Nonperturbative excitations in overoccupied gluon plasmas

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July 29, 2021

Talk mainly based on:

with Kurkela, Lappi, Mace, Peuron, Schlichting,
PRD 98, 014006 (2018), [1804.01966]
JHEP 05, 225 (2021), [2101.02715]
arXiv:2106.11319

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Motivation

The goal is to study

Microscopic properties of QCD out of equilibrium from first principles

Spectral functions $\rho(\omega, p)$ of gluons / quarks encode excitation spectrum!

- Our approach: *far from equilibrium, classical real-time lattice*
  - $\star$ Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)

  $\Rightarrow$ Then nonperturbative and perturbative methods available!

- Classical-statistical lattice simulations vs. HTL, kinetic theory

$$\text{class. covariant equ.: } D_\mu F^{\mu\nu} = 0, \qquad \text{HTL: } \Pi_{\mu\nu}^{\mathrm{HTL}}$$

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  class. covariant equ.: $D_\mu F^{\mu\nu} = 0$, HTL: $\Pi^{\mathrm{HTL}}_{\mu\nu}$
- Applications:
  - ⋆ Heavy-ion collisions at early times
  - ⋆ Comparison to thermal properties (general features?)
  - ⋆ Nonperturbative extensions of kinetic theory, anisotropic HTL

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Considered models and initial conditions

- $\mathrm{SU}(N_c)$ Yang-Mills theory (simulations: $N_c = 2$)

$$S_{\mathrm{YM}}[A] = -\frac{1}{4} \int d^{d+1}x \; F_a^{\mu\nu} F_{\mu\nu}^a$$

(in gauge-covariant formulation, with links $U_j \approx \exp\left(ig\, a_s A_j\right)$)

- Models of initial stages (here non-expanding geometry)
  1. 3+1D: isotropic 3+1D
  2. 2+1D
  3. Glasma-like 2+1D: add adjoint scalar ϕ to model 1

- Initial conditions (with $E = \partial_0 A$): highly occupied

$$f(t{=}0, p \lesssim Q) \sim \frac{1}{g^2} \gg 1 \quad \text{with} \quad f(t,p) \propto \frac{\langle |E_T(t,p)|^2 \rangle}{p}$$

Classical-statistical lattice simulations

1. Set initial conditions

$$\langle E_T^*(t_0,\vec{p})E_T(t_0,\vec{q})\rangle \propto p\, f(t_0,p)\,\delta_{jk}(2\pi)^3\delta(\vec{p}-\vec{q})$$

with $E_T^j p_j = 0$, initially $\langle E_L^*(t_0,\vec{p})E_L(t_0,\vec{q})\rangle = 0$, same for A

2. Restore Gauss law (algorithm: G.D. Moore, Nucl. Phys. B 480, 657 (1996))

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2. Restore Gauss law (algorithm: G.D. Moore, Nucl. Phys. B 480, 657 (1996))
3. Solve classical field equations on the lattice

$$\begin{aligned} U_j(t+\mathrm{d}t/2,\vec{x}) &= e^{i\mathrm{d}t\, a_s g E_a^j(t,\vec{x})} U_j(t-\mathrm{d}t/2,\vec{x}) \\ gE_a^i(t+\mathrm{d}t,\vec{x}) &= gE_a^i(t,\vec{x}) - \frac{\mathrm{d}t}{a_s^3}\sum_{j\neq i}\left[U_{ij}\left(t-\frac{\mathrm{d}t}{2},\vec{x}\right)+U_{i(-j)}\left(t-\frac{\mathrm{d}t}{2},\vec{x}\right)\right]_{\mathrm{ah}} \end{aligned}$$

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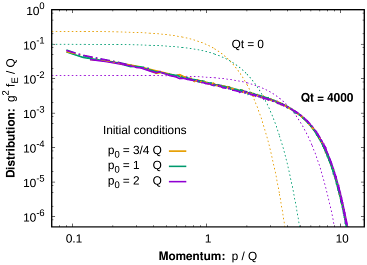
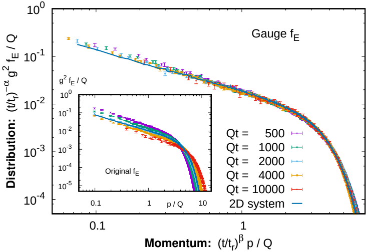
4. Evolve each initial configuration $\{U(t_0,\vec{x}), E(t_0,\vec{x})\}$ until $t > t_0$
5. Compute observable $O[U,E]$ that depends on the fields

$$O(t) = \frac{1}{\#k}\sum_k O[U(t),E(t)]$$

Self-similar turbulent attractors

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)





- The highly occupied systems approach self-similar attractors

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal exponents insensitive to details of initial conditions
  - ✓ 2+1D: $\beta = -1/5,\ \alpha = 3\beta,$ KB, Kurkela, Lappi, Peuron (2019)
  - ✓ 3+1D: $\beta = -1/7,\ \alpha = 4\beta,$ Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); ...
- We extract $\rho(t,\omega,p)$ of such typical states, we know their evolution!

Spectral and statistical correlation functions

- Spectral function ($\dot{\rho} = \partial_t \rho,\ E = \partial_t A$)

$$\rho(x', x) = \frac{i}{N_c^2 - 1} \left\langle \left[\hat{A}(x'), \hat{A}(x) \right] \right\rangle$$

- Statistical correlator $\langle EE \rangle$ ($\equiv \ddot{F}$), in general independent of $\dot{\rho}$

$$\langle EE \rangle(x', x) = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x'), \hat{E}(x) \right\} \right\rangle$$

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- Fix gauge: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j \big|_t = 0$ at time t
- Fourier transf. in $t' - t$ and $\vec{x}' - \vec{x}$ to frequency ω and momentum $\vec{p}$

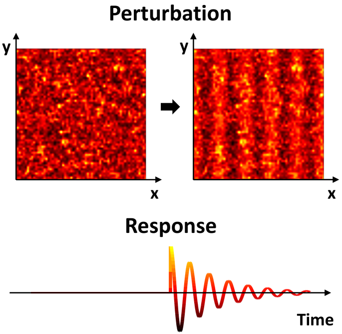
  Approximation: normally at fixed $\bar{t} = \frac{1}{2}(t + t')$, we hold $t \approx \bar{t}$
- In classical-statistical simulations

$$\langle EE \rangle(t', t, p) = \frac{1}{N_c^2 - 1} \left\langle E(t', \vec{p}) E^*(t, \vec{p}) \right\rangle$$

Nonperturbative computation of spectral function ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*, Editors' suggestion



- Split $A(t,\vec{x}) \mapsto A(t,\vec{x}) + \delta A(t,\vec{x})$, perturb $j(t',\vec{p}) = j_0(\vec{p})\,\delta\,(t'-t)$ at t
- Response $\langle \delta A(t',\vec{p}) \rangle = G_R(t',t,\vec{p})\, j_0(\vec{p})$
- Linearized EOM for $\delta A(t,\vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t'-t)\ \boxed{\rho(t',t,p)} = G_R(t',t,p)$

Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020); ...

NEW method for fermions: KB, Lappi, Mace, Schlichting, arXiv:2106.11319

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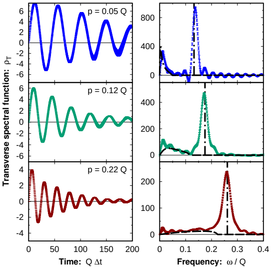
Gluon spectral function in isotropic 3+1D plasmas

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

$\rho_T(\omega, p)$



- Narrow Lorentzian q.p. peaks (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

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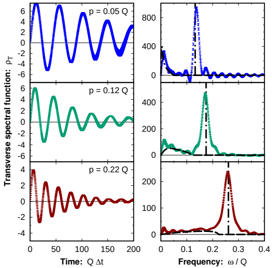
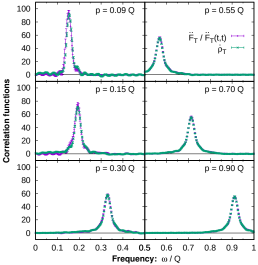


$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



- Narrow Lorentzian q.p. peaks (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
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- Generalized fluctuation dissipation relation (FDR)

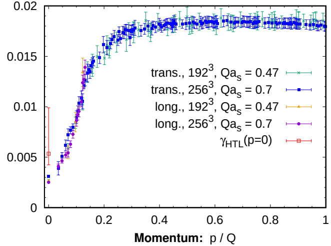
$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t{=}0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t{=}0, p)}$$

$\ddot{F} \equiv \langle EE \rangle$, $\alpha = T, L$ polarizations

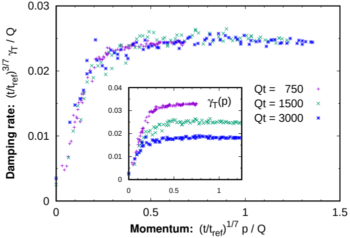
Damping rate in isotropic 3+1D plasmas

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

Width $\gamma_T(p)$

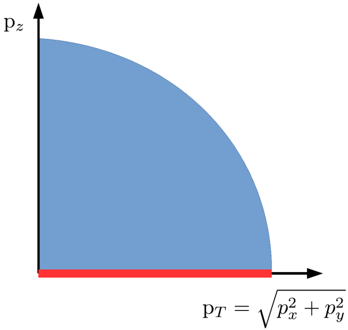


$\propto (Qt)^{3/7}\gamma_T(t,p)$



- Peak width $\gamma(t,p) \ll \omega(t,p)$ beyond HTL at LO
- First determination of full p dependence, finding $\gamma_T(t,p) \approx \gamma_L(t,p)$
  (vs. HTL prediction $\gamma(p{=}0)$, $\gamma(p{\to}\infty)$), Braaten, Pisarski (1990); Pisarski (1992))
- Use self-similar scaling $\Rightarrow \frac{\gamma_\alpha(t,p)}{\omega(t,p{=}0)} \sim (Qt)^{-2/7}$ decreases

3+1D → 2+1D

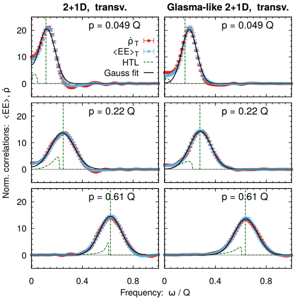


- Isotropic 3+1D: $f(t, p)$
- 2+1D: $f(t, p_T, p_z{=}0)$
- can be regarded as extreme momentum anisotropy

Vs. Gluon spectral function in 2+1D plasmas

KB, Kurkela, Lappi, Peuron, *JHEP 05, 225 (2021)*

$$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$$



✓ Generalized FDR, but:

- Broad non-Lorentzian peaks
- HTL curves (green) agree poorly
- Landau cut and q.p. peak not distinguishable

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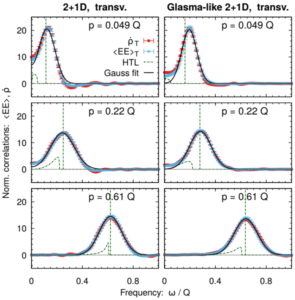
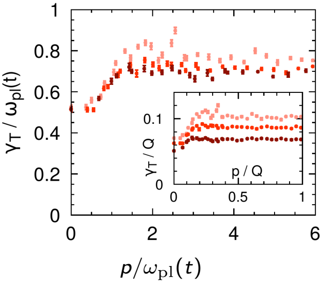


Width $\gamma_T(t,p)/\omega_{\rm pl}(t)$



✓ Generalized FDR, but:

- Broad non-Lorentzian peaks
- HTL curves (green) agree poorly
- Landau cut and q.p. peak not distinguishable

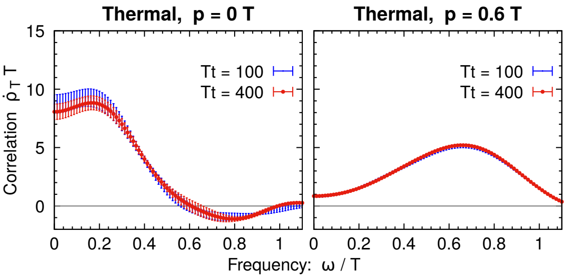
- Peak width $\gamma(t,p) \sim \omega_{\rm pl}(t)$

$(\omega_{\rm pl} \equiv \omega_T(p{=}0))$

$\Rightarrow$ $\boxed{\text{no quasiparticles for } p \lesssim \omega_{\rm pl}!}$

Gluon spectral function in 2+1D class. thermal equilibrium

KB, Kurkela, Lappi, Peuron, JHEP 05, 225 (2021)



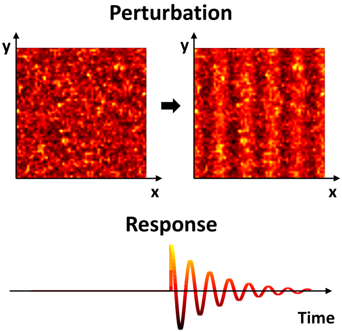
- Classical thermal equilibrium $f(p) \approx \frac{T}{\omega(p)}$
- $\rho(\omega, p)$ qualitatively similar as far from equilibrium
  - ✓ Broad gluonic excitations with $\gamma(p) \sim \omega_{\mathrm{pl}}$
  - ✓ HTL provides poor description
  - ✓ For $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low p
- Interpretion: Features seem generic in 2+1D gauge theories

Nonperturbative computation of **fermion** spectral function

Classical-statistical $\mathrm{SU}(N_c)$ simulations + Dirac equation

KB, Lappi, Mace, Schlichting, *arXiv:2106.11319*

$$\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$$



- Mode function expansion of ψ

  Aarts, Smit, *Nucl. Phys. B 555, 355 (1999)*

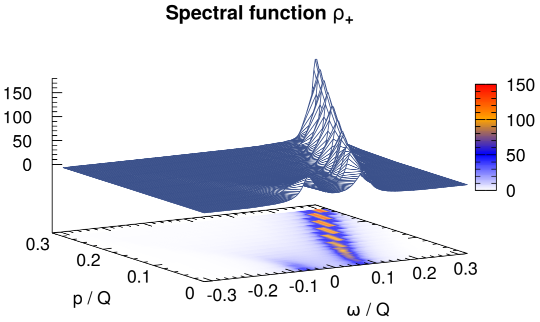
- At time t plane waves
- $t' > t$: solve Dirac & Yang-Mills EOM
- Compute fermion spectral function

  with spin & color indices $\lambda = 1, \cdots, 2N_c$

$$\rho^{\alpha\beta}(t', t, \vec{p}) =$$
$$\frac{1}{V} \sum_{\lambda} \left[\tilde{\phi}^{u,\alpha}_{\lambda,\vec{p}}(t', \vec{p}) u^{\dagger,\gamma}_{\lambda}(\vec{p}) + \tilde{\phi}^{v,\alpha}_{\lambda,-\vec{p}}(t', \vec{p}) v^{\dagger,\gamma}_{\lambda}(-\vec{p}) \right] \gamma_0^{\gamma\beta}$$

Fermion spectral function in isotropic 3+1D plasmas

KB, Lappi, Mace, Schlichting, arXiv:2106.11319



We find (see Backup or paper):

- Lorentzian quasiparticle peaks + Landau cut ($\omega < p$)
- very good description by HTL
- full p-dependence of $\gamma_+(t,p)$, where $\gamma_+(t,p{=}0) \sim m_F(t)$

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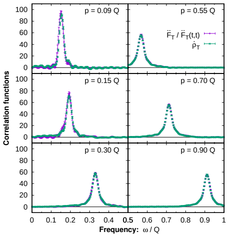
Conclusion

- We have developed tools to extract spectral functions in highly occupied plasmas
- Applied to classical self-similar attractors: gluonic 3+1D, gluonic 2+1D, fermionic 3+1D
- ρ's in 3+1D generally well described by HTL + first principles determination of damping rates
- Much broader peaks in gluonic 2+1D than in 3+1D
  $\Rightarrow$ anisotropic HTL may require nonperturbative contributions

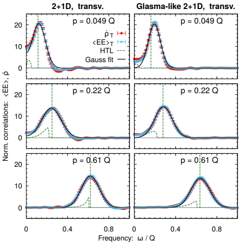
Outlook: applications to heavy-ion collisions

- ρ in Bjorken expanding systems
- Anisotropic / expanding kinetic theory, HTL
- Effects on transport coefficients $\Rightarrow$ talk by J. Peuron, Fr, 06:00

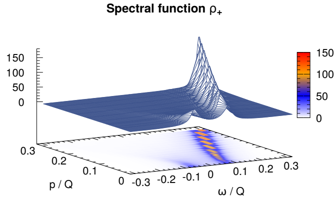


Gluonic 3+1D

Thank you for your attention!



Gluonic 2+1D

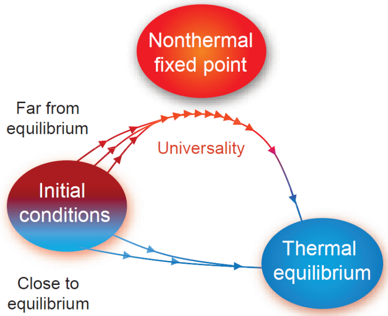


Fermionic 3+1D

Backup slides

Universal (classical) attractors

- Rich nonequilibrium dynamics in gauge and scalar systems
- Share similar universal features:



Nonthermal fixed point (NTFP)

★ Large initial occupancy
$\Rightarrow$ may approach attractor

★ System 'forgets' initial conditions

★ Self-similar dynamics

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

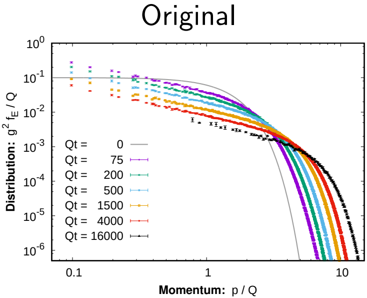
★ *Universal* α, β, $f_s(p)$

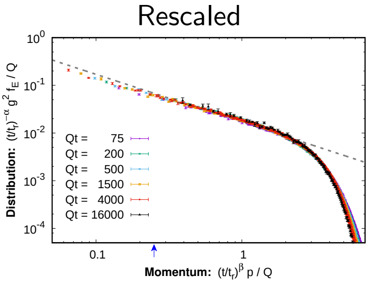
NTFP: Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

Universality: Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

Experimental observations: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)

Self-similarity of 2+1D theory (PRD 100, 094022 (2019))





- Self-similar evolution

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal scaling exponents

$$\beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(energy conserv.)}$$

Perturbative computation: HTL results

- Hard loop (HTL) framework requires $m_D/\Lambda \ll 1$;
  in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f\,\Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{\mathrm{pol}} N_{\mathrm{c}} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m^2 + p^2}} \sim g^2 f\,\Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\mathrm{HTL}}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\mathrm{HTL}}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

HTL spectral functions in 2+1D

- HTL polarization tensor $\Pi_\alpha(x)$, with $x = \omega/p$

$$\Pi_T(x) = m_D^2 \, x \left(x - \frac{x^2 - 1}{\sqrt{x+1}\sqrt{x-1}} \right)$$

$$\Pi_L(x) = m_D^2 \left(-1 + x \, \frac{x^2 - 1}{(x+1)^{3/2}(x-1)^{3/2}} \right)$$

$$\Pi_\phi(x) = m_D^2 = const$$

- Retarded propagator

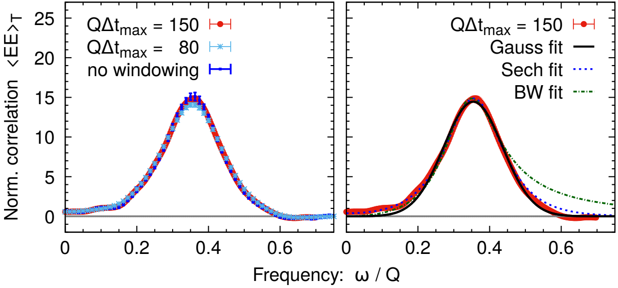
$$G_T^{\mathrm{HTL}}(\omega, p) = \frac{-1}{\omega^2 - p^2 - \Pi_T(\omega/p)}$$

$$G_L^{\mathrm{HTL}}(\omega, p) = \frac{p^2}{\omega^2} \frac{-1}{p^2 - \Pi_L(\omega/p)}$$

$$G_\phi^{\mathrm{HTL}}(\omega, p) = \frac{-1}{\omega^2 - p^2 - m_D^2}$$

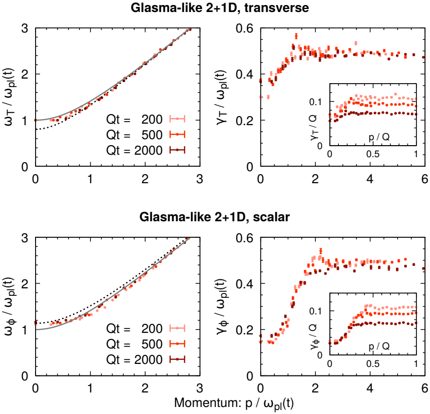
- Spectral function $\rho_\alpha^{\mathrm{HTL}}(\omega, p) = 2 \, \mathrm{Im} \, G_\alpha^{\mathrm{HTL}}(\omega, p)$

Gluon ρ in 2+1D: Shape of the excitation peak

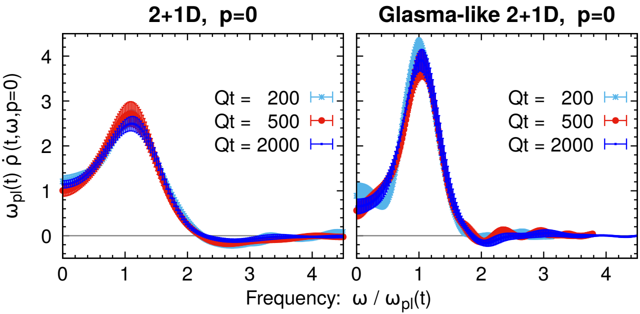


- *Left:* Different ways of computing the Fourier transform are consistent
- *Right:* Peak has non-Lorentzian shape (not Breit-Wigner)

Gluon ρ in 2+1D: Dispersion relations, damping rates

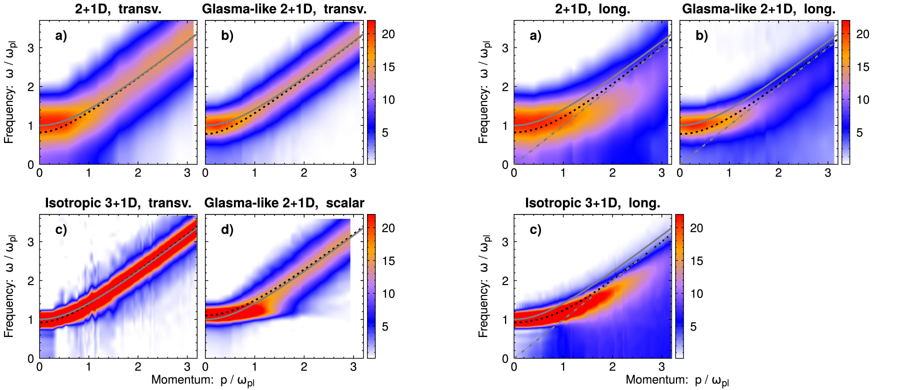


- *Left:* Dispersions $\omega_\alpha(t,p)/\omega_{\rm pl}(t)$
- *Right:* Peak width $\gamma_\alpha(t,p)/\omega_{\rm pl}(t)$
- As functions of $p/\omega_{\rm pl}(t)$ time independent $\Rightarrow \gamma(t,p) \sim \omega_{\rm pl}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same t dependence



- $\omega_{\rm pl}\,\dot{\rho}(t, \omega/\omega_{\rm pl}, p/\omega_{\rm pl})$ is time independent
- This implies $\boxed{\gamma_\alpha(t,p) \sim \omega_{\rm pl}(t)} \sim m_D \sim Q(Qt)^{-1/5}$
- No quasiparticles at low p seems to be quite general in 2+1D

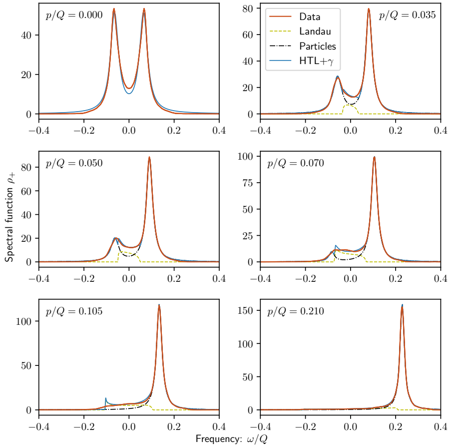
Summary of $\langle EE\rangle_\alpha(t{=}const, \omega, p)$



Black dashed: $\omega_\alpha^{\mathrm{HTL}}(p)$, gray: $\sqrt{\omega_{\mathrm{pl}}^2 + p^2}$, with $\omega_{\mathrm{pl}} \propto m_D$

Fermion ρ in 3+1D: Comparison with HTL

Spectral function $\rho_+(\omega, p)$



- Landau cut + q.p. peaks:

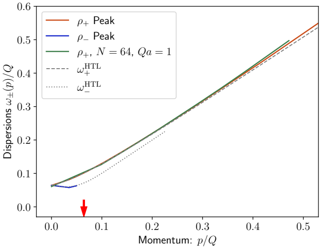
$$\rho_+^{\mathrm{HTL}}(\omega, p) = 2\pi\, \beta_+(\omega/p, p) + 2\pi\, [Z_+(p)\delta(\omega - \omega_+(p)) + Z_-(p)\delta(\omega + \omega_-(p))]$$

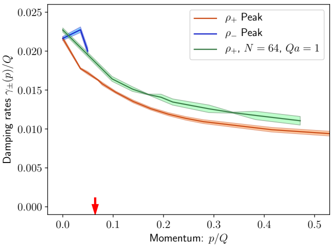
- HTL+γ extension

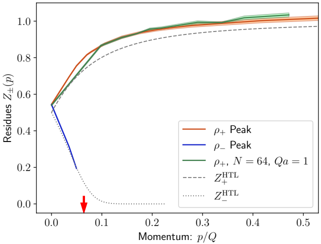
$$\rho_+^{\mathrm{HTL}}(\omega, p) = 2\pi\, \beta_+(\omega/p, p) + \frac{2Z_+(p)\gamma_+(p)}{(\omega - \omega_+(p))^2 + \gamma_+^2(p)} + \frac{2Z_-(p)\gamma_-(p)}{(\omega + \omega_-(p))^2 + \gamma_-^2(p)}$$

- Very good description, $Z_\alpha(t,p)$, $\omega_\alpha(t,p)$, $\gamma_\alpha(t,p)$ fits

Fermion ρ in 3+1D: ω_α, Z_α, γ_α







- HTL dispersions and residues agree well with data
- First principles insights into *p*-dependence of damping rates