Lattice QCD in strong magnetic background

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July 28^{th}, 2021

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Introduction I

Why study QCD in a magnetic field?

It affects physics in a variety of systems:

- Early Universe
- Neutron stars
- Non-central ion scattering

Introduction II

Magnetic field interacts with charged fermions

$$\overline{\psi}^f(x) D^f_\mu(x) \psi^f(x) = \overline{\psi}^f(x) \left(\partial_\mu + igG^a_\mu(x) T^a + iq^f A_\mu(x) + m^f \right) \psi^f(x)$$

The discretization can be performed through the following substitution in the usual LQCD quark action

$$U_{i;\mu} \rightarrow u^f_{i;\mu} U_{i;\mu}$$

$$u^f_{i;y} = e^{ia^2 q_f B_z \, i_x}, \qquad u^f_{i;x}\big|_{i_x = L_x} = e^{-ia^2 q_f L_x B_z i_y}$$

$$eB = \frac{6\pi b_z}{a^2 L_x L_y}, \quad \text{with} \quad b_z \in \mathbb{Z} \quad \text{and} \quad b_z < \frac{L_x L_y}{6}.$$

String tension

The QCD potential between a $Q\overline{Q}$ pair placed at distance r can be parameterized as

$$V(r) = V_0 + \frac{\alpha}{r} + \sigma r$$

σ is called **string tension**.
We can access this observable via quark-antiquark correlation function

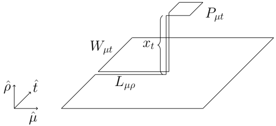
$$\left\langle \mathcal{O}^\dagger(0)\mathcal{O}(r,t) \right\rangle = A\, e^{-V(r)t} \left(1 + \sum_n e^{-\Delta V^{(n)}(r)} \right)$$

In the infinite quark mass limit this correlator can be represented with a Wilson loop, in the rt-plane. With a few manipulations

$$aV(r) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(r, an_t) \rangle}{\langle W(r, an_t + 1)) \rangle} \right)$$

We can then fit $V(r)$ and extract the string tension.

Flux tubes



$$\rho_{conn}^{\mu t}(x_t) = \frac{\left\langle Tr(W(an_\mu, an_t)LP^{\mu t}(x_t)L^\dagger)\right\rangle}{\langle Tr(W)\rangle} - \frac{\left\langle Tr(W(an_\mu, an_t))Tr(P^{\mu t}(x_t))\right\rangle}{3\left\langle Tr(W(an_\mu, an_t))\right\rangle}$$

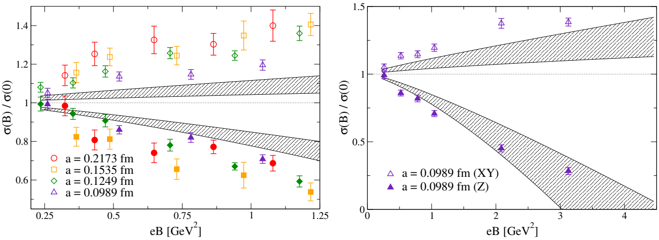
In the continuum limit it can be shown that it reduces to

$$\rho_{conn}^{\mu t} \simeq a^2 g_0 \frac{\left\langle Tr\left[iWLF^{\mu t}L^\dagger\right]\right\rangle}{\langle Tr\ W\rangle}$$

To compute its values gives us access to microscopic informations about chromoelectric field.

$$E_l(d, x_t) = \frac{1}{a^2}\sqrt{\frac{\beta}{6}}\rho_{conn}^{(\mu,t)}(d, x_t)$$

Zero temperature - Previous results I

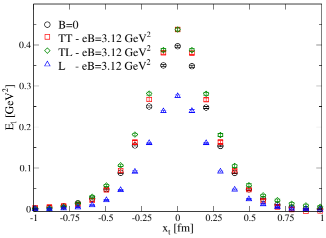


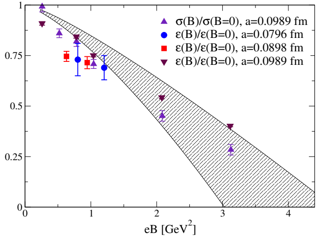
Anisotropy in the string tension is expected to grow with eB

Anisotropic deconfinement could be expected for strong enough fields.

C. Bonati, M. DElia, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo Phys. Rev. D **94**, 094007 (2016)

Zero temperature - Previous results II





We expect to see similar behavior also in flux tubes.

Linear energy density is defined as

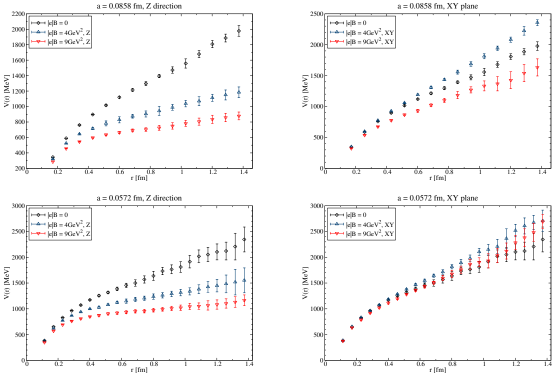
$$\epsilon = \frac{1}{2}\int E_l(d,x)^2 d^2 x_t$$

and it is expected to give results comparable with the string tension

C. Bonati, S. Cal, M. DElia, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo Phys. Rev. D **98**, 054501 (2018).

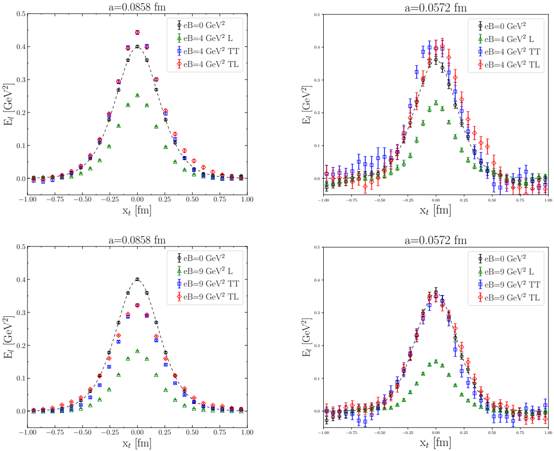
Zero temperature - New Results I

We performed measures at $|e|B = 4GeV^2$ and $|e|B = 9GeV^2$



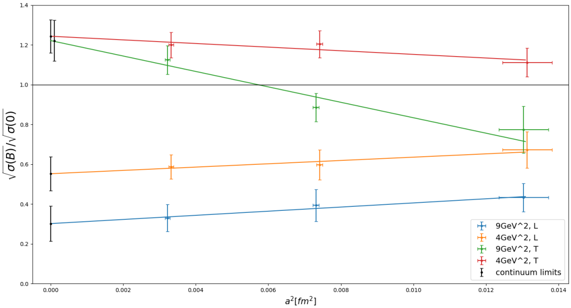
Static potential between a $Q\bar{Q}$ pair, at the two finest lattice spacing explored, along the z direction (left panels) and on the xy plane.

Zero temperature - New Results II



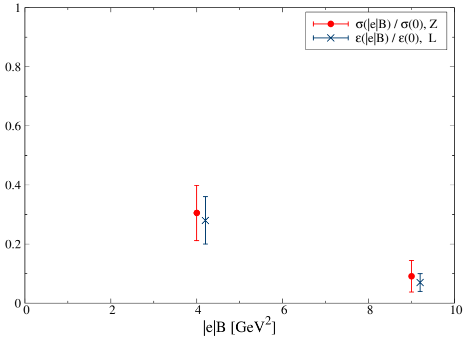
Flux tubes at $|e|B = 4GeV^2$ (upper panels) and $9GeV^2$ (lower panels) for different lattice spacings.

Zero temperature - New Results III



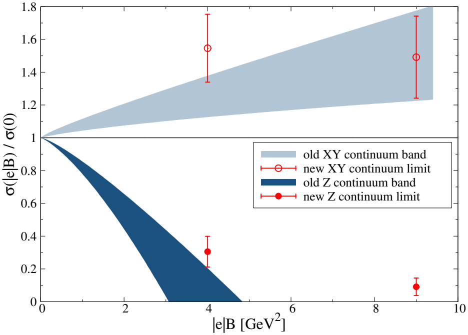
The continuum limits of the string tensions shows the strong dependence of the parameter on the magnetic field in the direction parallel to the magnetic field. On the orthogonal plane the results of the extrapolations are the same within errors.

Zero temperature - New Results IV



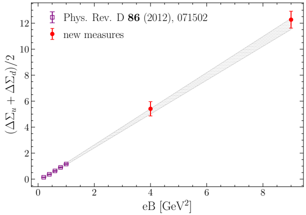
String tension and linear energy continuum extrapolations are in good agreement even for the strongest magnetic field.

Zero temperature - New Results V



The guess for an anisotropic deconfinement at $|e|B \simeq 4GeV^2$ is not confirmed by measurements. The orthogonal string tension behavior seems to be in accordance to previous results, but it is affected by big uncertainties, as already seen.

Zero temperature - New Results VI



$$\Sigma(B,T)_{u,d} = \frac{2m_l}{M_\pi^2 F^2}\left[\overline{\psi}\psi(B,T)_{u,d} - \overline{\psi}\psi(0,0)_{u,d}\right] + 1$$

$$\overline{\psi}\psi(B,T)_f = \frac{T}{V}\frac{\partial \log Z(B,T)}{\partial m_f}$$

Another interesting result at $T=0$ is the chiral condensate behavior. The mean value of the differences $\Delta\Sigma(B,0)_{u,d} = \Sigma(B,0)u,d - \Sigma(0,0)_{u,d}$ preserves linearity in $|e|B$ even at $|e|B = 9GeV^2$.

G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502 (2012) [arXiv:1206.4205 [hep-lat]].

Finite temperature

Methods

We use a fixed lattice spacing framework and vary N_t in order to change temperature, according to

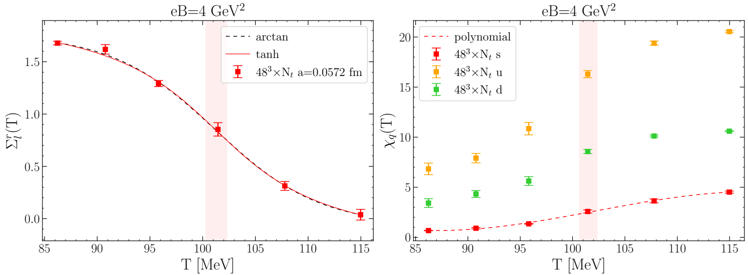
$$T = \frac{1}{aN_t}.$$

Goals

- Locate T_c at the same magnetic fields explored at $T = 0$
- Study the order of the transition

Till now only preliminary results for $|e|B = 4GeV^2$ are available. The $9GeV^2$ simulations are still in progress.

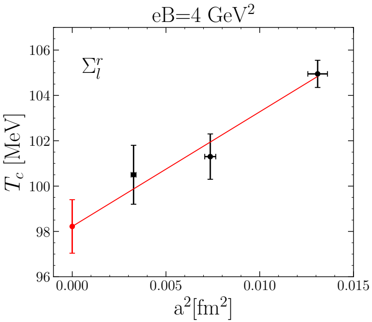
Finite temperature - Preliminary results I



$$\Sigma_l^r(B,T) = \frac{m_l}{M_\pi^4} \sum_{i=u,d} \left(\overline{\psi}\psi(B,T)_i - \overline{\psi}\psi(0,0)_i \right) \qquad \chi_q(T) = \frac{T}{VM_\pi^2} \frac{\partial^2 \log Z(B,T)}{\partial \mu_q^2}\bigg|_{\mu_q=0}$$

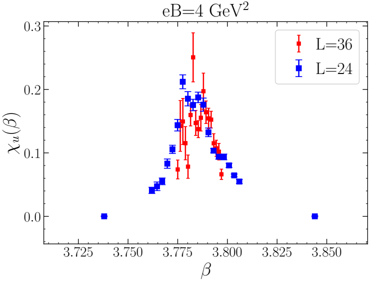
We have measured the chiral condensate and the quark number susceptibility for every lattice spacing and temperature, in order to extract critical temperatures by locating the inflection point of the curves as showed in the plot.

Finite temperature - Preliminary results II



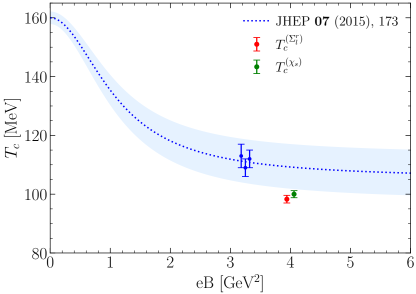
We computed the critical transition temperature at every lattice spacing with the light quarks chiral condensate and the strange quark number susceptibility. We find a drop with respect the $|e|B = 0$ case.

Finite temperature - Preliminary results III



$$\chi_u(\beta) = \frac{\partial^{disc}}{\partial m_f} \overline{\psi}\psi(B, T)$$

We performed a Multi Histogram interpolation of the disconnected chiral susceptibility, varying beta around a point near to the transition temperature. We observe no finite size scaling effects.



We find a critical temperature which is comparable with previous empirical extrapolations.

G. Endrodi, JHEP **1507**, 173 (2015) [arXiv:1504.08280 [hep-lat]].

Summary

$T = 0$:

- The string tension is not vanishing in the magnetic field direction for the $|e|B = 4GeV^2$ case, and it shows a $\sim 1.7\sigma$ tension with 0 in the $|e|B = 9GeV^2$ case;
- Chiral condensate exhibits a linear dependence on the magnetic field even at $|e|B = 9GeV^2$;

Finite T:

- $T_c \simeq 100MeV$ for $|e|B = 4GeV^2$;
- The transition is still a crossover;
- $|e|B = 9GeV^2$ simulations are still in progress.

Thanks for your attention!

Please ask your questions now, or in Gather at the end of the section.

Anyway you can contact me with an email at:

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