An exploration of sphaleron rate

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with

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Work done at Bielefeld University

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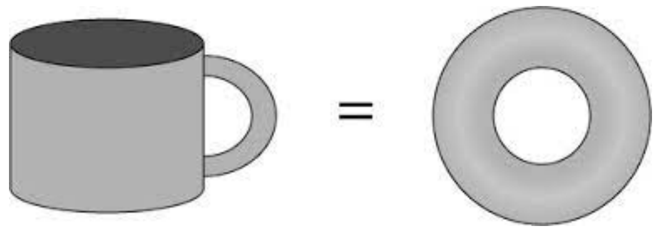
Topology

- Topology studies the general properties of objects



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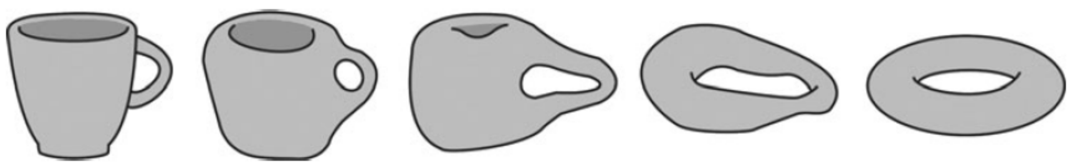
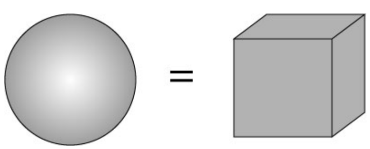
- Two objects are topologically equivalent if there exists a continuous, invertible mapping between them

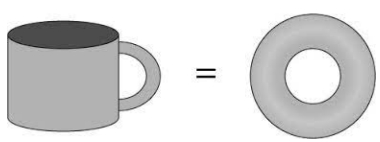


[https://skullsinthestars.com/2010/10/10/twisting-light-into-a-mobius-strip/]

- Objects can be classified by the topological number

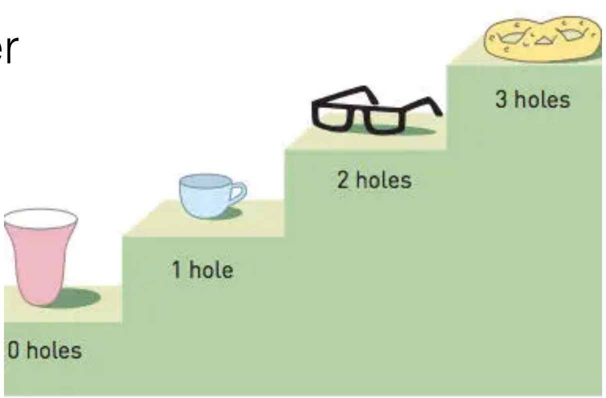


n=0



n=1

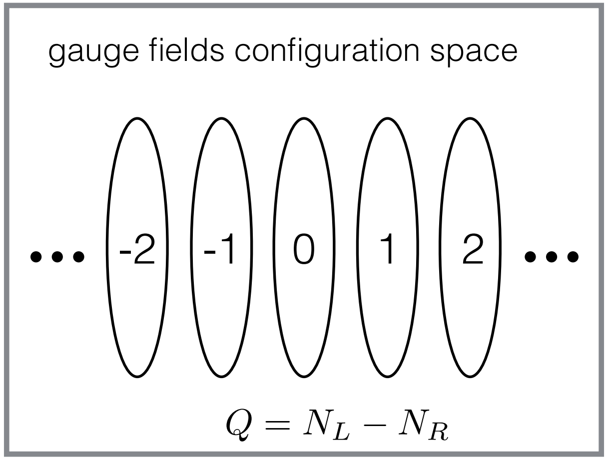
···⇨



Diffusion of topological charge in YM theory

- QCD vacua also have topological structure: energetically degenerate gauge configurations could be topologically distinct!

$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[F_{\mu\nu}(x)F_{\rho\sigma}(x)] \qquad Q = \int \mathrm{d}^4x\ q(x)$$



- Thermal activation between different topological sectors characterized by sphaleron rate

$$\Gamma_{\mathrm{sphal}} \equiv \lim_{t\to\infty}\frac{\Big(Q(t)-Q(0)\Big)^2}{Vt}$$

- Analytic continuation from Minkowski space to Euclidean space

$$\rho_{qq}(\omega) = \mathrm{Im}G^R_{qq}(\omega,\vec{k}=0) \quad \Rightarrow \quad G_{qq}(\tau) = \int \mathrm{d}\vec{x}\ \langle q(\vec{x},\tau)q(\vec{0},0)\rangle = \int \frac{d\omega}{\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_{qq}(\omega)$$

$$\Gamma_{\mathrm{sphal}}(T) = \lim_{\omega\to 0} 2T\frac{\rho_{qq}}{\omega}$$

- Sphaleron rate also establishes how fast the axial light quark number equilibrates

$$\partial_\mu J^\mu_5 = 2N_f q(x) \ \Rightarrow\ \frac{\mathrm{d}Q_5}{\mathrm{d}t} = -Q_5\frac{(2N_f)^2}{\chi_Q}\frac{\Gamma_{\mathrm{sphal}}}{2T} \qquad Q_5 = \sum_f \int \mathrm{d}^4x\ J^\mu_{5,f}$$

Gradient flow

- A diffusion equation along flow time *t* towards the stationary point of the action:

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} \sim -\frac{\delta S_G[B_\mu(x,t)]}{\delta B_\mu(x,t)} \sim D_\nu G_{\nu\mu}(x,t) \qquad B_\nu(x,t)|_{t=0} = A_\nu(x)$$

- LO solution shows a "smearing radius":

[Luscher & Weisz, JHEP1102(2011)051]
[Narayanan & Neuberger, JHEP0603(2006)064]

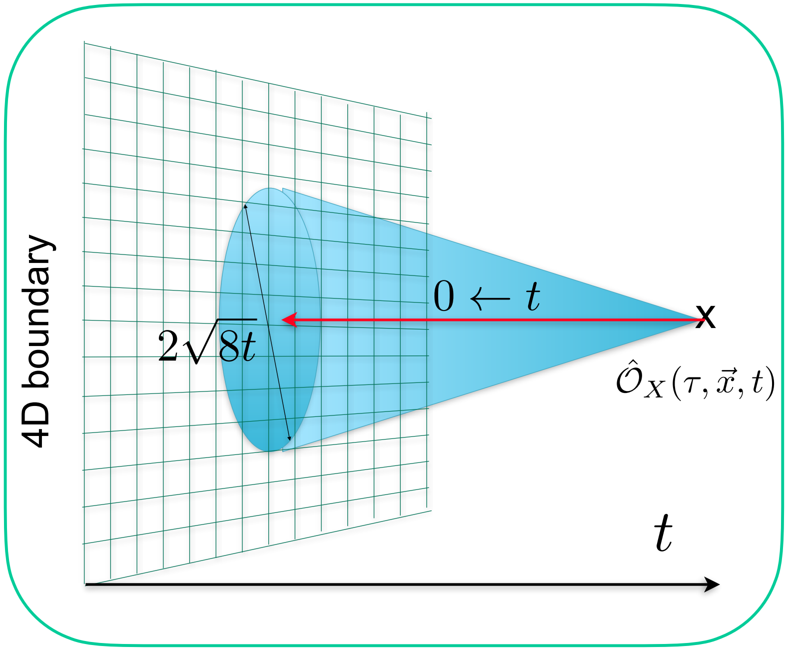
$$B_\nu^{\mathrm{LO}}(x,t) = \int \mathrm{d}y(\sqrt{2\pi}\sqrt{8t}/2)^{-4} \exp\left(\frac{-(x-y)^2}{\sqrt{8t}^2/2}\right) B_\nu(y)$$

- Small flow time expansion of operators:

$$\mathcal{O}(x,t) \xrightarrow{t\to 0} \sum_k c_k(t)\mathcal{O}_k^R(x)$$

- Applications: running coupling / defining operators / scale setting / noise reduction / …

- When working with gradient flow…
  - Must first $a\to 0$ and then $t\to 0$
  - Spectral reconstruction only valid on double extrapolated correlators



Lattice setup

β	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	#confs.
6.8736	0.026 (7.496)	64	16	1.50	10000
7.0350	0.022 (9.119)	80	20	1.50	10000
7.1920	0.018 (11.19)	96	24	1.50	10000
7.3940	0.014 (14.21)	120	30	1.50	10000
7.5440	0.012 (17.01)	144	36	1.50	10000

- Quenched approximation on large, fine, isotropic lattices
- Standard Wilson gauge action
- Large statistics
- Intensive discrete flow times
- Scale set by Sommer parameter

High precision for reliable $a \to 0$ extrapolation and $t \to 0$ extrapolation !

Construct the topological charge operator

- Gluonic definition of the topological charge density (cheaper than calculating the left/right-hand zero modes of the overlap Dirac operator)

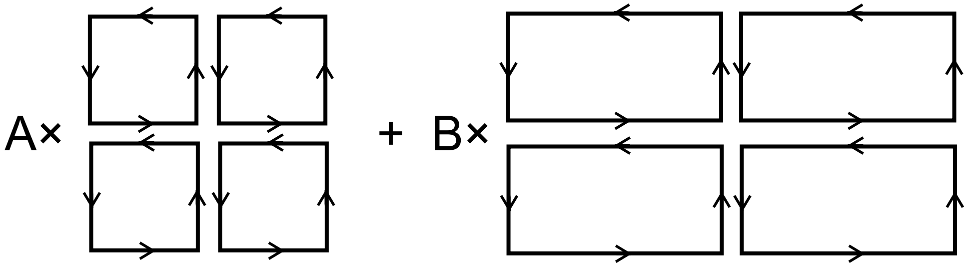
$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[F_{\mu\nu}(x)F_{\rho\sigma}(x)]$$

for comparison of different definitions, see e.g. [C. Alexandrou, et al, EPJC80(2020)5,424]

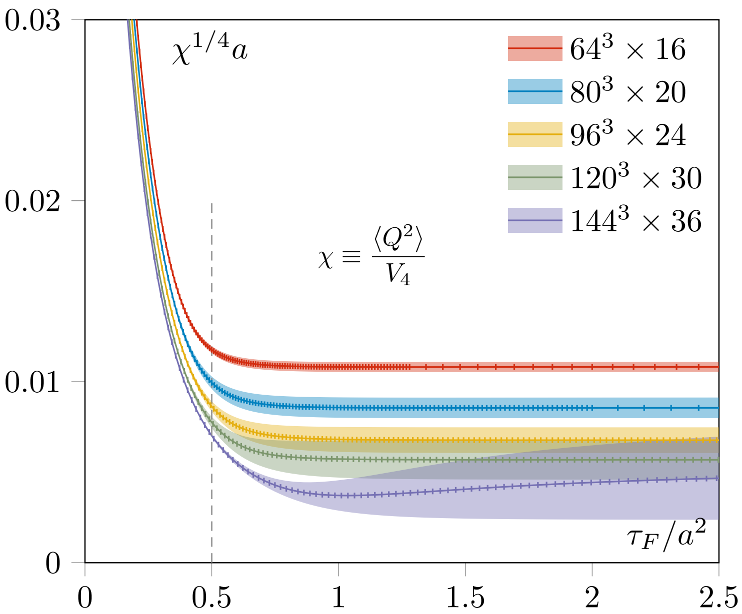
- Improved discretization of the field strength tensor operator

$$F^{\mathrm{Imp}}_{\mu\nu}(n) \equiv \frac{5}{3}C^{(1,1)}_{\mu\nu}(n) - \frac{1}{3}C^{(1,2)}_{\mu\nu}(n) = a^2(F_{\mu\nu}(x) + \mathcal{O}(a^4))$$

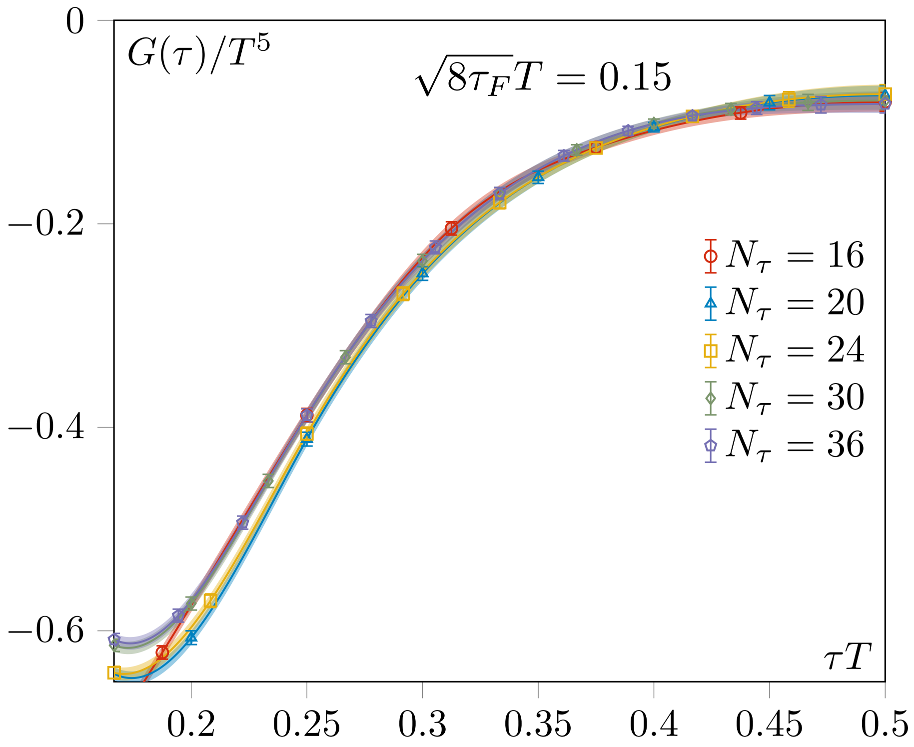
[M. G. Perez, et al., NPB 413 (1994) 535-552]
[G. D. Moore, PRD59,014503]
[S. O. Bilson-Thompson, et al, Annals Phys.304,1(2003)]

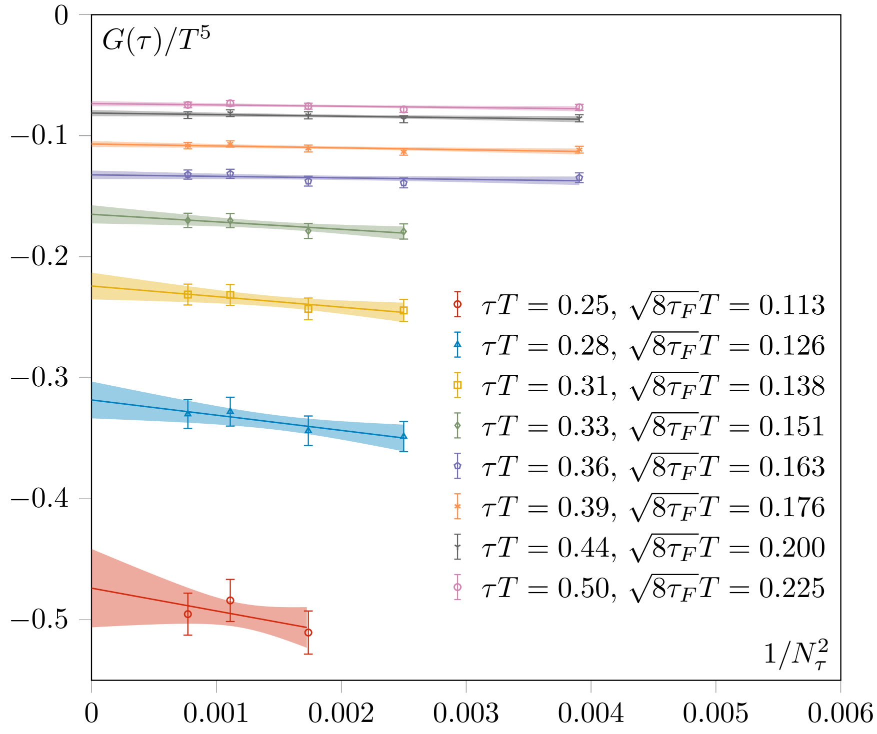


- Still not enough to capture accurate topology
➞ apply gradient flow for further smearing
  - Contributions from higher order operators removed after $\tau_F/a^2 = 0.5$
  - Set a baseline for flow time



Continuum extrapolation



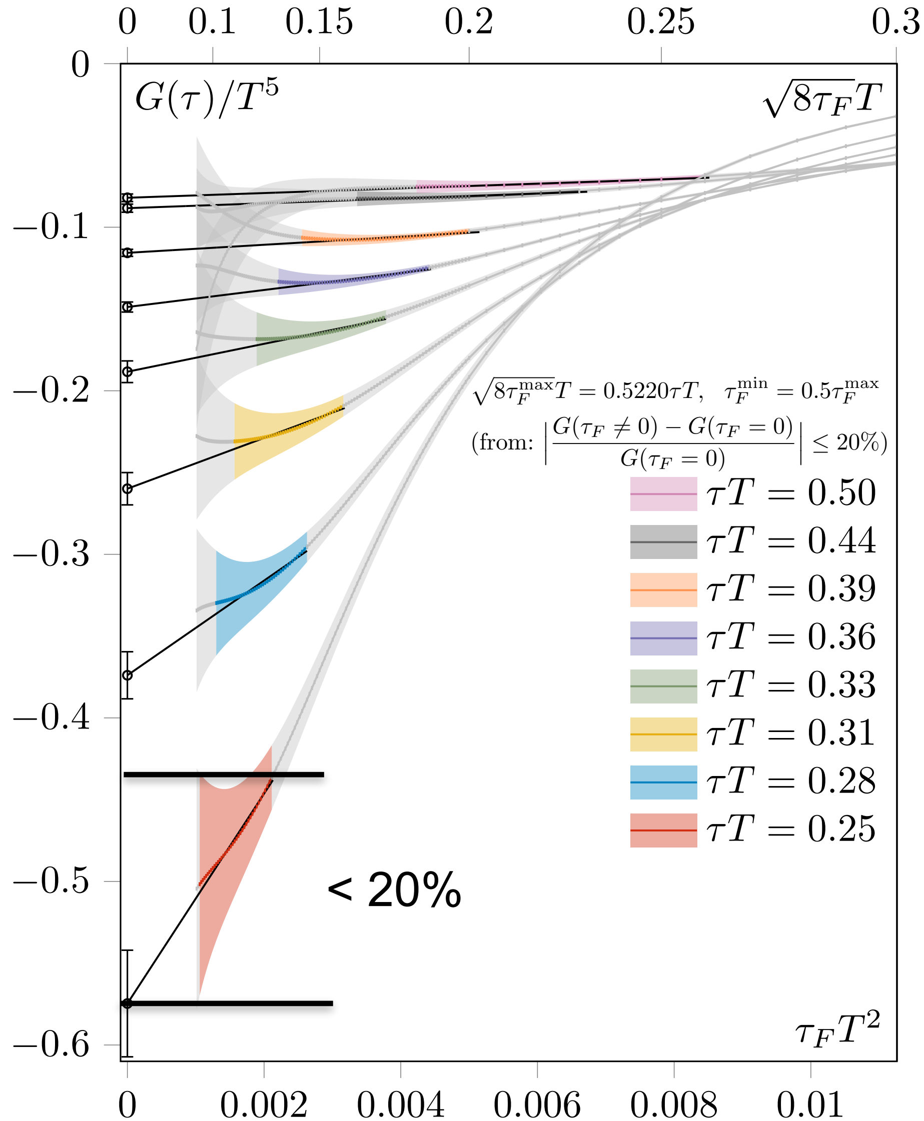


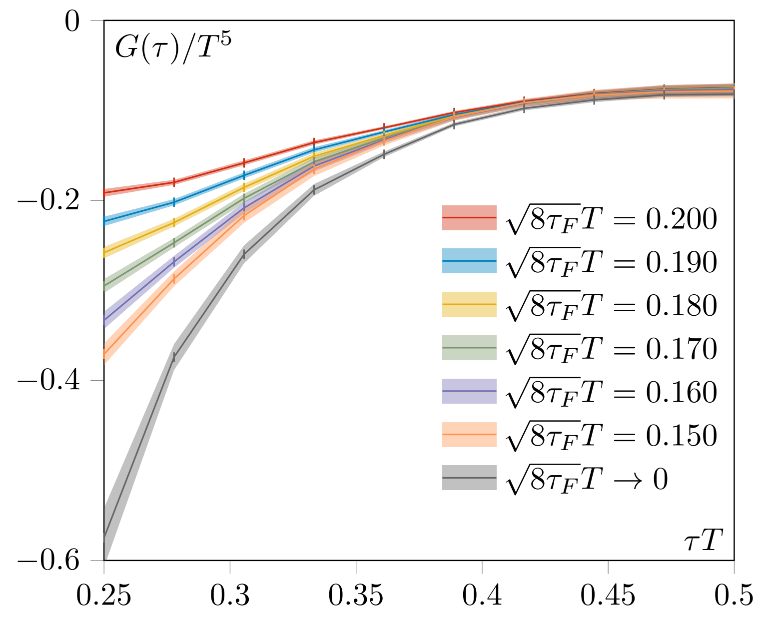
- Correlators on different lattices overlap

  ➙ Lattice cutoff effects are small under gradient flow
- Well controlled continuum extrapolation with ansatz respecting the lattice action

$$\frac{G_{\tau,\tau_F}(N_\tau)}{T^5} = m \cdot N_\tau^{-2} + b$$

Flow time extrapolation





- Minimum flow time as half the maximum one
- Linear flow time extrapolation applies

$$\frac{G_\tau(\tau_F)}{T^5} = c \cdot \tau_F T^2 + d$$

- Final correlators respect reflection positivity

[E. Vicari, Nucl. Phys. B554, 301(1999)]

Spectral function reconstruction

- Extract the spectral function from the correlators via kubo formula:

$$G_{qq}(\tau) = \int \mathrm{d}\vec{x}\ \langle q(\vec{x},\tau) q(\vec{0},0)\rangle = \int \frac{d\omega}{\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/2T)} \rho_{qq}(\omega)$$

- Large frequency part of the spectral function is available up to NLO:

$$\rho^{\mathrm{LO}}(\omega) = \frac{d_A c_\chi^2 \omega^4 g^4}{\pi} \coth\left(\frac{\omega}{4T}\right) \qquad \rho^{\mathrm{NLO}}(\omega) = \rho^{\mathrm{LO}}(\omega) + \frac{d_A c_\chi^2 \omega^4}{\pi} \coth\left(\frac{\omega}{4T}\right) \frac{g^6 N_c}{(4\pi)^2} \left[\frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\phi_T(\omega)\right]$$

  - uncertainty of fixing the renormalization point
  - the perturbative series not rapidly converging

[M. Laine, et al, JHEP 09, 084 (2011)]

- Lack of knowledge for the small frequency part ➙ try different possibilities:

$$\mathrm{M1}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} \delta\left(\frac{\omega}{T}\right) + B \frac{\rho^{\mathrm{LO}}(\omega)}{\omega T^3}$$

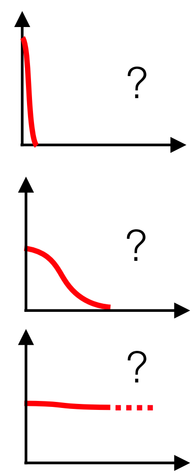
$$\mathrm{M2}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} \frac{CT^2}{C^2T^2 + \omega^2} + B \frac{\rho^{\mathrm{LO}}(\omega)}{\omega T^3}$$

$$\mathrm{M3}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} + B \frac{\rho^{\mathrm{LO}}(\omega)}{\omega T^3}$$

$$\mathrm{M4}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} \delta\left(\frac{\omega}{T}\right) + B \frac{\rho^{\mathrm{NLO}}(\omega)}{\omega T^3}$$

$$\mathrm{M5}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} \frac{CT^2}{C^2T^2 + \omega^2} + B \frac{\rho^{\mathrm{NLO}}(\omega)}{\omega T^3}$$

$$\mathrm{M6}: \frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^4} + B \frac{\rho^{\mathrm{NLO}}(\omega)}{\omega T^3}.$$



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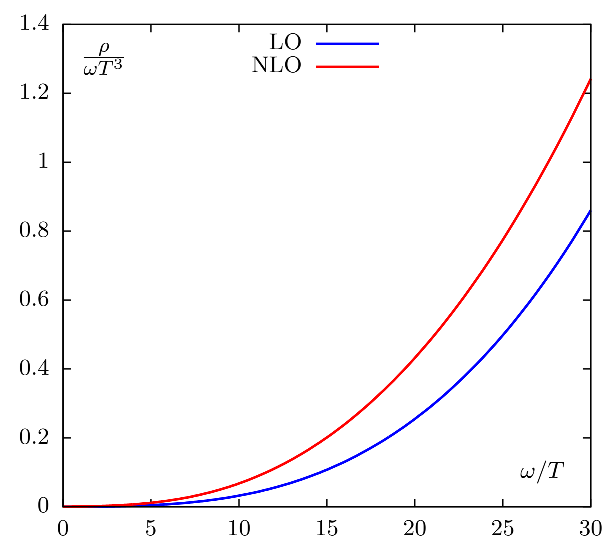
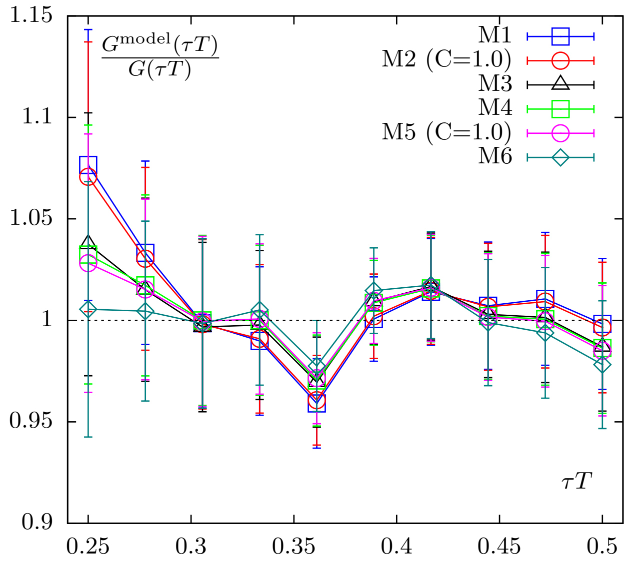
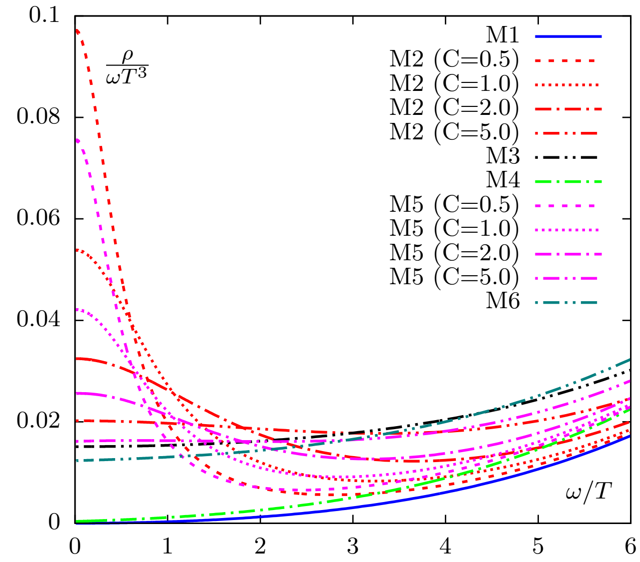
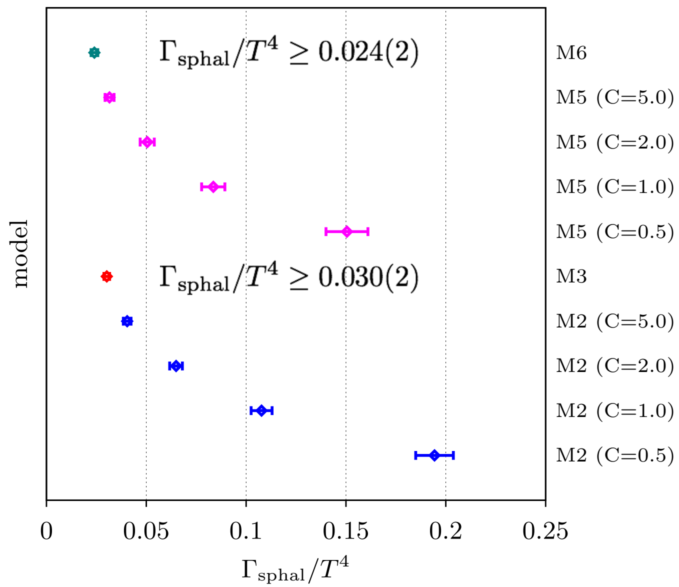


Sphaleron rate from modeling







- Models agree with data within errors
  ➙ all different models are possible
- Lower bound for the sphaleron rate can be obtained
- Need theoretical guidance for the transport peak to further constrain the sphaleron rate

Conclusion and outlook

✳ Gradient flow is introduced to study sphaleron rate on the lattice

✳ Gradient flow helps to capture the correct topology in quenched approximation

✳ Gradient flow helps to reduce the noise of topological charge density correlators

✳ A methodology of using gradient flow properly has been developed to study transport coefficients on the lattice

✳ Extend to full QCD using large and fine 2+1-flavor HISQ lattices

$\Lambda_{\overline{\rm MS}}\big|_{N_f=0} \approx 255{\rm MeV}$

$T_c\big|_{N_f=0} \approx 1.24\Lambda_{\overline{\rm MS}}\big|_{N_f=0}$

$\alpha_s^{\rm EQCD}\big|_{T\simeq T_c} \simeq 0.2$

$1^{\rm st}$ order deconfinement transition

$\Lambda_{\overline{\rm MS}}\big|_{N_f=3} \approx 340{\rm MeV}$

$T_c\big|_{N_f=3} \approx 0.45\Lambda_{\overline{\rm MS}}\big|_{N_f=3}$

$\alpha_s^{\rm EQCD}\big|_{T\simeq T_c} > 0.3$

chiral crossover

Backup:Previous studies for sphaleron rate

- Precise determination at weak-coupling limit in QCD:

  $$\Gamma_{\rm sphal} \propto \alpha_s^5 T^4$$ **[G.D. Moore and M. Tassler, JHEP02,105(2011)]**

- "Desperate extrapolation" to finite coupling:

  $$\Gamma_{\rm sphal} \sim 40\alpha_s^4 T^4 \quad (\text{at } \alpha_s = 0.3)$$ **[G.D. Moore and M. Tassler, JHEP02,105(2011)]**

  - Heavy ion physics relevant coupling corresponds to lattices too coarse for topology study
  - $\mathcal{O}$(50%) systematic corrections are expected

- Thermal $\mathcal{N}$= 4 SU(N) supersymmetric Yang-Mills theory:

  $$\Gamma_{\rm sphal} = \frac{(g^2N)^2}{256\pi^3} T^4$$ **[D.T. Son and A.O. Starinets JHEP09(2002)042]**

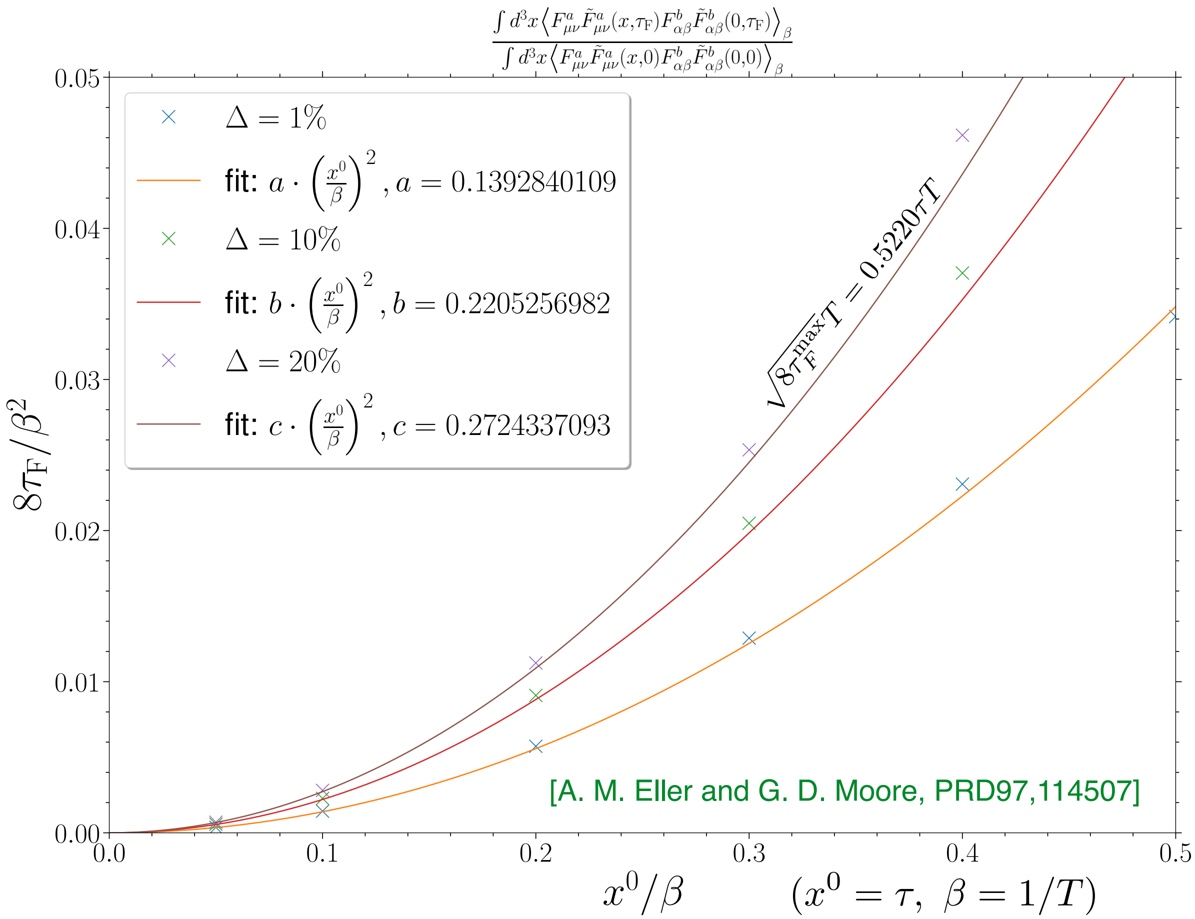
- Improved Holographic QCD (large-N SYM theory):

  $$1.64 \leq \Gamma_{\rm sphal}/T_c^4 \leq 2.8$$ **[U. Gürsoy, et al, 10.1007/JHEP02(2013)119]**

- SU(3) gluodynamics on the lattice using gradient flow:

  $$\Gamma_{\rm sphal}/T^4 = 0.12(3) \text{ at } T = 1.24T_c$$ **[A. Yu. Kotov, JETP Letters 108, 352-355(2018)]**

Backup: Range of available flow time from pQCD



- Maximum available flow time determined by at most 20% deviation of flowed correlators to unflowed ones in LO calculations

Backup: working with gradient flow

- Order of extrapolation is important: first $a \to 0$ and then $t \to 0$; opposite order runs into lattice-spacing issues

corrections	taken care by
a^2/τ^2	$a \to 0$
a^2/t	$a \to 0$
t/τ^2	$t \to 0$

- Spectral reconstruction only valid on double extrapolated correlators

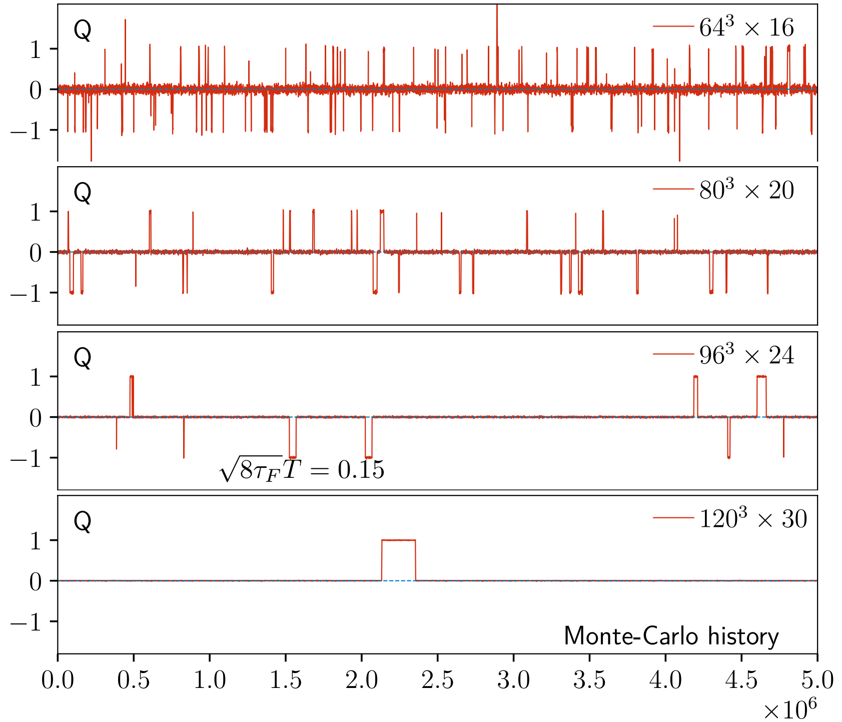
$$\rho_{EE}^{\mathrm{LO}}(\omega, t \neq 0) = \rho_{EE}^{\mathrm{LO}}(\omega, t = 0) \qquad G_{EE}(t \neq 0) \neq G_{EE}(t = 0)$$

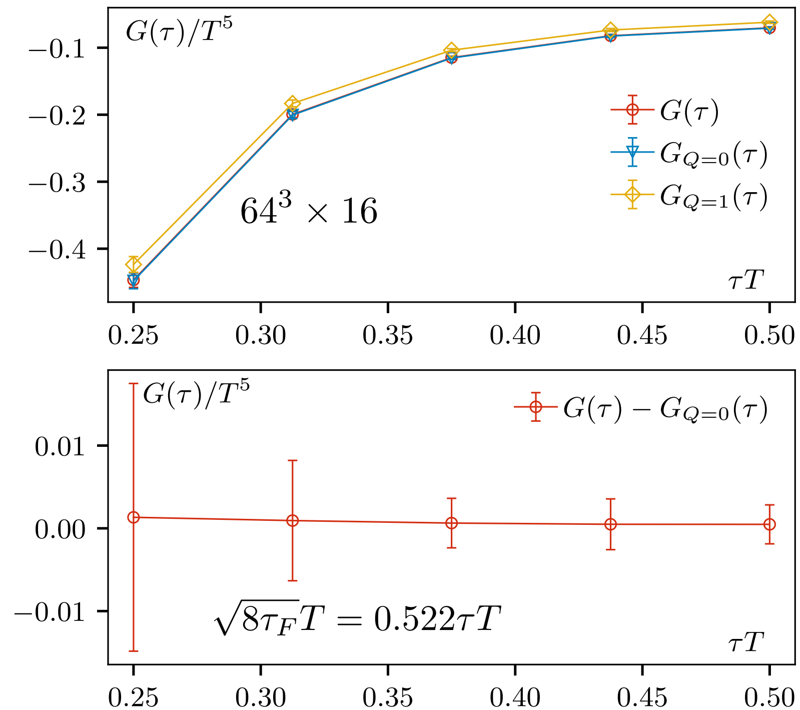
[L. Altenkort, A. M. Eller et al., PRD103(2021)1,014511]

$$G_X(\tau, \vec{p}) = \int d^3\vec{x}\; e^{-i\vec{p}\cdot\vec{x}} \langle \hat{\mathcal{O}}_X(\tau, \vec{x}) \hat{\mathcal{O}}_X(0, \vec{0}) \rangle = \int_0^\infty \frac{d\omega}{2\pi} K(\omega, \tau, T) \rho_X(\omega, \vec{p})$$

only holds at t=0 !

Backup: examine the topology





- Q could return integer number now, but topological freezing?
  - Q=0 sector and Q≠0 sectors do not agree
  - But the contribution of Q≠0 sectors is negligible