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Heavy quark momentum diffusion from the lattice

using gradient flow

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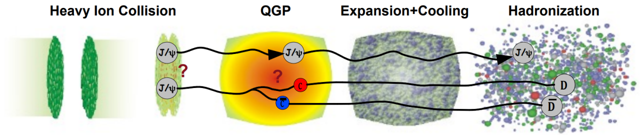
Heavy quarkonia in heavy ion collisions



How fast do heavy quarks thermalize in a hot medium?

- Hydrodynamics → kinetic equilibration time $\tau_{\text{kin}}^{\text{heavy}} \simeq \frac{M}{T}\, \tau_{\text{kin}}^{\text{light}}$
- But: significant collective motion (v_2)! → $\tau_{\text{kin}}^{\text{heavy}} \overset{?}{\approx} \tau_{\text{kin}}^{\text{light}}$

Can we calculate τ_{kin} from first principles?

- Consider non-relativistic limit $(M \gg \pi T)$:

$$\tau_{\text{kin}} = \eta_D^{-1}$$

$$\eta_D = \frac{\kappa}{2 M_{\text{kin}} T}\left(1 + \mathcal{O}\left(\frac{\alpha_s^{3/2} T}{M_{\text{kin}}}\right)\right)$$

$$D = 2T^2/\kappa$$

- **Problem:**
  perturbative series for D or κ ill-behaved!
  → need for **non-perturbative ab-initio** calculation
    → **lattice QCD**

Figure: Steffen Bass, mod. by O. Kaczmarek

Investigating HQ diffusion on the lattice

How to calculate diffusion coefficients from the lattice?

- Linear response theory ⇒ diffusion physics ⇔ **in-equilibrium spectral functions** (SPF)

⇒ SPF of HQ vector current
$$\boldsymbol{\rho^{ii}}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{i\omega t} \int \mathrm{d}^3\mathbf{x} \left\langle \frac{1}{2}\left[\hat{\mathcal{J}}^i(\mathbf{x},t), \hat{\mathcal{J}}^i(\mathbf{0},0)\right]\right\rangle$$

- Lattice: reconstruct SPF from **Euclidean correlation functions**

A. Spatial diffusion, hadronic correlators

- reconstruct $\boldsymbol{\rho^{ii}}(\omega)$ from hadron corr.

⇒ $\boldsymbol{\rho^{ii}}(\omega)$ encodes spatial diffusion coeff. D through **Kubo-formula**:

$$D = \frac{1}{3\chi^{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\boldsymbol{\rho^{ii}}(\omega)}{\omega}$$

- difficult to resolve transport peak at $\omega \to 0$

⇒ see recent talk by H.-T. Shu at SQM '21

B. Momentum diffusion, gluonic correlators

- start from $\boldsymbol{\rho^{ii}}(\omega)$ but utilize HQET (nonrelativistic limit $M \gg \pi T$)

⇒ construct **"color-electric correlator"** whose SPF encodes momentum diff. coeff. $\boldsymbol{\kappa}$:

$$\boldsymbol{\kappa} = \lim_{\omega\to 0} 2T \frac{\boldsymbol{\rho}(\omega)}{\omega}$$

- smooth $\omega \to 0$ limit expected: **no transport peak!**

⇒ **this talk**

From HQET to EE correlator to κ

- HQET: do Foldy-Wouthuysen trans. of S_{QCD} ⇛ decouple quarks and anti-quarks (up to $\mathcal{O}(1/M^2)$)
- insert leading-order $(1/M)$ currents in SPF, . . . , translate to Euclidean
  - ⇛ find gluonic "$\boldsymbol{EE}$ **correlator**" Caron-Huot et al. 2009

$$G(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,[\mathrm{tr}\,[\; U(\beta,\tau)\;\; gE_i(\mathbf{0},\tau)\;\; U(\tau,0)\;\; gE_i(\mathbf{0},0)\;\;]]\rangle}{\langle \mathrm{Re}\,[\mathrm{tr}[\;\; U(\beta,0)\;\;]]\rangle}$$

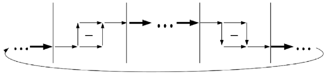
$$= \int_0^\infty \mathrm{d}\omega\, \frac{\cosh(\omega(\tau-\beta/2))}{\sinh(\omega\beta/2)}\, \boldsymbol{\rho}(\omega)$$

- encodes transport physics of **static heavy quark** in thermalized medium

$$\Rightarrow \boldsymbol{\kappa} = \lim_{\omega\to 0} 2T\frac{\boldsymbol{\rho}(\omega)}{\omega}$$

← no transport peak, smooth limit expected

Lattice discretization



- **Problem:** IR part of $\boldsymbol{\rho}(\omega)$ encoded in **large-τ part of** $G(\tau)$
  - ⇛ underlying signal overshadowed by UV fluctuations
    - ⇛ large statistical errors!
  - ⇛ need noise reduction (= gauge smoothing) method

Gradient flow for EE correlator

Solution to noise problem: gradient flow Lüscher 2010

- applicable to dynamical QCD (nonlocal action)
- introduces *flow time* $\tau_\mathrm{F} \equiv ta^2$ with dimensionless parameter t

$$\frac{\mathrm{d}A_\mu(x,\tau_\mathrm{F})}{\mathrm{d}\tau_\mathrm{F}} \sim \frac{-\delta S_G[A_\mu(x,\tau_\mathrm{F})]}{\delta A_\mu(x,\tau_\mathrm{F})}, \quad A_\mu(x,\boldsymbol{\tau_\mathrm{F}=0}) = A_\mu(x)$$

- evolves gauge fields $A_\mu(x)$ towards minimum of the action S_G
- smears them over Gaussian envelope (LO); width: *flow radius* $\simeq \sqrt{8\tau_\mathrm{F}}$

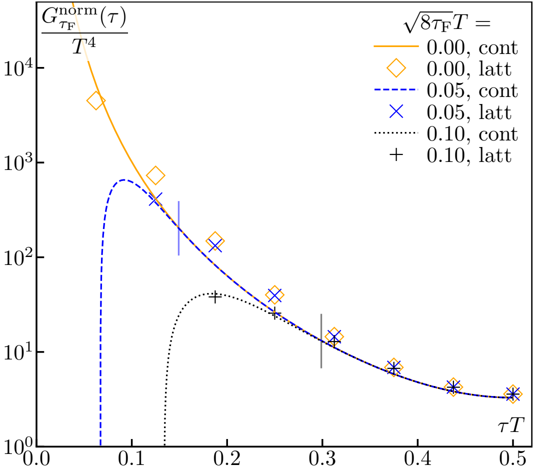
$$A_\mu^\mathrm{LO}(x,\tau_\mathrm{F}) = \int \mathrm{d}y \left(\sqrt{2\pi}\sqrt{8\tau_\mathrm{F}}/2\right)^{-4} \exp\left(\frac{-(x-y)^2}{\sqrt{8\tau_\mathrm{F}}^2/2}\right) A_\mu(y)$$

- **improves signal & produces renormalized fields...**

  ...but **contaminates** EE correlator $G(\tau)$ for $\sqrt{8\tau_\mathrm{F}} \gtrsim \tau/3$ according to LO pert. theory Eller, Moore 2018

  → flow up to **flow limit** $\sqrt{8\tau_\mathrm{F}} \approx \tau/3$, extrapolate back to $\tau_\mathrm{F} = 0$

Leading-order perturbative EE correlator under Wilson flow



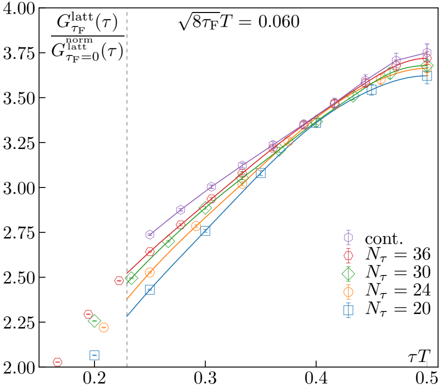
continuum corr. from Eller, Moore 2018 ,
lattice corr. from Eller, Moore, LA et al. 2021

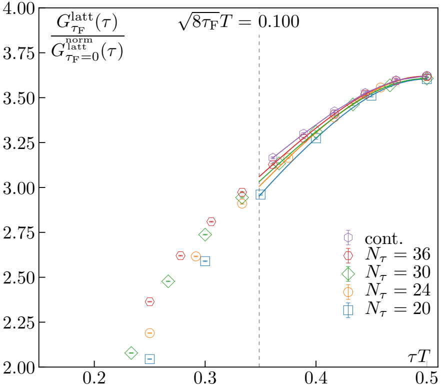
- **flow limit:**
  cont. correlator deviates $< 1\%$ for
  $\tau T \gtrsim 3\sqrt{8\tau_F}T$ → vertical lines

Use to enhance nonpert. lattice results:

- filter out τ^{-4} behavior via $G^{\text{nonpert}}/G^{\mathrm{norm}}$
  → increases visibility of details
- comparison of LO cont. and LO latt. correlators
  → approx. remove tree-level discretization errors

Nonpert. EE correlator (quenched approximation, Wilson action, $T \approx 1.5\,T_c$)



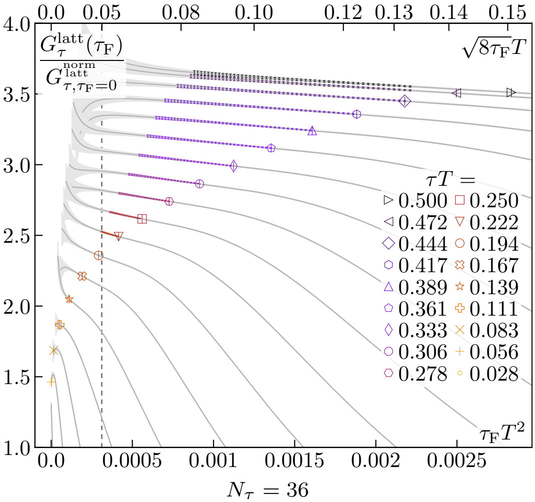


$N_\sigma^3 \times N_\tau$	a [fm]
$80^3 \times 20$	0.0213
$96^3 \times 24$	0.0176
$120^3 \times 30$	0.0139
$144^3 \times 36$	0.0116

- 10000 quenched conf. each
- well-separated: 500 sweeps of (1 HB, 4 OR)
- $\mathcal{O}(a^2)$-improved "Zeuthen flow"
- 3rd-order RK with adaptive stepsize

- data normalized to pert. lattice correlator
  - ⇒ dominant τ^{-4} behavior filtered out, tree-level improvement
- dashed line: flow limit as lower bound for separation $\boxed{\tau T \gtrsim 3\sqrt{8\tau_F}T}$
  - ⇒ more flow = higher precision, but smaller window of noncontaminated data
- interpolation through cubic splines (no smoothing)

EE correlator as a function of flow time

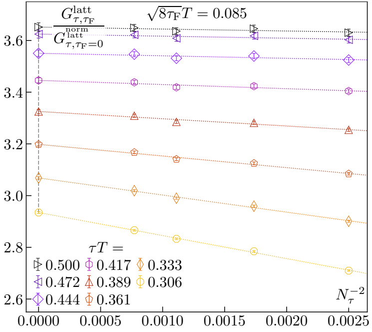


- flow limit $\boxed{\sqrt{8\tau_F}T \lesssim \tau T/3}$ ⇛ markers
- for large τT: modest flow dependence
  ⇛ extrapolation to $\tau_F = 0$
- need some flow to get signal, but too much contaminates the physics
- initial rising behavior (visible for $\tau T = 0.056$): discretization-induced tadpole renormalization effect also found in pert. NLO lattice QED

LA et al. 2021

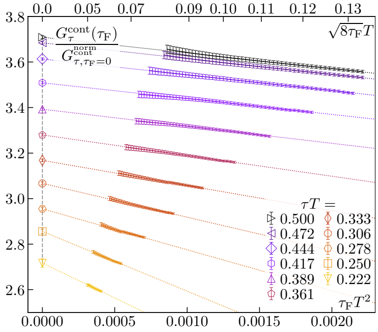
Continuum and flow-time-to-zero extrapolation

- 1. Continuum extrapolation (linear in N_τ^{-2})



- ansatz motivated by gauge action discretization
- taken separately for each flow time
- removes a^2/τ^2-type discretization errors
- a^2/τ_F-type errors only vanish if **continuum limit** is taken **first!**

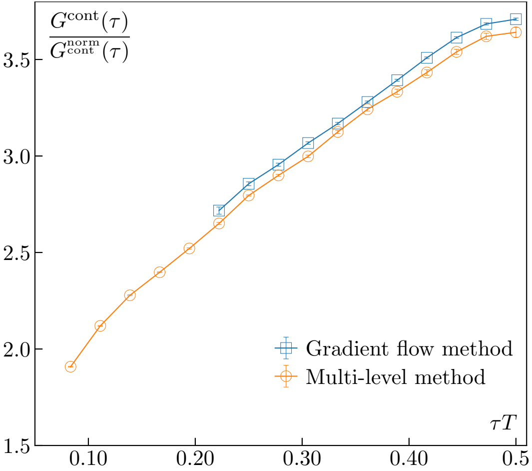
- 2. Flow-time-to-zero extrapolation (linear in τ_F)



- ansatz motivated by NLO pert. theory 🔗 Eller 2021
- removes τ_F/τ^2-type effects
- flow time window depends on:
  - signal-to-noise ratio
  - $\sqrt{8\tau_F} \gtrsim a$ (renormalization, suppression of latt. artifacts)
  - $\sqrt{8\tau_F} \lesssim \tau/3$ (flow limit)

🔗 LA et al. 2021

Renormalized continuum EE correlator



- Nonpert.-renormalized continuum EE correlator after $a \to 0$ and $\tau_{\mathrm{F}} \to 0$ extrapolations
- Shape consistent with previous (only pert. renorm.) results
  Francis et al. 2015 , Christensen, Laine 2016
- Overall shift due to
  - nonperturbative renormalization
  - difference in statistical power of gauge conf.
  - systematic uncertainty introduced by flow extr.
- Only large-τ of correlator can be obtained
  ⇒ not a problem for diffusion physics!

LA et al. 2021

Spectral reconstruction through pert. model fits

- for details see LA et al. 2021

- Reminder: $G(\tau) = \int_0^\infty d\omega \, \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)} \boldsymbol{\rho}(\omega),$

$$\boldsymbol{\kappa} = \lim_{\omega \to 0} 2T \frac{\boldsymbol{\rho}(\omega)}{\omega}$$

⇒ integral inversion problem; valid only at $\tau_F = 0$ Eller 2021

- Strategy: constrain allowed form of $\rho(\omega)$ to

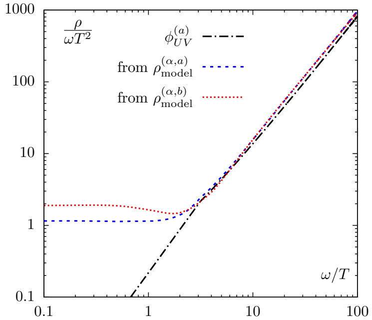
$$\rho^{(\mu,i)}_{\rm model}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\rm max}} c_n e_n^{(\mu)}(y)\right] \sqrt{\left[\phi_{\rm IR}(\omega)\right]^2 + \left[\phi^{(i)}_{\rm UV}(\omega)\right]^2}$$

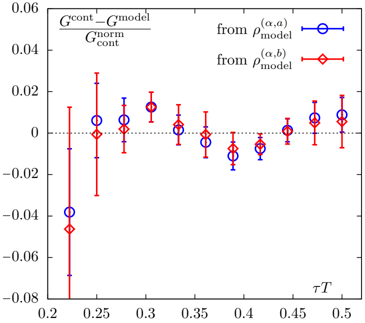
using IR and UV asymptotics:

$$\phi_{\rm IR}(\omega) \equiv \frac{\kappa\omega}{2T}, \quad \phi^{(a)}_{\rm UV}(\omega) \equiv \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}, \quad \ldots$$

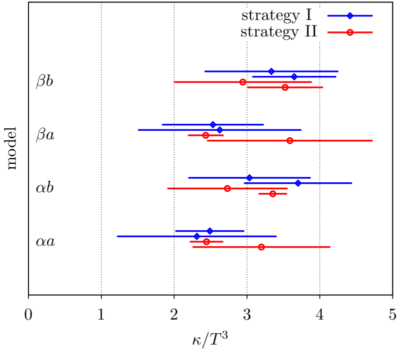
⇒ well-defined fit with parameters $\boldsymbol{\kappa}/T^3$ and c_n via

$$\chi^2 \equiv \sum_\tau \left[\frac{G^{\rm cont}(\tau) - G^{\rm model}(\tau)}{\delta G^{\rm cont}(\tau)}\right]^2$$





HQ momentum diffusion coefficient κ at $T = 1.5\,T_c$



κ/T^3-value similar / slightly larger compared to previous study Francis et al. 2015
(using quenched-only multi-level method + pert. renorm.)

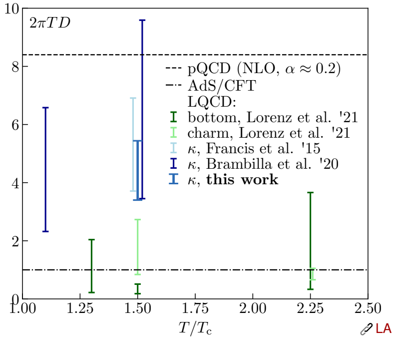
→ We find $\kappa/T^3 = 2.31 \ldots 3.70$

and (for $M \gg \pi T$ using $D = 2T^2/\kappa$):

$$2\pi T D = 3.40 \ldots 5.44$$

→ kinetic equilibration time:

$$\tau_{\rm kin} = \eta_D^{-1} = (1.63 \ldots 2.61) \left(\frac{T_c}{T}\right)^2 \left(\frac{M}{1.5\,\mathrm{GeV}}\right)\ \mathrm{fm/c}$$



LA et al. 2021

Recap

What do we want?

- a first-principles nonpert. estimate from dynamical QCD for the **HQ momentum diffusion coefficient** κ (or in turn D, τ_{kin})

Why?

- phenomenology: explain experimental data for HQ
- crucial input for transport simulations

What did we achieve so far?

- proof-of-concept for gradient flow method in quenched QCD
  - no restrictions for application to dynamical QCD!
  - high-prec. data for IR part of EE correlator (nonpert. renorm.)
  - consistent results for κ from reconstructed spectral function (pert. model fits)

What to do next?

- measure dynamical QCD lattices (HISQ) **[in progress]**
- determine finite mass correction (color-magnetic correlator) Bouttefeux, Laine 2021 **[in progress]**