Funny business from the large N_c finite temperature crossover

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Outline

- The issue
- Its resolution

Talk based on Phys. Rev. D **103** 094513 (2021) [arXiv:2102.01150 [hep-lat]].

Large N_c QCD on the lattice

Why study large N_c QCD?

- QCD simplifies at large N_c
- QCD idealizes at large N_c

Why do it on the lattice?

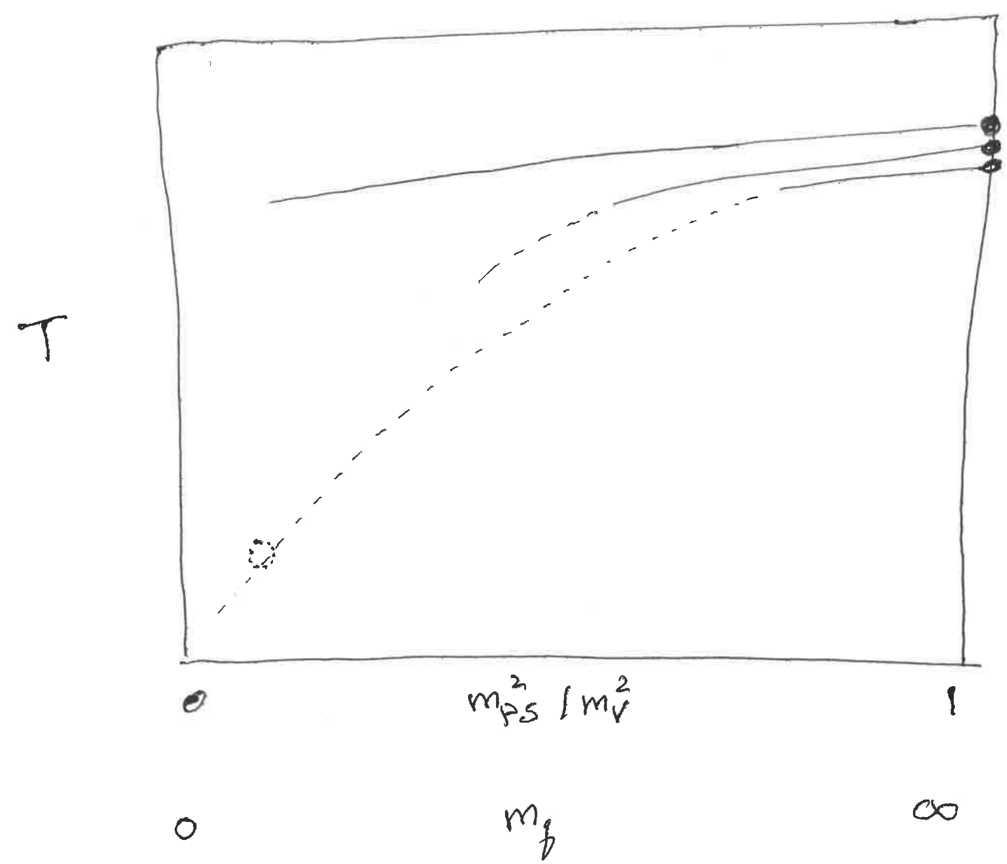
- Many large N_c predictions are actually nonperturbative, nice to do nonperturbative tests

How to do a lattice large N_c calculation:

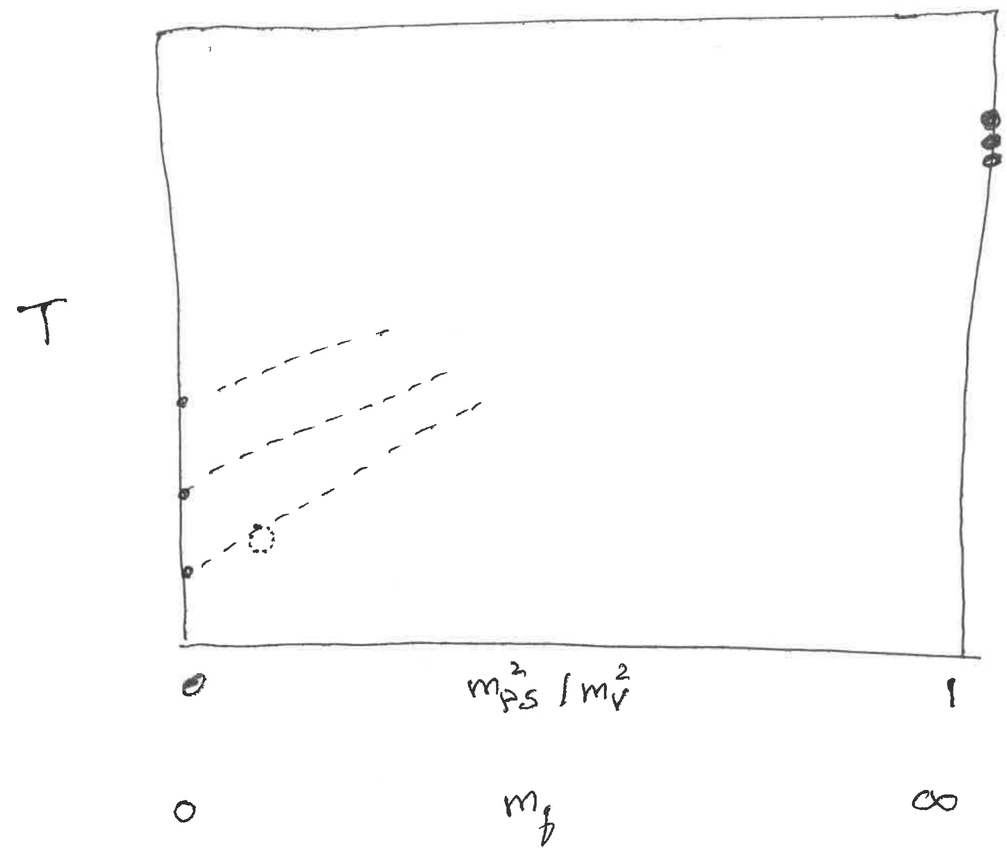
- Simulate at fixed bare $\lambda = g^2 N_c$ or $\beta \propto N_c^2$
- Measure the same observables across N_c
- Scale your observable appropriately ($f_{PS} \propto \sqrt{N_c}$)
- Observe curve collapse (vs m_q, m_q/N_c, $m_q N_c$, $\ldots$)

Usually, there is a simple story for any observable at large N_c (Ex: $f_{PS} \propto \sqrt{N_c}$, $\Sigma \propto N_c \ldots$)

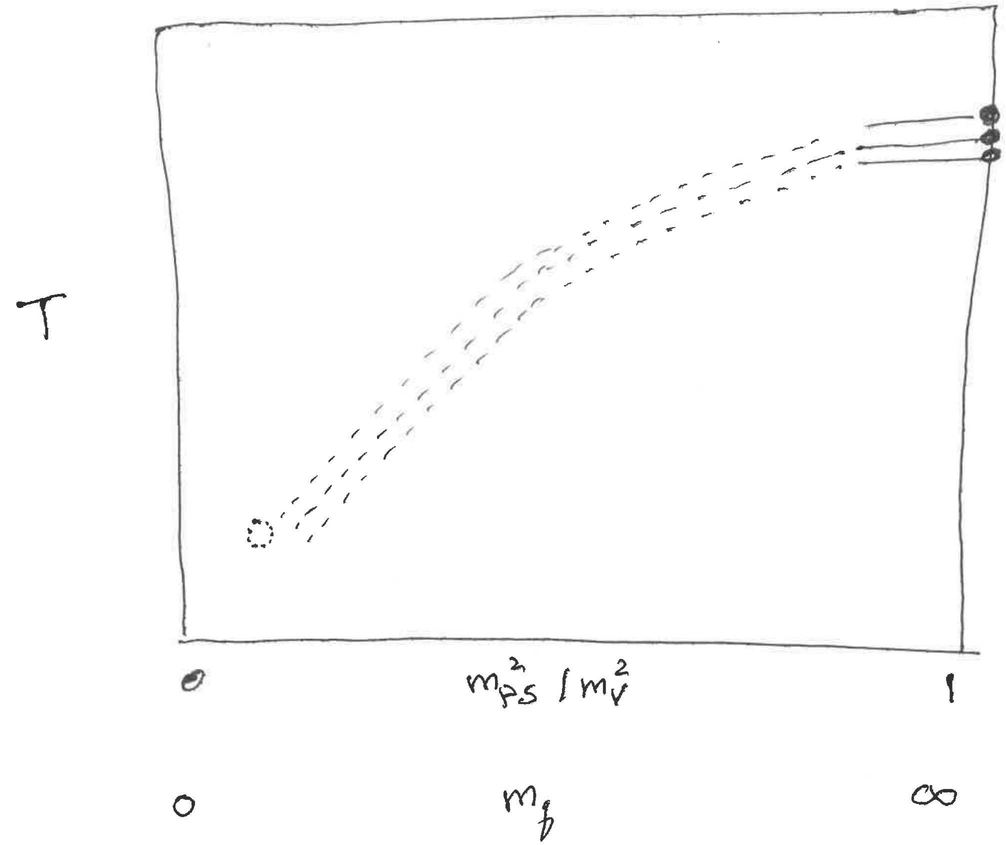
But not for the finite temperature crossover temperature – there are at least three stories



Possibility 1: fermions become irrelevant as $N_c \to \infty$



Possibility 2: Linear sigma model AKA Pisarski-Wilczek: $T_c \propto f_{PS} \propto \sqrt{N_c}$



Possibility 3: Same meson spectrum equals same Hagedorn temperature, same crossover temperature

The observable – the temperature-dependent condensate

So I did a test...(nHYP clover fermons, blah)

Wilson fermion condensate is awkward, instead measure difference ("finite temperature condensate")

$$\Sigma(T) = \langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle_{T} - \langle\bar{\psi}\psi\rangle_{T=0} \quad (1)$$

Dimensionless, rescaled ($\Sigma \propto N_c$) version is the integrated pseudoscalar correlator

$$\frac{3}{N_c} t_0^{3/2} \Sigma(T) = \frac{3}{N_c} t_0^{3/2} \times m_q(\Delta_{PP}(T) - \Delta_{PP}(T=0)) \quad (2)$$

with some lattice to continuum (tadpole improvement) rescaling

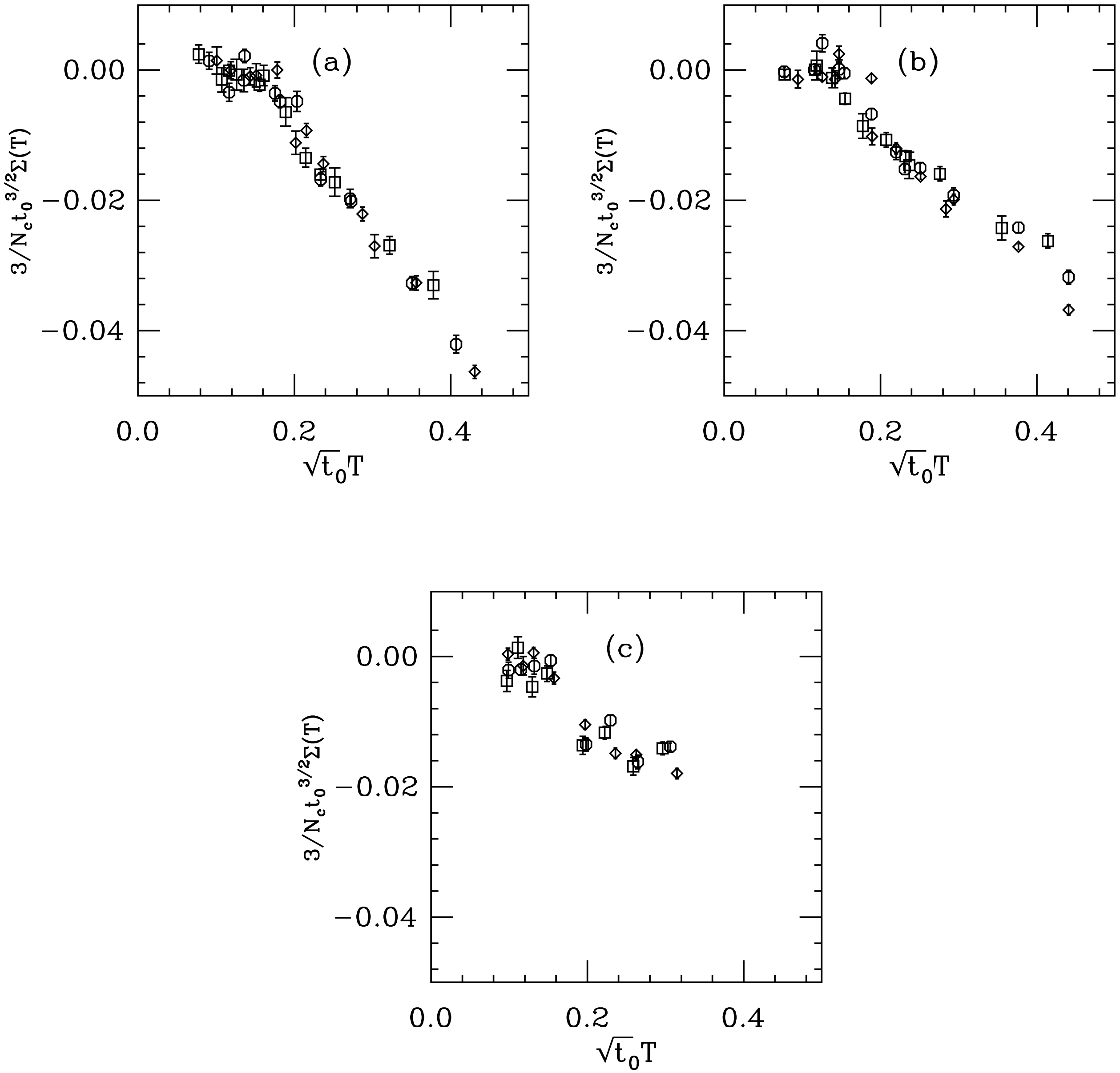
$$\Delta_{PP}(T) = \hat{\Delta}_{PP}(N_t)(1 - \frac{3\kappa}{4\kappa_c})^2. \quad (3)$$

and the lattice correlator is

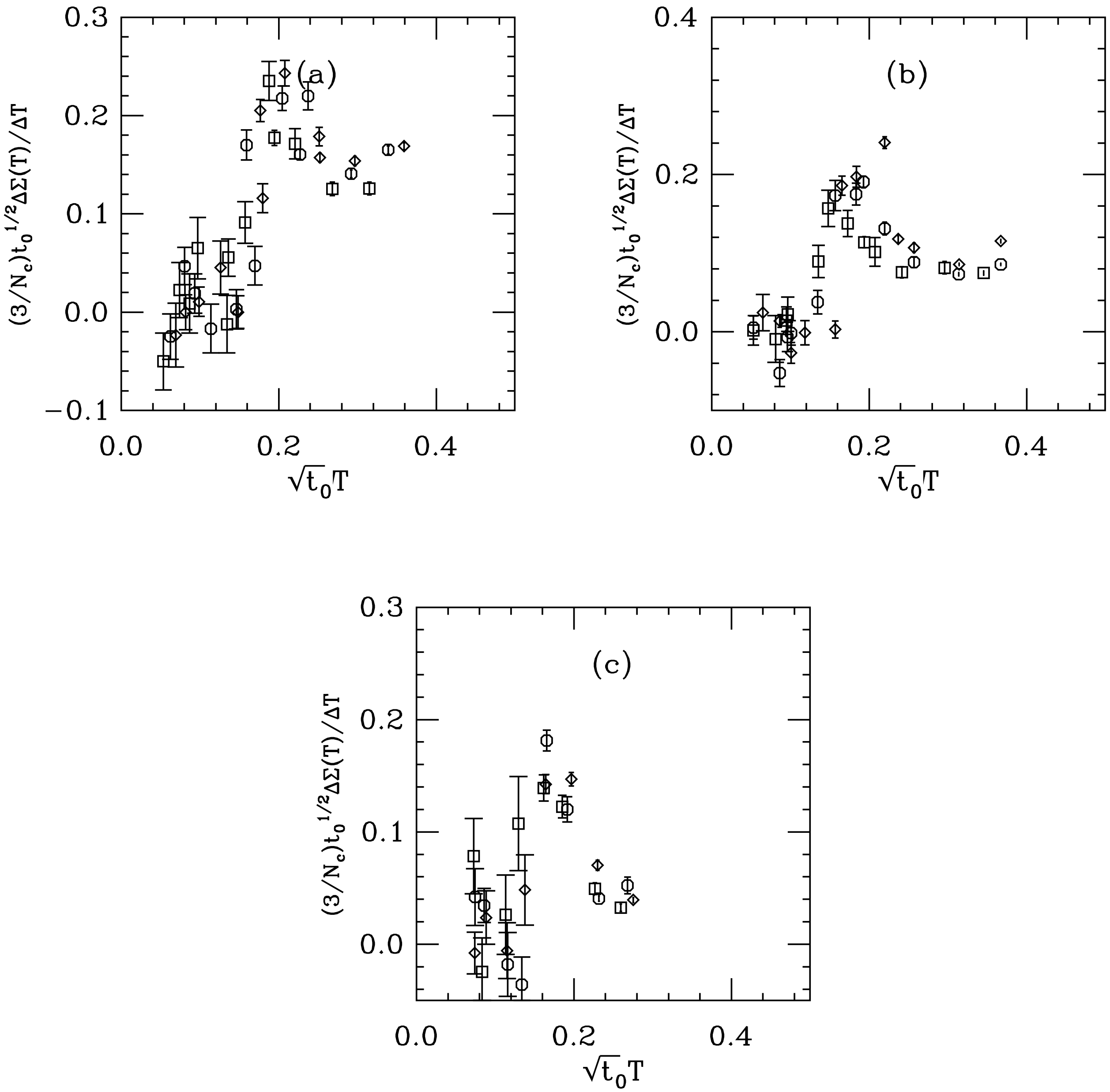
$$\hat{\Delta}_{PP}(N_t) = \sum_{t=0}^{N_t} \sum_{x} \langle P(x,t)P(0,0)\rangle \ . \quad (4)$$

where $P(x,t) = \bar{\psi}(x,t)\gamma_5\psi(x,t)$

($\Sigma = m_{PS}^2 f_{PS}^2/(4m_q)$, $\langle 0|P|PS\rangle = m_{PS}^2 f_{PS}/(2m_q)$)



Temperature dependent condensate, 3 different $(m_{PS}/m_V)^2$ values, squares, octagons, and diamonds label $N_c = 3$, 4, and 5.



Rescaled $d\Sigma/dT$, 3 different $(m_{PS}/m_V)^2$ values, squares, octagons, and diamonds label $N_c = 3$, 4, and 5.

How dull can things be?

Across N_c, at intermediate fermion masses ($(m_{PS}/m_V)^2 \sim 0.63$, 0.5, 0.25) I saw

- $\Sigma(T)$ shows broad featureless crossover, temperature independent of N_c
- $d\Sigma(T)/dT$ shows broad bump, peak at temperature independent of N_c
- This T_c is about halfway in between the quenched value and the low quark mass value

And no sign of anything first order, either (at least, where I simulated)

Hmm...

No first order transition?

- Apparently, fermions are still sometimes important degrees of freedom at large N_c!

What about Pisarski - Wilczek?

- Pions in QCD NOT described by linear sigma model (G & L 1984)
- Critical temperature isn't universal, only critical exponents are universal

What about a connection with the spectrum?

- Can't test Hagedorn directly (yet)
- But basically all $\bar{\psi}\Gamma\psi$ mesons (S-wave and P-wave mesons) have an N_c-independent spectrum
- $f_{PS} \propto \sqrt{N_c}$ means that $A(\pi\pi \to \pi\pi) \propto 1/N_c$: pions (like other hadrons) don't interact at large N_c
- Large N_c scaling says $\langle\gamma|\rho\rangle \propto \sqrt{N_c}$ and vector dominance says $g_{\rho\pi\pi} \propto 1/\langle\gamma|\rho\rangle \propto 1\sqrt{N_c}$ so again hadrons don't interact at large N_c
- So what else is left but the density of states?
- A test: compare trace anomaly $(\epsilon - 3P)/T^4$ for $T < T_c$ to Hadron Resonance Gas
- All N_c's should match

Summary

- My numerics were lousy but the effect was so obvious it didn't matter
- Maybe very interesting to do large N_c thermo "right"?
  - "right" means staggered fermions
  - Most continuum QGP phenomenology (ex. AdS/CFT) is large N_c based

If you are a "lost" phenomenologist, my advice is, you can trust large N_c scaling for your project