Critical behavior towards the chiral limit at vanishing and non-vanishing chemical potentials

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(HotQCD Collaboration)

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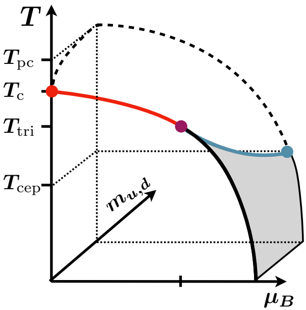
UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC–TR 211
Strong-interaction matter under extreme conditions

Chiral phase transition in 2+1 QCD

$$T_{pc} = 156.5(1.5) \text{ MeV}$$

[HotQCD, PLB 795 (2019) 15–21]



[F. Karsch, arxiv:1905.03936]

$$T_c = 132^{+3}_{-6} \text{ MeV}$$

[HotQCD, PRL 123, 062002 (2019)]

- ➔ **Expected** to belong to the **universality class** of $\boldsymbol{3d\ O(4)}$ spin model

  [R.D. Pisarski, F. Wilczek, PRD 29 338 (1984)]

- ➔ Important for understanding the phase diagram at physical mass (A. Lahiri : Plenary talk)

- ➪ Imprint of the criticality on energy-like and mixed observables

Lattice Setup

- Gauge ensembles generated with HISQ fermion discretization and Symanzik-improved gauge action, used in chiral T_c determination [HotQCD, arxiv:1905.11610].
- Ensembles for smaller-than-physical quark (up, down) masses $m_l = ms/27, m_s/40, m_s/80, m_s/160$, keeping strange quark mass m_s *fixed at physical value*.
- Corresp. pion masses : 140 MeV, 110 MeV, 80 MeV, 55 MeV
- Thermodynamic and continuum limit not yet performed. Measurements done at the largest simulated volumes for each mass at fixed time extent $N_\tau = 8$.
- Computing resources : Jülich, Piz Daint, JLAB, Bielefeld and Wuhan supercomputing facilities.

Critical behavior of thermodynamic quantities

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(T, \vec{\mu}, m_l) = \boldsymbol{h^{(2-\alpha)/\beta\delta} f_f(z)} + f_r(T, \vec{\mu}, m_l)$$

infinite volume

singular **regular**

$$\bar{t} \equiv t t_0 = \left(\frac{T - T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T} \right)^2 \right)$$

"energy-like" coupling

Scaling variable

$$\boldsymbol{z = z_0 \bar{t} / H^{1/\beta\delta}}$$

$$z_0 = h_0^{1/\beta\delta} / t_0$$

$$H \equiv h h_0 = \frac{m_l}{m_s}$$

"magnetic-like" coupling

$3d\ O(2)$ critical exponents : HISQ at finite lattice spacing

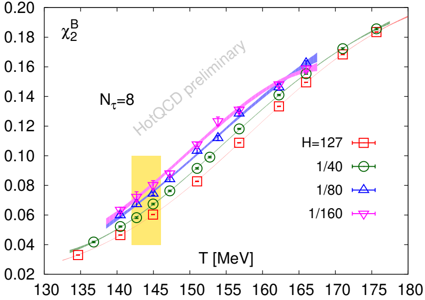
Derivatives of free energy density / pressure

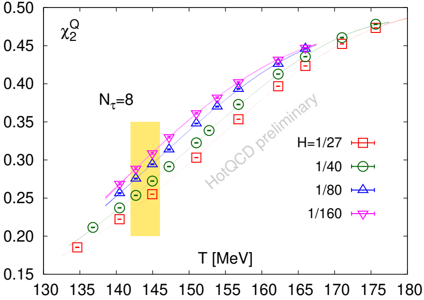
Conserved charge fluctuations are energy-like w.r.t to chiral phase transition (also Polyakov loop, see David's talk)

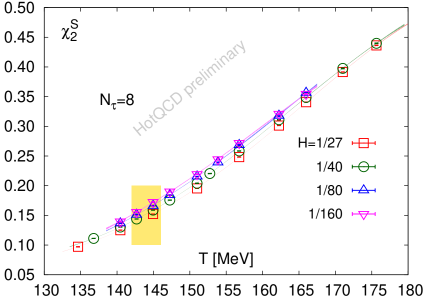
	magnetic-like	**mixed**	**energy-like**
	$\frac{\partial^2 \ln Z}{\partial H^2}$	$\frac{\partial^2 \ln Z}{\partial H \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
	$\sim H^{1/\delta - 1}$	$\sim H^{(\beta-1)/\beta\delta}$	$\sim H^{-\alpha/\beta\delta}$
	$\sim H^{-0.79}$	$\sim H^{-0.34}$	$\sim H^{+0.11}$
divergence :	**strong**	**moderate**	**vanishes**

Conserved charge fluctuations at $\mu = 0$ (Singular part) :

$$\chi^X_{2n} = -\frac{\partial^{2n} f/T^4}{\partial(\mu_X/T)^{2n}}\bigg|_{\mu_X=0} \sim -(2\kappa^X_2)^n \, H^{(2-\alpha-n)/\beta\delta} f^{(n)}_f(z)$$





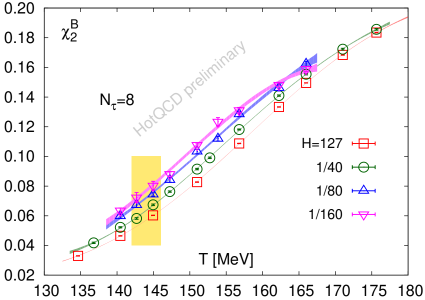


– similar features as energy density

$T_c = 144(2)$ MeV at $N_\tau = 8$ (yellow band)

Estimation of singular contribution to χ_2

$$\chi_2^X(T_c, H) \sim -\kappa_2^X H^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(H^2)$$



– expect straight line fit for $\chi_2(T_c, H)$ vs $H^{0.61}$ if scaling holds ($O(2)$ exponents)

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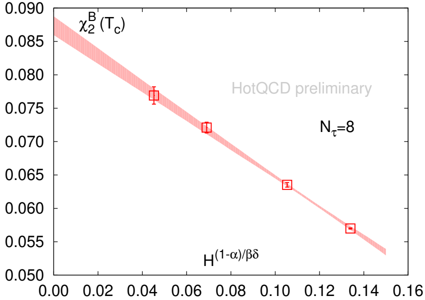

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$$\chi_2^X(T_c, H = 0) - \chi_2^X(T_c, H = 1/27) = \text{Singular part of } \chi_2^X(T_c)$$

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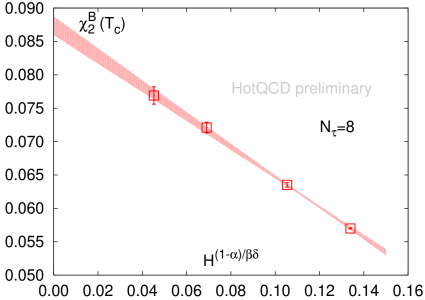

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Singular contribution to χ_2^B at physical masses $\sim$ 50%

Estimation of singular contribution to χ_2

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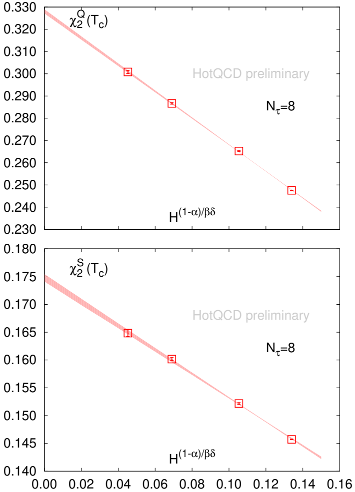

singular contribution at physical mass

$\chi_2^B(T_c)$	$\sim 50\%$
$\chi_2^Q(T_c)$	$\sim 30\%$
$\chi_2^S(T_c)$	$\sim 20\%$

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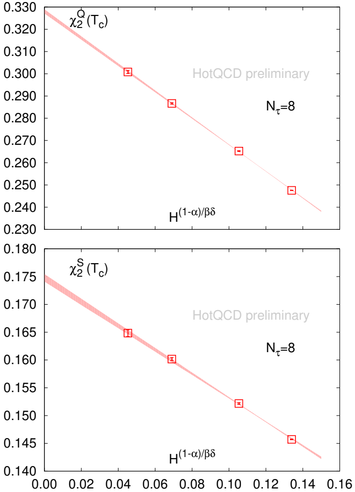

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– ratio of singular parts = ratio of κ_2

$\kappa_2^Q/\kappa_2^B \quad \sim 2.6$

$\kappa_2^B/\kappa_2^S \quad \sim 1.0$

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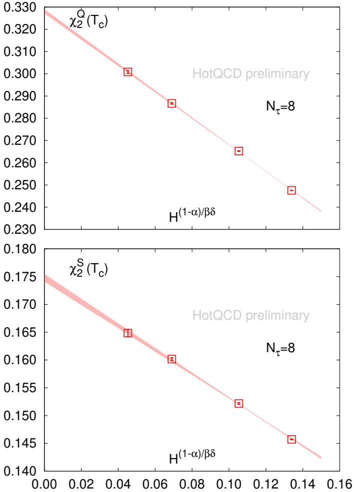

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– ratio of singular parts = ratio of κ_2

$\kappa_2^Q/\kappa_2^B \quad \sim 2.6 \quad 1.8(8)^*$

$\kappa_2^B/\kappa_2^S \quad \sim 1.0 \quad 0.9(4)^*$

Close to results for physical mass

*[HotQCD, Phys. Lett. B 795 (2019) 15]

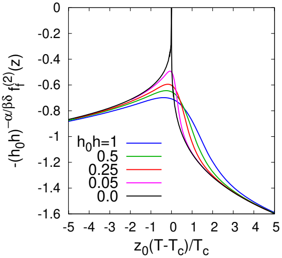
Singular part :

$$\chi_4^Q \sim H^{-\alpha/\beta\delta} f_f^{(2)}(z)$$

$$O(4) : -\alpha/\beta\delta = 0.116$$

Not divergent

– but pronounced spike



[Friman et al, Eur. Phys. J. C (2011) 71:1694]

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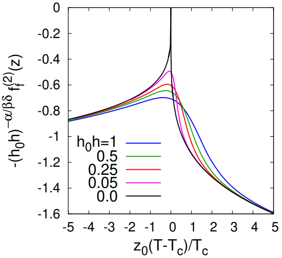

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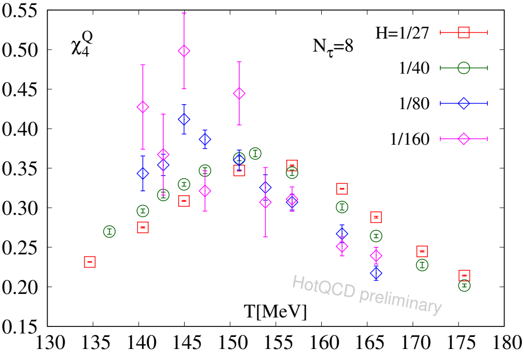
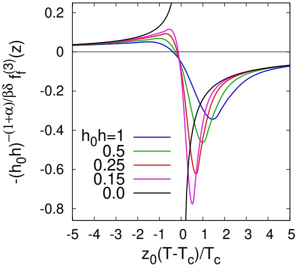


➪ Spike seems to develop already!

Singular part :

$$\chi_6^Q \sim H^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z)$$
$$\sim H^{-0.429}$$

Moderate divergence



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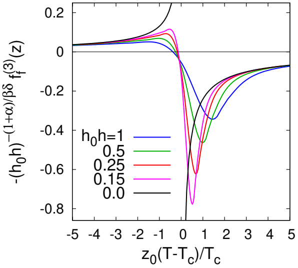

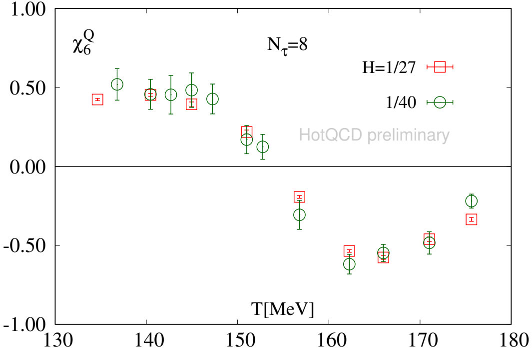

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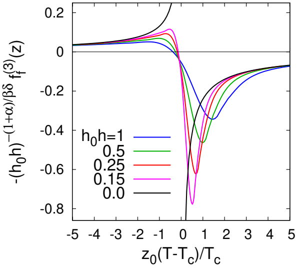
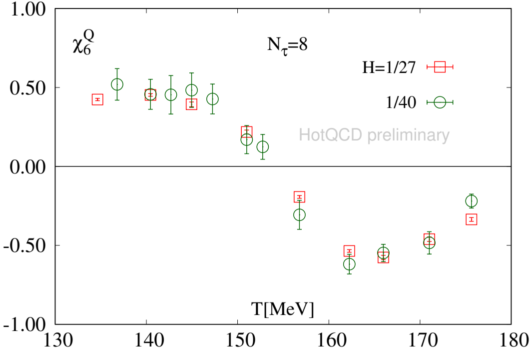




Ratio of peak heights expected from scaling : $(\chi_6^Q)^{max}_{1/40}/(\chi_6^Q)^{max}_{1/27} \sim 1.18$

H-derivatives of energy-like observables

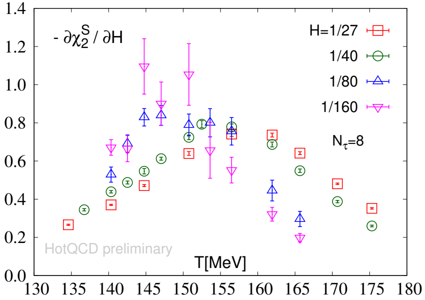
$$\chi_2^S \sim -\kappa_2^S H^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\rm reg}$$

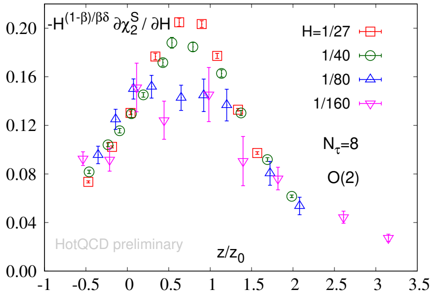
$$H = \frac{m_l}{m_s}$$

$$\frac{\partial \chi_2^S}{\partial H} \sim \kappa_2^S H^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial f_{\rm reg}}{\partial H}$$

$$\frac{\beta-1}{\beta\delta} = -0.39, O(2)$$

Divergent already for 2nd order





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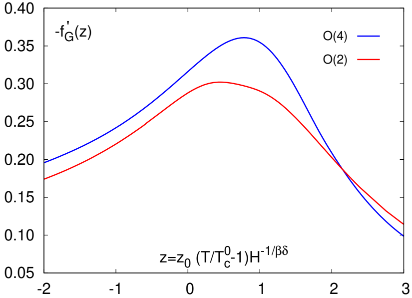

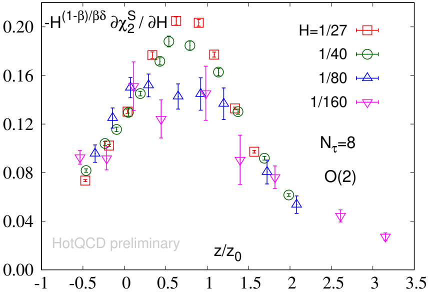

Dominant singular part

More mixed observables ...

Strange chiral condensate $\Rightarrow$ *energy-like* observable

$$\langle\bar{\psi}\psi\rangle_s = AH^{(1-\alpha)/\beta\delta} f'_f(z) + f_{\text{reg}}$$

$$\chi_{ls} \equiv \frac{\partial\langle\bar{\psi}\psi\rangle_s}{\partial H} = -AH^{(\beta-1)/\beta\delta} f'_G(z) + \frac{\partial f_{\text{reg}}}{\partial H}$$

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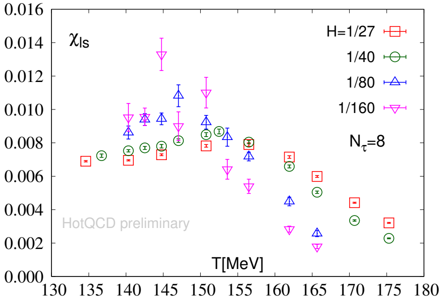
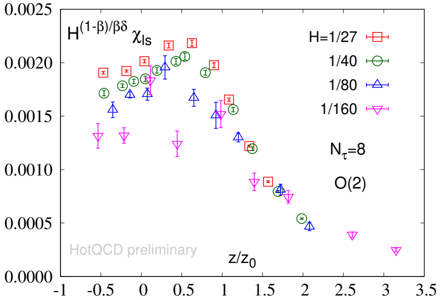
Divergent at 2nd order





Similar divergence, different regular contribution

Mixed observables contd..

Subtracted chiral condensate $\Rightarrow$ *magnetic-like* observable

$$\Sigma = \frac{1}{f_K^4}\left[m_s \left\langle \bar{u}u + \bar{d}d \right\rangle - (m_u + m_d)\left\langle \bar{s}s \right\rangle\right]$$

$$\frac{\partial^2 \Sigma}{\partial(\mu_B/T)^2} \sim -\kappa_2^B H^{(\beta-1)/\beta\delta} f'_G(z) + \text{reg.}$$

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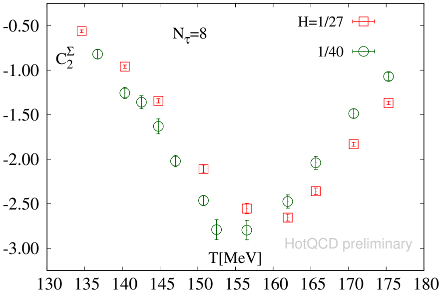
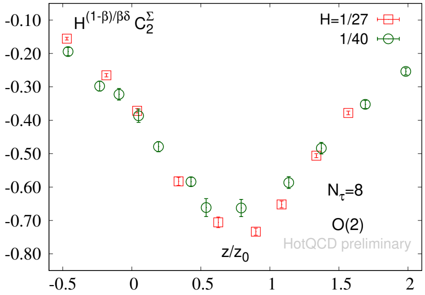
Divergent at 2nd order





Curvature in the chiral limit [Preliminary]

$$t = \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T}\right)^2\right)$$

$$\kappa_2^B \simeq \frac{T^2 \frac{\partial^2}{\partial \mu_B^2} f}{2T \frac{\partial}{\partial T} f}$$

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divergent after another derivative

$$f = \Sigma \equiv \frac{1}{f_K^4}\left[2m_s \left\langle \bar{\psi}_l \psi_l \right\rangle - 2m_l \left\langle \bar{\psi}_s \psi_s \right\rangle\right]$$

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HotQCD, Phys. Lett. B 795 (2019) 15

Physical mass
$\kappa_2^B[H = 1/27] = 0.015(4)$

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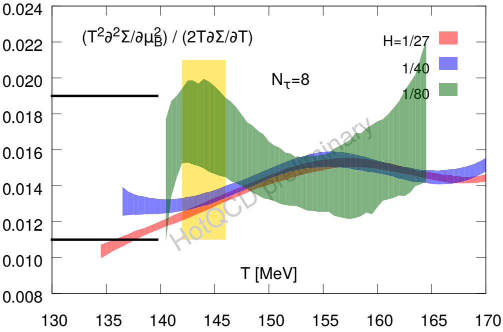

Curvature doesn't change towards the chiral limit

Conclusions and Outlook

- Consistent with $O(4)$ or $O(2)$ universality class in the chiral limit
  - → strangeness fluctuations and strange quark condensate behave as energy-like quantities
- Singular part at physical mass can be extracted
  - → Ratio of curvature in chiral limit remains similar
- Preliminary estimate of curvature in chiral limit - consistent with physical mass
- Scaling analysis to be peformed
- Future comparison with HRG with smaller masses
- Continuum and thermodynamic limits

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