

# Finite Temperature QCD with Physical (u/d, s, c) Domain-Wall Quarks

Ting-Wai Chiu (趙挺偉)



Academia Sinica



National Taiwan Normal University



National Taiwan University

In collaboration with Yu-Chih Chen and Tung-Han Hsieh  
(for the TWQCD collaboration)

Lattice 2021, July 26-30, 2021

# Outline

- Introduction
- Actions and algorithms
- Gauge ensembles
- Hadron masses on the  $64^4$  lattice with a  $\sim 0.064$  fm
- Topological charge via the Wilson flow
- Topological susceptibility for  $T \sim 155\text{--}516$  MeV
- Remarks

# Introduction

- The topologically susceptibility  $\chi_t = \lim_{V_4 \rightarrow \infty} \langle Q_t^2 \rangle / V_4$  is a vital quantity to measure the quantum fluctuations of the topology of **QCD** vacuum. For  $T < T_c$ ,  $\chi_t$  is related to chiral condensate  $\Sigma = -\langle \bar{\psi}\psi \rangle$ , which is the origin of spontaneous chiral symmetry breaking, giving the majority of the visible (non-dark) mass in the universe today.
- If the Pecci-Quinn mechanism is realized in Nature, the evolution of  $\chi_t(T)$  from the early universe to  $T_\gamma$  determines the axion relic energy density, a promising candidate of the dark matter.  
axion mass:  $m_A(T) = \sqrt{\chi_t(T)} / f_A$ ,  $f_A$ : the scale of breaking the  $U(1)_{PQ}$  sym.
- Compute  $\chi_t(a, T)$  of 15 ensembles with physical ( $u/d, s, c$ ) DW quarks, for  $T \approx 155 - 516$  MeV and 3 lattice spacings (0.064, 0.068, 0.075) fm, to extract  $\chi_t(T)$  in the continuum limit.

# Actions and Algorithms

- Quarks: optimal DWF [TWC, PRL 2003] with  $N_s = 16$ ,  $\lambda_{\max} / \lambda_{\min} = 6.20 / 0.05$ .  
Gluons: plaquette gauge action at  $\beta = 6 / g^2 = (6.20, 6.18, 6.15)$
- For the one-flavor, use the Exact One-Flavor pseudofermion Action (EOFA)  
[Y.C. Chen & TWC, Phys. Lett. B738 (2014) 55; TWC, Phys. Lett. B744 (2015) 95]
- For the 2-flavor, use the two-flavor action for DWF.  
[TWC, T.H. Hsieh, Y.Y. Mao, Phys. Lett. B702 (2012) 131]

# Optimal Domain-Wall Fermion

VOLUME 90, NUMBER 7

PHYSICAL REVIEW LETTERS

week ending  
21 FEBRUARY 2003

## Optimal Lattice Domain-Wall Fermions

Ting-Wai Chiu\*

*Department of Physics, University of Washington, Seattle, Washington 98195-1560*

*Department of Physics, National Taiwan University, Taipei, Taiwan 106, Taiwan*

(Received 30 October 2002; published 19 February 2003)

I show that the conventional formulations of lattice domain-wall fermion with any finite  $N_s$  (in the fifth dimension) do not preserve the chiral symmetry optimally and propose a new action which preserves the chiral symmetry optimally for any finite  $N_s$ .

$$\omega_s = \frac{1}{\lambda_{\min}} \sqrt{1 - \kappa'^2 \operatorname{sn}^2(v_s; \kappa')}, \quad s = 1, \dots, N_s$$

Effective 4D Dirac op. of the optimal DWF is exactly equal to the Zolotarev optimal rational approx. of the overlap Dirac op.

Optimal DWF with  $R_5$  Symmetry [ TWC, Phys. Lett. B 744 (2015) 95-100 ]

$$N_s = 2n \text{ (even)} \quad \omega_s = \omega_{N_s+1-s} = \frac{1}{\lambda_{\min}} \sqrt{1 - \kappa'^2 \operatorname{sn}^2\left(\frac{(2s-1)K'}{N_s}; \kappa'\right)}$$

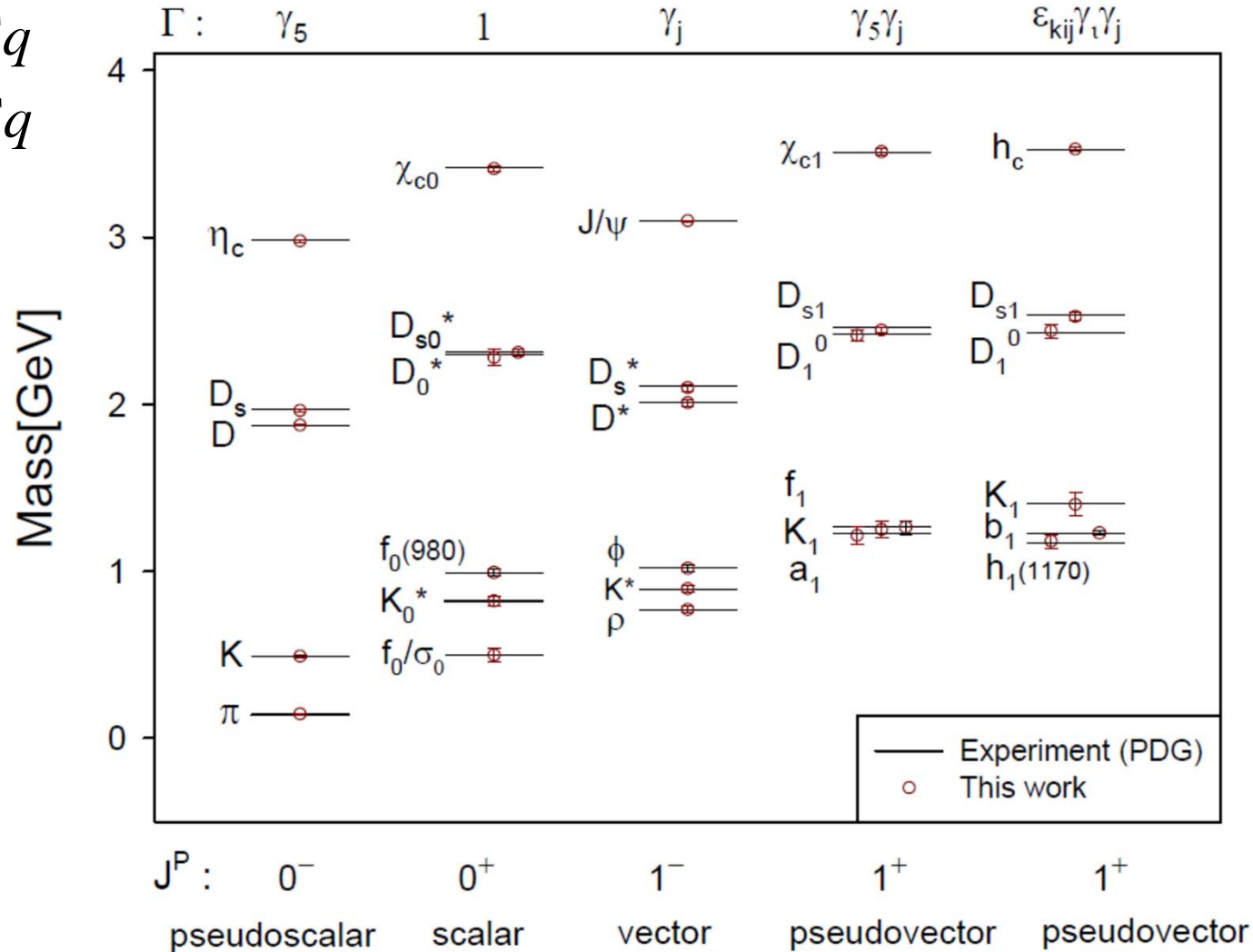
# Gauge Ensembles

- Lattice sizes :  $64^3 \times (64, 20, 16, 12, 10, 8, 6)$
- Lattice spacings :  $a \simeq (0.064, 0.068, 0.075)$  fm,  
 $a^{-1} \approx (3.1, 2.9, 2.6)$  GeV
- Spatial volume :  $L^3 > (4 \text{ fm})^3$ ,  $M_\pi L > 3$
- Temperatures :  $\sim 0 - 520$  MeV
- Statistics :  $\sim 500 - 3000$  configurations per ensemble
- The lattice spacings are determined by the Wilson flow,  
using  $t^2 \langle E \rangle \Big|_{t=t_0} = 0.3$  with  $\sqrt{t_0} = 0.1416(8)$  fm.
- The masses of  $s$  and  $c$  quarks are fixed by the masses of  $\phi(1020)$  and  $J/\psi(3097)$  respectively, while the mass of  $u/d$  quarks by  $M_\pi(140)$ , on the  $64^4$  lattices.
- The HMC simulations are performed on 10-20 units of Nvidia DGX-1 at 3 institutions/organizations in Taiwan, since January 2018.

# The Spectrum of the Lowest-lying Mesons

$64^4$ ,  $\beta = 6.20$ ,  $a = 0.064$  fm, 400 confs      TWC, arXiv:2020.06126

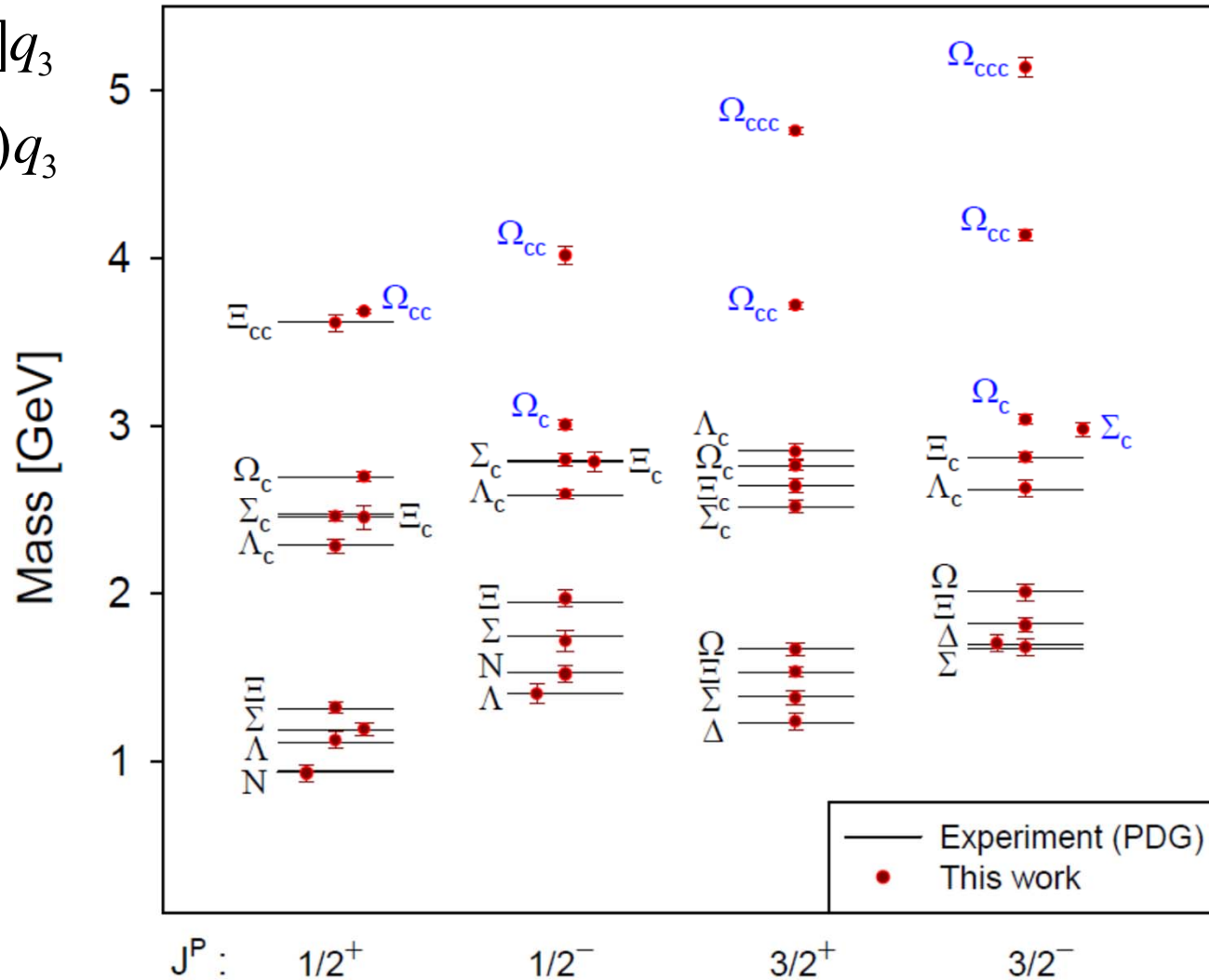
$\bar{Q}\Gamma q$   
 $\bar{q}\Gamma q$



# The Spectrum of the Lowest-lying Baryons

$64^4$ ,  $\beta = 6.20$ ,  $a = 0.064$  fm, 400 confs TWC, arXiv:2020.06126

$[q_1 C \Gamma q_2] q_3$   
 $(q_1 C \Gamma q_2) q_3$





# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

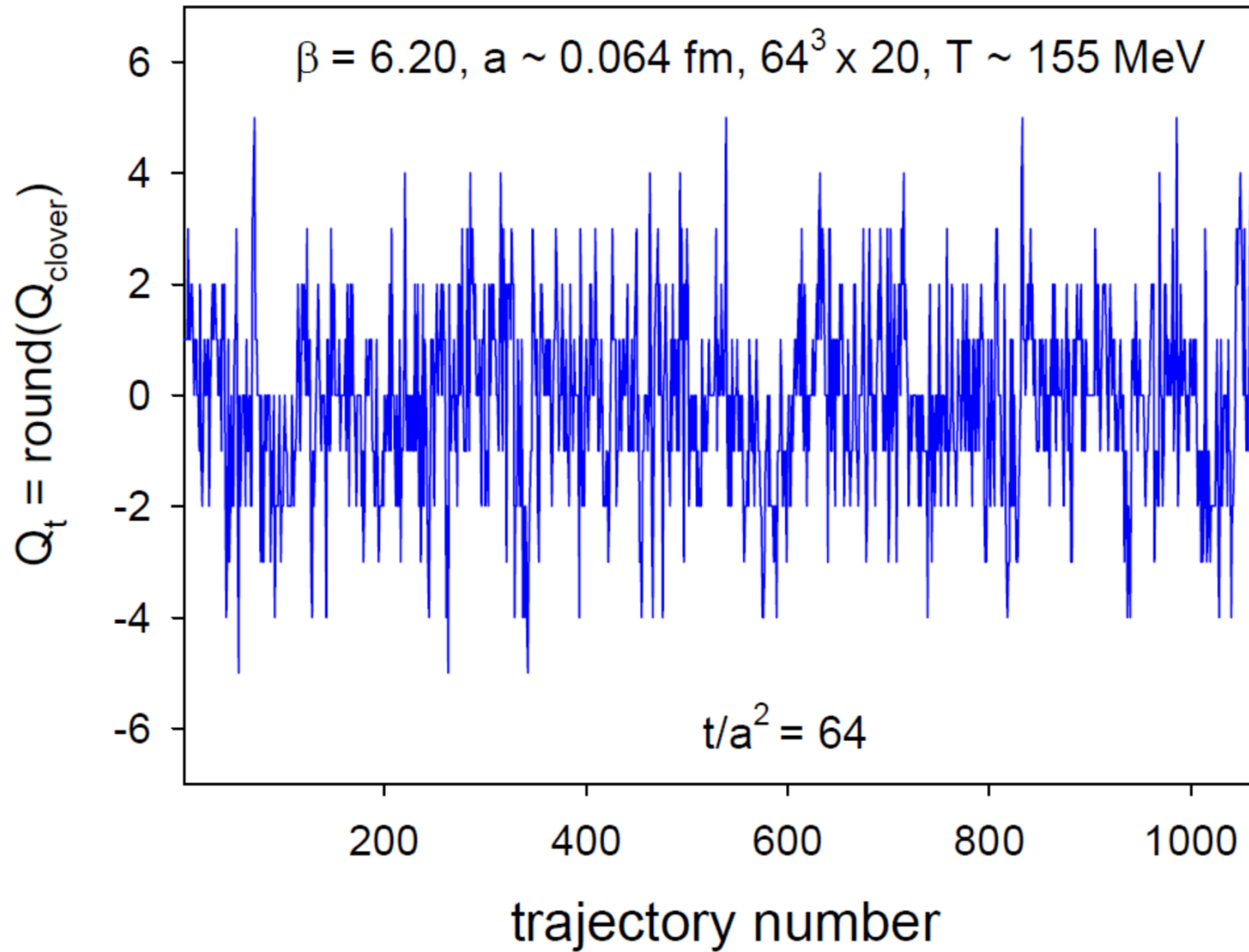
## 15 Ensembles

$\beta$	$a$ [fm]	$N_x$	$N_t$	$T = (N_t a)^{-1}$ [MeV]	# confs
6.20	0.064	64	6	516	3038
6.20	0.064	64	8	387	2665
6.20	0.064	64	10	310	2547
6.20	0.064	64	12	258	1684
6.20	0.064	64	16	193	2230
6.20	0.064	64	20	155	2491
6.18	0.068	64	6	479	2908
6.18	0.068	64	8	360	2088
6.18	0.068	64	10	288	1629
6.18	0.068	64	12	240	649
6.18	0.068	64	16	180	565
6.15	0.075	64	6	438	2454
6.15	0.075	64	8	329	2109
6.15	0.075	64	10	263	831
6.15	0.075	64	12	219	811

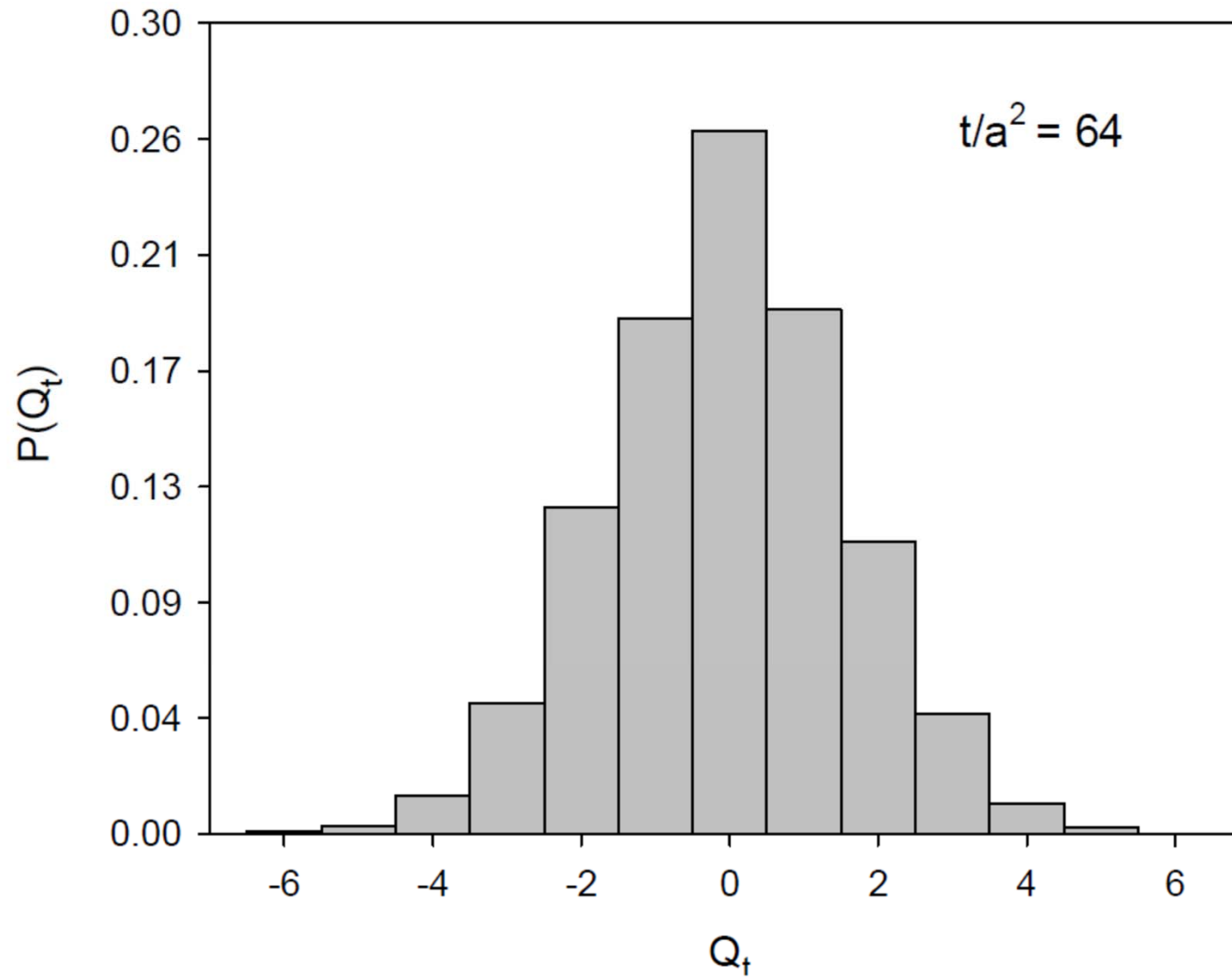
# Topological Charge and Topological Susceptibility

- The topological charge  $Q_t$  of each configuration is measured via the Wilson flow, using the clover definition.
- The Wilson flow equation is integrated from  $t / a^2 = 0$  to 256 with the step size  $\Delta t / a^2 = 0.01$ .
- In order to extrapolate  $\chi_t = \langle Q_t^2 \rangle / V_{4D}$  to the continuum limit,  $Q_t$  is required to be measured at the same  $t$  (in physical units) for all ensembles, which is chosen to be  $t = 0.8192 \text{ fm}^2$  in this study.

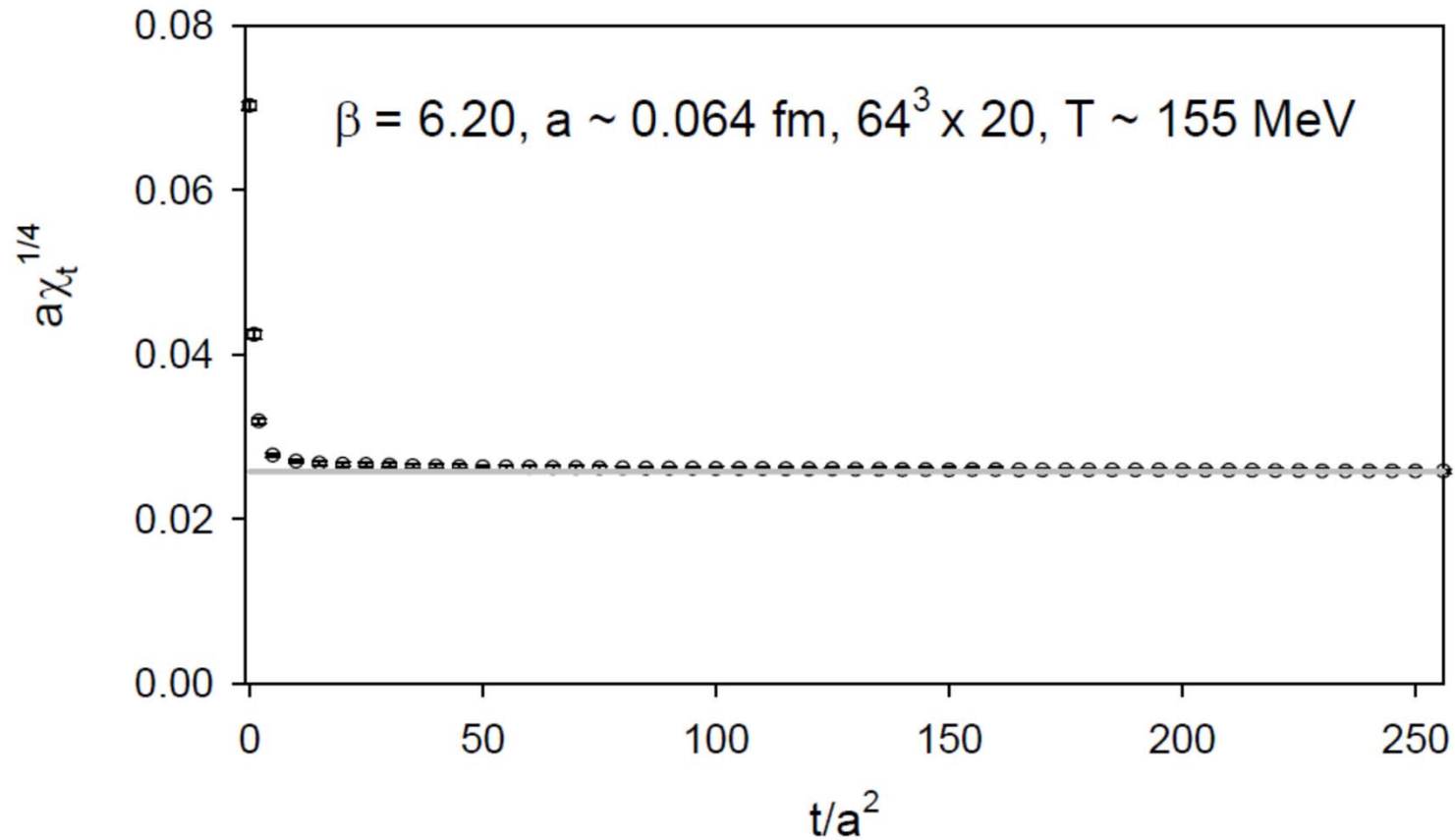
# Evolution of $Q_t$ in HMC at $T=155$ MeV



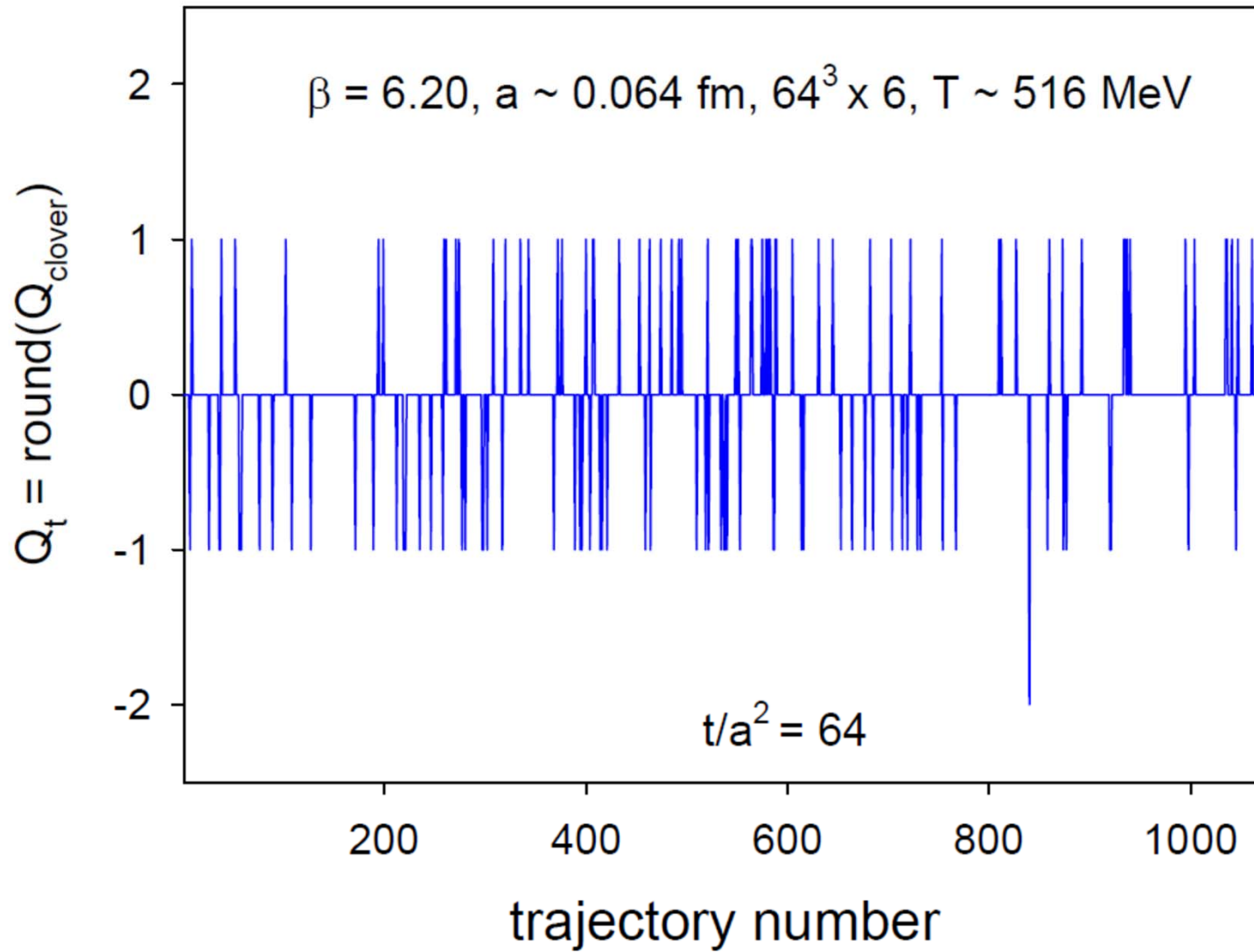
# Histogram of $Q_t$ at $T=155$ MeV



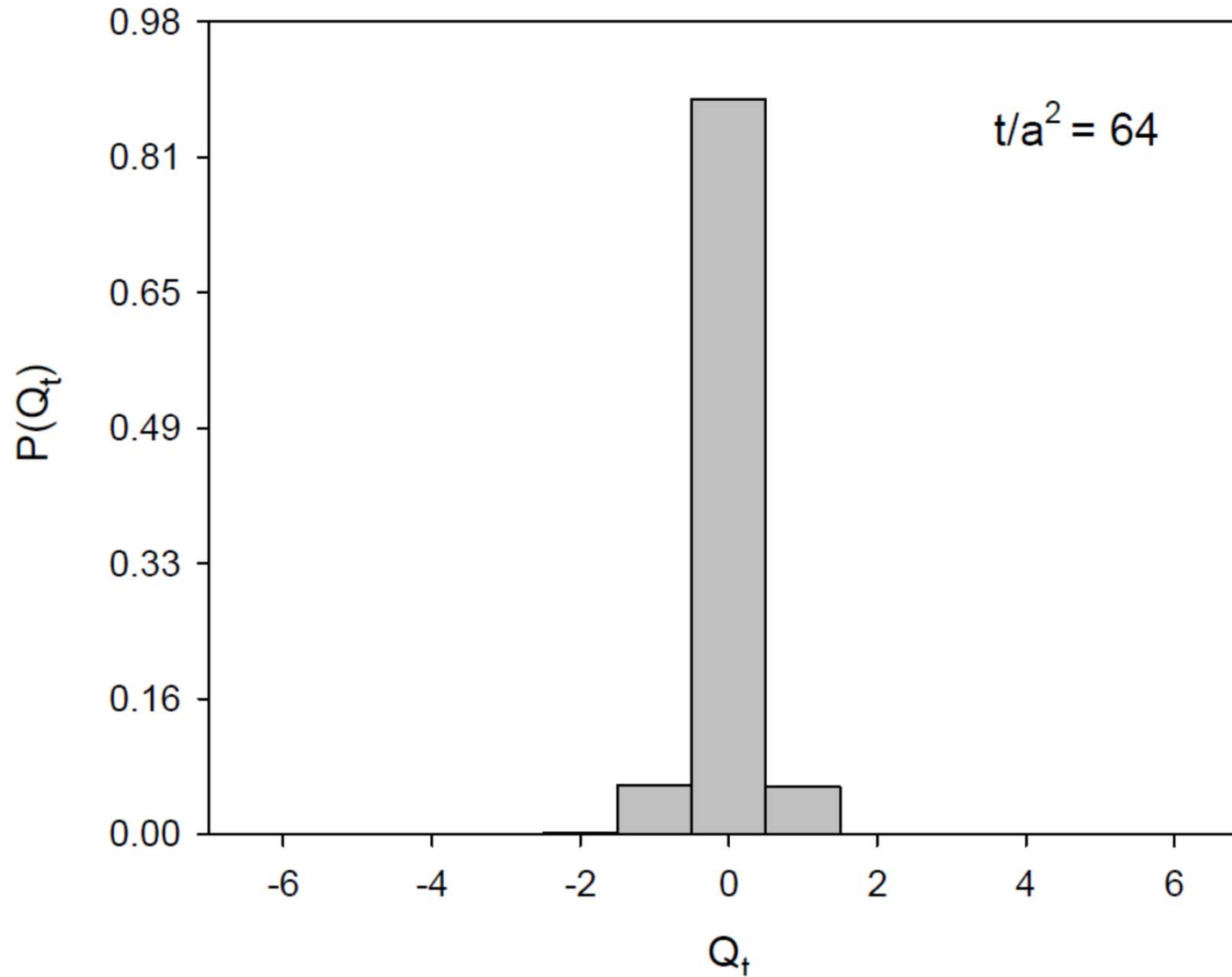
# Topological Susceptibility at $T=155$ MeV



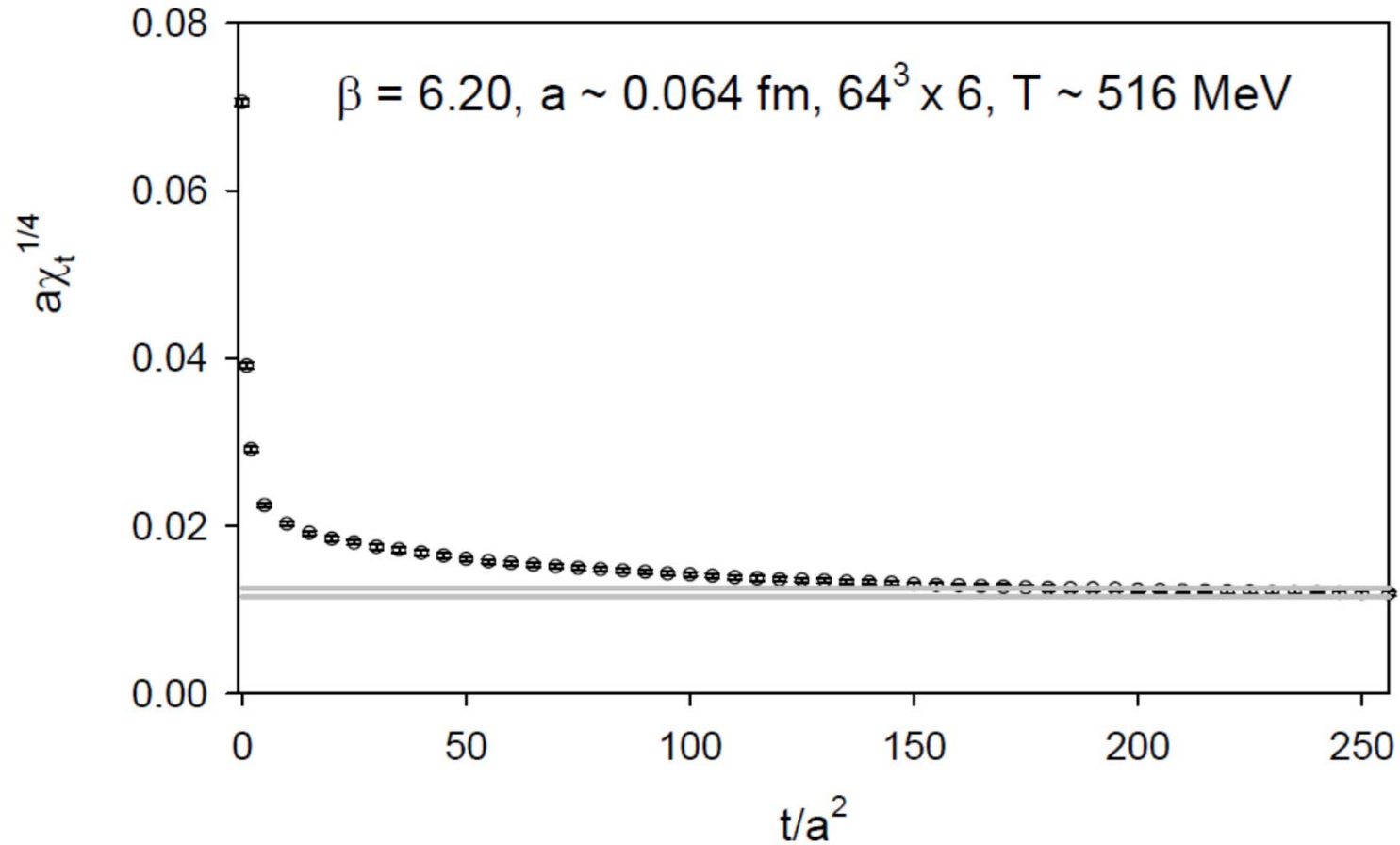
# Evolution of $Q_t$ in HMC at $T=516$ MeV



# Histogram of $Q_t$ at $T=516$ MeV



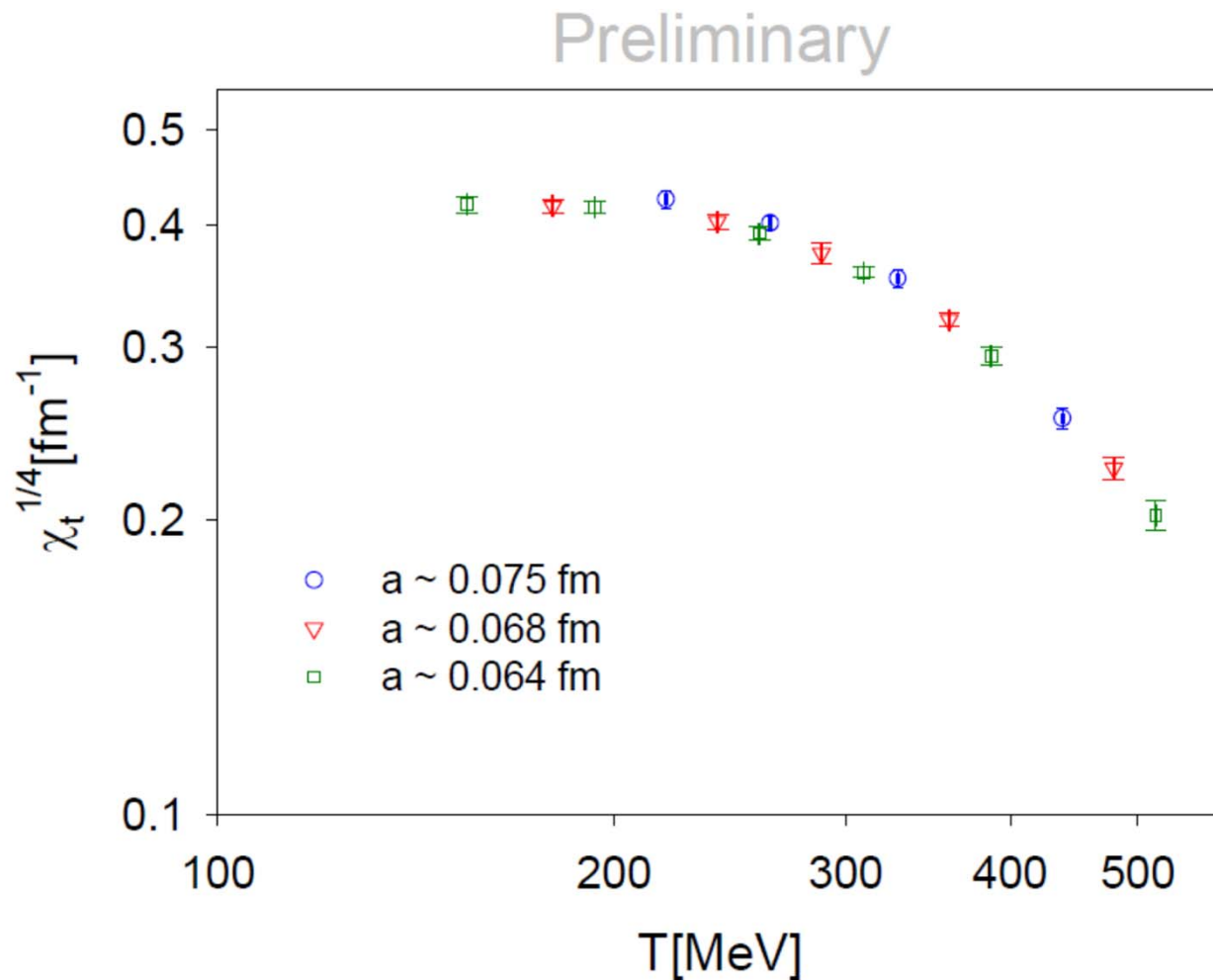
# Topological Susceptibility at $T=516$ MeV





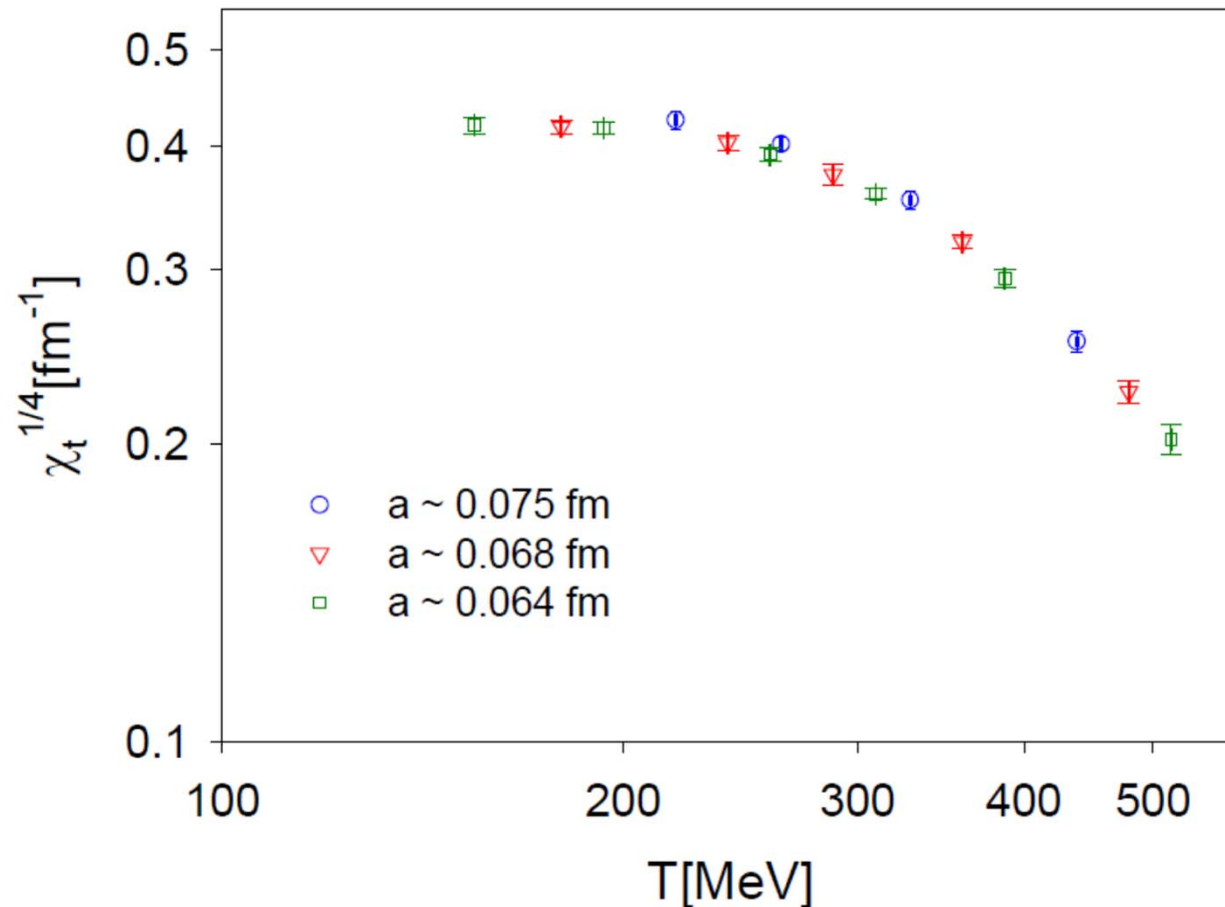
# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

How to use the data points of  $\chi_t(a, T)$  to extract  $\chi_t(T)$  in the continuum limit ?



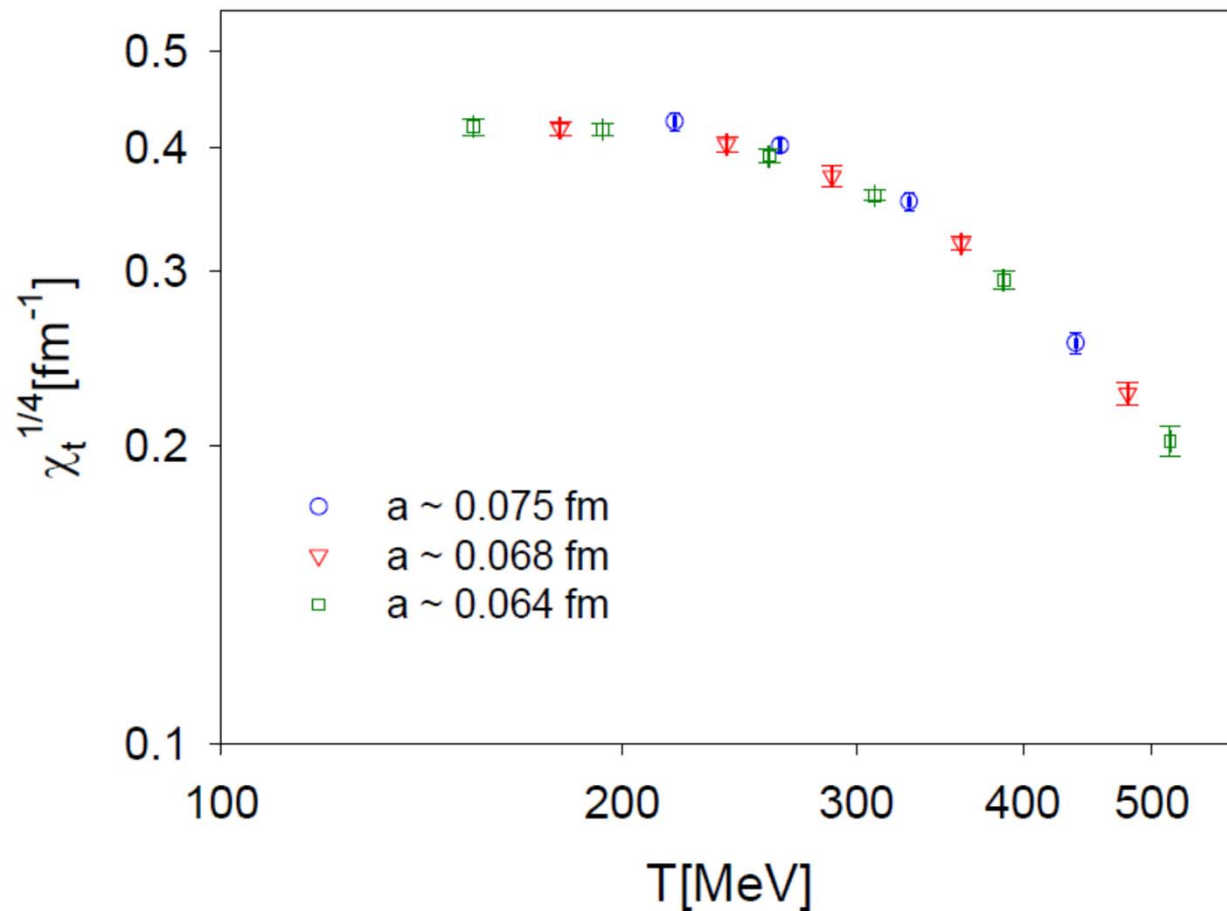
# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

power law  $\chi_t^{1/4}(a, T) = (c_0 + c_1 a^2) \left( \frac{T_c}{T} \right)^p$  can only fit data points at high  $T$



# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

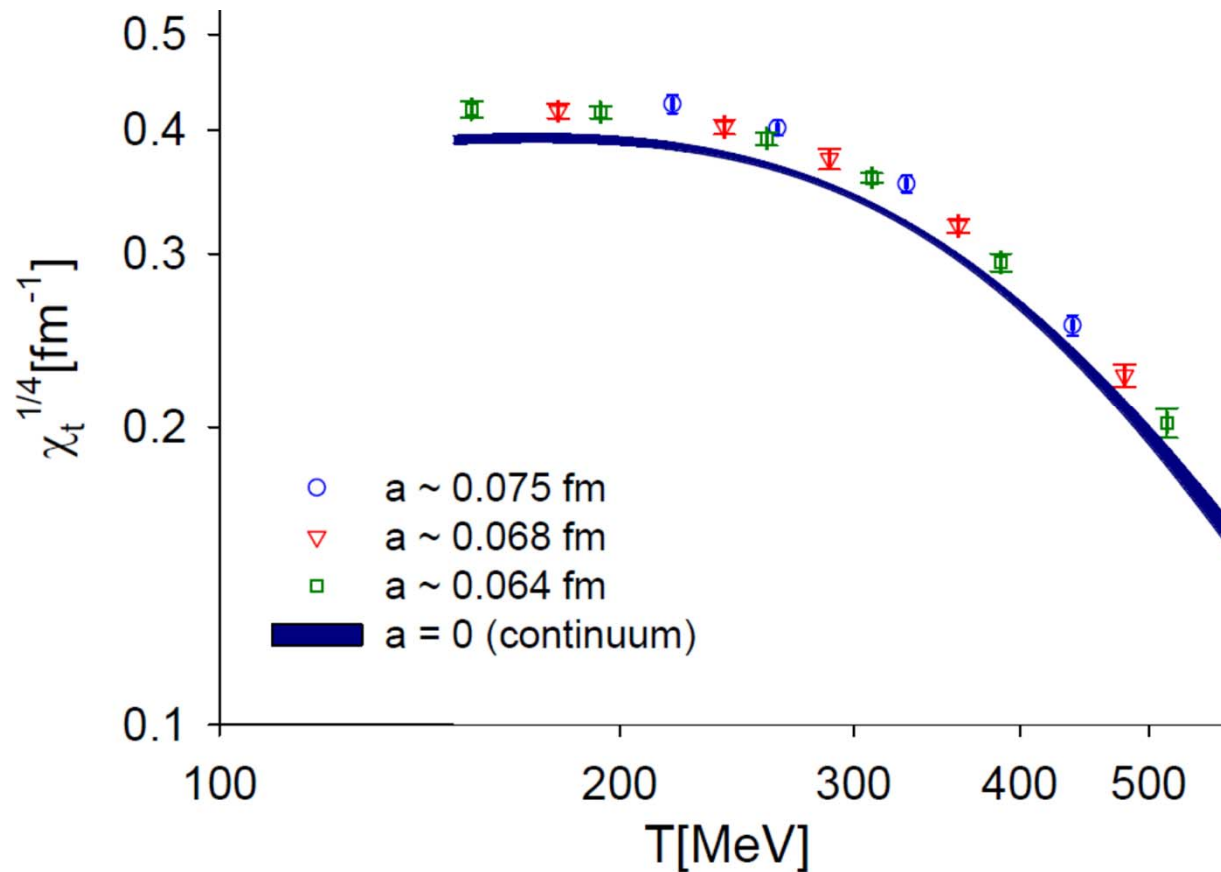
Fitting to  $\chi_t^{1/4}(a, T) = (c_0 + c_1 a^2) \frac{x^p}{1 + bx + cx^2}$ ,  $x = \frac{T_c}{T}$ ,  $T_c = 155$  MeV



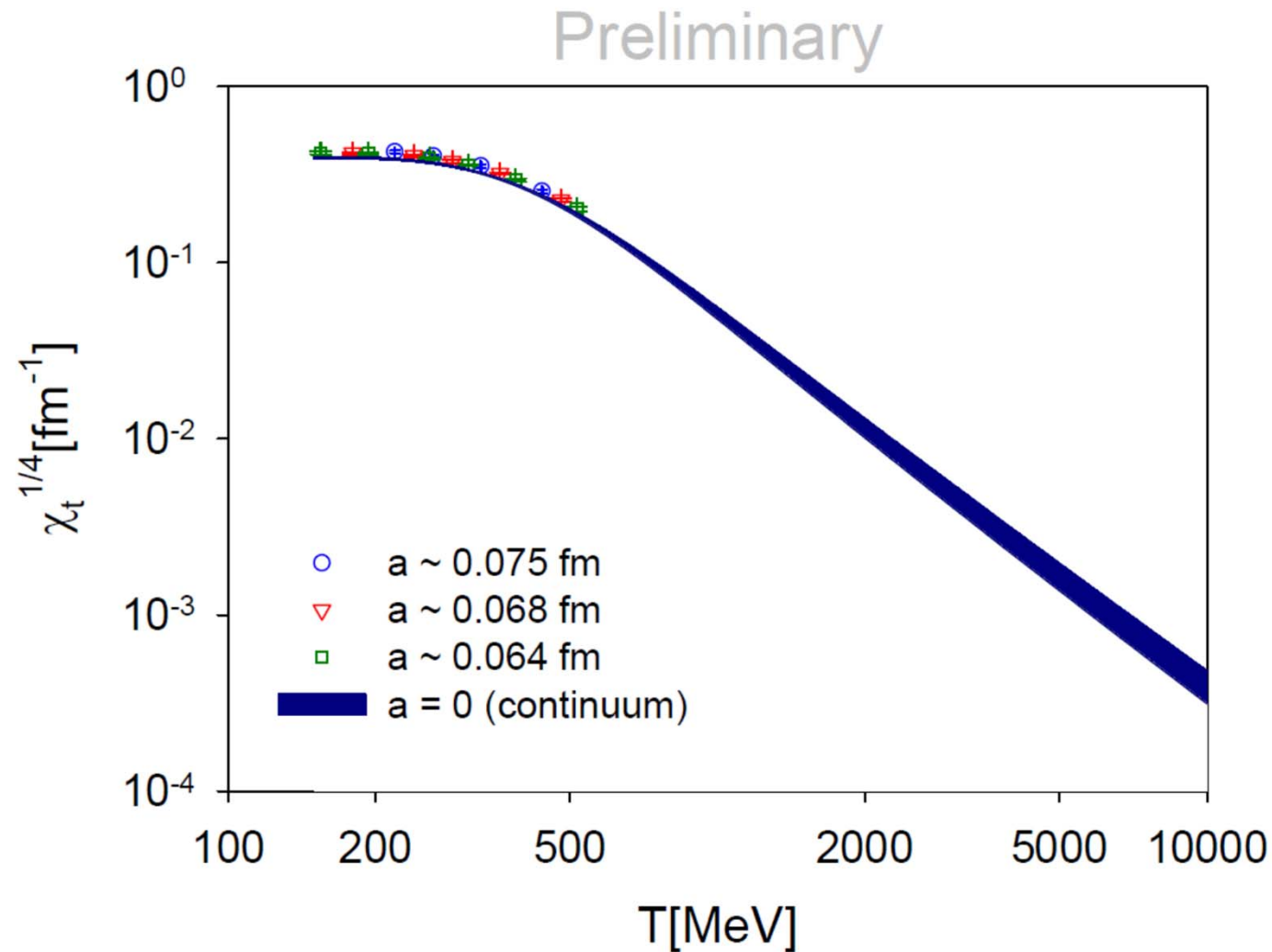
# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

Fitting to  $\chi_t^{1/4}(a, T) = (c_0 + c_1 a^2) \frac{x^p}{1 + bx + cx^2}$ ,  $x = \frac{T_c}{T}$ ,  $T_c = 155$  MeV

$c_0 = 1.89(3)$ ,  $c_1 = 32.2(6.8)$ ,  $p = 2.03(5)$ ,  $b = -2.42(19)$ ,  $c = 6.25(14)$

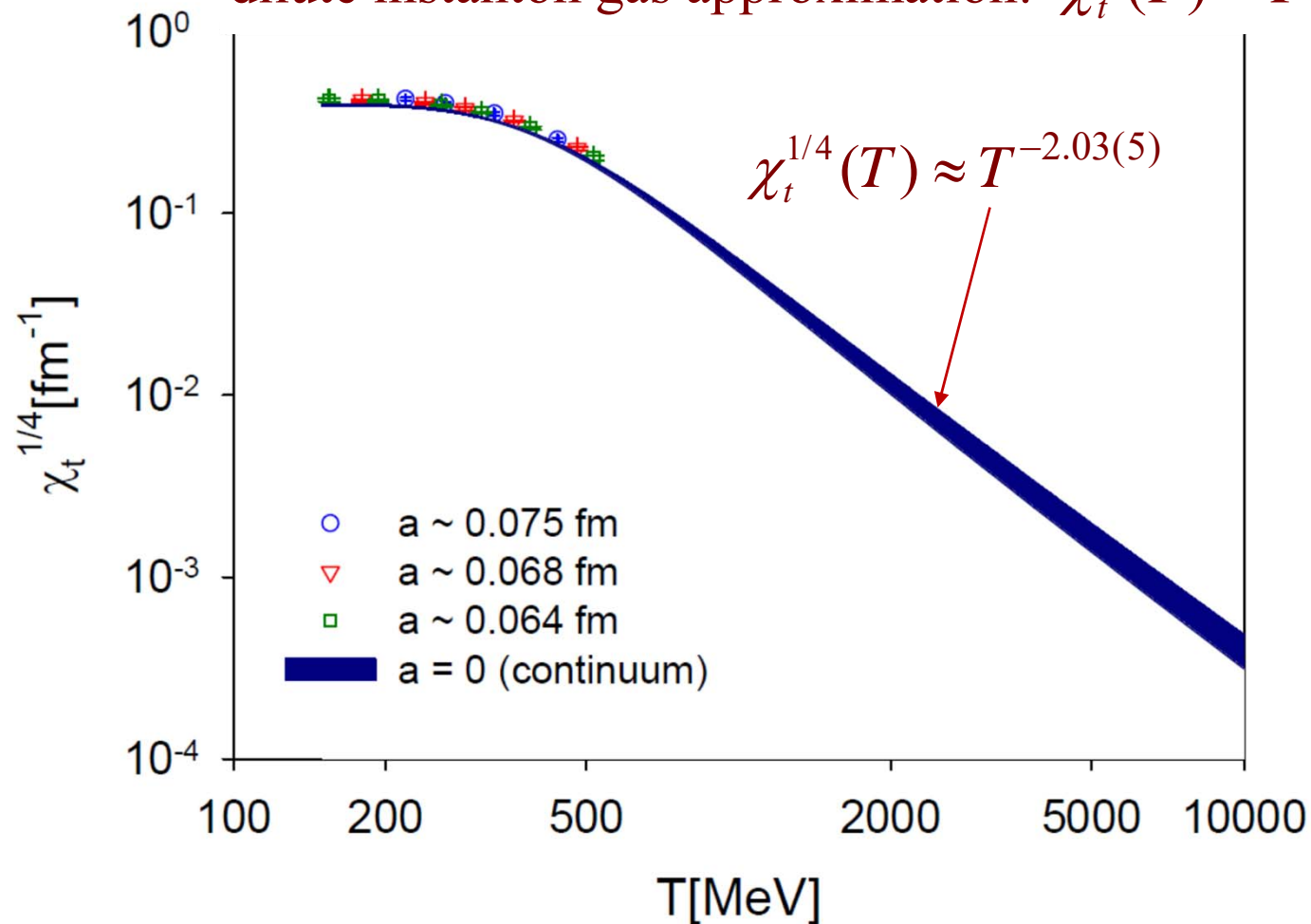


# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

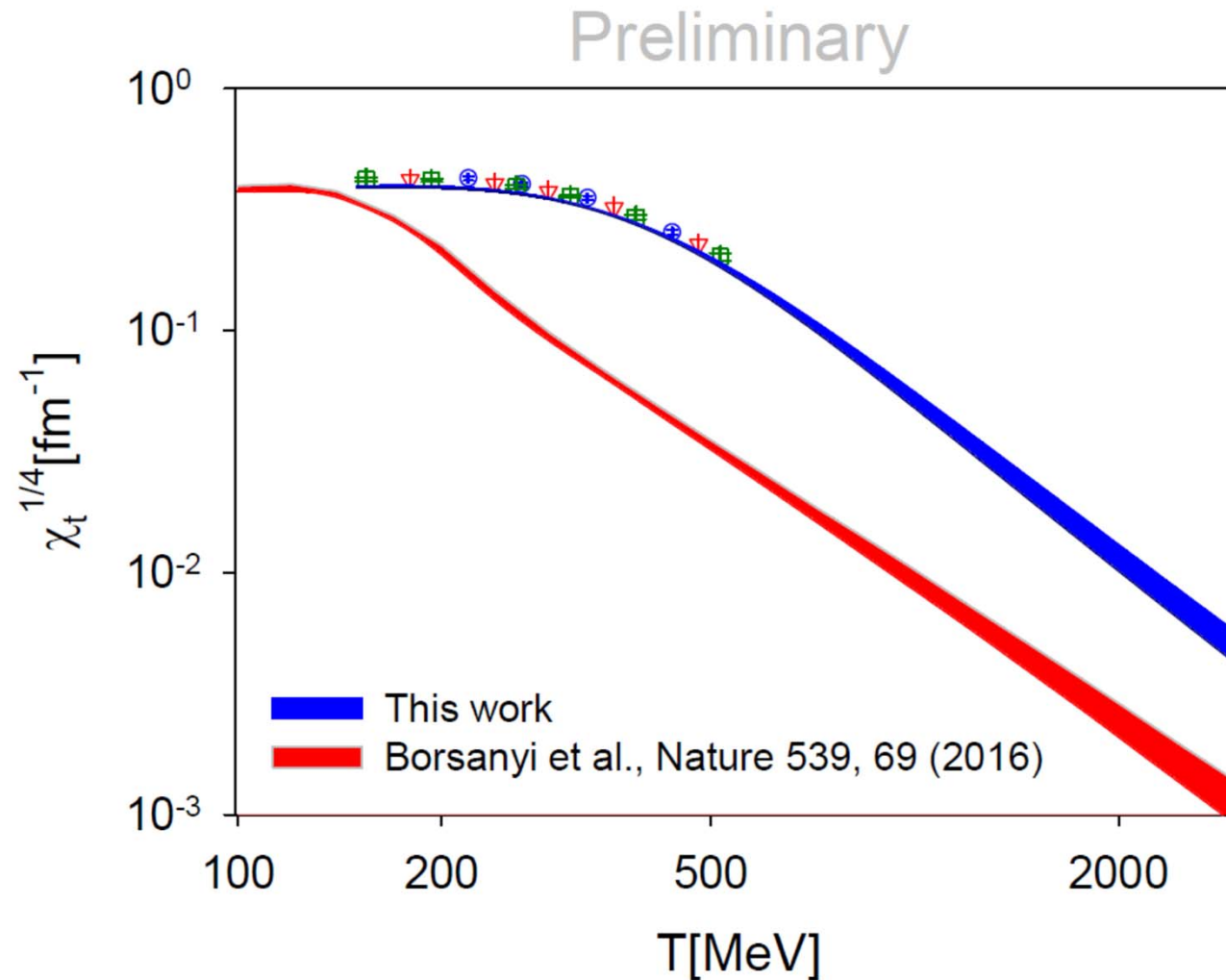


# Topological susceptibility $\chi_t(T)$ of lattice QCD with physical (u/d, s, c) domain-wall quarks

For  $T \gg T_c$ , its  $T$  dependence agrees with dilute instanton gas approximation:  $\chi_t(T) \approx T^{-8}$



# Comparison with other lattice QCD results with physical (u/d, s, c) quarks



# Remarks

What would be the causes for the discrepancy between this work and Borsanyi et al., Nature 539, 69 (2016) ?

- Different lattice fermions ?

But, presumably, in the continuum limit, they should give consistent results, if they are both in the same universality class of Dirac fermion.

- Borsanyi et al. used the eigenvalue reweighting and the fixed sector integral techniques rather than direct simulations.

It is unclear to what extent these techniques are valid ?

- To clarify these issues, independent studies using the DWF are needed, as well as direct simulations with the staggered fermion.

**Thank you for your attention !**



# Acknowledgement



Academia Sinica



National Taiwan Normal University

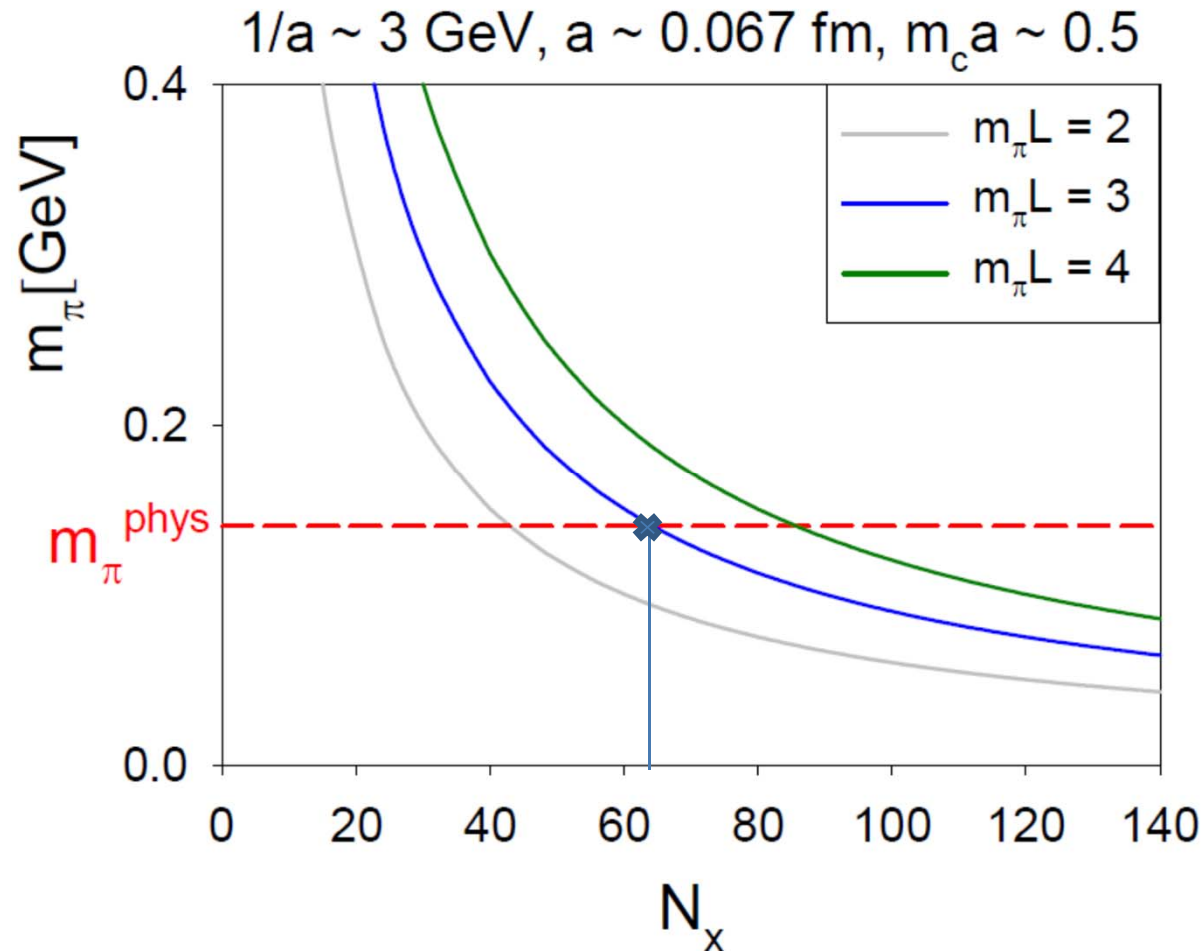


National Taiwan University

# Backup Slides

# Design lattice QCD with physical (u,d,s,c) quarks

TWC, arXiv:1811.08095



For the  $64^3 \times 64$  lattice,  $M_\pi L \approx 3$ ,  $M_\pi \approx 140 \text{ MeV}$ ,  $L \approx 4.3 \text{ fm}$

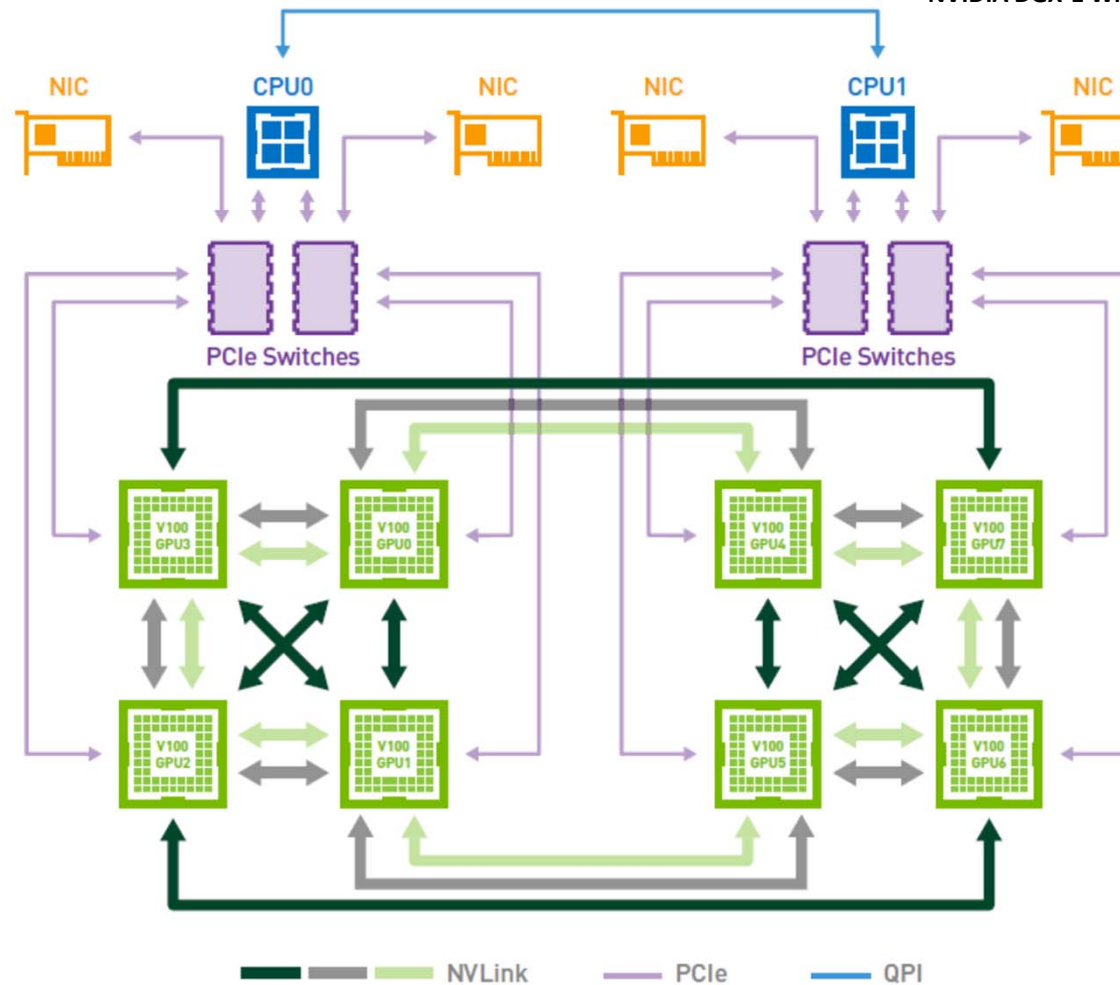
# Nvidia DGX-1



- Jan 2018 – Oct 2018, the **DGX-1 in the Nvidia Taipei office** is used for the initial thermalization of the  $64^4$  lattice at  $\beta = 6.20$  with  $a=0.064$  fm.
- Since Dec 2018, **4 units of DGX-1 (HP clone)** at Academia Sinica Grid Computing are used for production runs.
- From 2019-2020, used 10-20 units of **DGX-1 in Taiwan 2 at NCHC** for production runs.

# Nvidia DGX-1(8 V100+NVLink)

Figure taken from the White Paper  
NVIDIA DGX-1 With Tesla V100 System Architecture



NVLink 2.0, data rate ~ 300 GB/s

## 臺灣杉二號 ( TAIWANIA 2 )

2,016 NVIDIA Tesla V100 32GB GPU = 252 units of DGX-1



TOP500 #28 (9 PFLOPS) Green500 #19 (11.285 GFLOPS/W)

# Topological Susceptibility of Lattice QCD with Physical (u/d, s, c) DW quarks at zero temperature

TWC, arXiv:2002.06126

