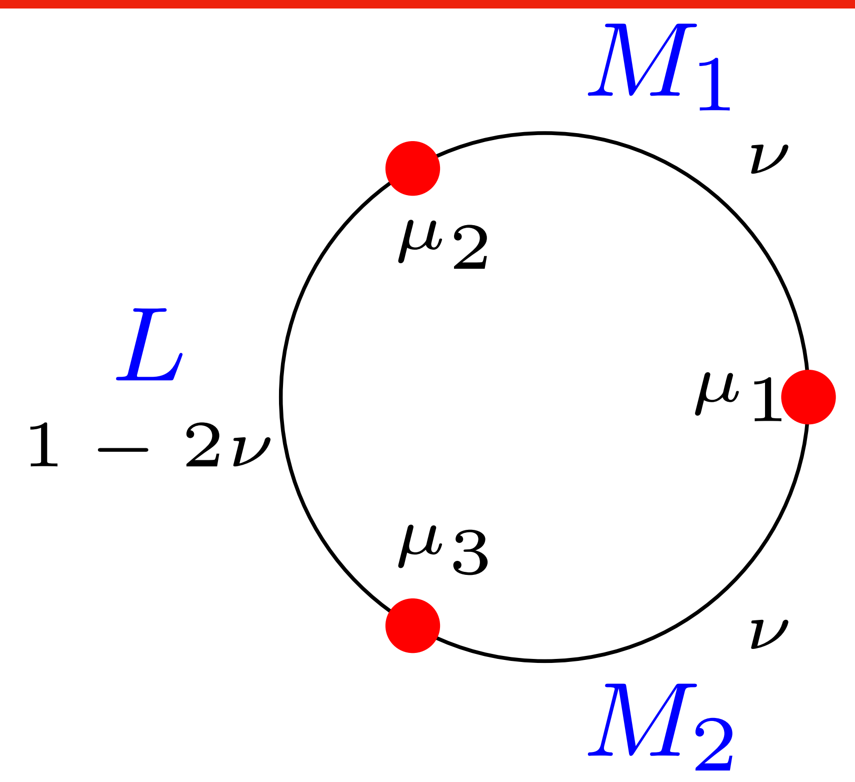
holonomy

$$\nu = 0 \ is \ the \ trivial \ case$$

$$\nu = 1/2 \ confining$$

So, as a function of the dyon density the potential changes its shape and confinement takes place

red dots move to the right at higher T



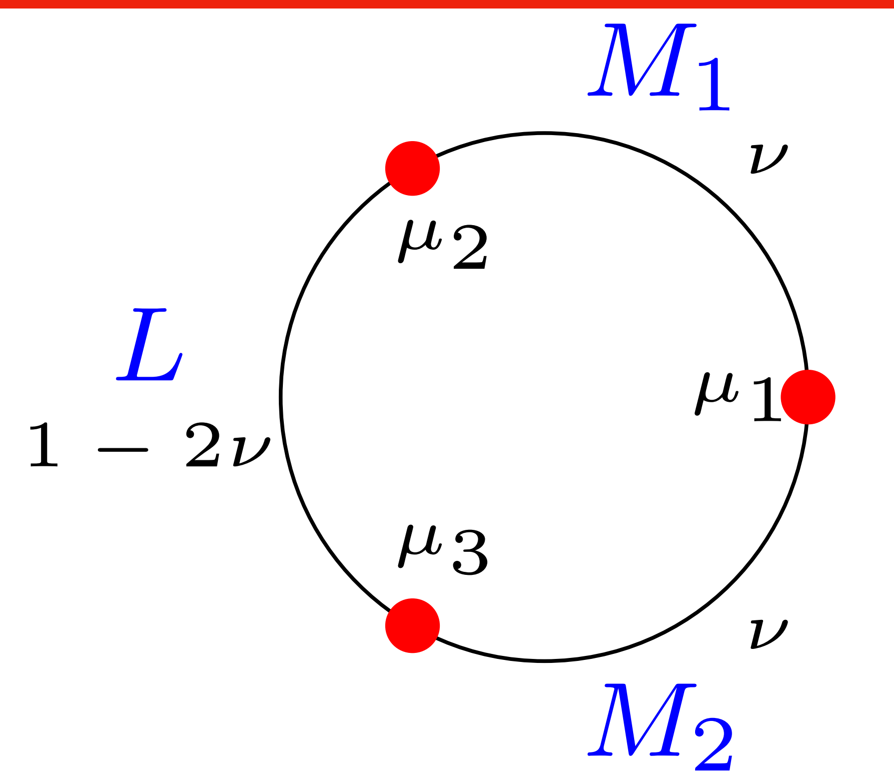
Deconfinement Phase Transition in the $SU(3)$ Instanton-dyon Ensemble

Dallas DeMartini and Edward Shuryak
Center for Nuclear Theory, Department of Physics and Astronomy,
Stony Brook University, Stony Brook NY 11794-3800, USA

arXiv:2102.11321v1 [hep-ph] 22 Feb 2021

critical:
jump in
holonomy

red dots move to the right at higher T



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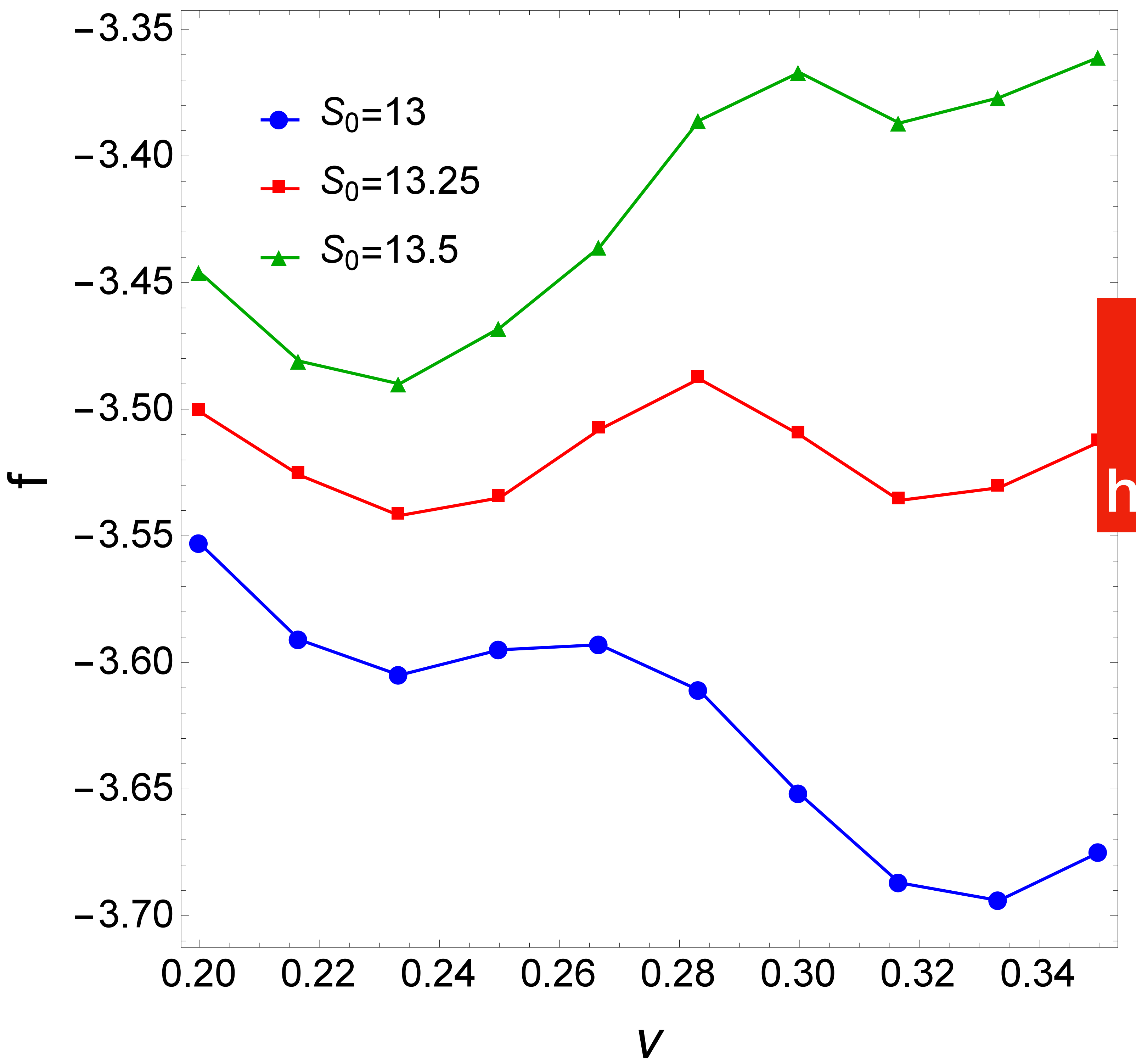
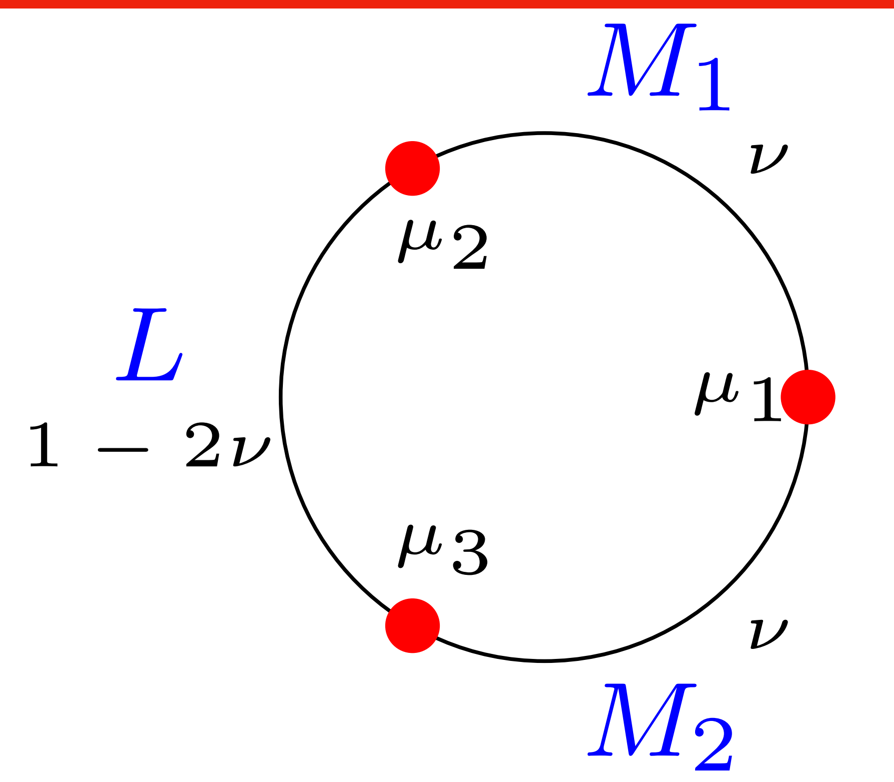


critical: jump in holonomy

FIG. 4. (Color online) Holonomy dependence of the minimum free energy density near the phase transition. Error bars not

red dots move to the right at higher T



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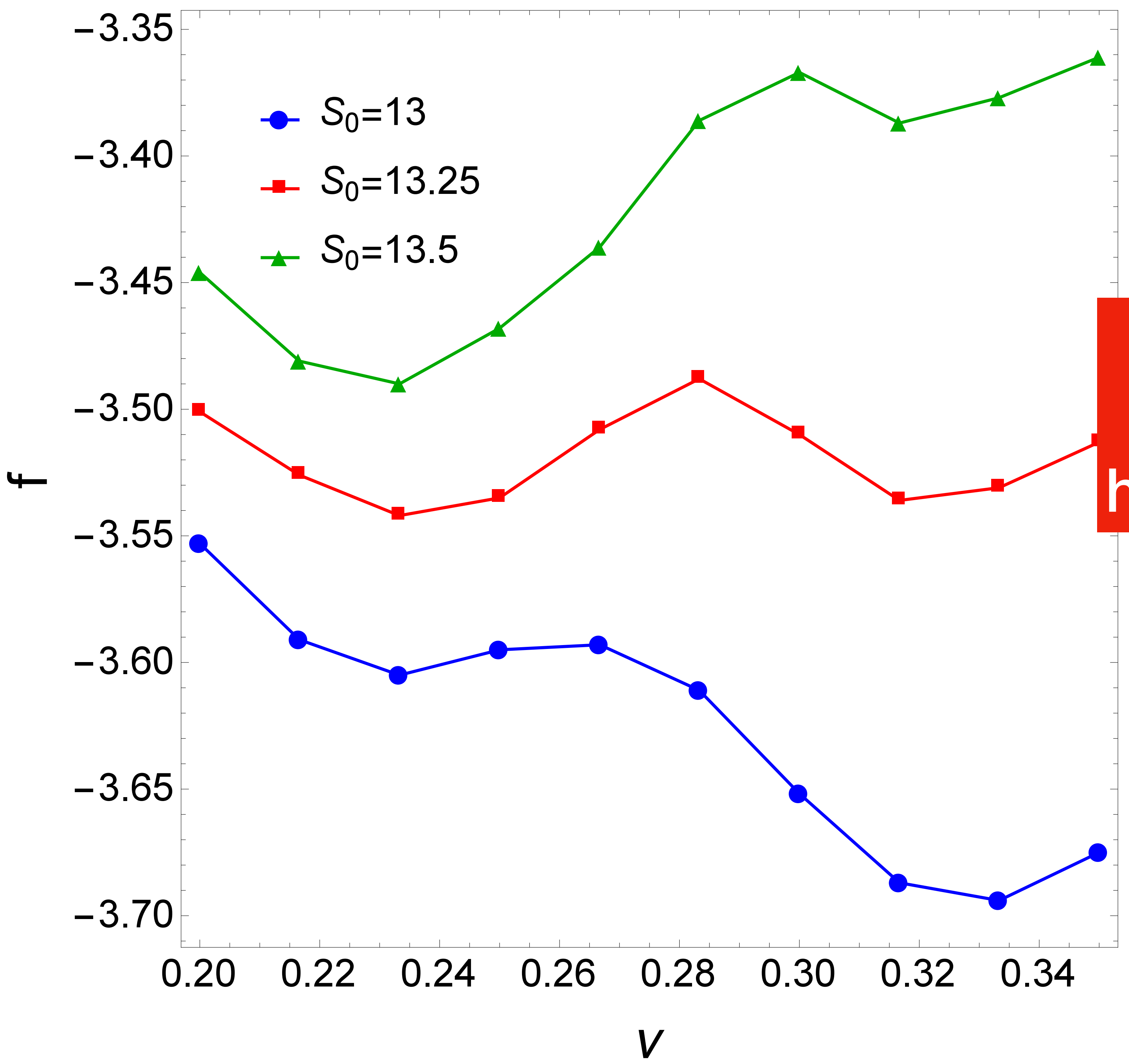

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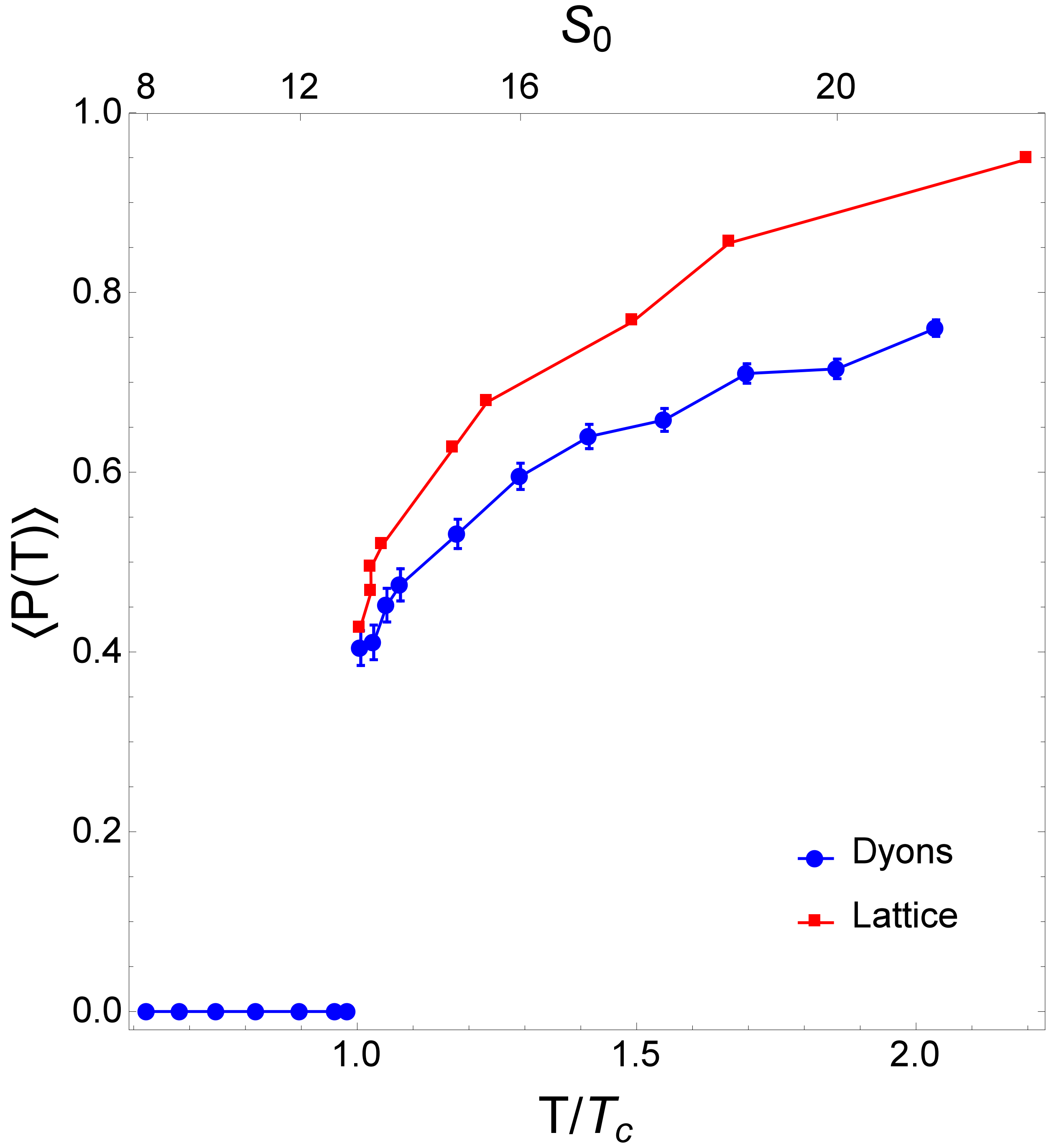
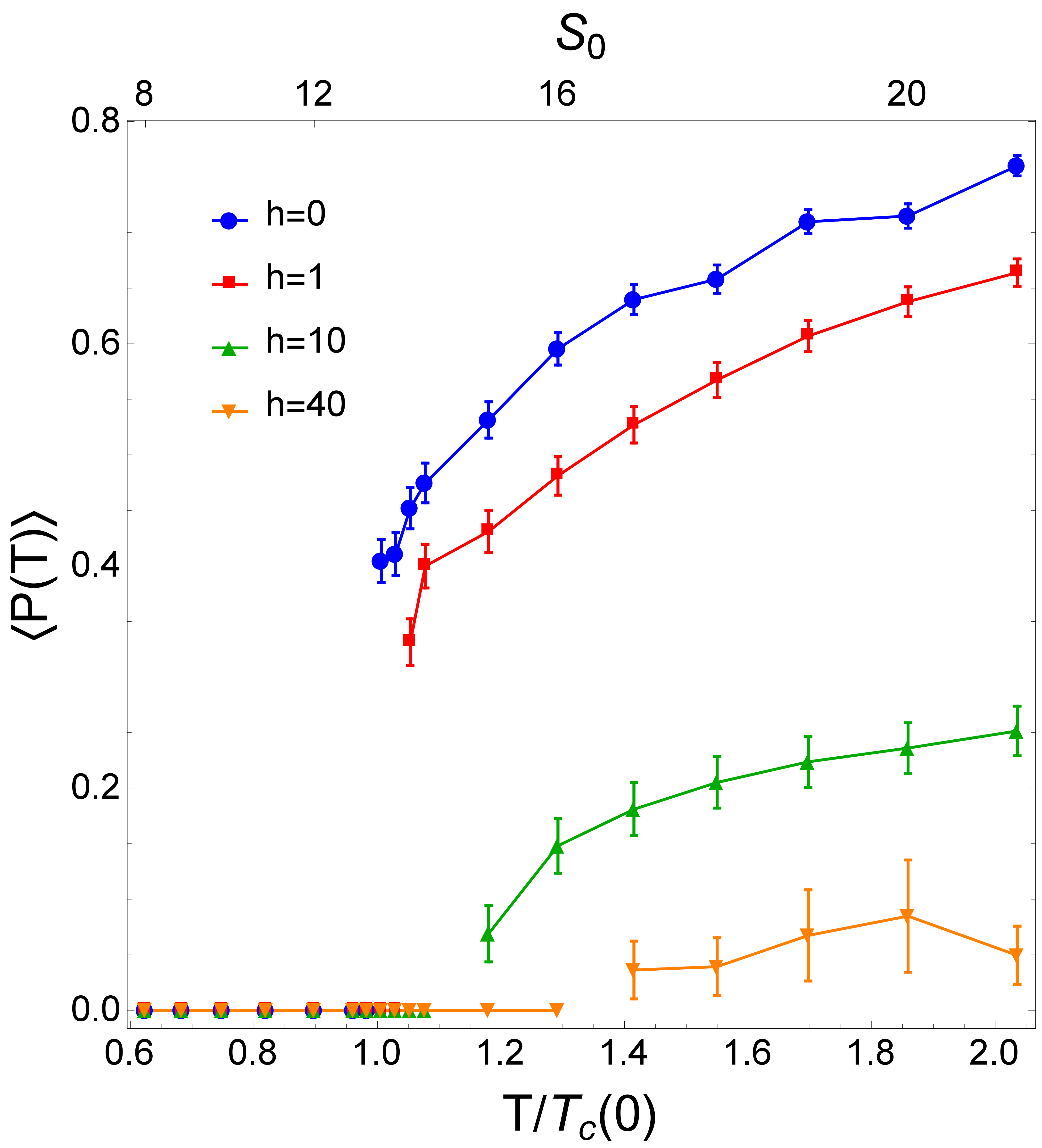
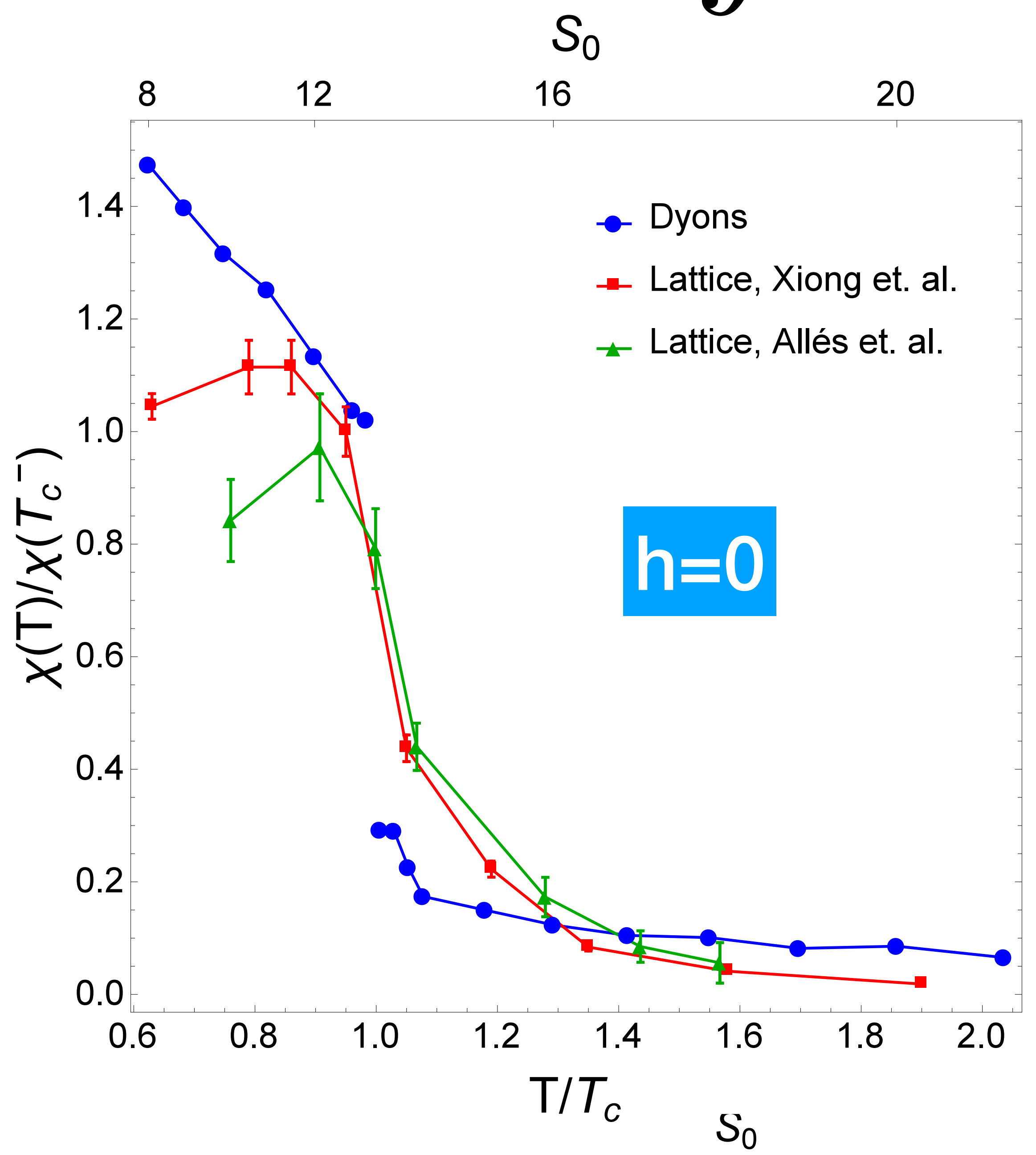


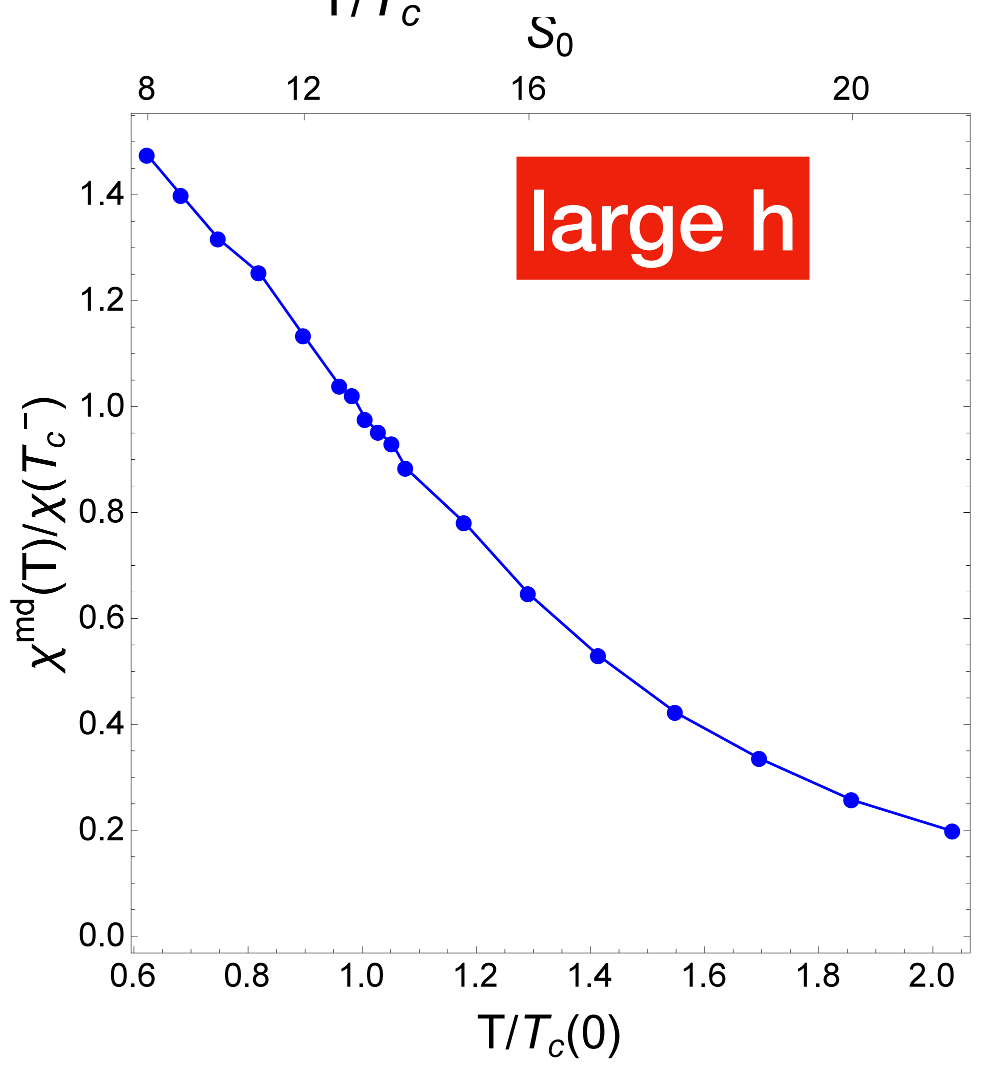
FIG. 6. (Color online) Temperature dependence of the average Polyakov loop of the dyon ensemble. Lattice data taken from Ref. [21] and shown without error bars. Error on lattice data has magnitudes comparable to dyon data.

deformation by powers of P in the action push deconfinement to higher T

$$\Delta S_{def} = h \int d^3x |P(\vec{x})|^2$$





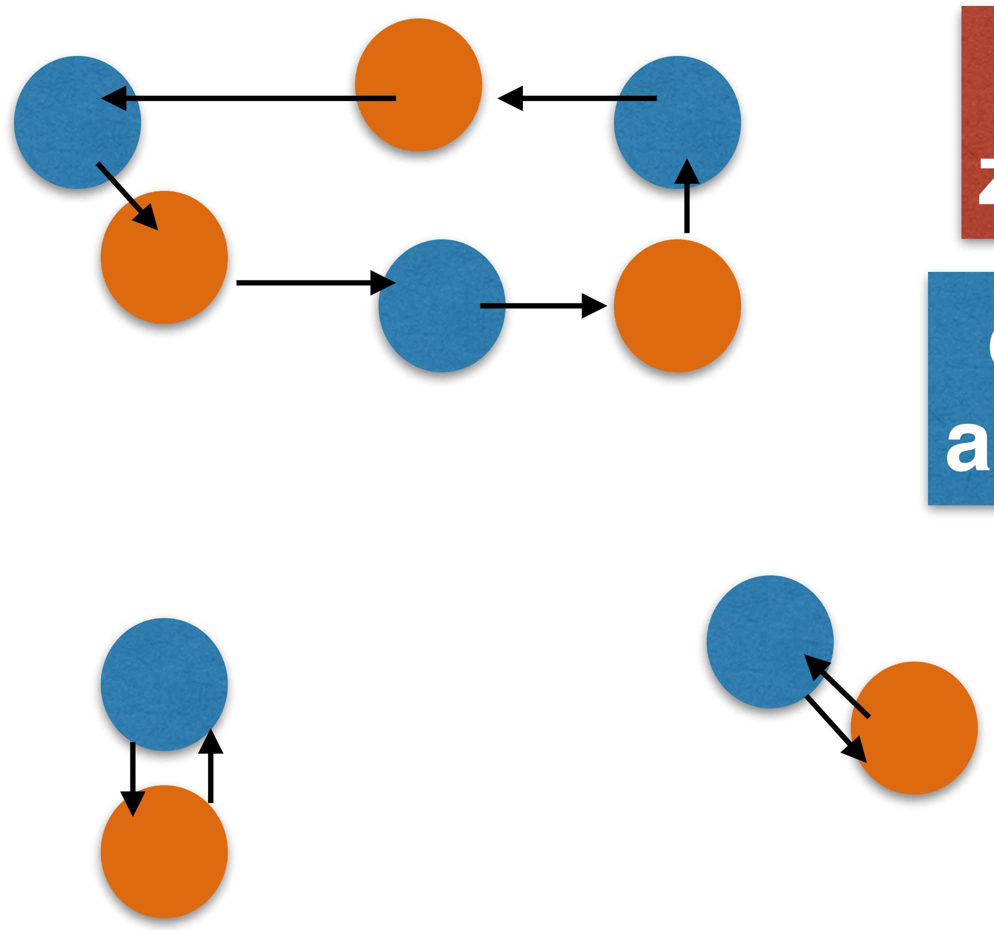


Instanton-dyon Ensemble with two Dynamical Quarks: the Chiral Symmetry Breaking

Rasmus Larsen and Edward Shuryak
Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA

This is the second paper of the series aimed at understanding of the ensemble of the instanton-dyons, now with two flavors of light dynamical quarks. The partition function is appended by the fermionic factor, $(detT)^{N_f}$ and Dirac eigenvalue spectra at small values are derived from the numerical simulation of 64 dyons. Those spectra show clear chiral symmetry breaking pattern at high dyon density. Within current accuracy, the confinement and chiral transitions occur at very similar densities.

$$|<\bar{\psi}\psi>| \; = \; \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty}$$



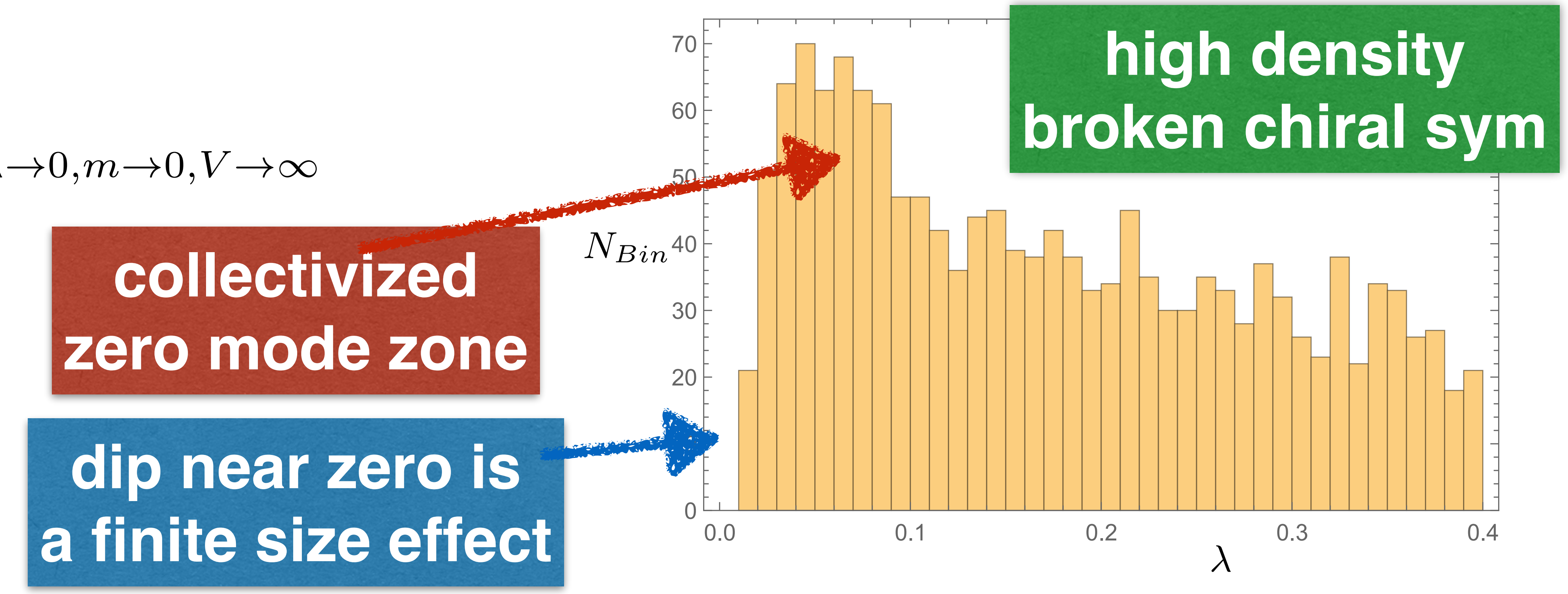


FIG. 1: Eigenvalue distribution for $n_M = n_L = 0.47$, $N_F = 2$ massless fermions.

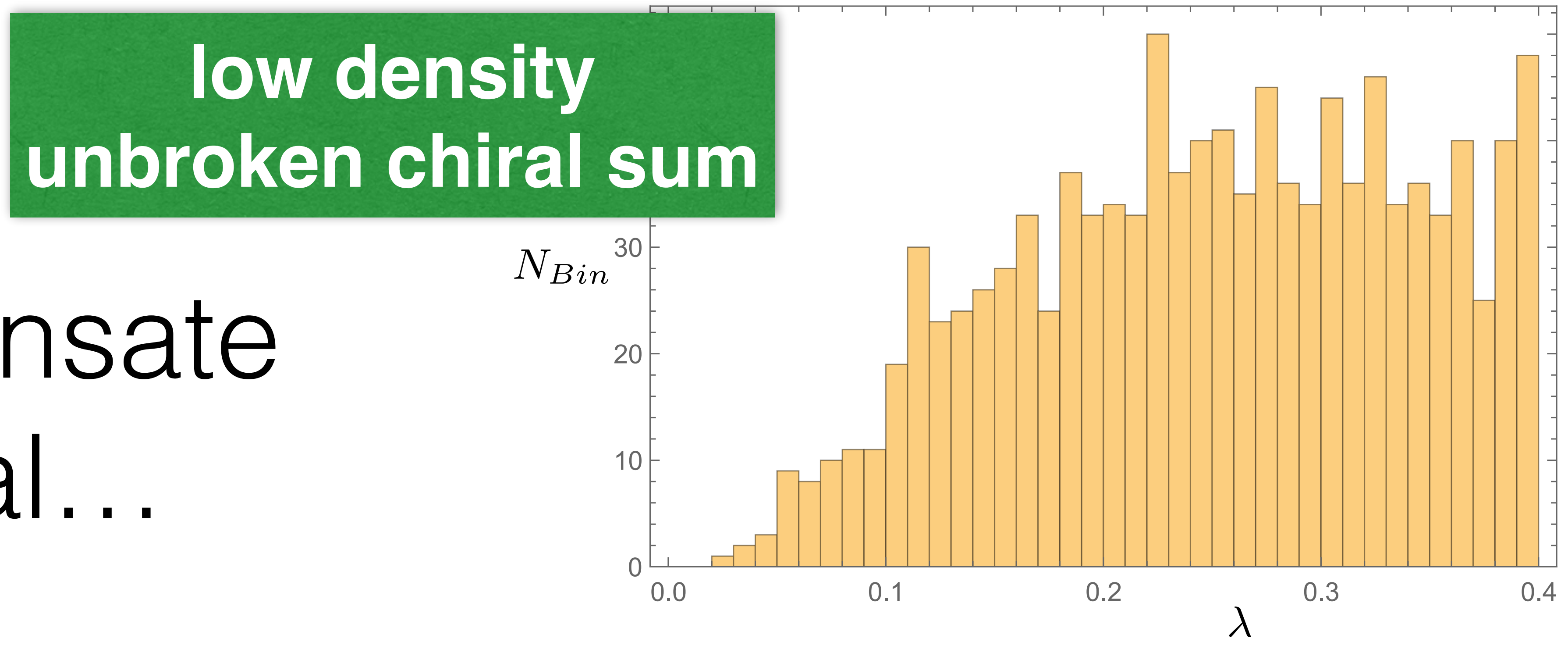


FIG. 2: Eigenvalue distribution for $n_M = n_L = 0.08$, $N_F = 2$ massless fermions.

extracting condensate
is far from trivial…

Chiral Symmetry Breaking and Confinement from an Interacting Ensemble of Instanton-dyons in Two-flavor Massless QCD

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Stony Brook University, Stony Brook NY 11794-3800, USA

Casher-Banks quark condensate is obtained by linear extrapolation to 0

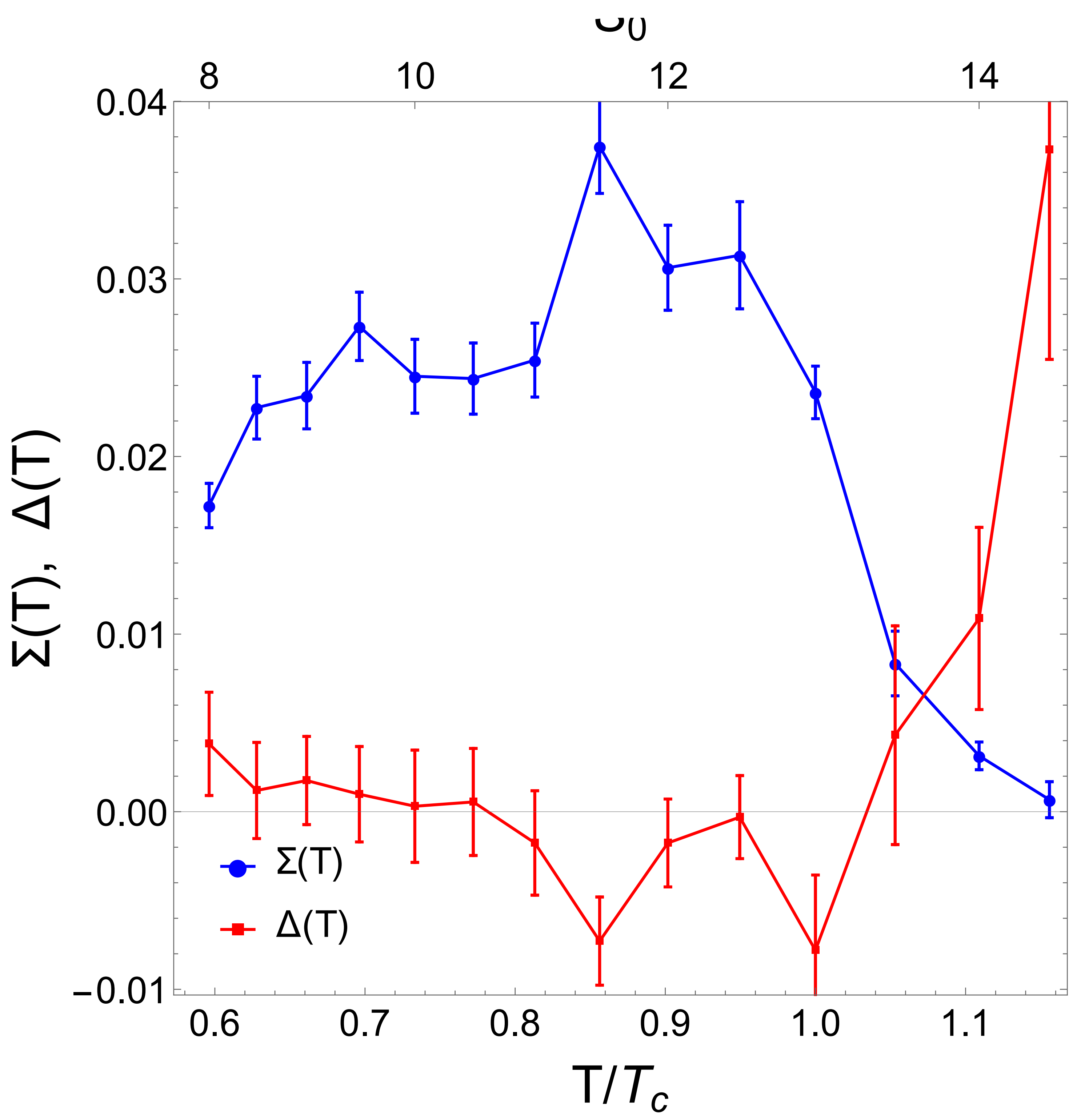


FIG. 8. (Color online) [Preliminary] The chiral quark condensate $\Sigma(T)$ and the eigenvalue gap $\Delta(T)$ as functions of the temperature.

$\langle \bar{q} q \rangle$

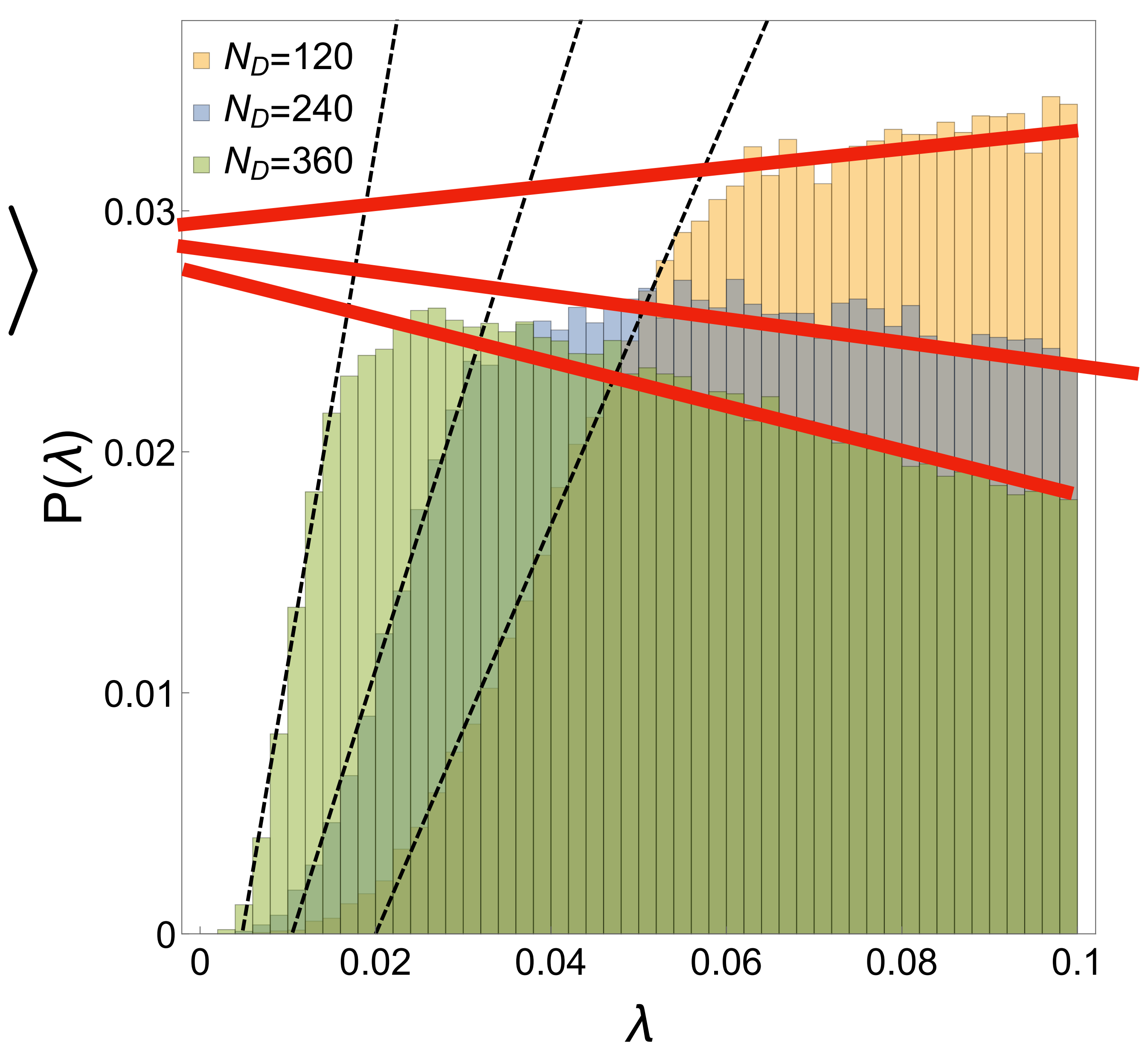


FIG. 7. (Color online) Eigenvalue distributions at $S_0 = 8.5$ for three different ensemble sizes. Dashed lines represent fits to the approximately-linear portion of the distribution near zero. The eigenvalue gaps are given by the x-intercepts of the fits. Note that the relative normalization of the distributions does not affect results.

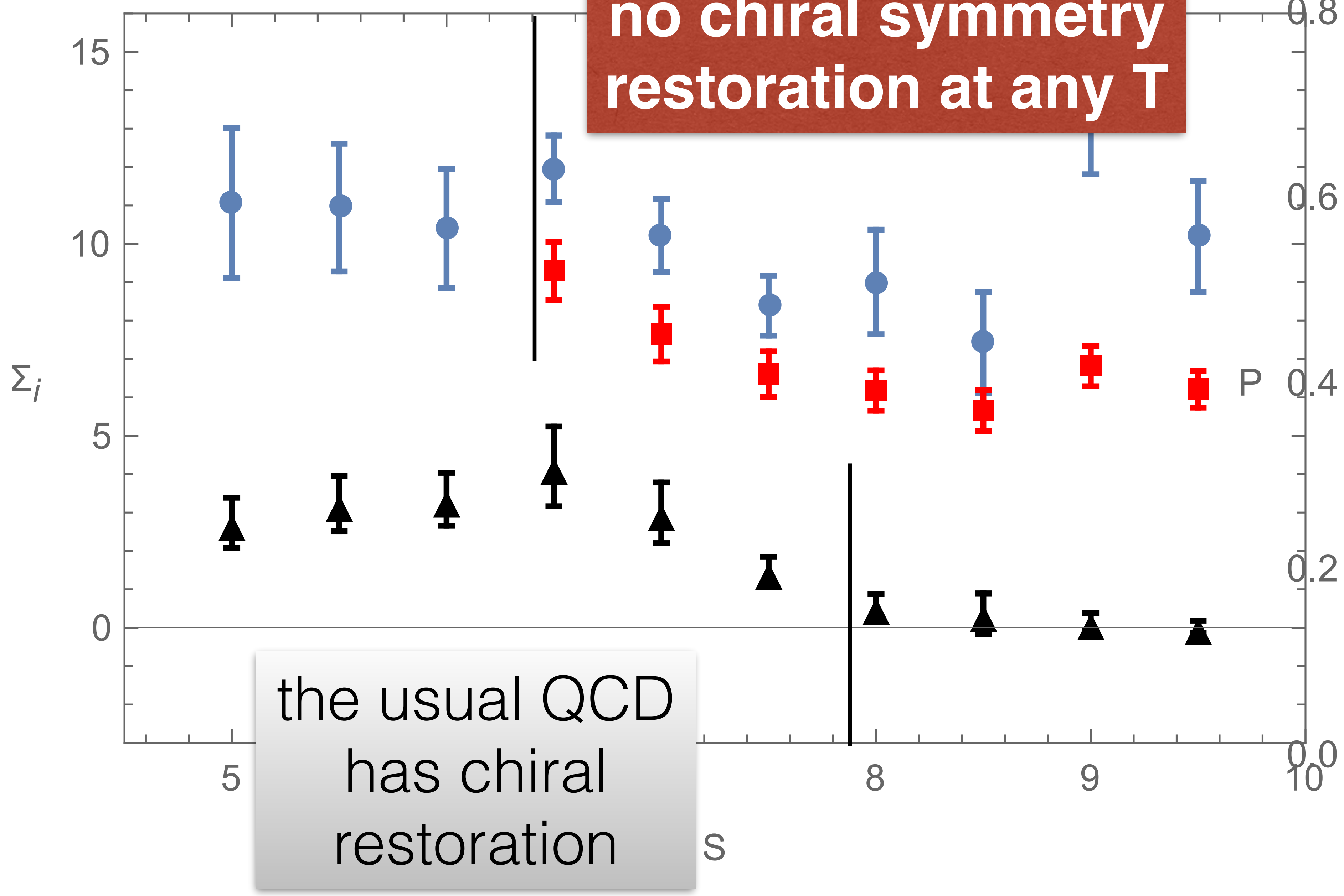
the gap scales as 1/V and is therefore a purely finite volume effect

Second deformation: QCD2 and Z2QCD are dramatically different!

symmetric phase

u=>M
d=>L
$< \bar{u}u > \neq < \bar{d}d >$

no chiral symmetry restoration at any T



the usual QCD has chiral restoration

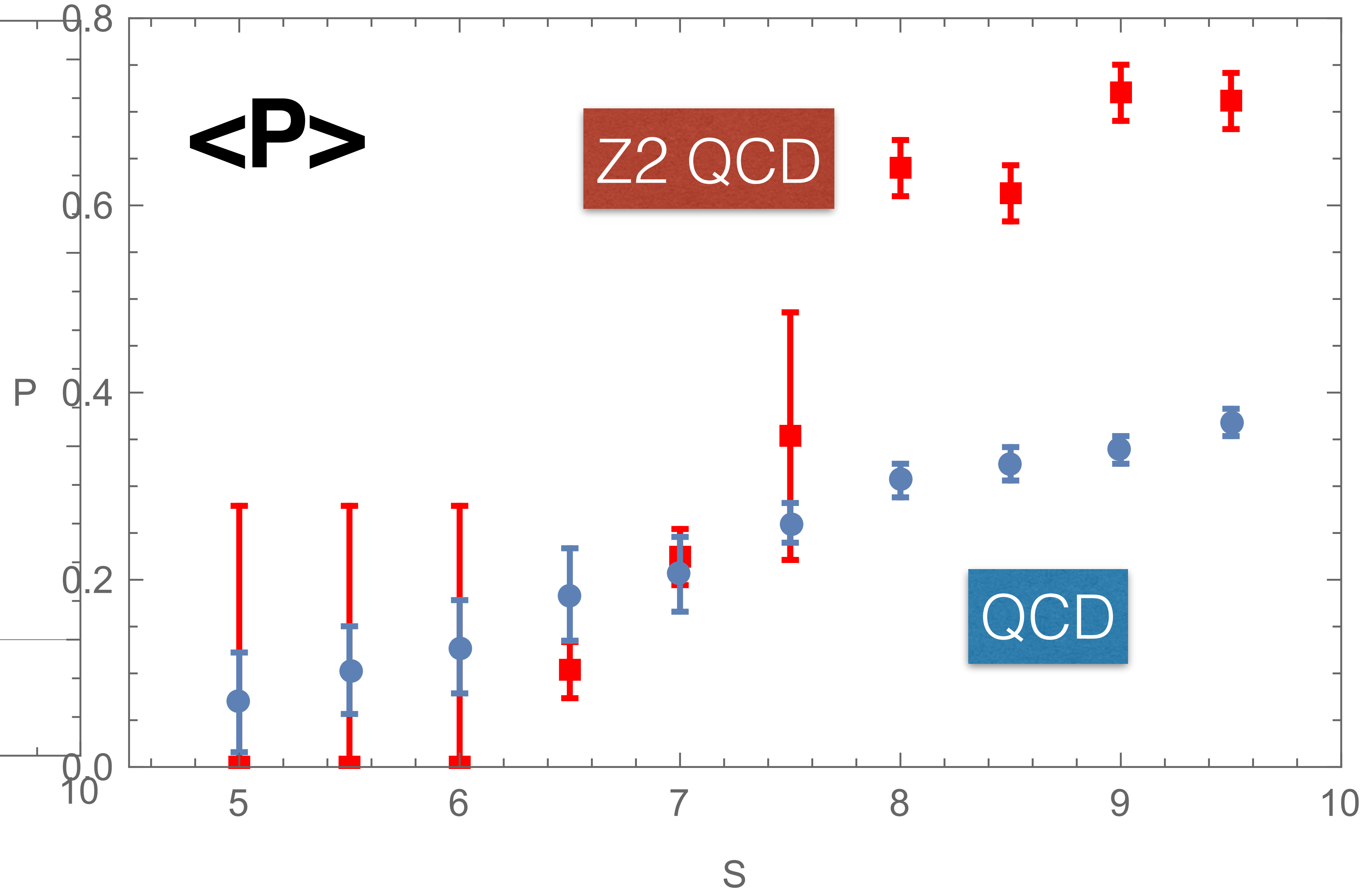


FIG. 6: Chiral condensate generated by u quarks and L dyons (red squares) and d quarks interacting with M dyons (blue circles) as a function of action S, for the Z_2-symmetric model. For comparison we also show the results from II for the usual QCD-like model with $N_c = N_f = 2$ by black triangles.

confining phase gets much more robust: strong first order mixed phase (flat F) is observed at medium densities

note the condensate is much larger for Z2?

Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

Adith Ramamurti,[*] Edward Shuryak,[†] and Ismail Zahed[‡]

instanton-dyons with winding number n

The twisted solution is obtained in two steps. The first is the substitution

$$v \to n(2\pi/\beta) - v\,, \tag{13}$$

and the second is the gauge transformation with the gauge matrix

$$\hat{\Omega} = \exp\left(-\frac{i}{\beta} n\pi\tau\hat{\sigma}^3\right), \tag{14}$$

where we recall that $\tau = x^4 \in [0, \beta]$ is the Matsubara time. The derivative term in the gauge transformation adds a constant to A_4 which cancels out the unwanted $n(2\pi/\beta)$ term, leaving v, the same as for the original static monopole. After "gauge combing" of v into the same direction, this configuration – we will call L_n – can be combined with anv other one. The solutions are all

$$S_n = (4\pi/g^2)|2\pi n/\beta - v|$$

Found first in N=4 SYM theory, by Dorey Parnachev simple toy examplein this paper

$$\sum_{n=-\infty}^{\infty} f(\omega + nP) = \sum_{l=-\infty}^{\infty} \frac{1}{P}\tilde{f}\left(\frac{l}{P}\right) e^{i2\pi l\omega/P}$$

Poisson summation formula can be used to derive the monopole Z

$$Z_{\text{inst}} = \sum_n e^{-\left(\frac{4\pi}{g_0^2}\right)|2\pi n - \omega|}$$

$$Z_{\text{mono}} \sim \sum_{q=-\infty}^{\infty} e^{iq\omega - S(q)}$$

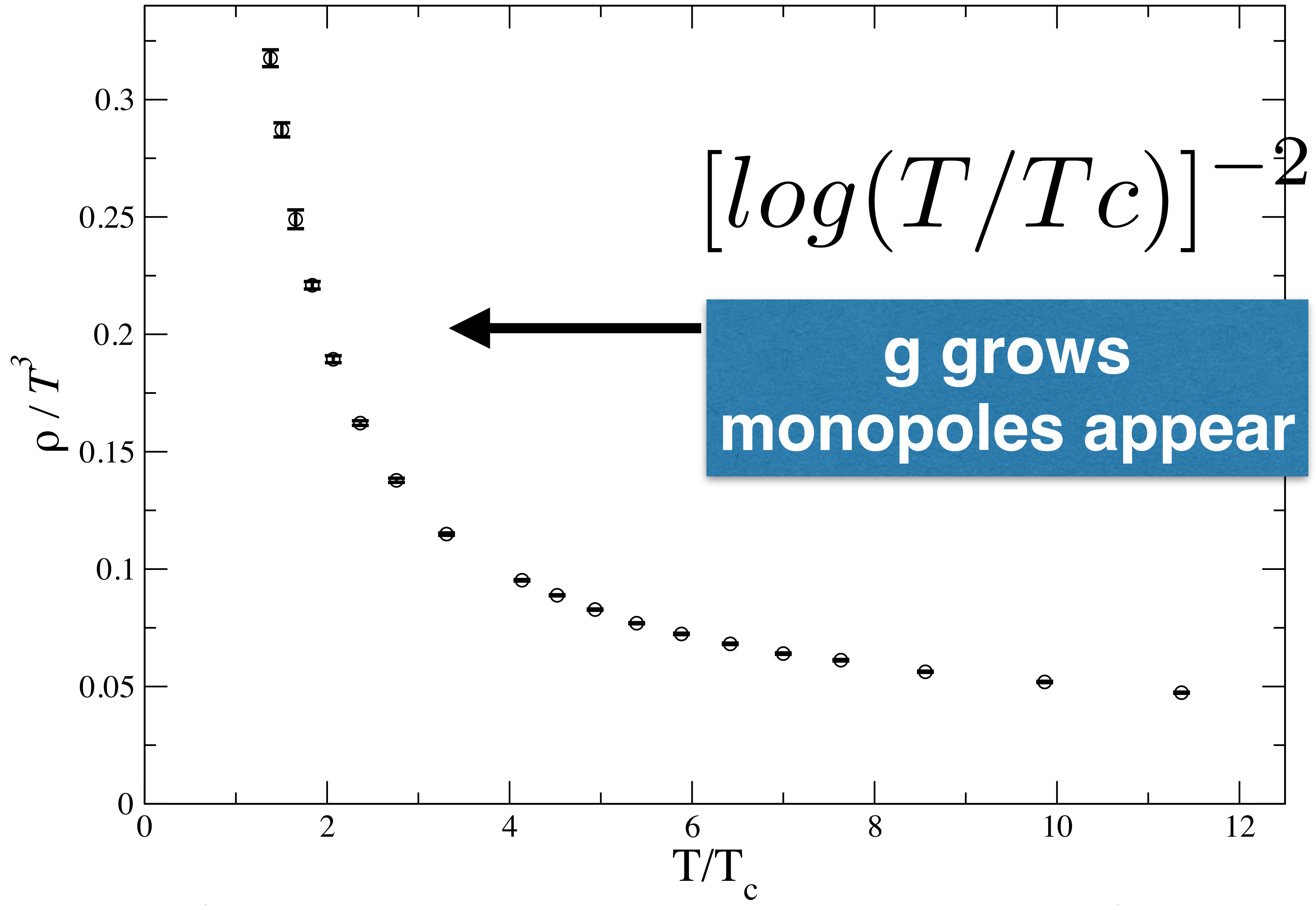
$$S(q) = \log\left(\left(\frac{4\pi}{g_0^2}\right)^2 + q^2\right)$$

$$\approx 2\log\left(\frac{4\pi}{g_0^2}\right) + q^2\left(\frac{g_0^2}{4\pi}\right)^2 + \ldots$$

q is angular momentum of rotating monopole, so it is electric charge

Therefore we now understand why
The density of monopoles is well fitted by an inverse power of log(T) , not power of T =>
It is because they are not really semiclassical objects!

$$S_{mono} \sim log(const/g^2) = log\big(log(T/Tc)\big)$$



$$[log(T/Tc)]^{-2}$$

g grows
monopoles appear

Fig. 2.6 The normalized monopole density ρ/T^3 for the $SU(2)$ pure gauge theory as a function of the temperature, in units of the critical temperature T/T_c, above the deconfinement transition.

D'Alessandro, A. and D'Elia, M. (2008). Magnetic monopoles in the high temperature phase of Yang-Mills theories. Nucl. Phys., B799:241–254. 0711.1266.

For instantons and dyons it is different

$$exp(-S) \sim exp(-const/g^2) = exp(-const' * log(T)) = 1/T^{power}$$

Summary

- Semiclassical objects at finite T are instanton-dyons, fractions of instantons. Their interactions and ensembles for SU(2) and SU(3) gauge theories, with and without quarks studied
- Very cleanly they are seen in lattice configurations via Dirac zero (and quasizero) eigenmodes. Even when overlapping, the lattice shapes follow semiclassical formulae very accurately (?)
- in QCD deconfinement and chiral transitions are close, but
- can be moved by deformations: (1) Polyakov line suppression;
- (2) changes of fermion periodicity phases
- Poisson duality for monopoles to instanton-dyons explains
- the monopole density(T) and why monopoles are not semiclassical
-

extra slides

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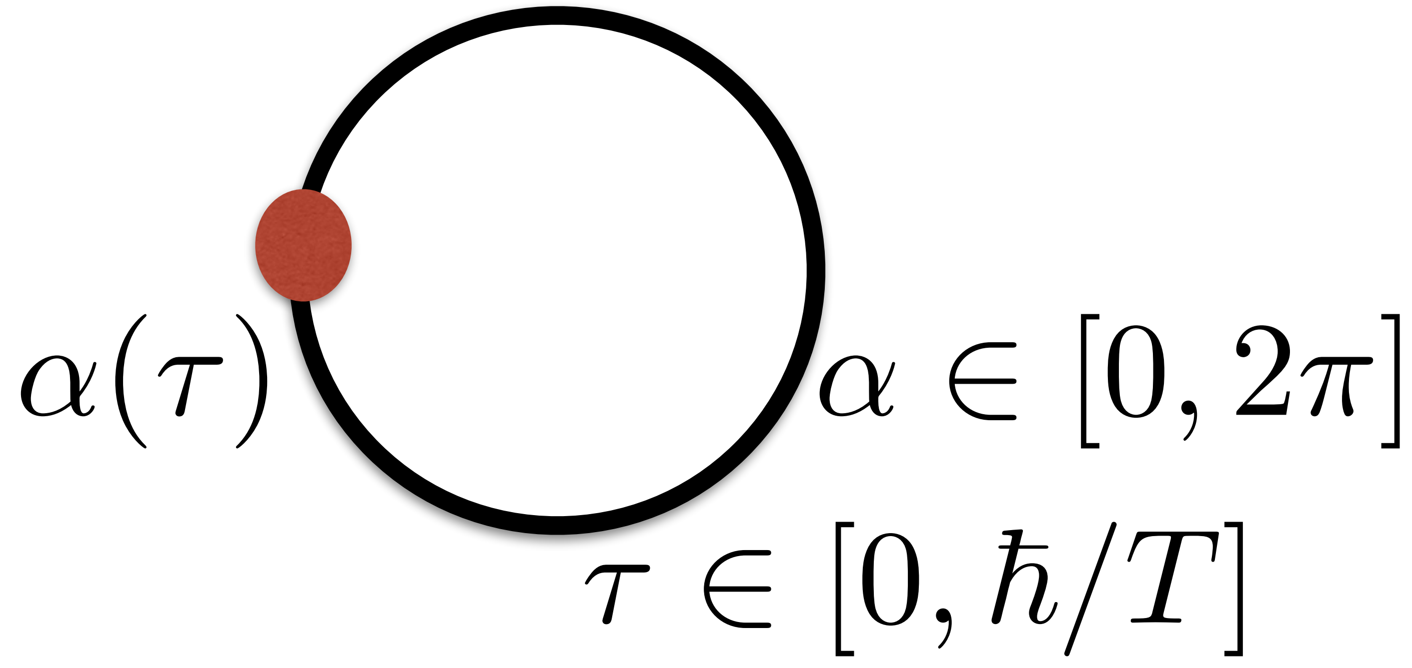

The same phenomenon in much simpler setting:
quantum particle on a circle at finite T

A Hamiltonian vs Lagrangian approaches

$$Z_1 = \sum_{l=-\infty}^{\infty} \exp\left(- \frac{l^2}{2\Lambda T} + il\omega \right)$$

moment of inertia

Aharonov-Bohm phase

$$Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T} \exp\left(- \frac{T\Lambda}{2} (2\pi n - \omega)^2 \right).$$

Matsubara winding number

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta},$$

based on classical paths

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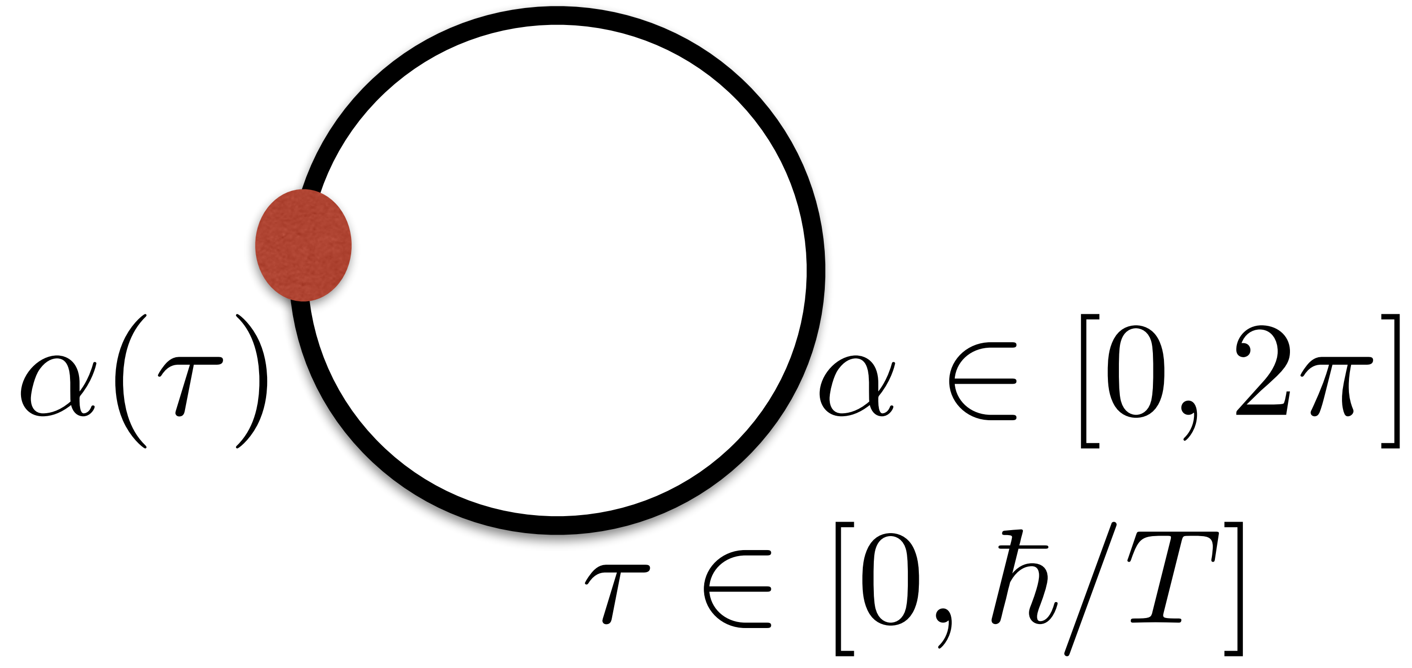

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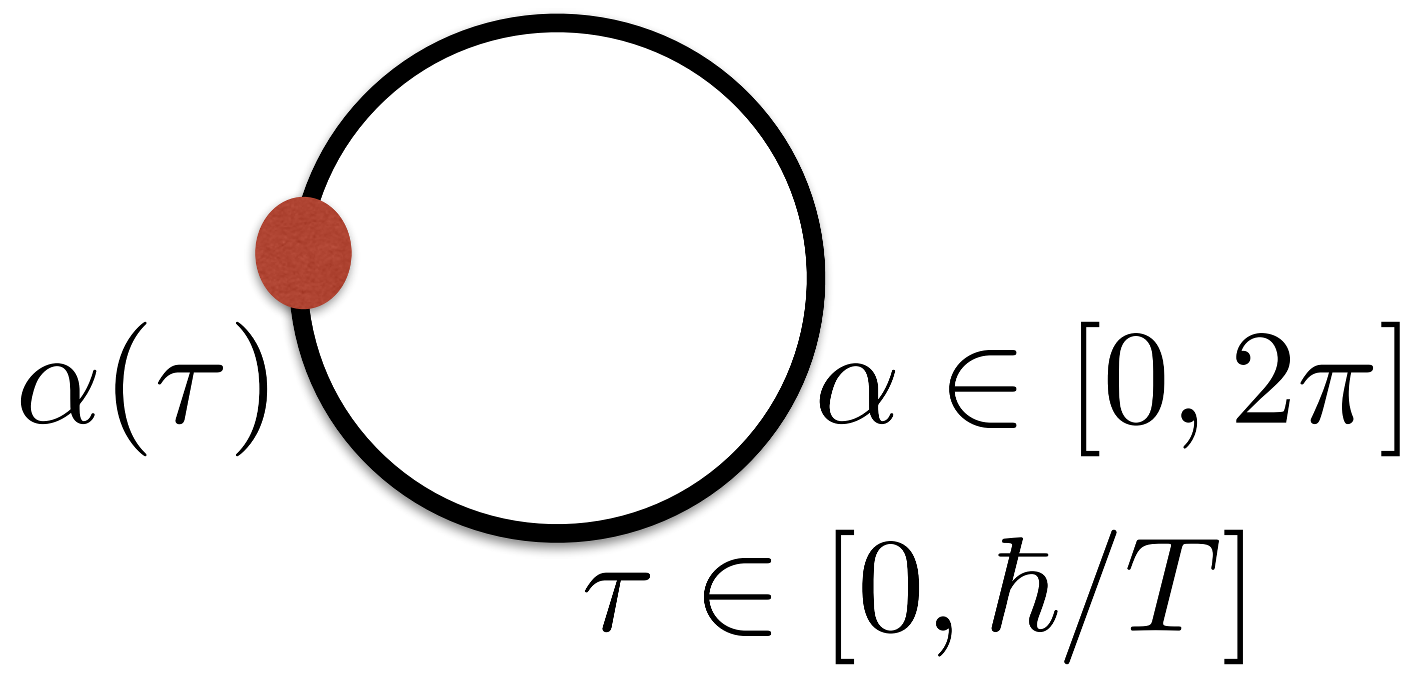

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based on classical paths

Note completely different dependence on T and holonomy omega

And yet, they are the same!
(elliptic theta function of the 3 type)

$$Z_1 = Z_2 = \theta_3\left(- \frac{\omega}{2}, \exp\left(- \frac{1}{2\Lambda T} \right) \right)$$

Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

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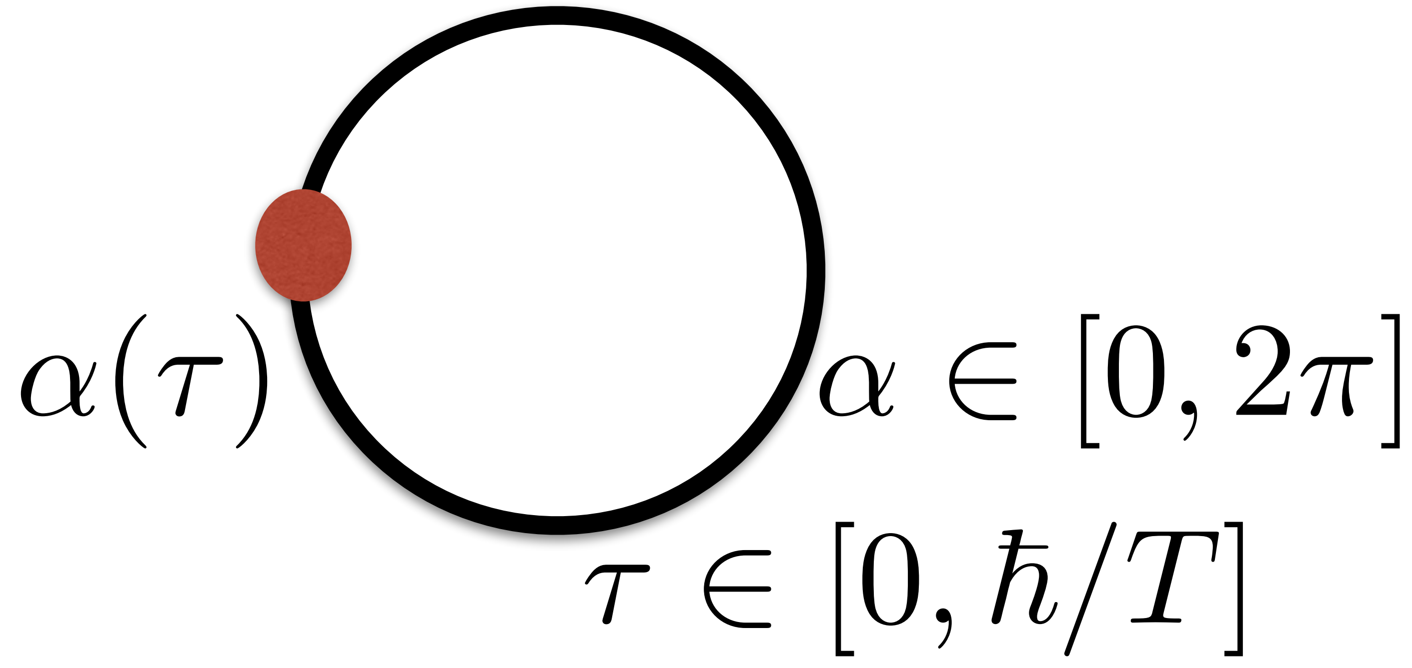

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