

Latent heat and pressure gap at the first-order deconfining phase transition of SU(3) Yang–Mills theory using the small flow-time expansion method

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Shirogane et al. (WHOT-QCD Collab.), Prog.Theor.Exp.Phys. 2021, 013B01 [arXiv: 2011.10292]

We study latent heat $\Delta\epsilon$ and the pressure gap Δp between the hot and cold phases at the first-order deconfining transition of SU(3) Yang–Mills using the small flow-time expansion (SFtX) method. In the continuum limit, we find $\Delta\epsilon/T^4 = 1.117 \pm 0.040$ for $N_s/N_t = 8$ and 1.349 ± 0.038 for $N_s/N_t = 6$ at T_c . We also confirm $\Delta p \approx 0$ as expected. From hysteresis curves of ϵ , we show that ϵ in the deconfined phase

is sensitive to the spatial volume, while that in the confined phase is insensitive. Furthermore, we examine effects of alternative procedures in the SFtX method -- the order of the continuum and the vanishing flow-time extrapolations, the renormalization scale, and higher-order corrections in the matching coefficients. We confirm that the final results are all consistent with each other.

I. SFtX method based on GF

Gradient flow: modification of fields in terms of a fictitious time t .

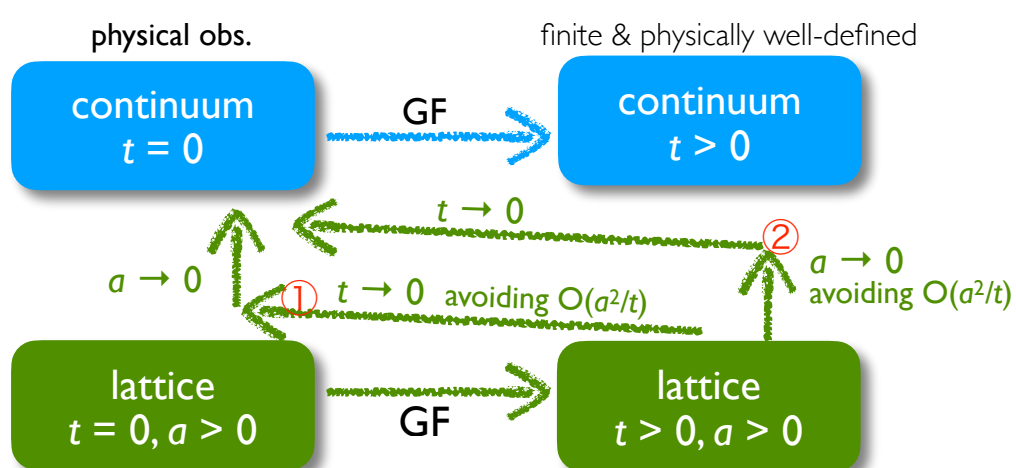
$$\dot{B}_\mu(t, x) = D_\nu G_{\nu\mu}(t, x), \quad B_\mu(0, x) = A_\mu(x) \quad \text{Narayanan-Neuberger(2006), Lüscher(2010)}$$

Flowed field $B_\mu \approx A_\mu$ smeared over a physical range $\sqrt{(8t)}$.

Operators of flowed fields have no UV divergences nor short-distance singularities at $t > 0$. Lüscher-Weisz (2011)

SFtX method: a general method to correctly calculate any renormalized observables on the lattice

H. Suzuki, PTEP 2013, 083B03 (2013) [E:2015, 079201]



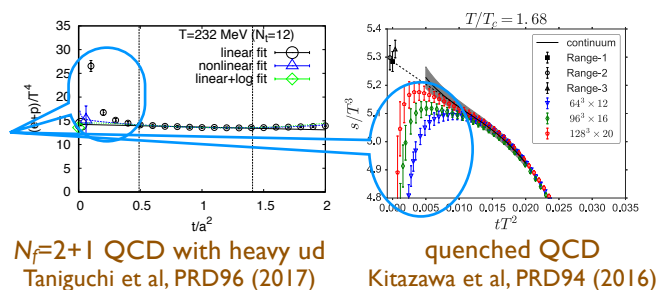
Directly evaluate corresponding operator on the lattice. No renormalization required.

SFtX works also when a basic symmetry is broken by the lattice regularization, thus helps to avoid complicated renormalizations.

We are applying SFtX to QCD with Wilson-type quarks to remove chiral violation problems. Our results show powerfulness of SFtX.

Taniguchi et al., PRD96, 014509 (2017); D95, 054502 (2017), D102, 014510 (2020)

To carry out $(a, t) \rightarrow 0$, we need to avoid $O(a^2/t)$ lattice artifacts.



We call the resulting fit range as "linear window".

Two methods of the double extrapolation $(a, t) \rightarrow 0$:

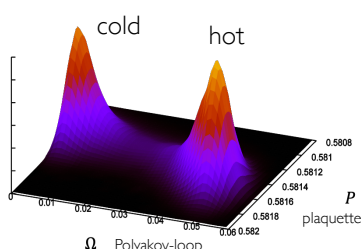
- ① $t \rightarrow 0$ then $a \rightarrow 0$
- ② $a \rightarrow 0$ then $t \rightarrow 0$

When the $O(a^2/t)$ lattice artifacts are correctly removed, the two methods should agree with each other.

2. $\Delta\epsilon$ and Δp at T_c in SU(3) YM

Shirogane et al., PTEP 2021, 013B01 (2021)

- $N_t = 8, 12, 16$; $N_s = 48-128$ ($N_s/N_t = 6, 8$)
- At each (N_s, N_t) , 3–6 β 's are combined by multipoint histogram to fine-tune to β_c
- $a = (N_t T_c)^{-1}$, $V = (N_s a)^3 = (N_s/N_t)^3 T_c^{-3}$
- Config. separation into hot and cold phases at β_c using Polyakov loop Ω with time-smearing.



2.1 Energy-momentum tensor by SFtX

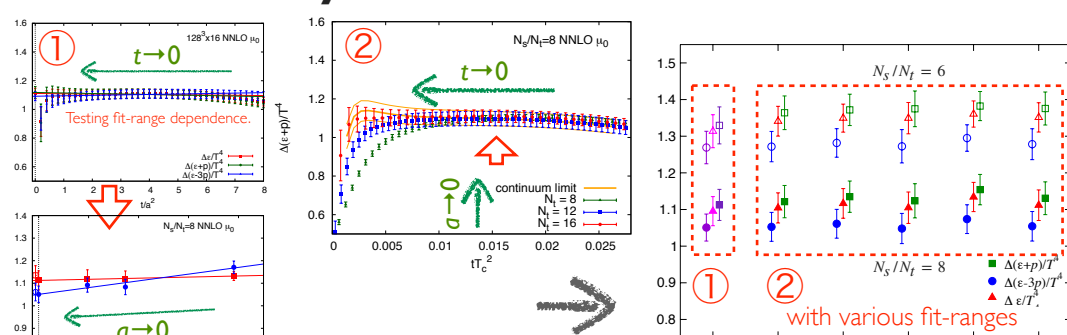
$$T_{\mu\nu}(x) = \lim_{(a,t) \rightarrow 0} \left\{ c_1(t) U_{\mu\nu}(t, x) + 4c_2(t) \delta_{\mu\nu} [E(t, x) - \langle E(t, x) \rangle_0] \right\}$$

$$U_{\mu\nu}(t, x) \equiv G_{\mu\rho}(t, x) G_{\nu\rho}(t, x) - \frac{1}{4} \delta_{\mu\nu} G_{\rho\sigma}(t, x) G_{\rho\sigma}(t, x) \quad E(t, x) \equiv \frac{1}{4} G_{\mu\nu}(t, x) G_{\mu\nu}(t, x)$$

$c_1(t)$ $c_2(t)$: matching coefficients determined by perturbation theory.
We show results of NNLO matching coeff's with the μ_0 renorm. scale.

$$\epsilon = -\langle T_{00} \rangle, \quad p = \frac{1}{3} \sum_i \langle T_{ii} \rangle$$

2.2 Consistency of the methods ① and ②



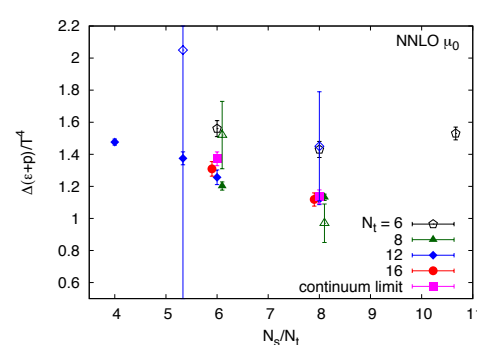
★ Methods ① and ② are well consistent.

We also compared with the results of NLO matching coefficients and/or the μ_d renormalization scale. => They are all consistent with each other.

2.3 Results of $\Delta\epsilon$ and Δp in the $(a, t) \rightarrow 0$ limit

★ $\Delta\epsilon/T^4 = 1.117(40)$ [$N_s/N_t = 8$], $1.349(38)$ [$N_s/N_t = 6$].

Systematic errors due to the extrapolations are smaller than the statistic errors.

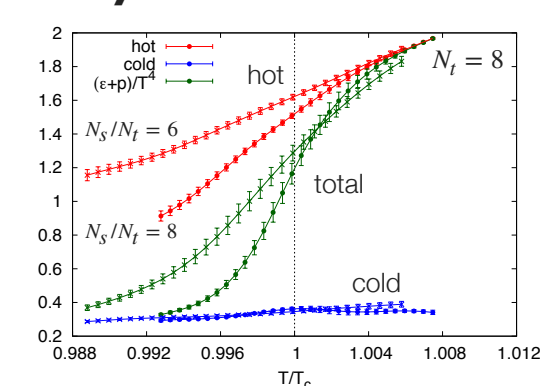


★ Consistent with the derivative method (open symbols), except for $N_t = 6$. SFtX has smaller errors

★ Volume dependence visible.

★ We confirm: Δp is consistent with 0 within errors both in ① and ②.

2.4 Hysteresis around T_c



★ Clean signal by SFtX.

★ Volume dependence visible in the metastable hot phase, while not in the cold phase.

3. Conclusions

★ Methods ① and ② as well as other variations of SFtX agree well with each other. => SFtX powerful and reliable.

★ $\Delta\epsilon$ obtained with small errors by SFtX. $\Delta p \approx 0$ confirmed.

★ Volume dependence of $\Delta\epsilon$ traced back to that in the hot phase.