What is chiral susceptibility probing? Hidenori Fukaya (Osaka U.)

S. Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki [JLQCD collaboration], arXiv:2103.05954

For more details, see our seminar slide

1. Introduction

QCD partition function

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)}$$

$$N_f = 2 \quad (m_u = m_d = m)$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V}\frac{\partial}{\partial m}\ln Z(m)$$

chiral susceptibility

$$\chi(m) = -\frac{\partial}{\partial m}\langle \bar{q}q \rangle(m)$$

probe for $SU(2)_L \times SU(2)_R$

breaking/restoration (at m=0) :

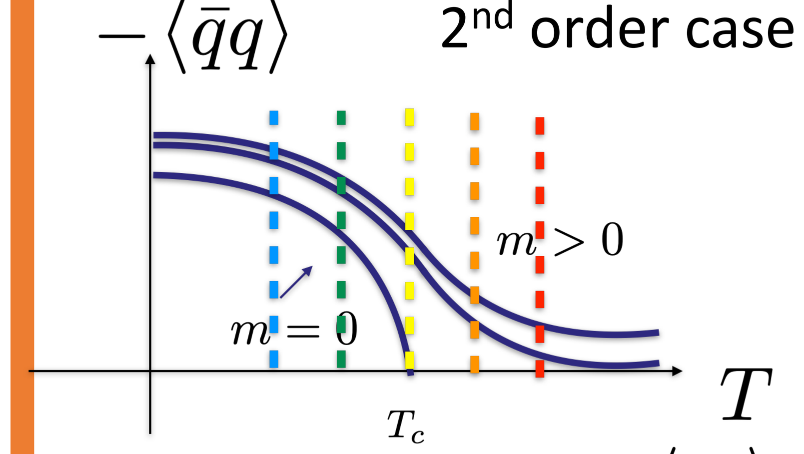
T<Tc (~150MeV) , T>Tc,

$\langle \bar{q}q \rangle \neq 0$ $\quad$ $\langle \bar{q}q \rangle = 0$

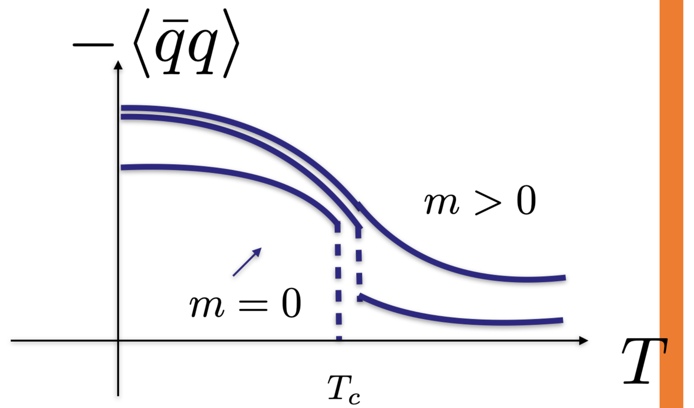
chiral phase transition

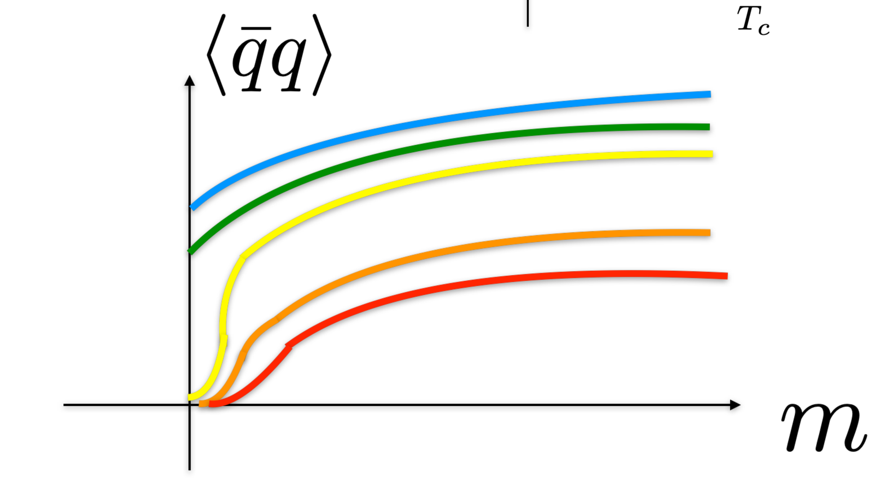
2. T and m dependence

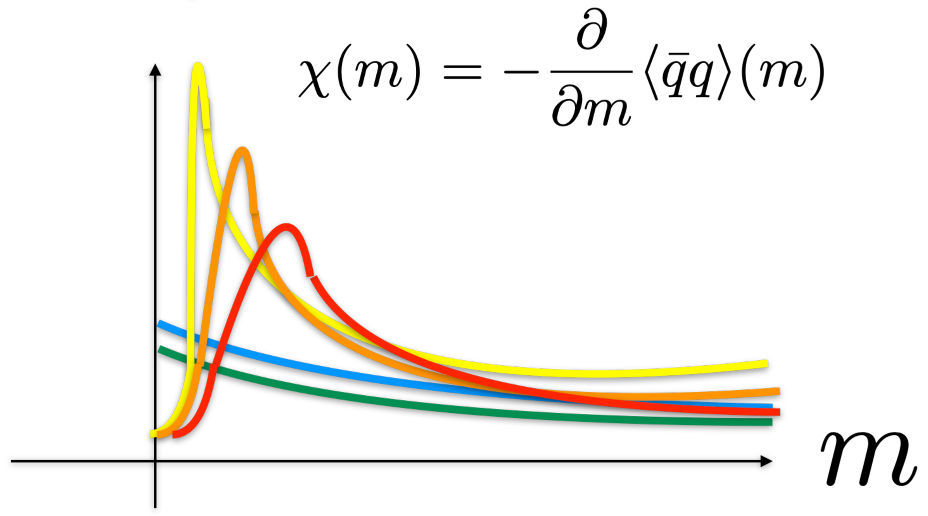
2nd order case



1st order case (but will not be so sharp on finite lattices)







3. Question

condensate breaks both

$SU(2)_L \times SU(2)_R$ and $U(1)_A$

How much of

$$\chi(m) = -\frac{\partial}{\partial m}\langle \bar{q}q \rangle(m)$$

comes from $U(1)_A$ breaking?

Cf. Callan-Dashen-Gross 1978
suggested instanton effect
= $U(1)_A$ anomaly
= trigger of SU(2)xSU(2) breaking.
It may indicate that
instanton disappears
= U(1)A anomaly disappears
= SU(2)xSU(2) restoration.

Let us examine this in lattice QCD w/ chiral fermions.

4. Separating $U(1)_A$ part

$$\chi(m) = \chi^{\rm con.}(m) + \chi^{\rm dis.}(m)$$

$$\chi^{\rm con.}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{U(1)_A \text{ anomaly contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\rm sub.}(m)}{m}}_{\text{mixed}}$$

$$\chi^{\rm dis.}(m) = \frac{N_f}{m^2}\chi_{\rm top.}(m) \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{SU(2)\times SU(2) \text{ breaking}}$$

$$\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0) \rangle$$

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0) \rangle$$

$$\Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0) \rangle$$

$$\chi_{\rm top.}(m) = \frac{\langle Q^2 \rangle}{V} \qquad Q(A) : \text{topological charge}$$

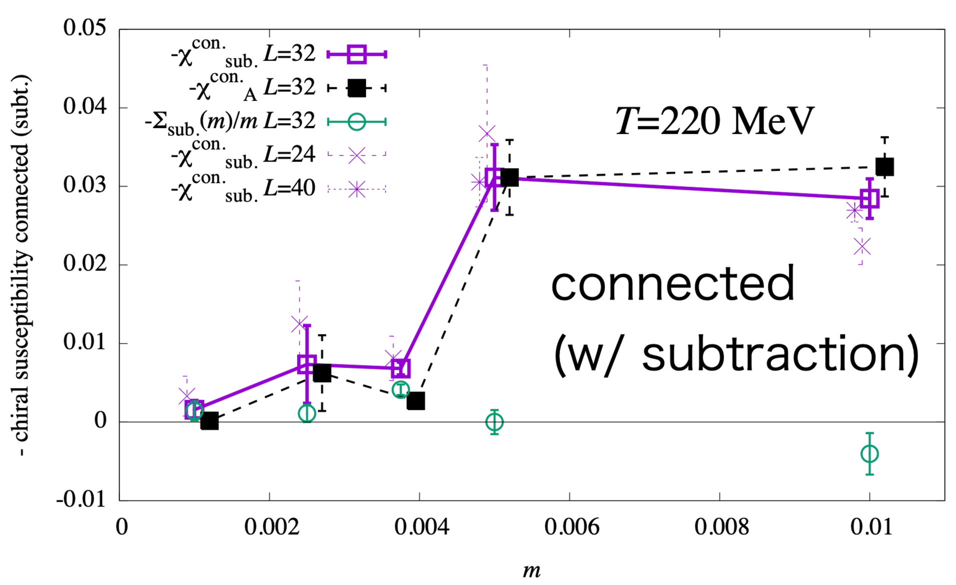
The formulas are known in continuum but true on a lattice only with overlap fermions.

[LLNL/RBC 2014, Nicola et al. 2018,2020]

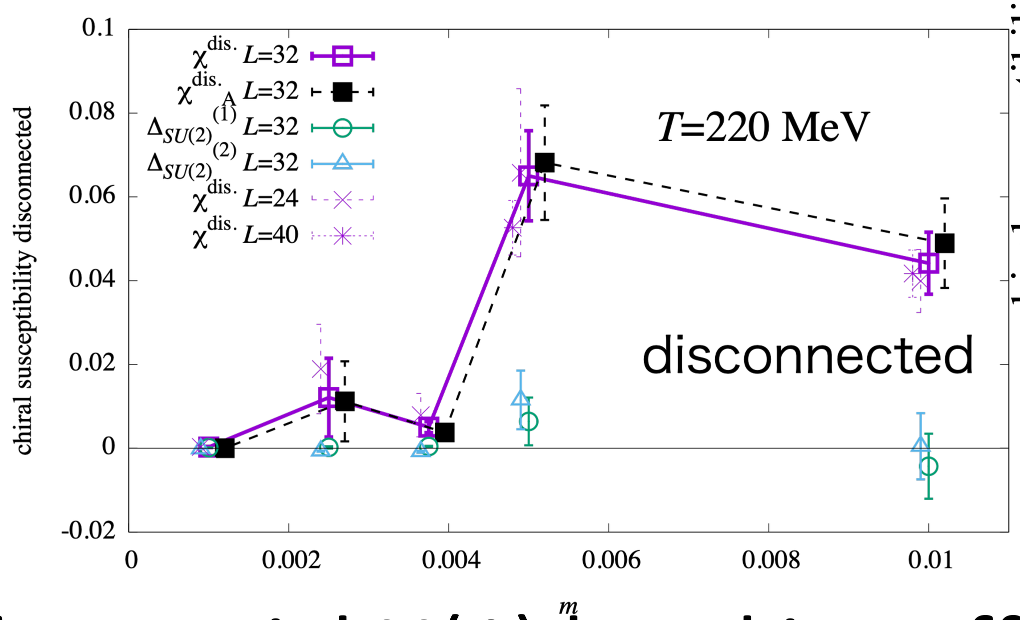
5. Lattice set-up

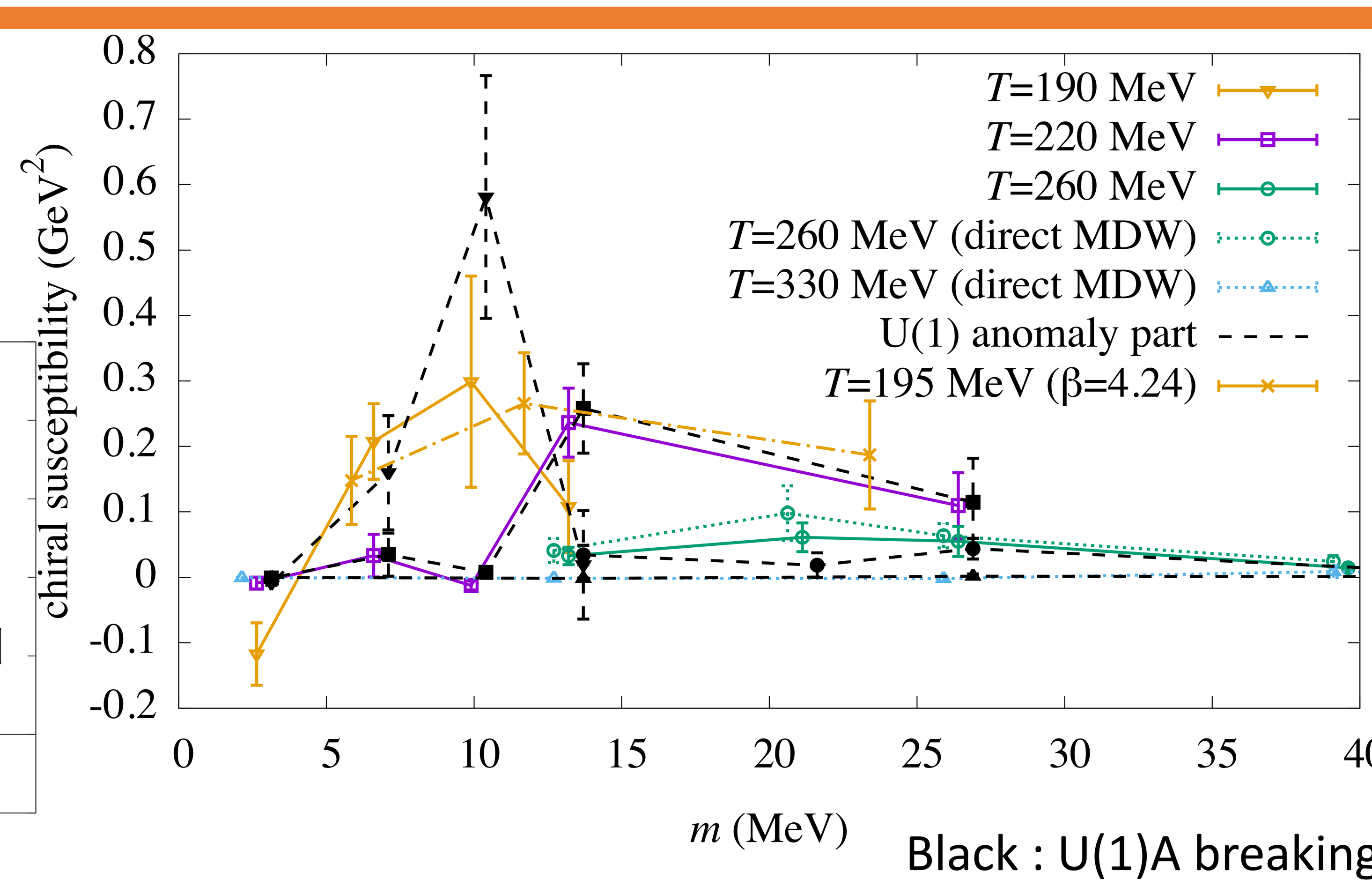
Nf=2 flavor QCD
T=190(~1.1Tc), 220, 260, 330 MeV.
(Lt=8,10,12,14)
1/a = 2.6 GeV (0.075fm)
L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)
Mobius domain-wall fermion + reweighted overlap fermion
Quark mass from 3MeV
(< phys. pt. ~4MeV) to 30MeV.
Measurement is w/ spec. decomposition.

6. Numerical Results



Black : U(1)A breaking effect
Purple : total contribution





Black : U(1)A breaking effect
Colored : total contribution

~95% of signals comes from axial U(1) breaking effect.

7. Summary

Chiral susceptibility is dominate by U(1) anomaly at T>=190MeV.
Conn. part ~ $U(1)_A$ susceptibility
Discon. part ~ top. susceptibility

Numerical simultions were performed on IBM System Blue Gene Solution at KEK under a support of its Large Scale Simulation Program (No. 16/17-14) and Oakforest-PACS at JCAHPC under a support of the HPCI System Research Projects (Project IDs: hp170061, hp180061, hp190090, and hp200086), Multidisciplinary Cooperative Research Program in CCS, University of Tsukuba (Project IDs: xg17i032 and xg18i023) and K computer provided by the RIKEN Center for Computational Science. We used Japan Lattice Data Grid (JLDG) for storing a part of the numerical data generated for this work.
This work is supported in part by the Japanese Grant-in-Aid for Scientific Research (No. JP26247043, JP16H03978, JP18H01216, JP18H03710, JP18H04484, JP18H05236), and by MEXT as “Priority Issue on Post-K computer” (Elucidation of the Fundamental Laws and Evolution of the Universe) and by Joint Institute for Computational Fundamental Science (JICFuS).