# Persistent homology analysis for QCD effective models

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**Collaborators**

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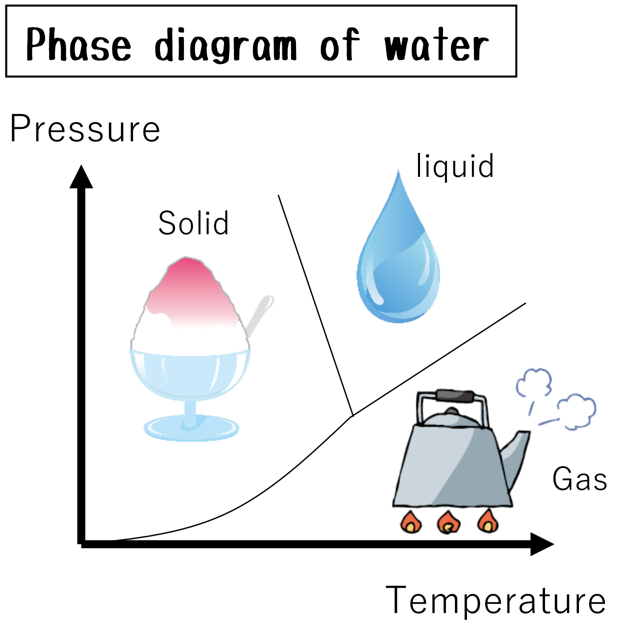
Takehiro Hirakida (Izumi Chuo High School)

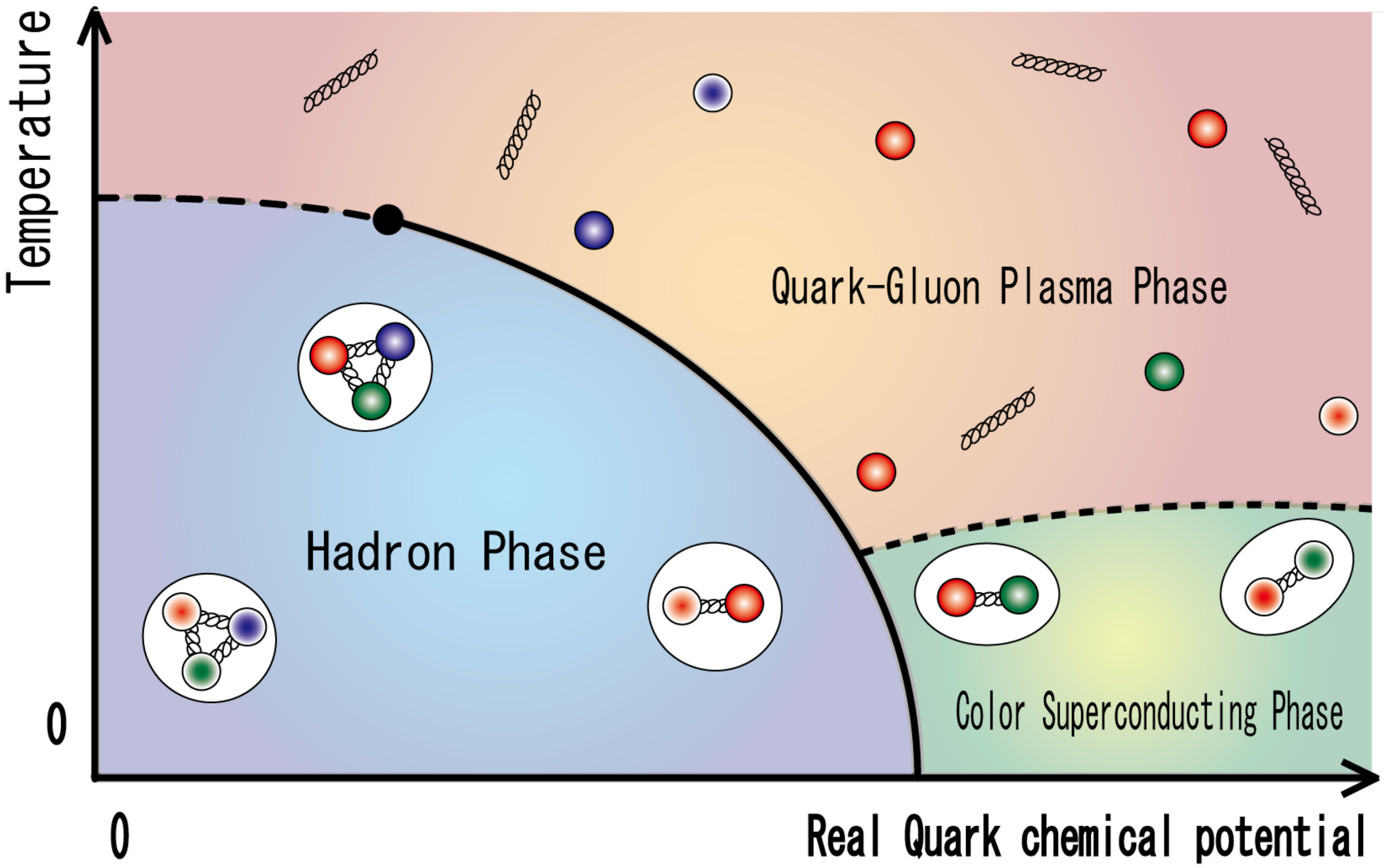
- T. Hirakida, **K.K.**, J. Sugano, J. Takahashi, H. Kouno. M. Yahiro, Int. J. Mod. Phys. A 35 (2020) 2050049
- **K.K.**, T. Hirakida, H. Kouno, arXiv:2103.11579

# Introduction: QCD phase diagram




Exploring the phase diagram is a first step to understand structures of the theory

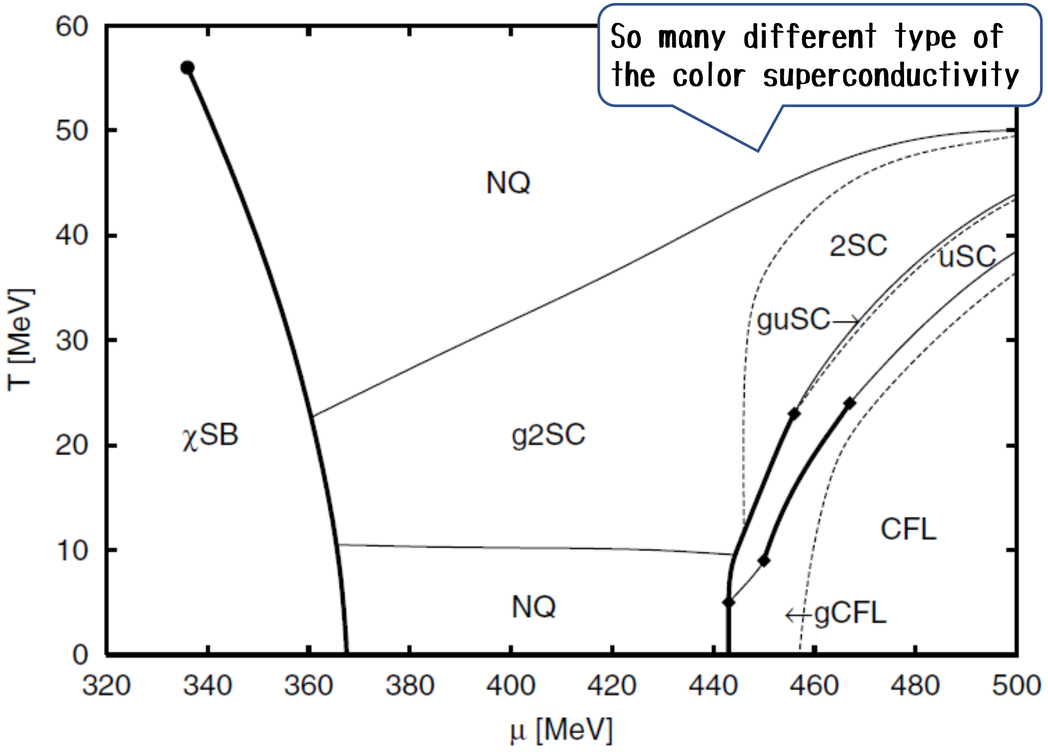
Schematic QCD phase diagram (rough sketch)



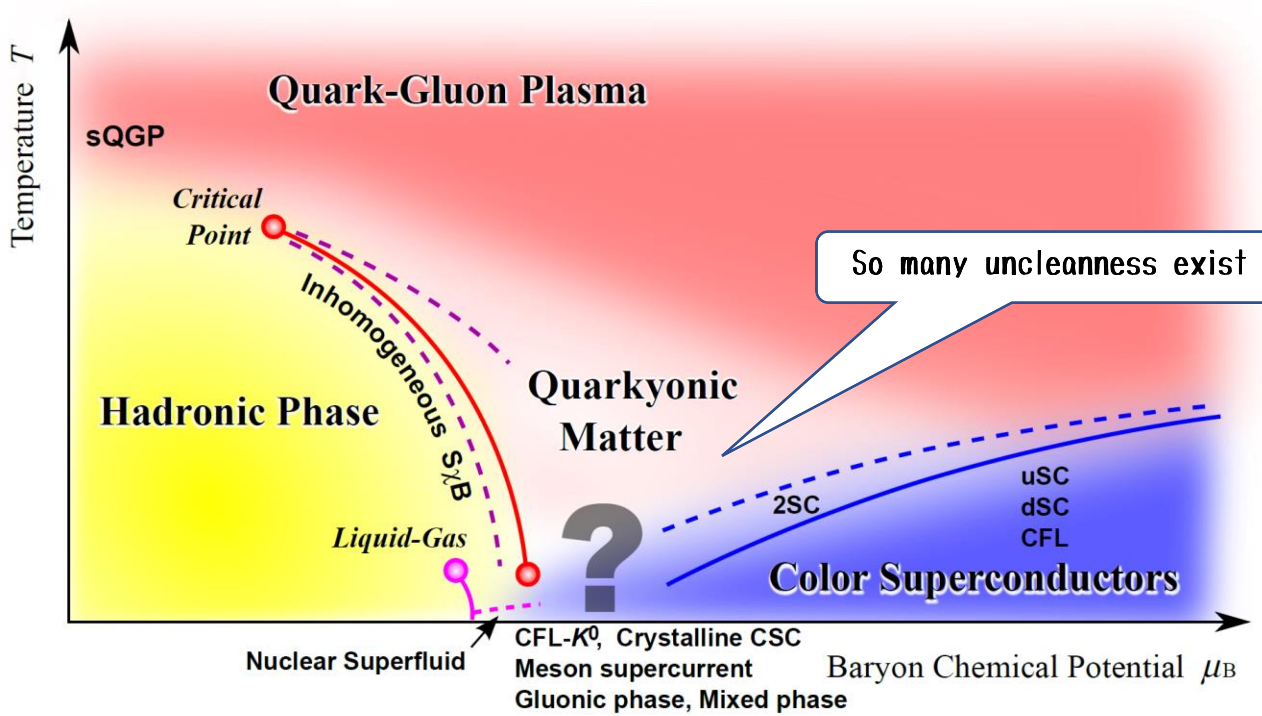
# Introduction: QCD phase diagram

## Expected QCD phase diagram (rough sketch)

S. B. Rüster, et al., Phys. Rev. D 72 (2005) 034004




K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2011) 014001

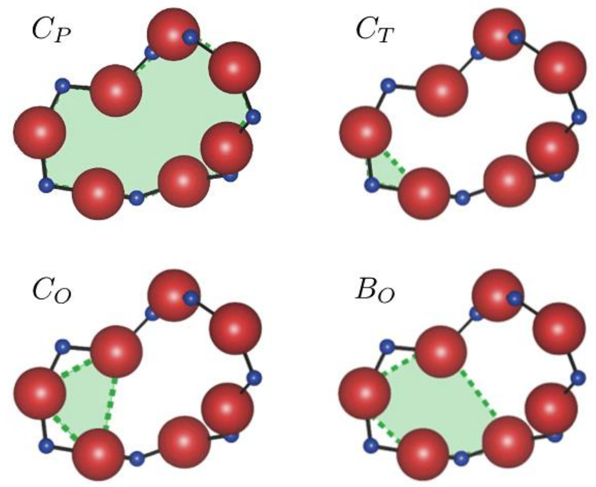



Detailed structure of QCD phase diagram at finite density is still an **open question**

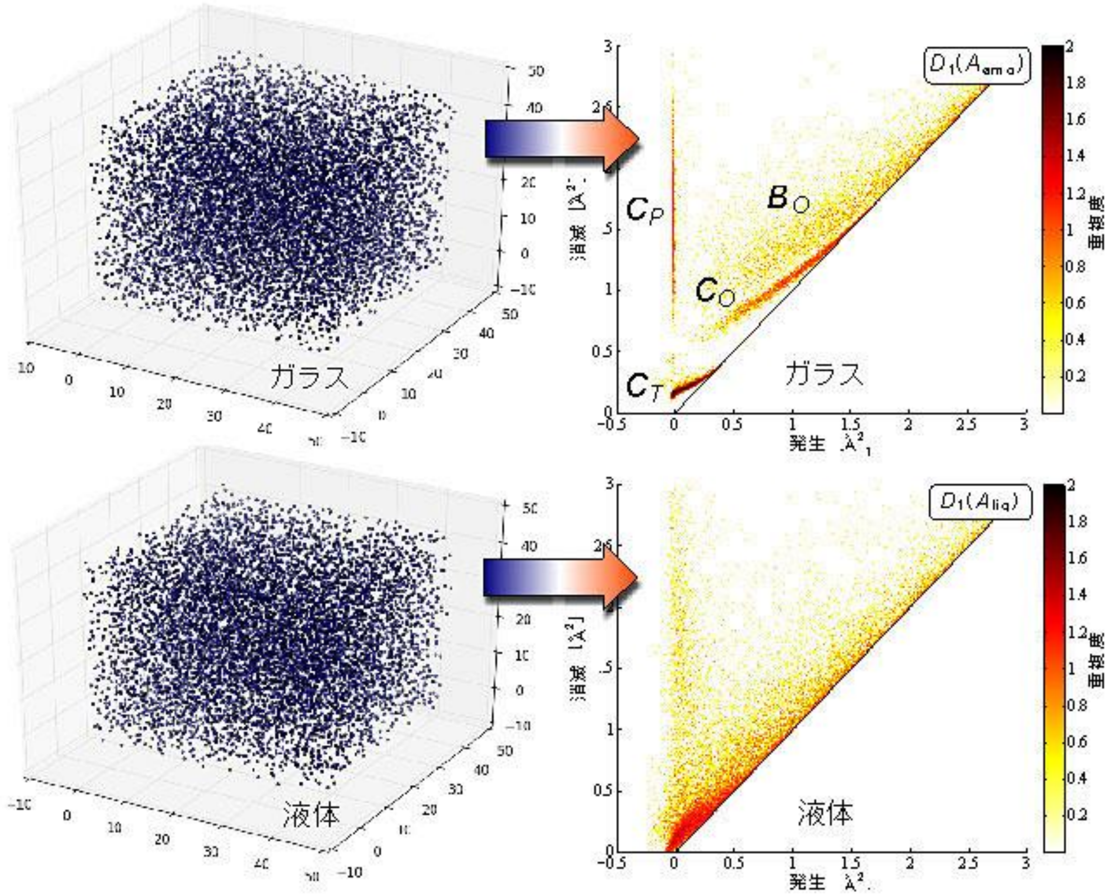
# Introduction: Persistent homology

**Investigate (topological) data structure**

Classification of **amorphous solid** and **liquid** states




Input configuration : Atomic configuration




From the persistent diagram, we can clarify the liquid and glass state clearly from hole structure

Persistent homology:
H. Edelsbrunner and J. Harer: Computational Topology: An Introduction (AMS), (2010)

https://www.jst.go.jp/pr/announce/20160614/index.html

See Y. Hiraoka, et al., PNAS 113 (2016) 26 for detailed

# Introduction: Persistent homology

## Procedure of computation for point cloud

We use HomCloud : https://homcloud.dev

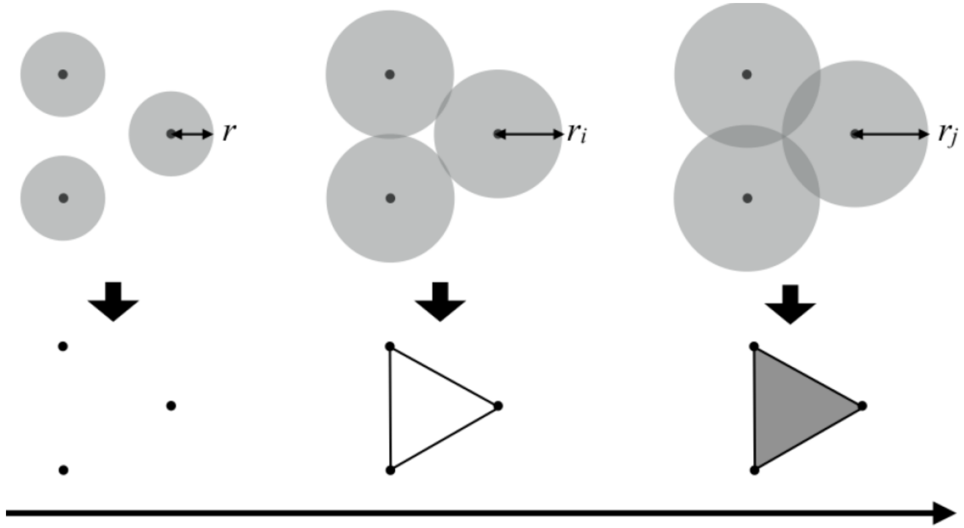
① Point cloud : Prepare **data set** and consider the balls

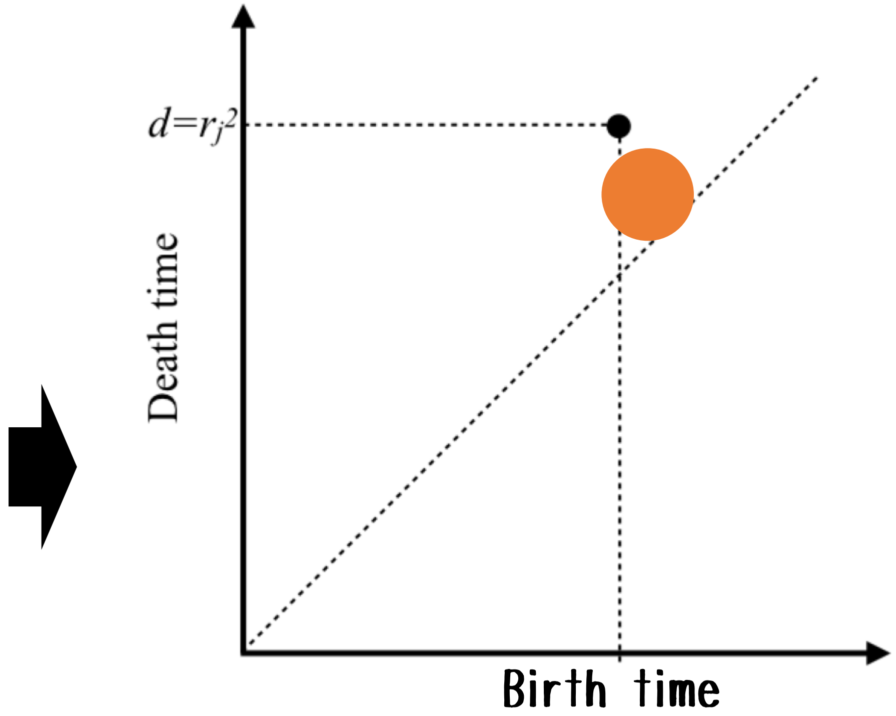
② Filtration : Increase **radius** of balls which are controlled by fictitious time

③ Birth time : Detect **holes** surrounded by balls

④ Death time : Detect **disappearance of holes**

⑤ Result : Plot persistent diagram



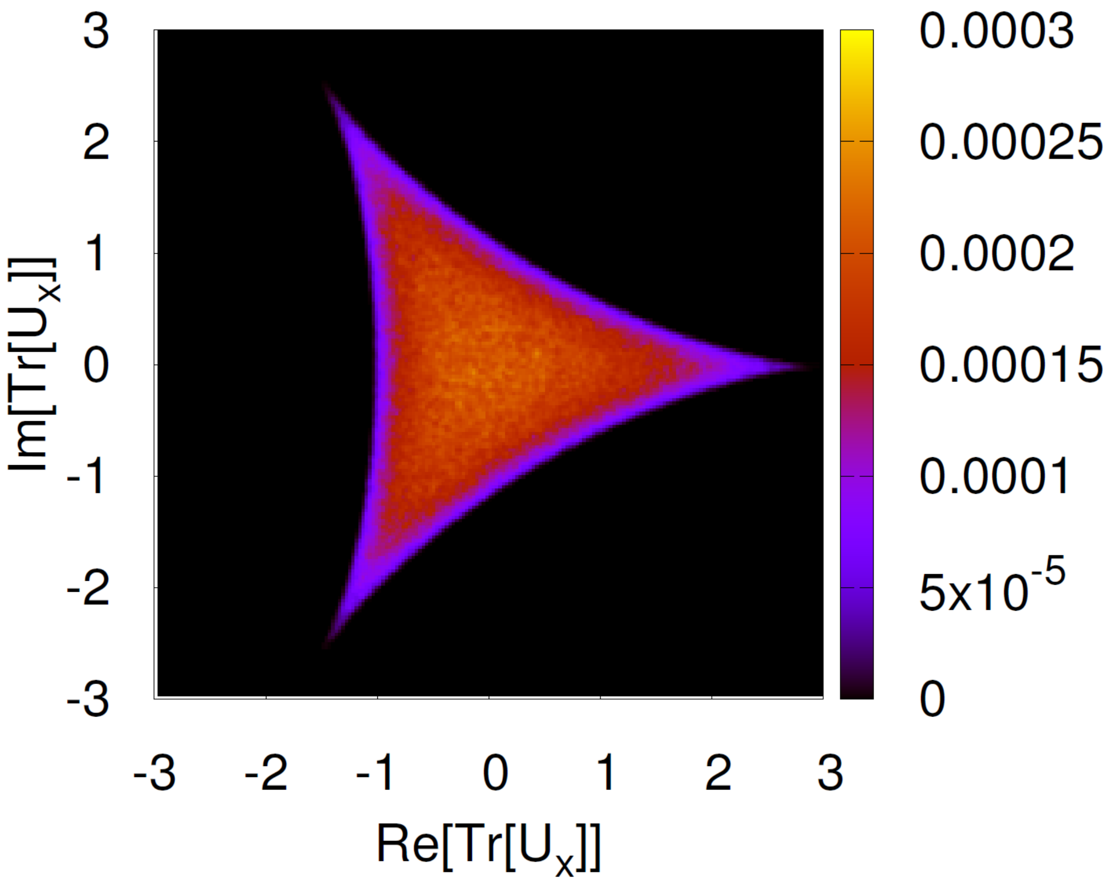




It is, of course, not trivial what happen in persistent homology for QCD phase transitions

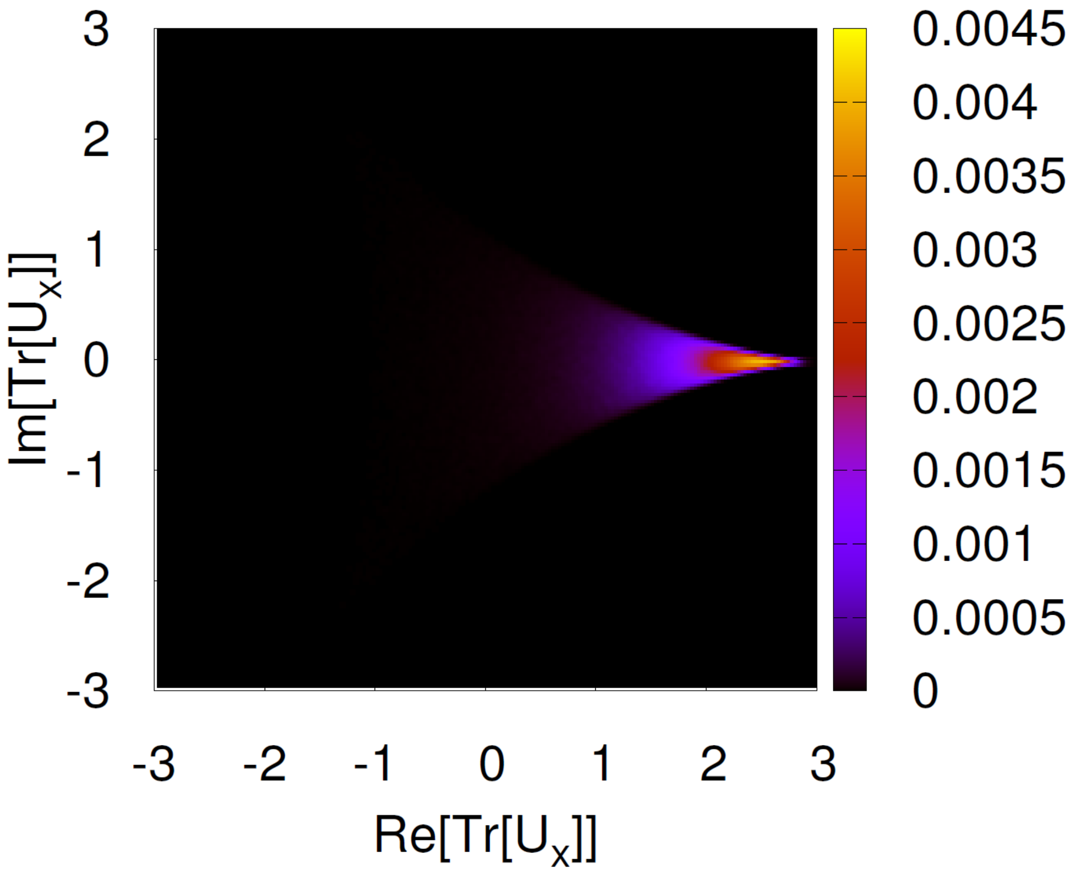
## Introduction: Polyakov loop

**Probability density for Polyakov-loop in QCD**

Low T

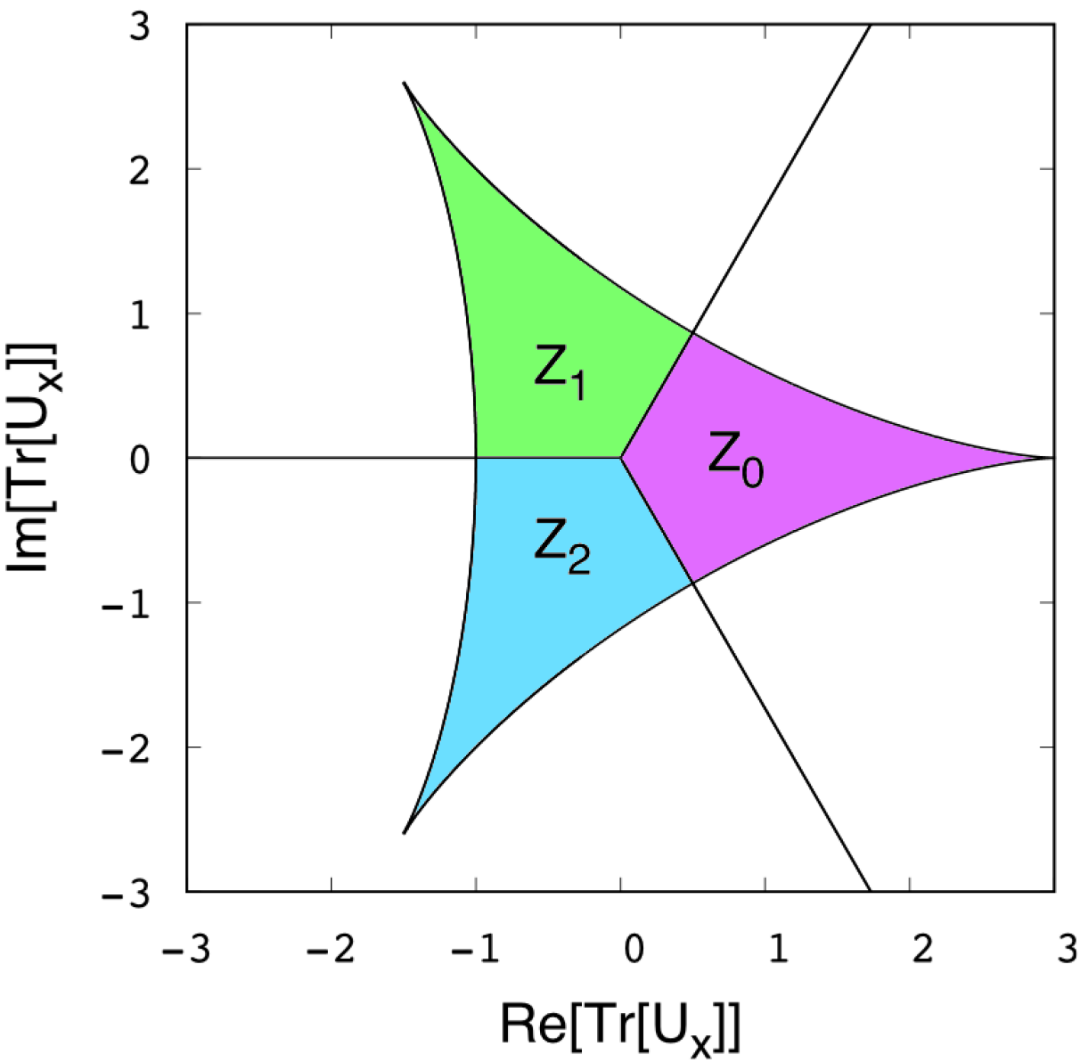



High T

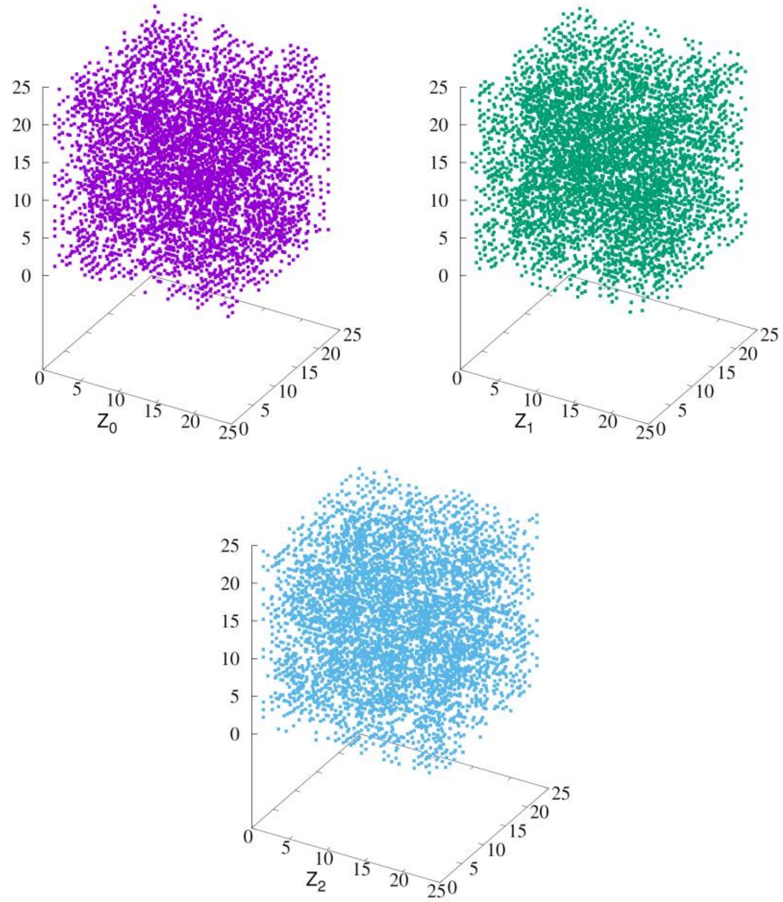


# Introduction: Polyakov loop

## Example : Point cloud approach




Center domain based data division




Input configuration:

Polyakov-loop at each cite in one configuration

# Results

**Effective Polyakov-line model (QCD effective model with heavy quarks)**

**Partition function** $\mathcal{Z} = \int \mathcal{D}U \, e^{-(S_{\mathrm{G}}+S_{\mathrm{Q}})}$

$\kappa$ is treated as the effective **temperature**

$$S_{\mathrm{Q}} = \sum_{\mathbf{x}} \mathcal{L}_{\mathrm{Q}}(\mathbf{x})$$

$$S_{\mathrm{G}} = -\kappa \sum_{\mathbf{x}} \sum_{k=1}^{3} \left( \mathrm{Tr}[U_{\mathbf{x}}] \, \mathrm{Tr}\left[U^{\dagger}_{\mathbf{x}+\hat{k}}\right] + \mathrm{c.c.} \right)$$

$$\mathcal{L}_{\mathrm{Q}} = -\ln\left[\det\{1 + e^{-\beta M} U_{\mathrm{x}}\}^{2N_{\mathrm{f}}} \det\{1 + e^{-\beta M} U^{\dagger}_{\mathrm{x}}\}^{2N_{\mathrm{f}}}\right]$$

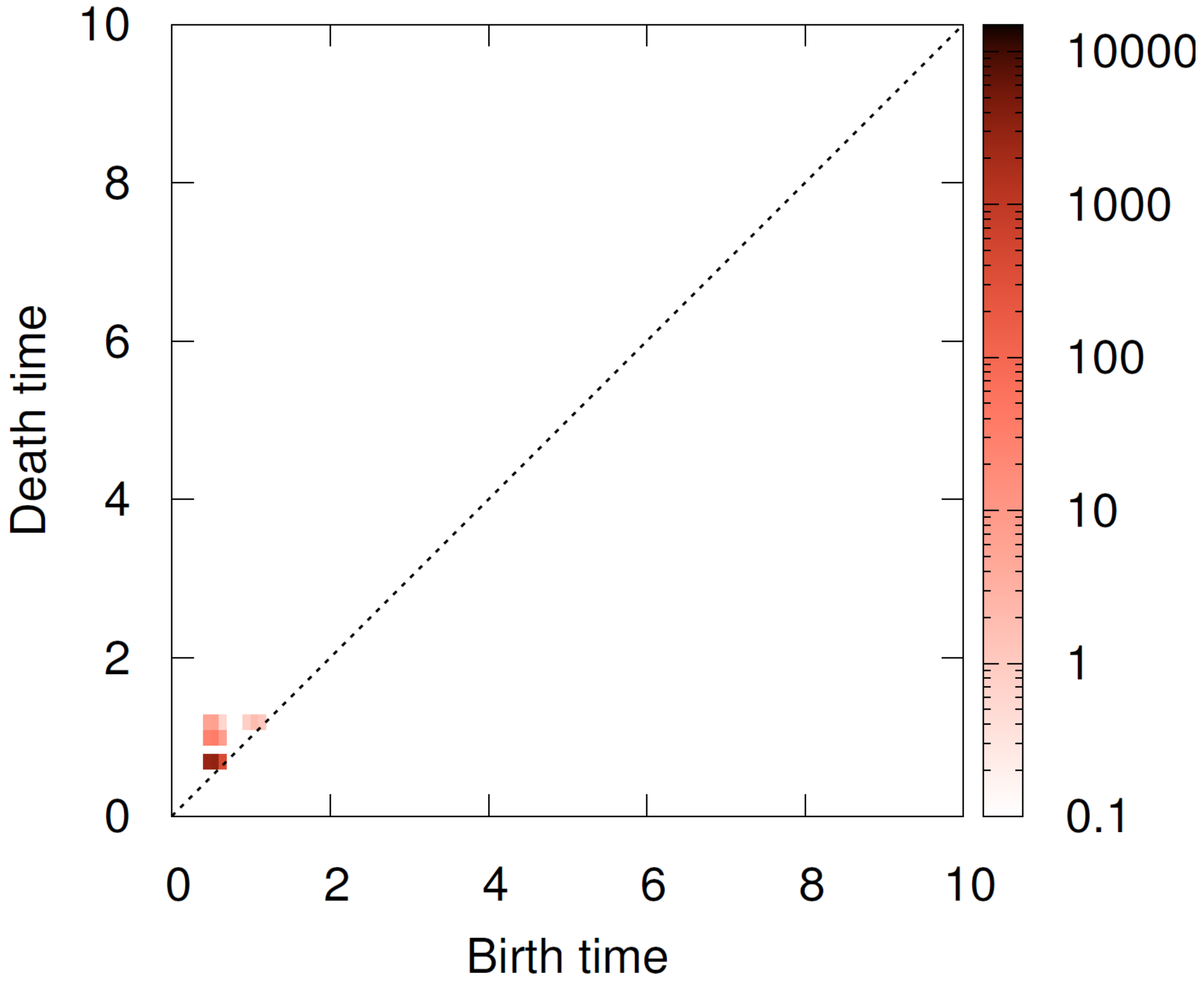
## Results

**Effective Polyakov-line model**

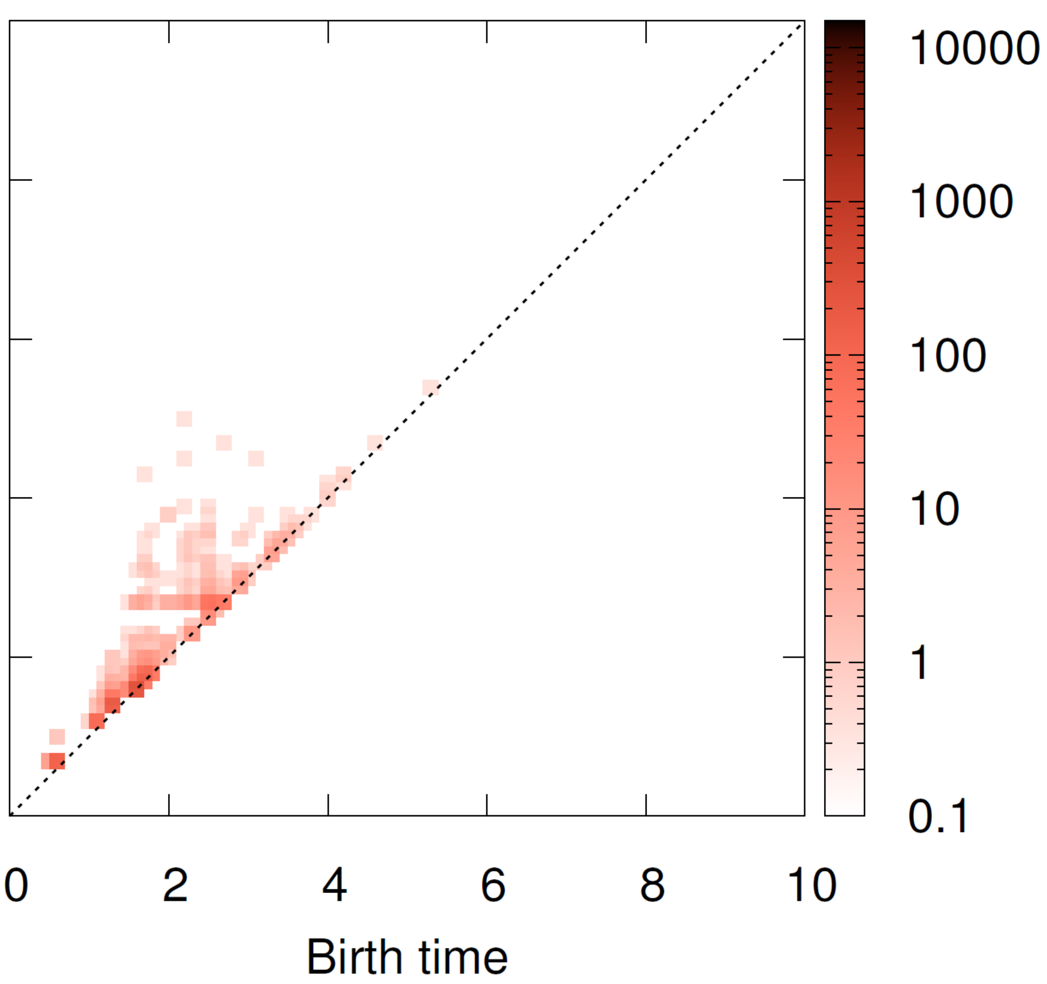
**Persistent diagram**

T. Hirakida, **K.K.**, J. Sugano, J. Takahashi, H. Kouno. M. Yahiro, Int. J. Mod. Phys. A 35 (2020) 2050049

**Low T**




**High T**

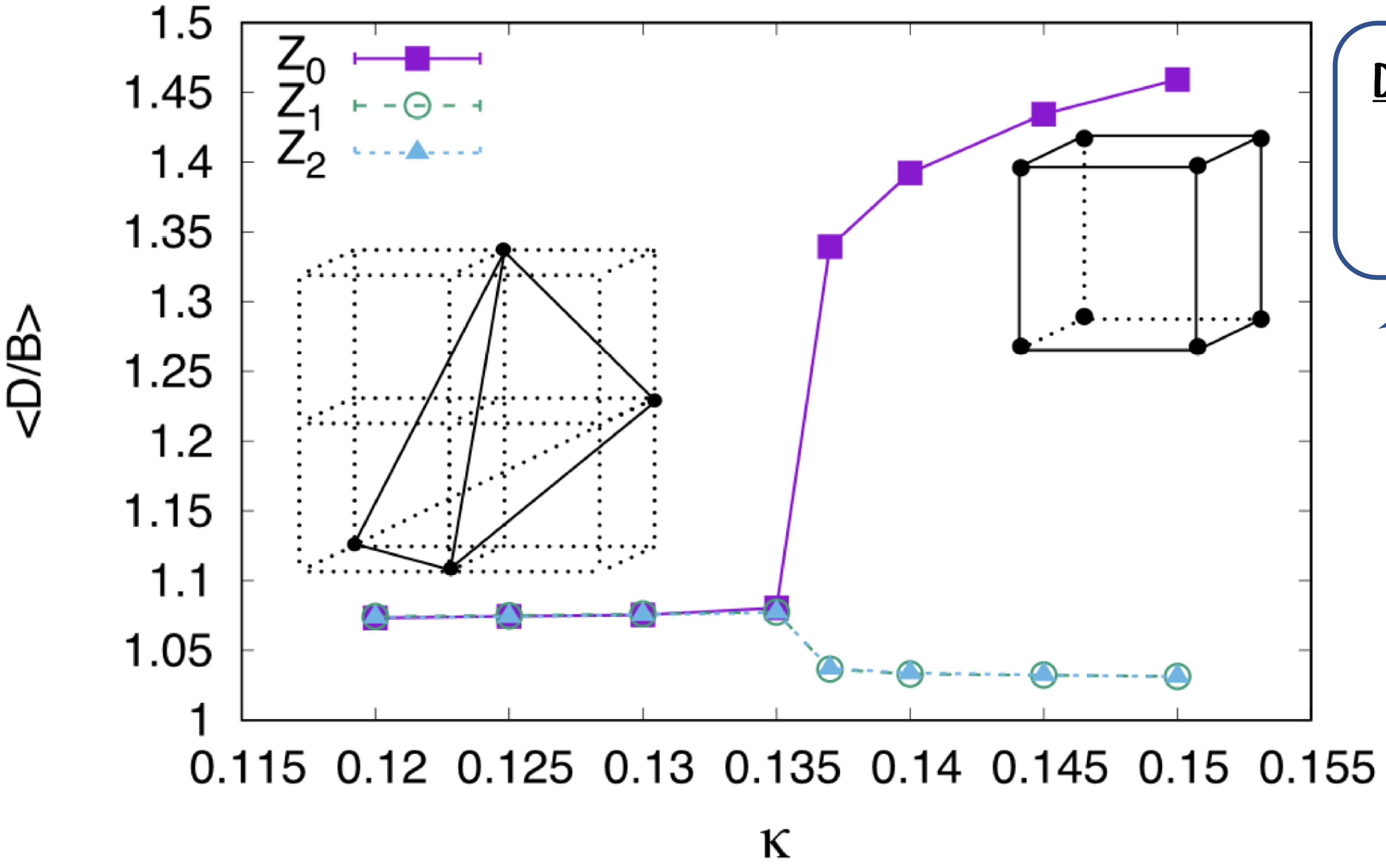


# Results

## Effective Polyakov-line model

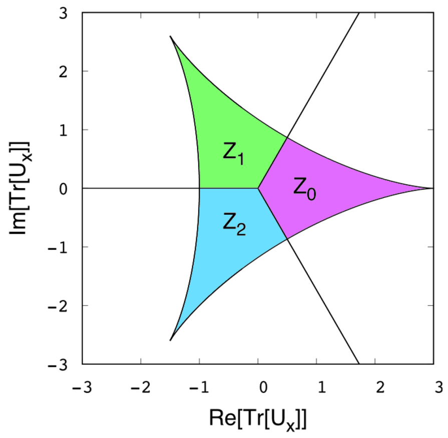
T. Hirakida, **K.K.**, J. Sugano, J. Takahashi, H. Kouno. M. Yahiro, Int. J. Mod. Phys. A 35 (2020) 2050049

## Averaged ratio




**Definition**

$$D/B \equiv \frac{1}{N_{\mathrm{hole}}} \sum_{i}^{N_{\mathrm{hole}}} \frac{d_i}{b_i}$$



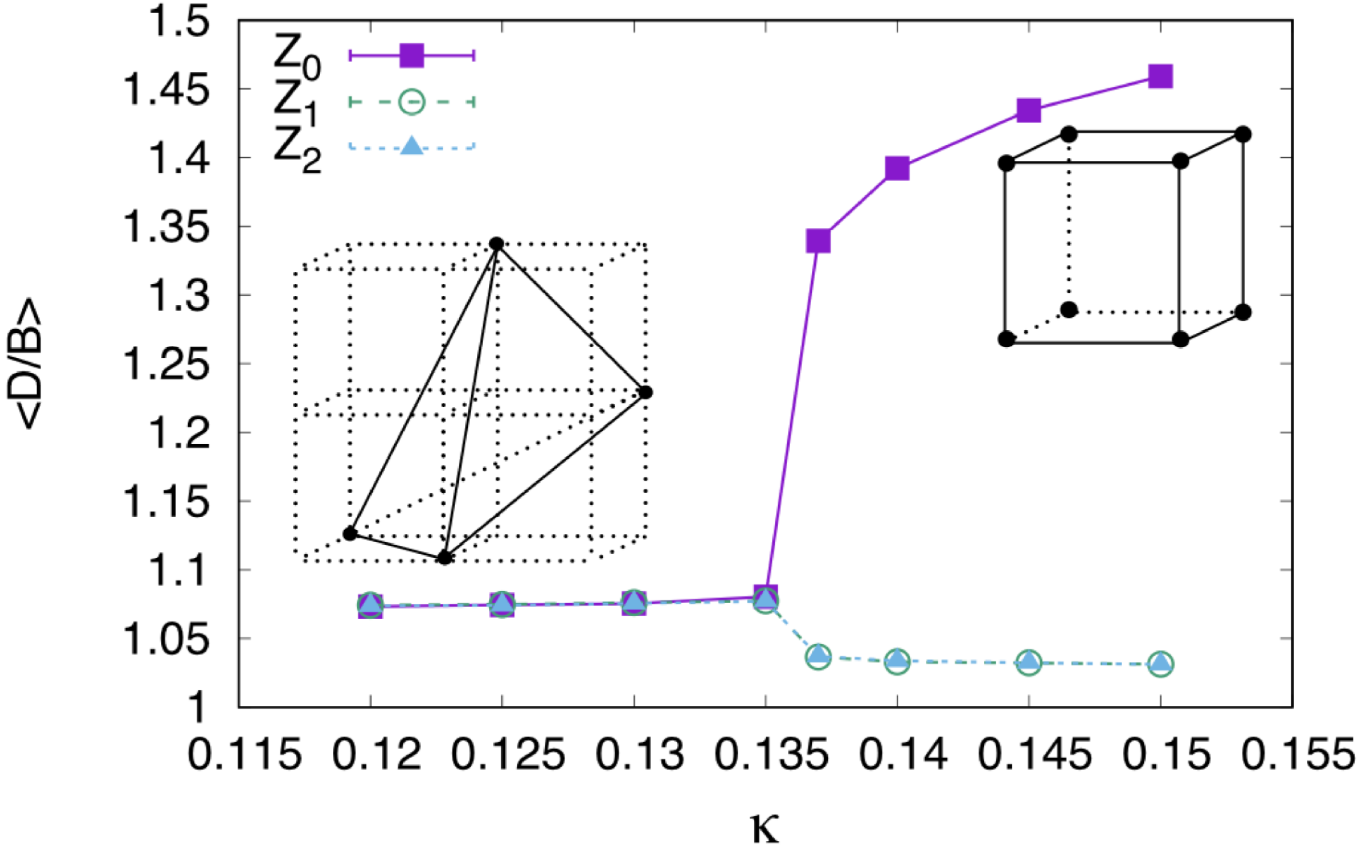
## Results

### Effective Polyakov-line model

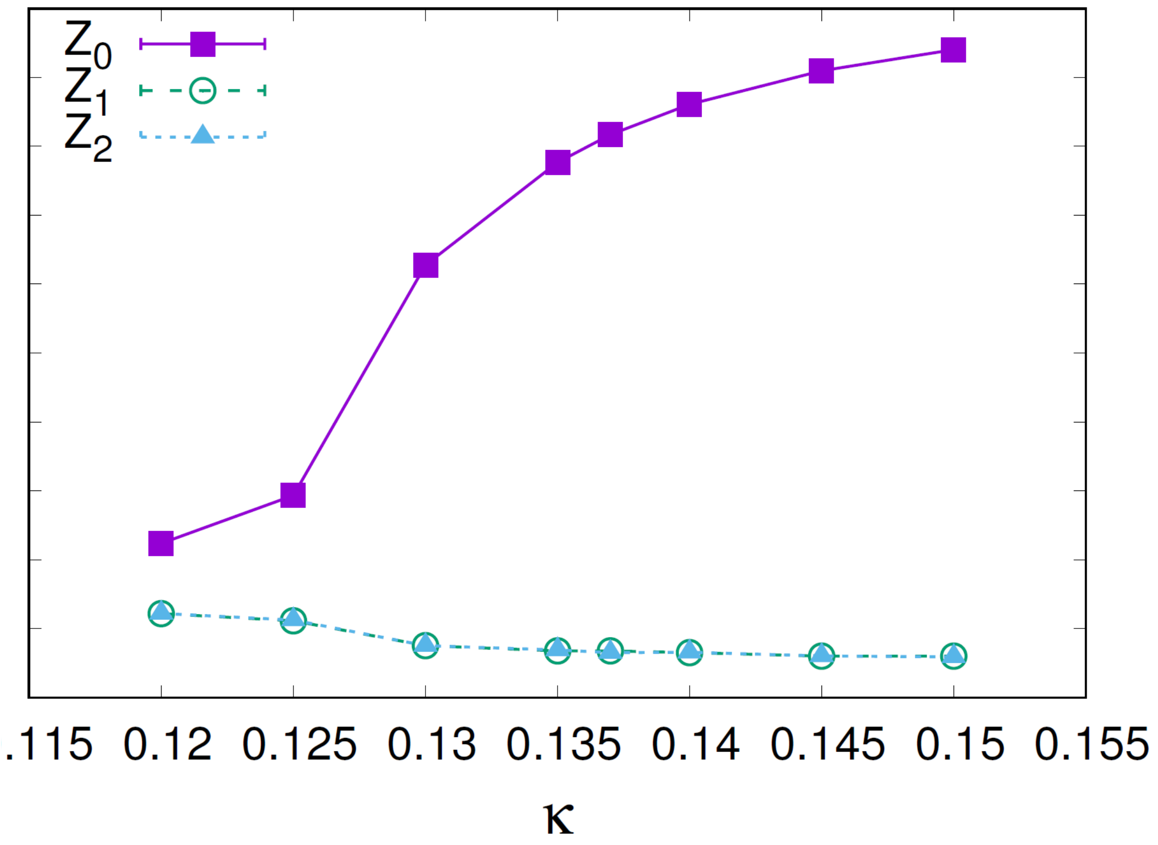
T. Hirakida, **K.K.**, J. Sugano, J. Takahashi, H. Kouno. M. Yahiro, Int. J. Mod. Phys. A 35 (2020) 2050049

### Averaged ratio

#### Large mass




#### small mass



# Results

**QCD-like Potts model**

**u-d quark contributions** in QCD-like Potts model

**Energy**
$$E_{\rm iso} = -\kappa \sum_{\mathbf{x},\mathbf{i}} \delta_{\Phi_{\mathbf{x}}\Phi_{\mathbf{x}+\mathbf{i}}} - \sum_{\mathbf{x}} \left[ (h_+\Phi_{\mathbf{x}} + h_-\bar{\Phi}_{\mathbf{x}}) + (h_-\Phi_{\mathbf{x}} + h_+\bar{\Phi}_{\mathbf{x}}) \right]$$
$$= -\kappa \sum_{\mathbf{x},\mathbf{i}} \delta_{\Phi_{\mathbf{x}}\Phi_{\mathbf{x}+\mathbf{i}}} - 2 \sum_{\mathbf{x}} \left[ (h_+ + h_-) \cos(\Phi_{\mathbf{x}}) \right]$$

**Polyakov loop** $\Phi_{\rm x} = e^{i\frac{2\pi}{3}k_{\rm x}}$, $\bar{\Phi}_{\rm x} = e^{-i\frac{2\pi}{3}k_{\rm x}}$

**External field** $h_{\pm} = e^{-\beta(E-\mu)}$

Numerical results are obtained by using simple **Metropolises algorithm**

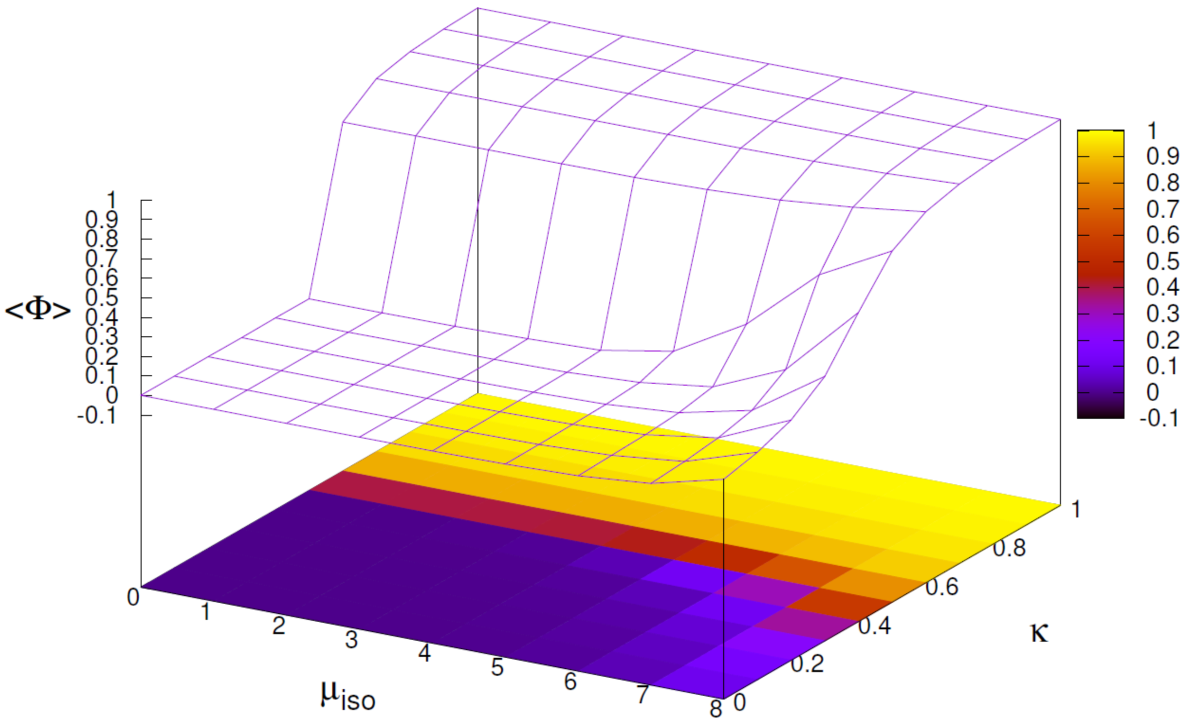
# Results

**Potts model with QCD like external field**

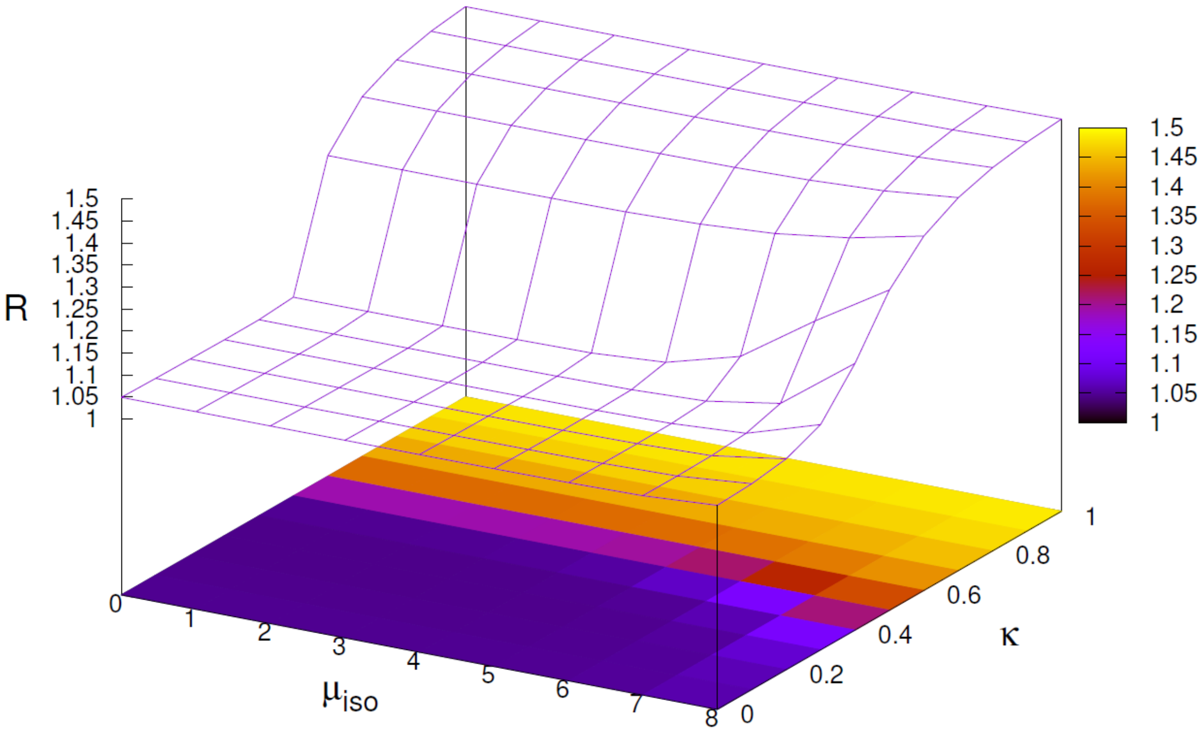
**Potts model with isospin chemical potential**

**K.K.**, T. Hirakida, H. Kouno, arXiv:2103.11579

## Polyakov loop




## Averaged birth-death time Ratio




The averaged ratio **shares the same tendency** with the Polyakov loop

# Results

## Potts model with QCD like external field

## Potts model with isospin chemical potential

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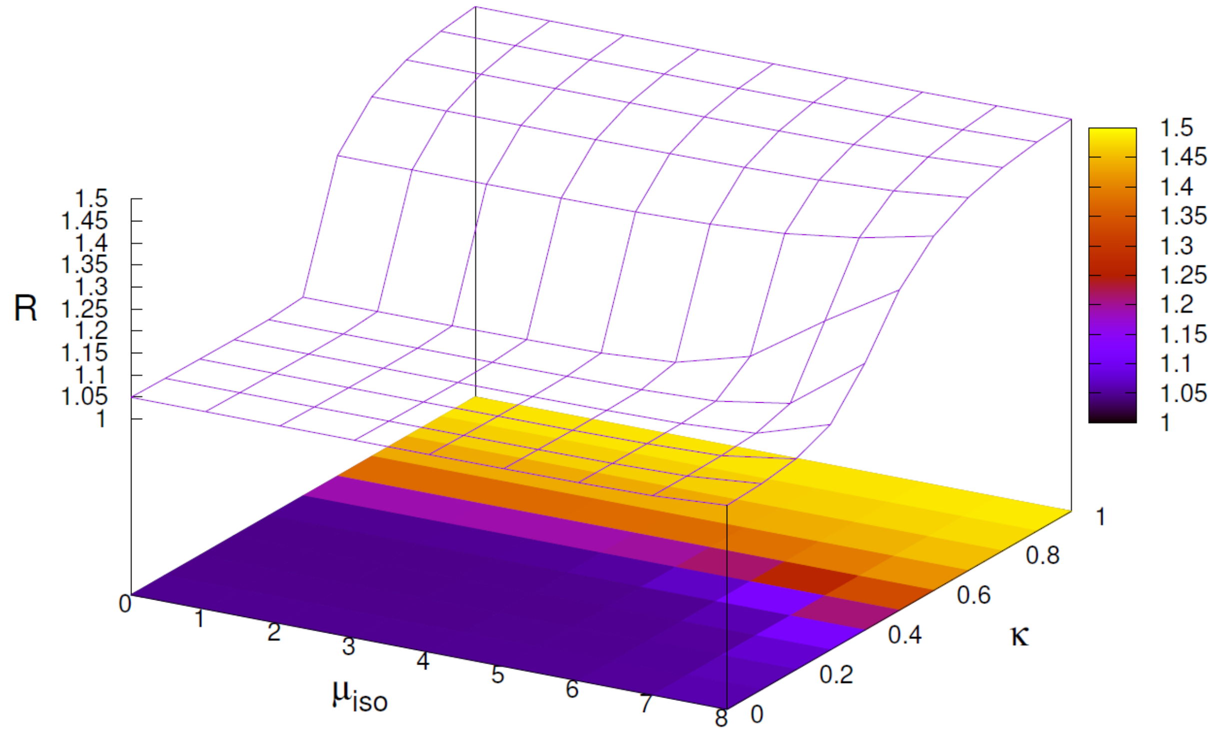
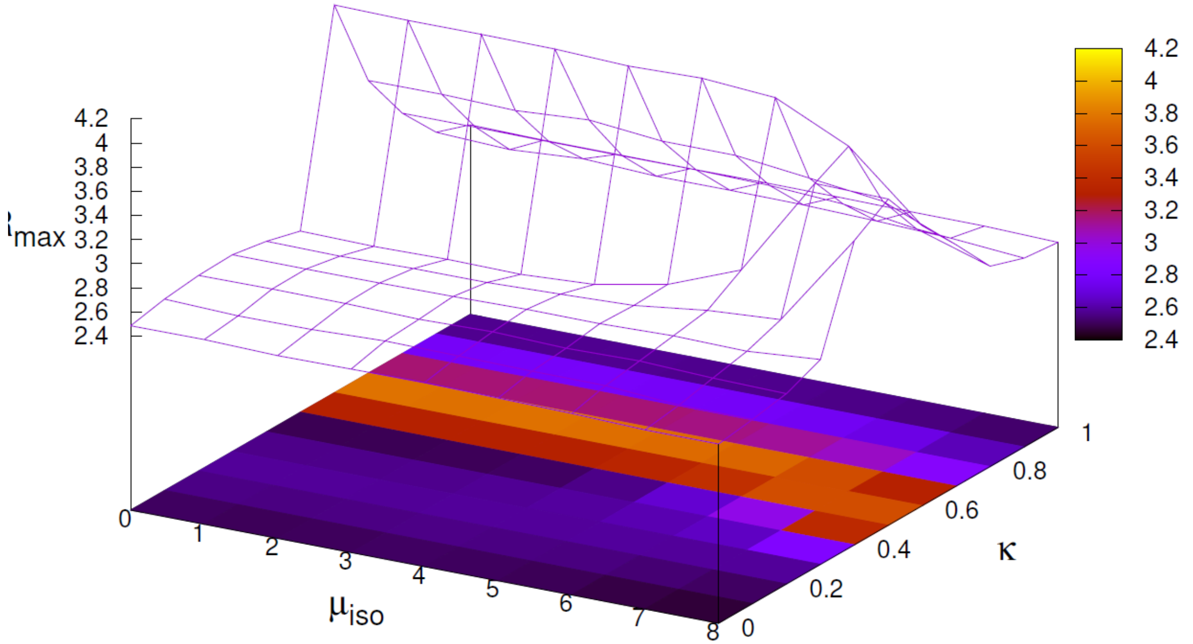
### Averaged birth-death time Ratio

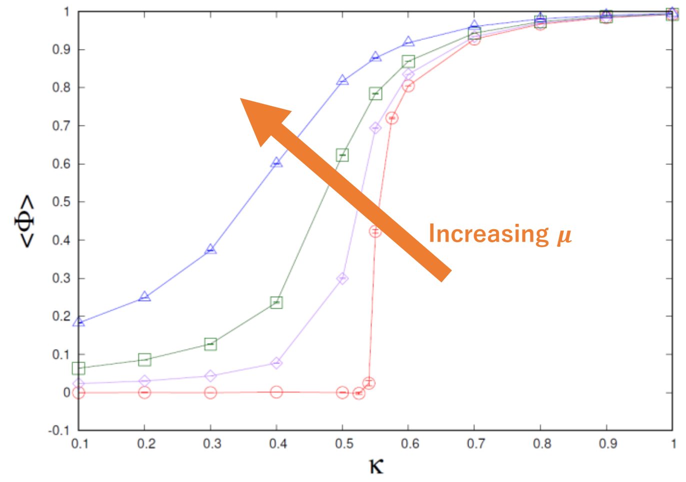


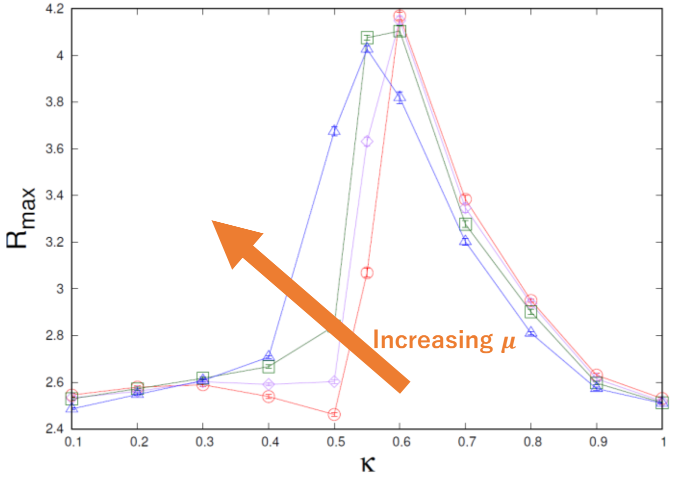

### Maximum birth-death time Ratio

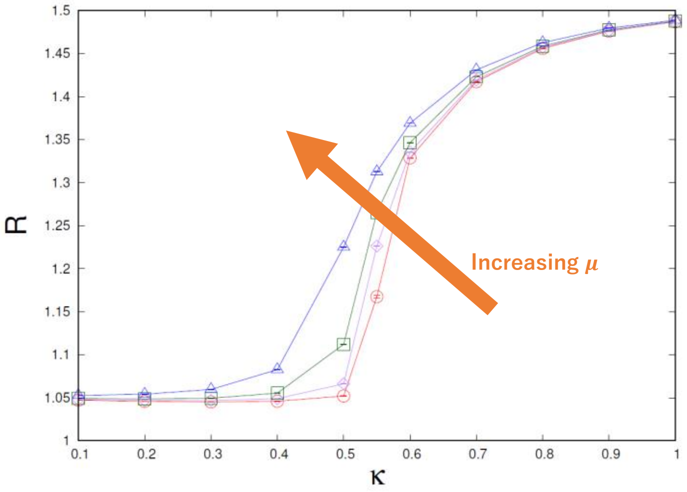



The maximum ratio has **more information** than the averaged one as we expected

# Results



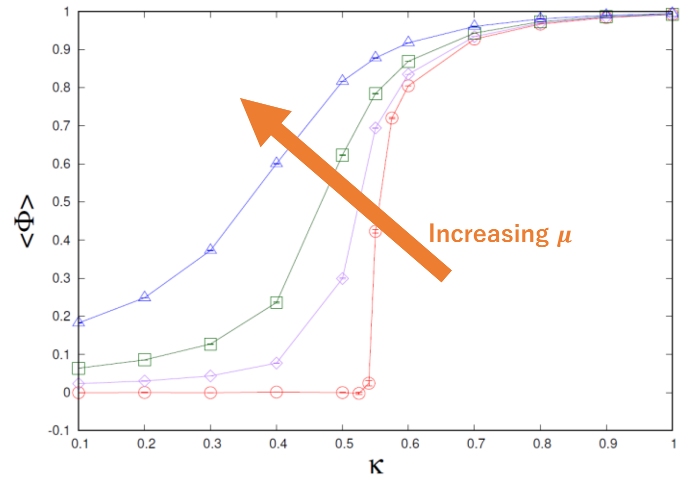



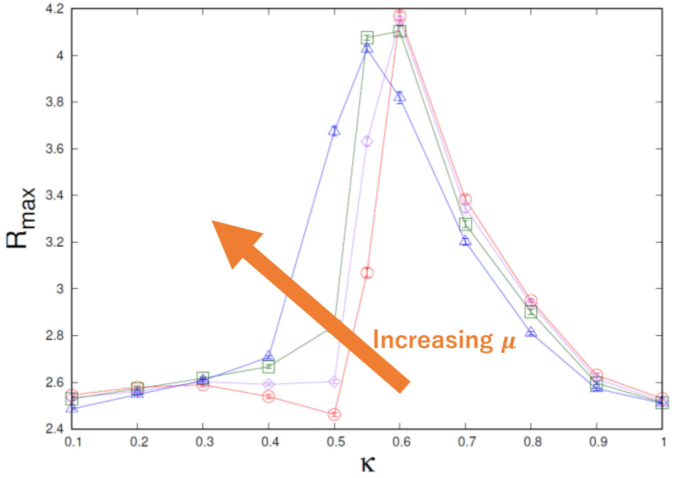

## Potts model with isospin chemical potential

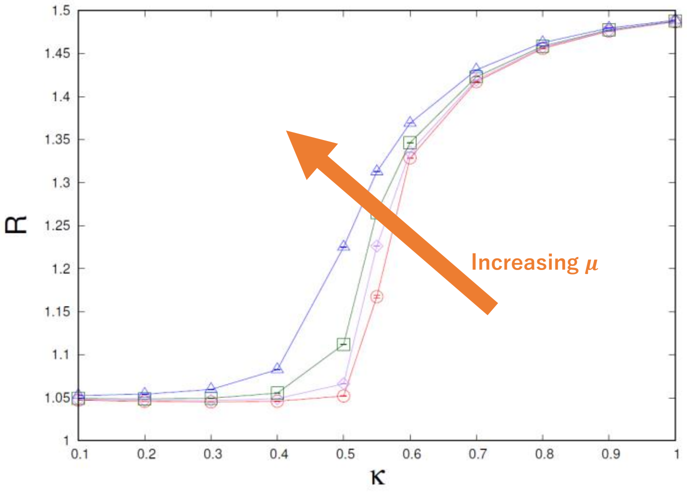
**K.K.**, T. Hirakida, H. Kouno, arXiv:2103.11579

- **Maximum ratio** has mor information near the phase transition
- **Averaged ratio** shares the same tendency with the Polyakov loop

# Summary




## Potts model with isospin chemical potential

**K.K.**, T. Hirakida, H. Kouno, arXiv:2103.11579

- **Maximum ratio** has mor information near the phase transition
- **Averaged ratio** shares the same tendency with the Polyakov loop

**We have applied the persistent homology analysis to QCD effective models**

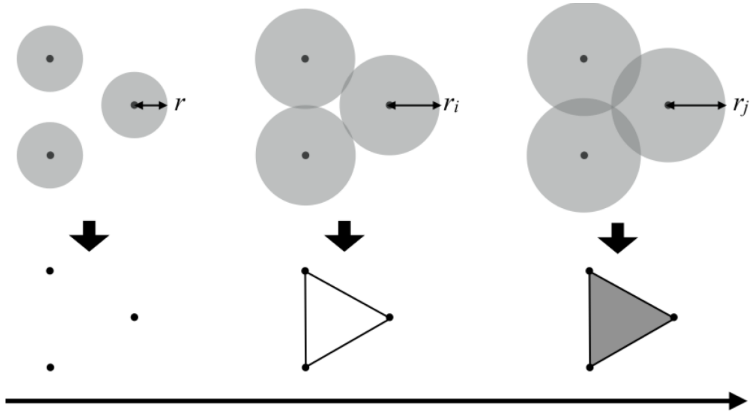
**Properties of the averaged and the maximum birth-death time are clarified**

**The effective Polyakov line model and the Potts model are too simple effective models of QCD with heavy quarks**

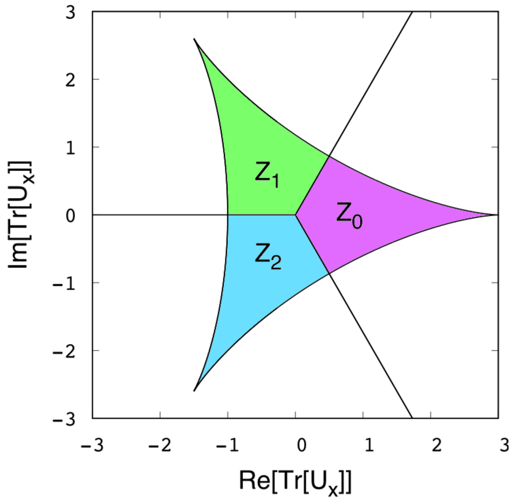
**We need some more extension to use the persistent homology to QCD**

# Extension

## Point cloud




This work




## Pixel data

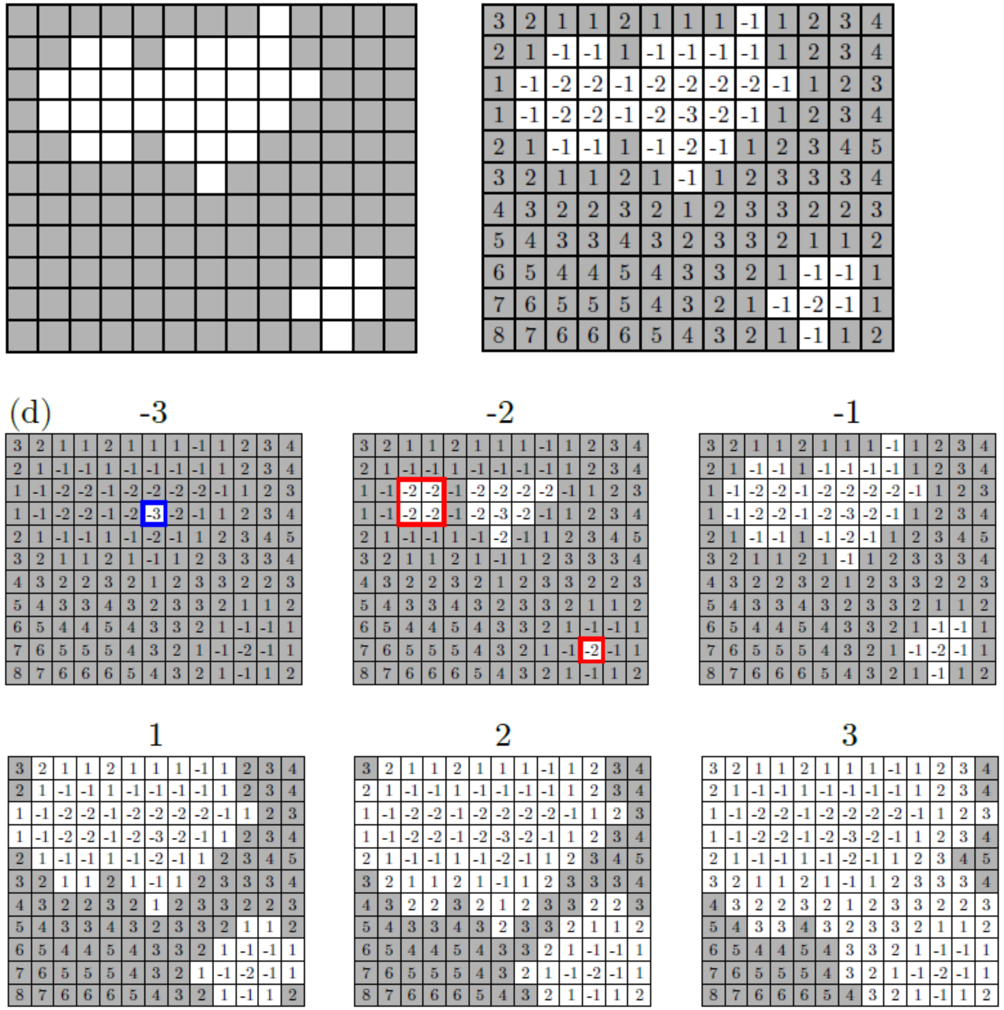


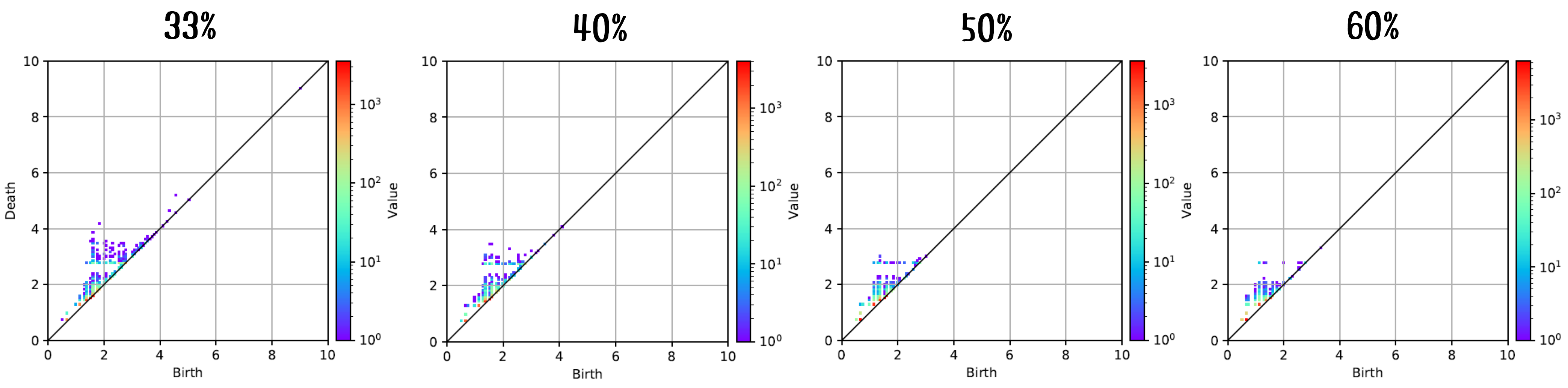

Figure: O. Ippei, Y. Hiraoka, and M. Kimura, Journal of Applied and Computational Topology 1.3-4 (2018): 421.

# Backup slides

# Introduction: Persistent diagram

**Typical results**

**Random** distribution (no nontrivial spatial structure)




**Low T** → **High T**

If there are **deviations** from the random behavior, it indicates **nontrivial structure** in the data space which is difficult to investigate via the ordinary order parameters

# Typical small hole structures

## Example of (relatively small) possible structures