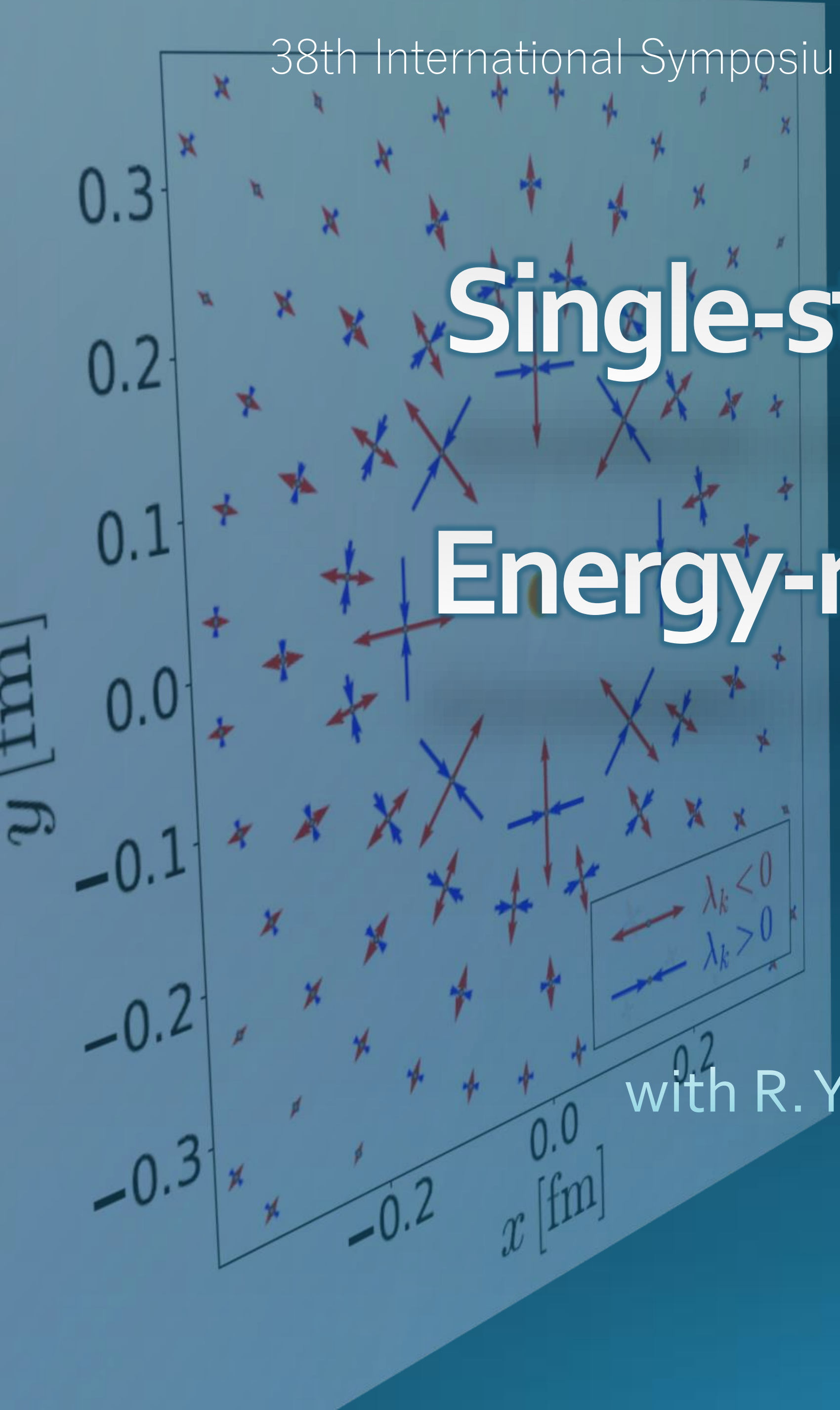
38th International Symposium on Lattice Field Theory, Jul. 27, 2021



Single-static-quark system above T_c investigated by Energy-momentum tensor in SU(3) Yang-Mills theory

Masakiyo Kitazawa
(Osaka U.)
with R. Yanagihara, M. Asakawa, T. Hatsuda
FlowQCD Collaboration

Phys. Rev. **D102**, 114522 (2020)

Static Charge

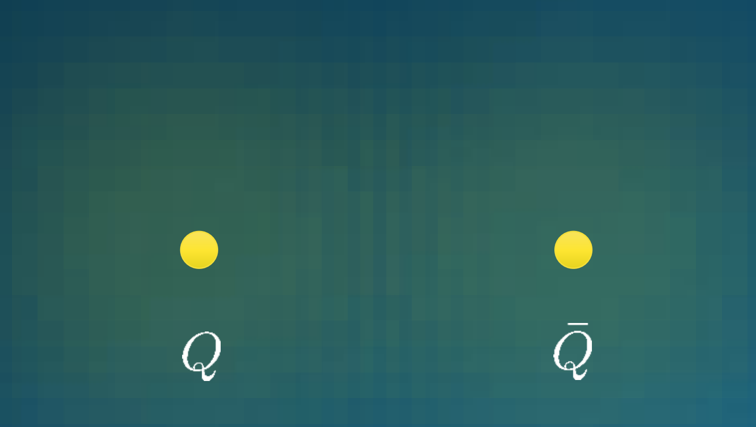
Fundamental tool to study field theories

Static Charge

Fundamental tool to study field theories

□ $Q\bar{Q}$

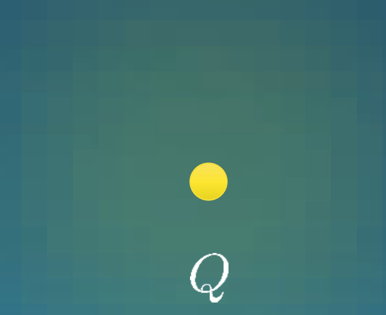
- Flux tube formation



This study

□ Single Q

- Heavy-light meson @ $T<T_c$
- Debye screening @ $T>T_c$



Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

stress

- The most fundamental quantity in physics.
- All components are important quantities.

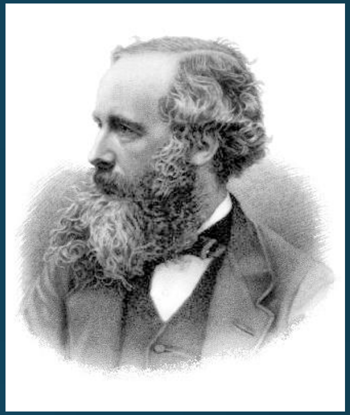
EMT in Static Q systems

Combine 2 fundamental tools!

Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

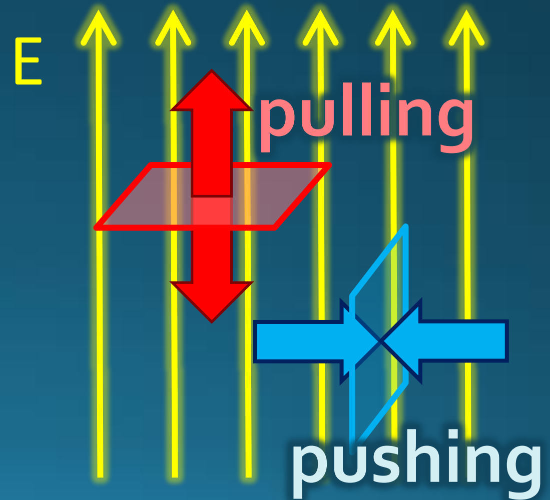


Maxwell

$$\vec{E} = (E, 0, 0)$$

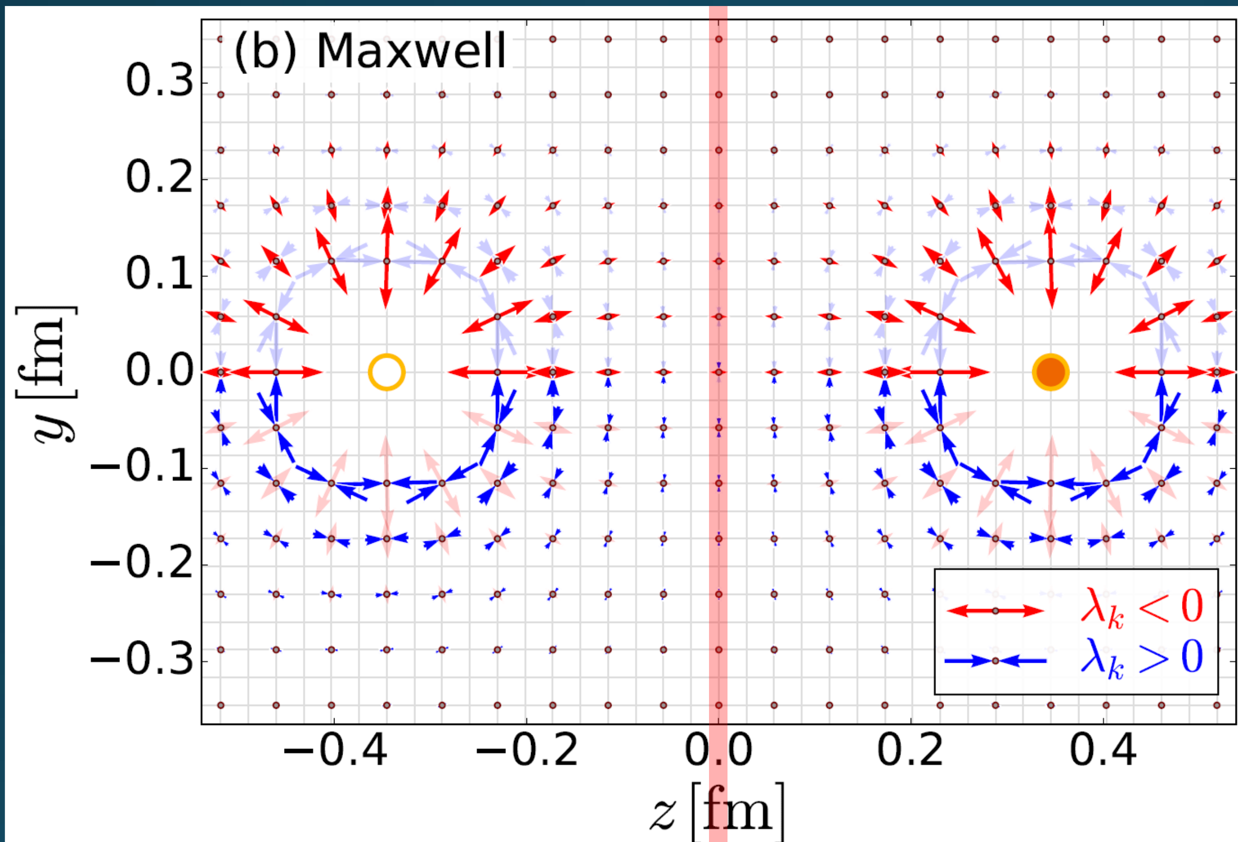
$$T_{ij} = \frac{1}{2}\begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

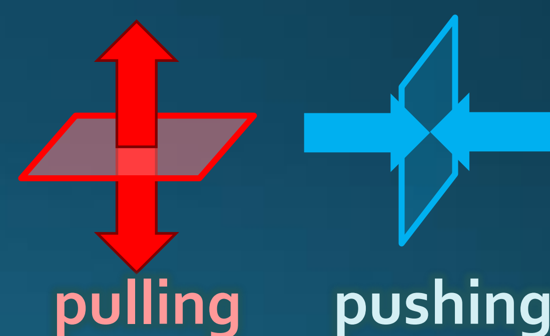
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

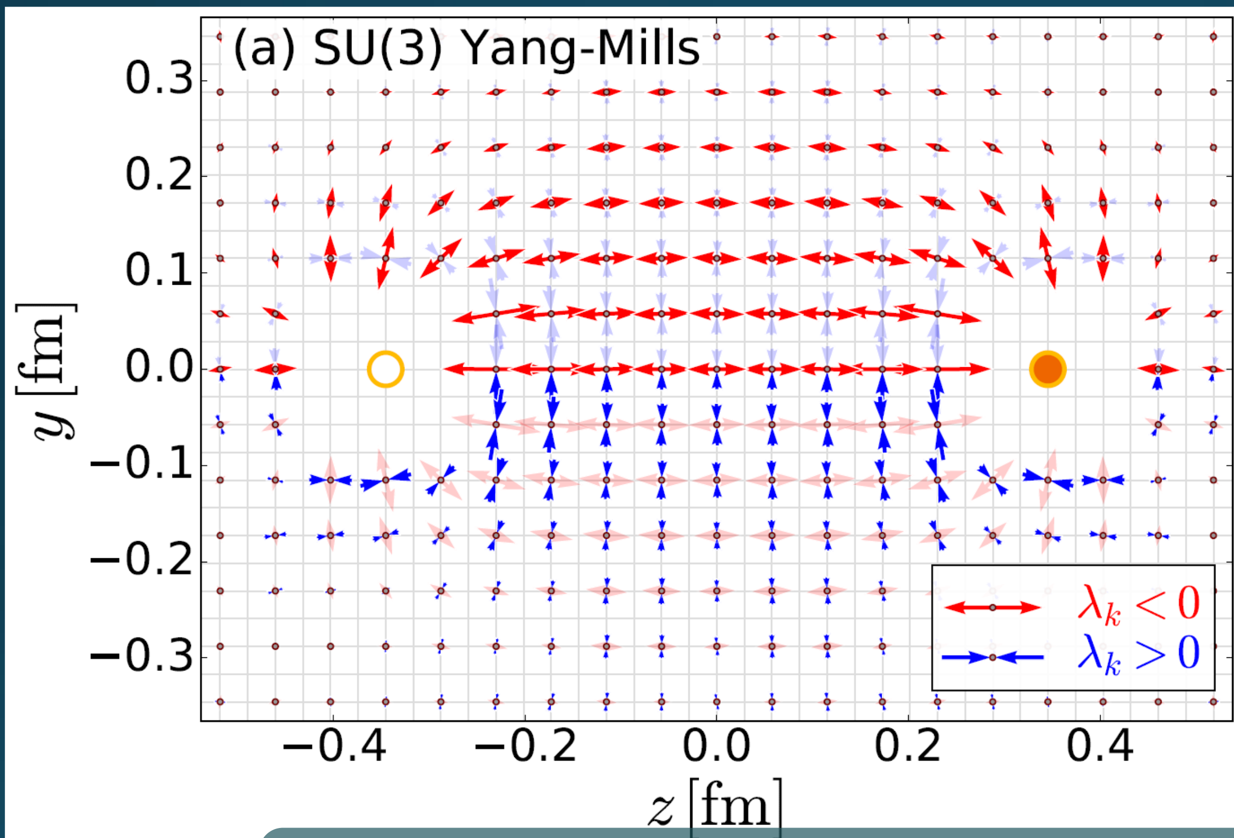


Definite physical meaning

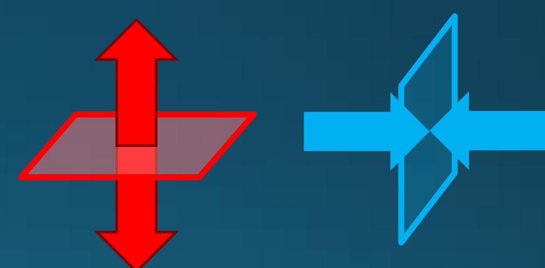
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Stress Tensor in $Q\bar{Q}$ System

FlowQCD, PLB (2019)



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0



pulling **pushing**

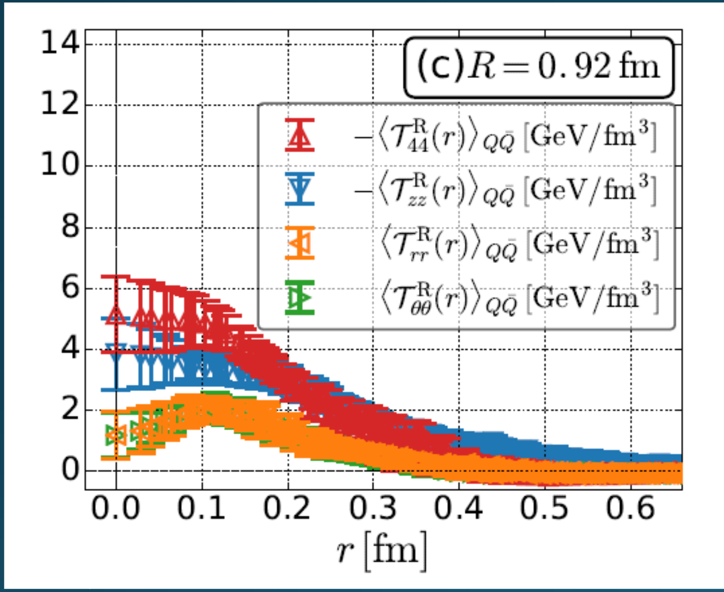
Definite physical meaning

- ☐ Distortion of field, line of field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

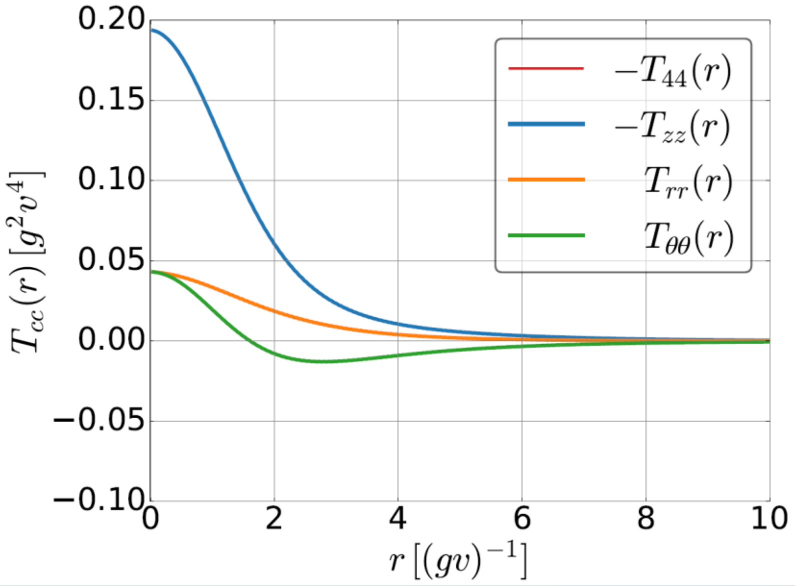
EMT in Abelian-Higgs Model

Yanagihara, MK (2019)

☐ Lattice



☐ AH model

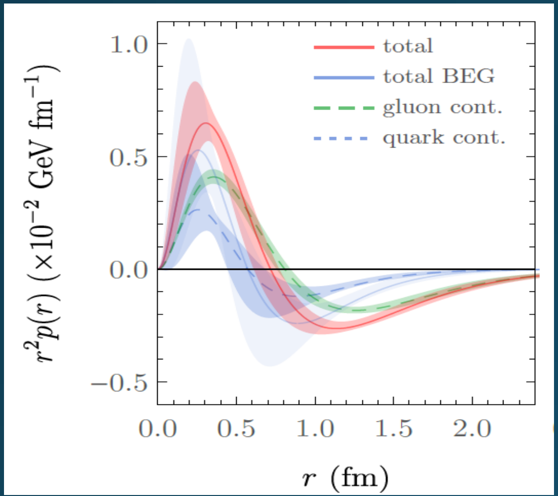


☐ EMT has 4 components / conservation law

➡ Insight into detailed property of the system

EMT Distribution Inside Hadrons

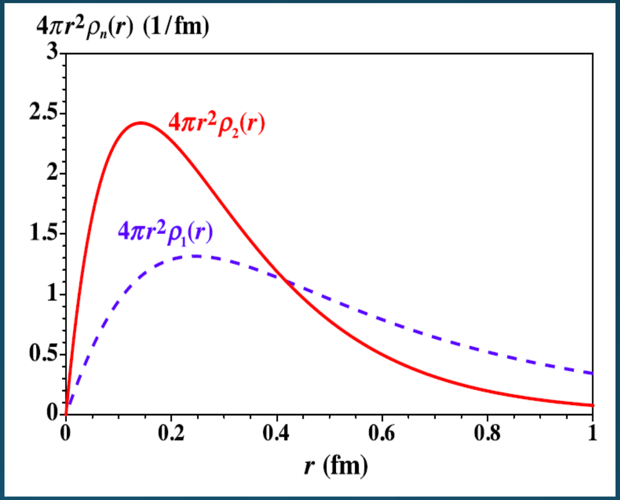
☐ Proton



Nature **557**, 396 (2018)

Lattice:
Shanahan, Detmold PRL (2019)

☐ Pion



Kumano+, PRD**97**, 014020 (2018)

☐ Static-Q system below T_c = Heavy-light meson

➡ Gravitational structure / dissociation

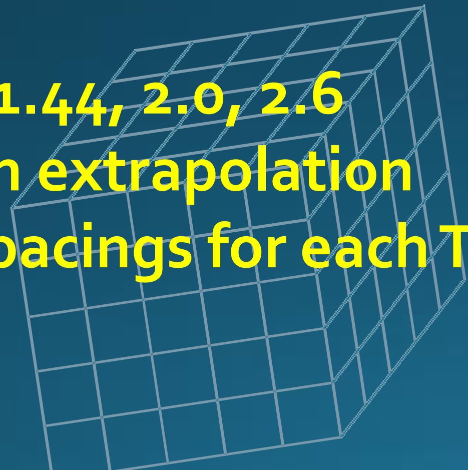
Lattice Setup

FlowQCD, PRD (2020)

- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- EMT around a Polyakov loop
- Noise reduction by multi-hit
- Gauge invariant

- **T/Tc=1.2, 1.44, 2.0, 2.6**
- **continuum extrapolation**
- **5 lattice spacings for each T**



T/T_c	N_s	N_τ	β	a [fm]	$N_{\rm conf}$
1.20	40	10	6.336	0.0551	500
	48	12	6.467	0.0460	650
	56	14	6.581	0.0394	840
	64	16	6.682	0.0344	1,000
	72	18	6.771	0.0306	1,000
1.44	40	10	6.465	0.0461	500
	48	12	6.600	0.0384	650
	56	14	6.716	0.0329	840
	64	16	6.819	0.0288	1,000
	72	18	6.910	0.0256	1,000
2.00	40	10	6.712	0.0331	500
	48	12	6.853	0.0275	650
	56	14	6.973	0.0236	840
	64	16	7.079	0.0207	1,000
	72	18	7.173	0.0184	1,000
2.60	40	10	6.914	0.0255	500
	48	12	7.058	0.0212	650
	56	14	7.182	0.0182	840
	64	16	7.290	0.0159	1,000
	72	18	7.387	0.0141	1,000

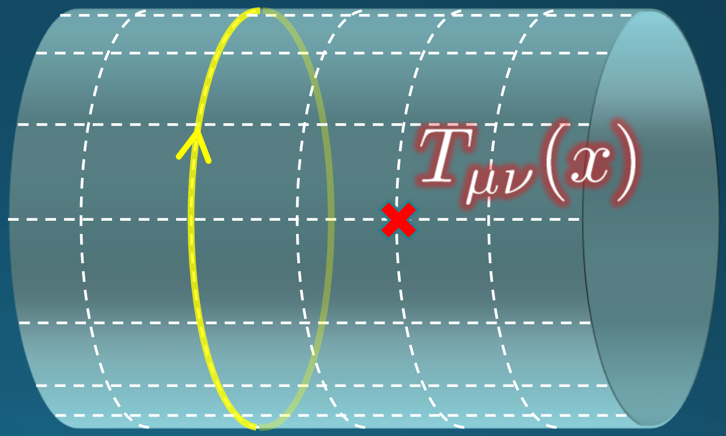
EMT Around a Static Q

$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q}} = \frac{\langle T_{\mu\nu}(x)\mathrm{Tr}\Omega(0)\rangle}{\langle \mathrm{Tr}\Omega(0)\rangle} - \langle T_{\mu\nu}(x)\rangle$$

Ω: Polyakov loop

- ☐ EMT-Polyakov loop correlation
- ☐ Gauge invariant
- ☐ Z3 symmetry has to be broken

- ☐ EMT by SF*t*X method



Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

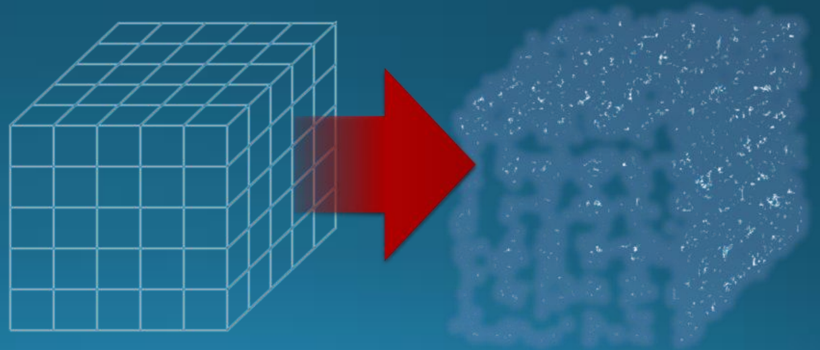
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length²]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at $t > 0$

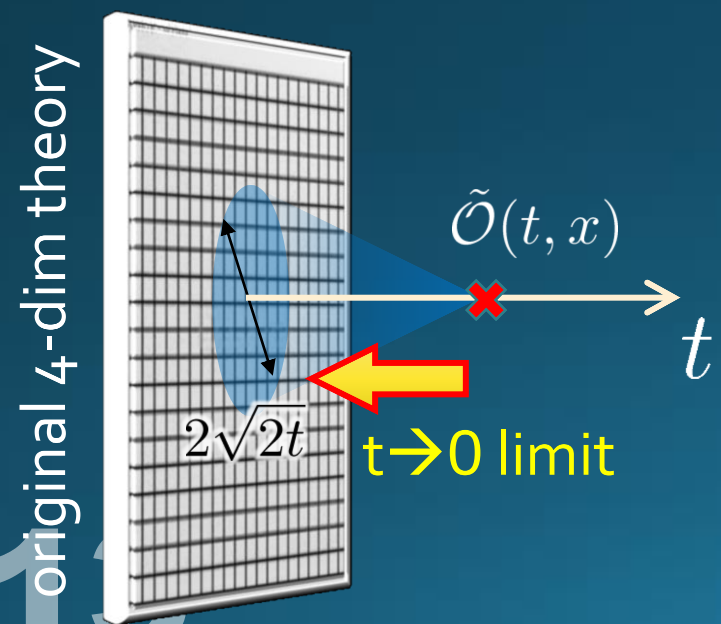


Small Flow-*t*ime Expansion

"SF*t*X method"

Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$



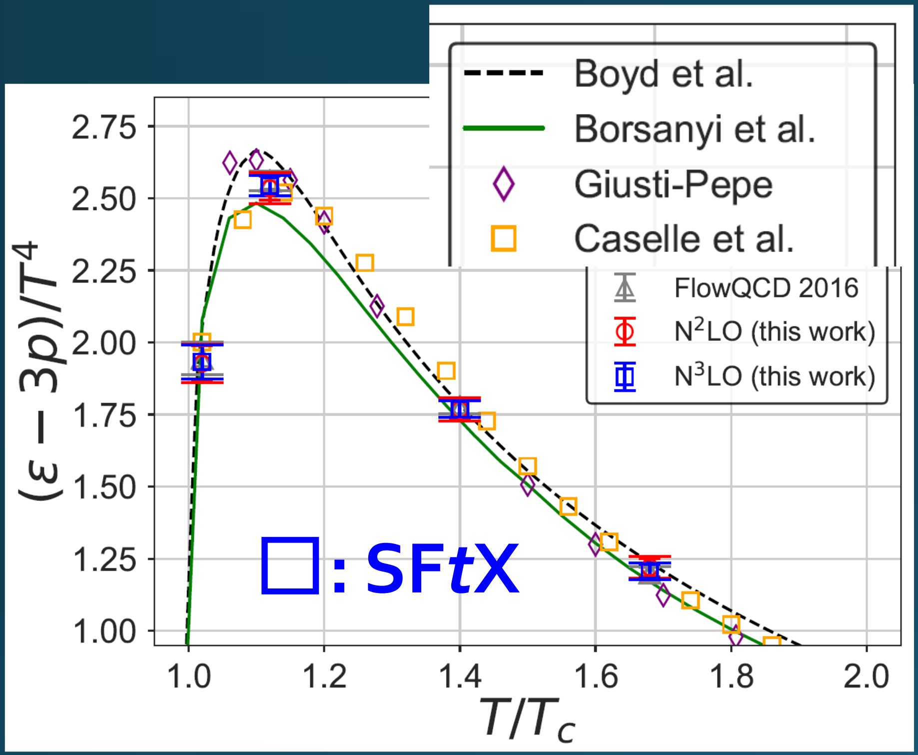
Operators at $t > 0$

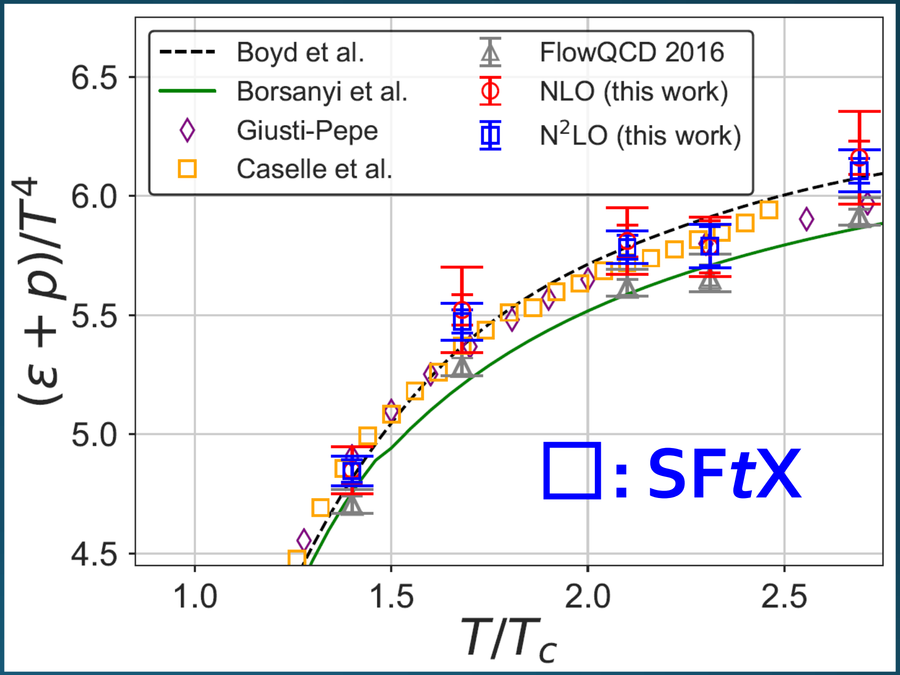
$$\begin{cases} E(t,x) = \dfrac{1}{4}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \delta_{\mu\nu}E(t,x) \end{cases}$$

Luescher, Weisz, 2011
Suzuki, 2013

Thermodynamics: $\varepsilon = \langle T_{00} \rangle,\ p = \langle T_{11} \rangle$

Iritani, MK, Suzuki, Takaura, 2019



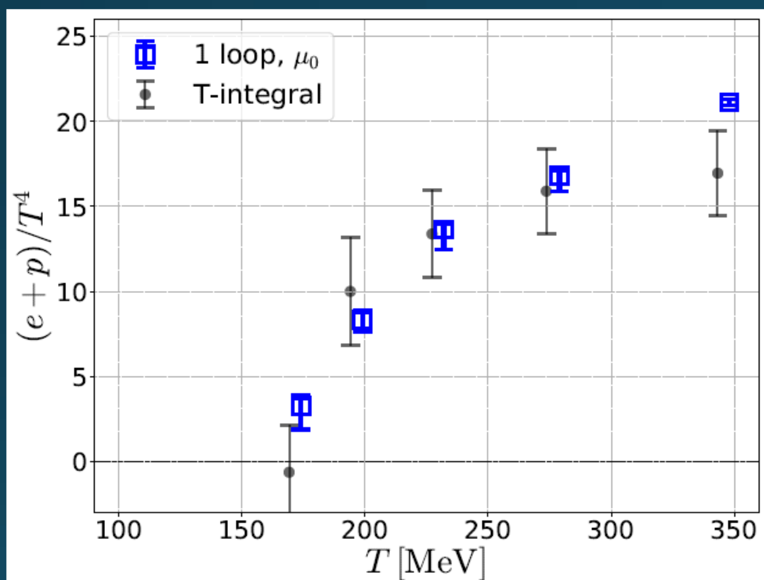


- Agreement with other methods within 1% level!
- Suppression of errors thanks to smearing nature

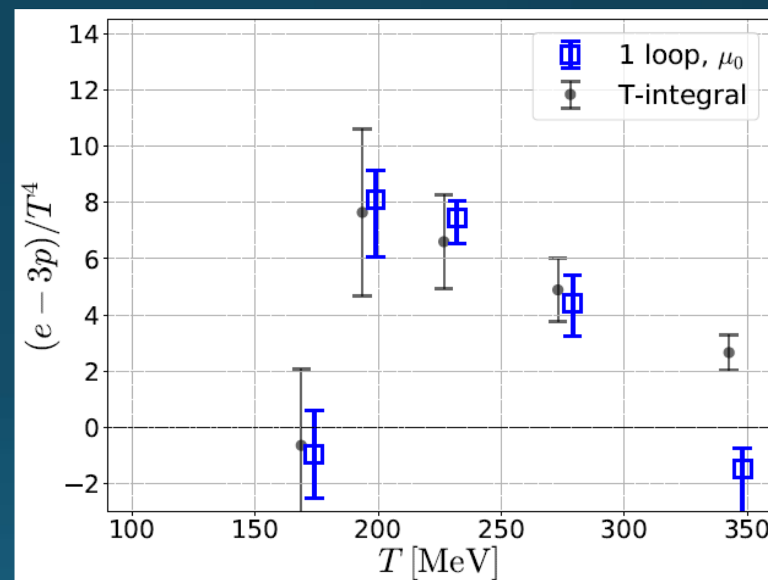
2+1 QCD EoS from Gradient Flow

WHOT-QCD, PRD**96** (2017); PRD**102** (2020)

$\langle T_{00}\rangle + \langle T_{11}\rangle$



$\langle T_{00}\rangle - 3\langle T_{11}\rangle$



- ☐ Agreement with integral method
- ☐ Substantial suppression of statistical errors

$m_{PS}/m_V \approx 0.63$

Physical mass: Kanaya+ (WHOT-QCD), 1910.13036

Double Extrapolation

$$t \to 0, a \to 0$$

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{latt}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{phys}} + C_{\mu\nu} t + D_{\mu\nu}(t)\frac{a^2}{t}$$

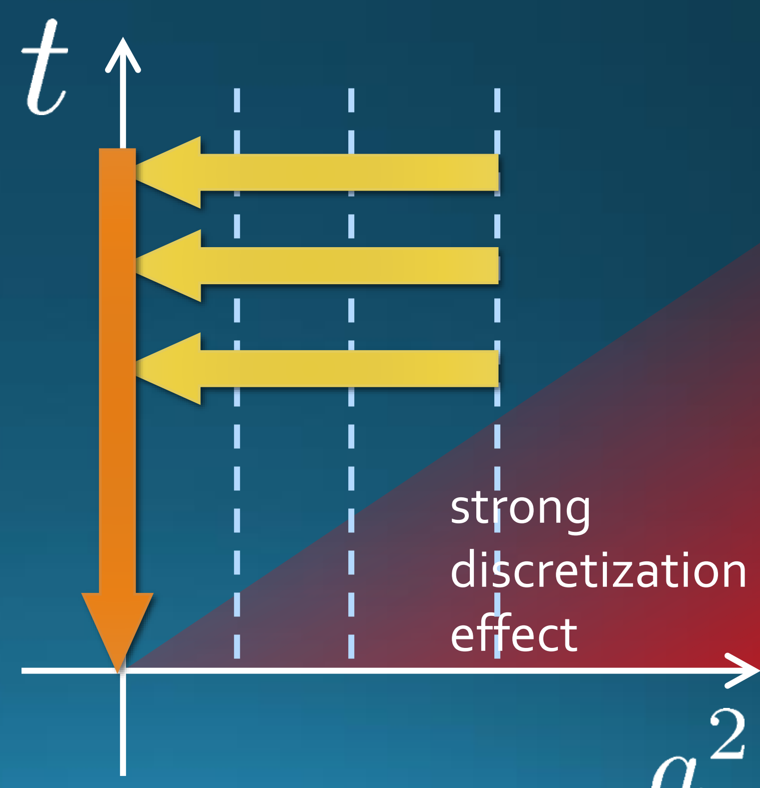
O(t) terms in SFTE   lattice discretization

Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{cont}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{lat}} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C' t$$

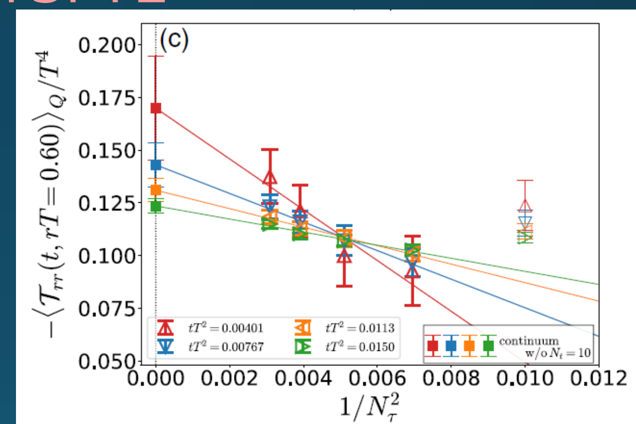


Double Extrapolation

$t \to 0, a \to 0$

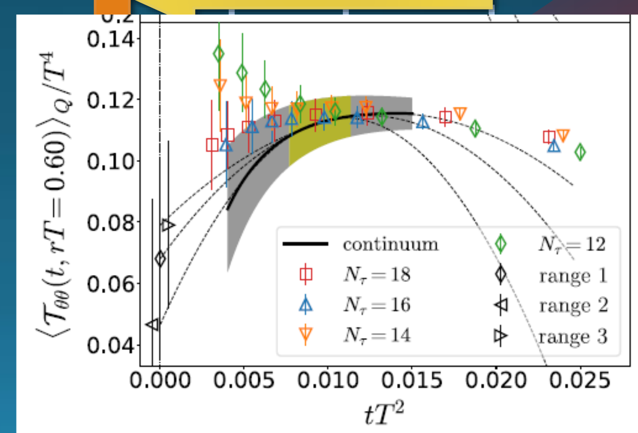
$$\langle T_{\mu\nu}(t)\rangle_{\text{latt}} = \langle T_{\mu\nu}(t)\rangle_{\text{phys}} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE lattice discretization



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\text{cont}} = \langle T_{\mu\nu}(t)\rangle_{\text{lat}} + C(t)a^2$$



Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

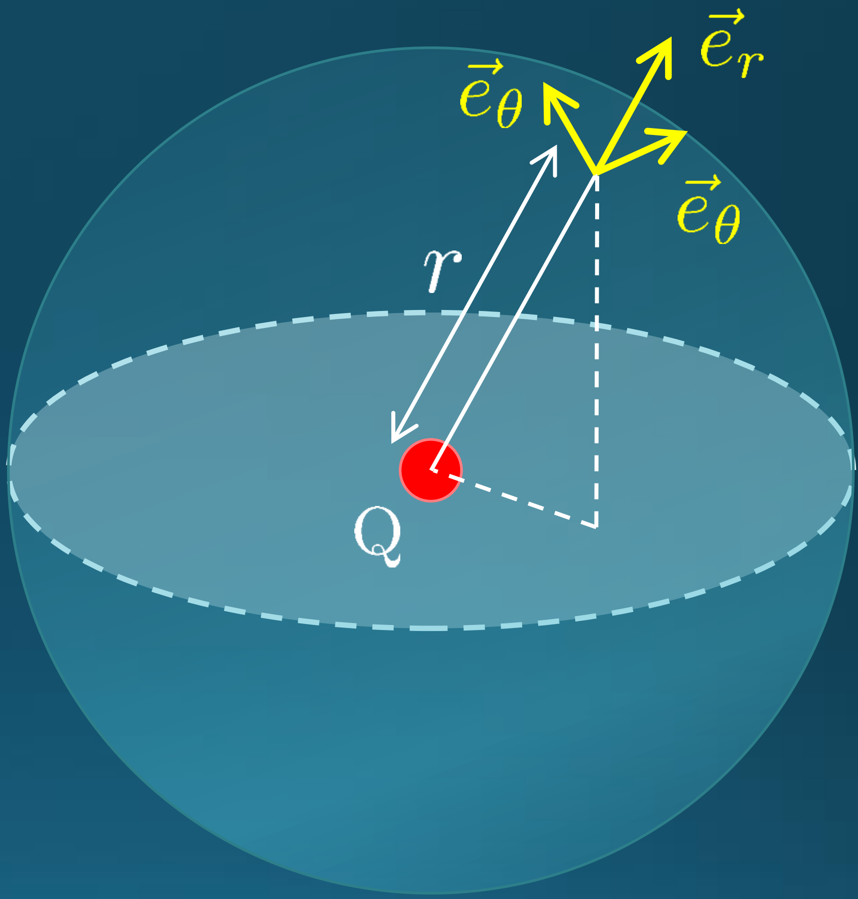
tion

a^2

Spherical Coordinates

EMT is diagonalized
in Spherical Coordinates

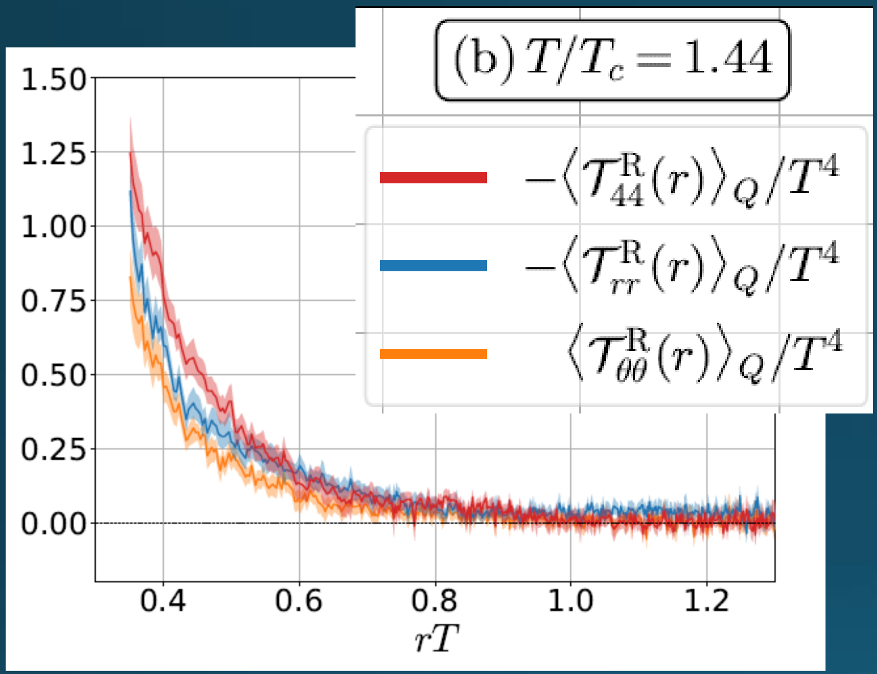
$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{\theta\theta} & \\ & & & T_{44} \end{pmatrix}$$



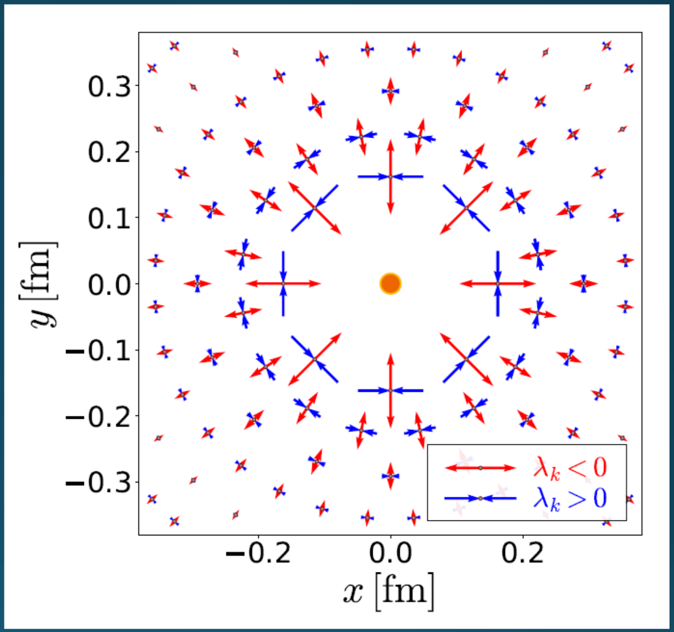
- Maxwell theory

$$T_{44} = T_{rr} = -T_{\theta\theta} = -\frac{|\boldsymbol{E}|^2}{2} = -\frac{\alpha}{8\pi}\frac{1}{r^4}$$

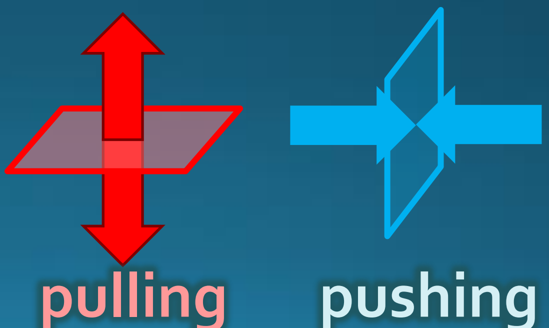
Stress Tensor Around a Quark



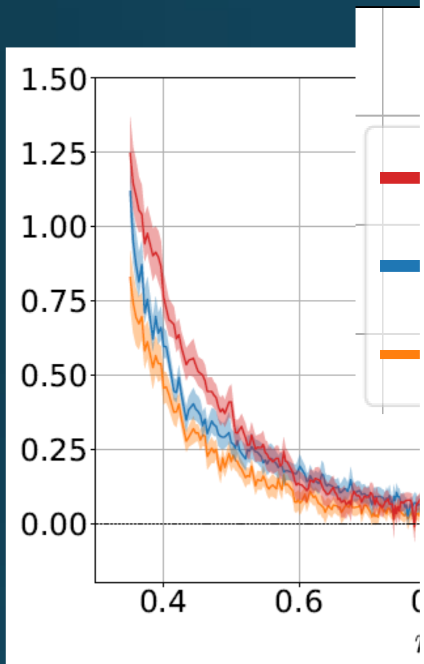
$T=1.44T_c$

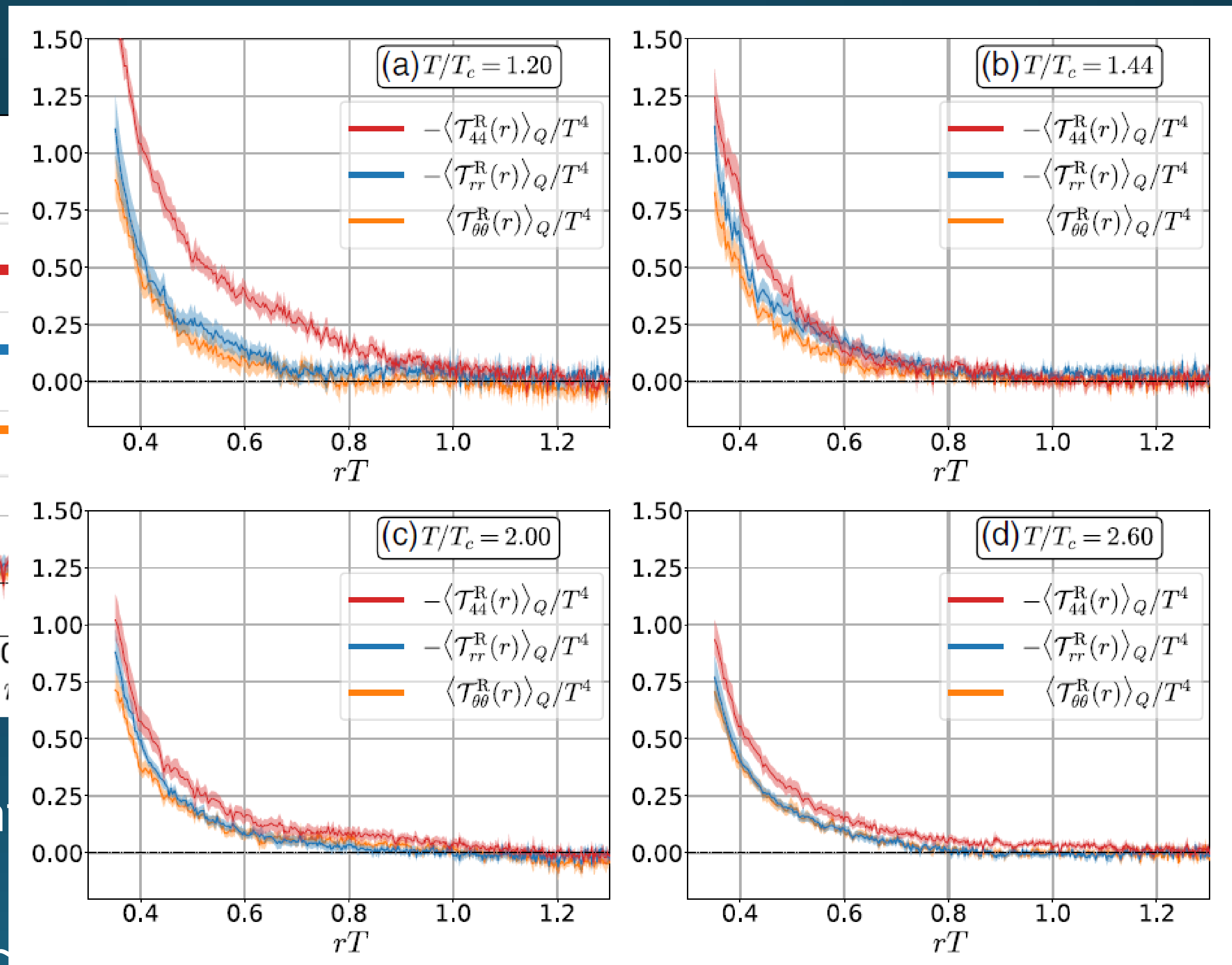


- ☐ Suppression at large distance
- ☐ Separation of different channels
- ☐ $|T_{44}| > |T_{rr}| \sim |T_{\theta\theta}|$



Stress Tensor Around a Quark





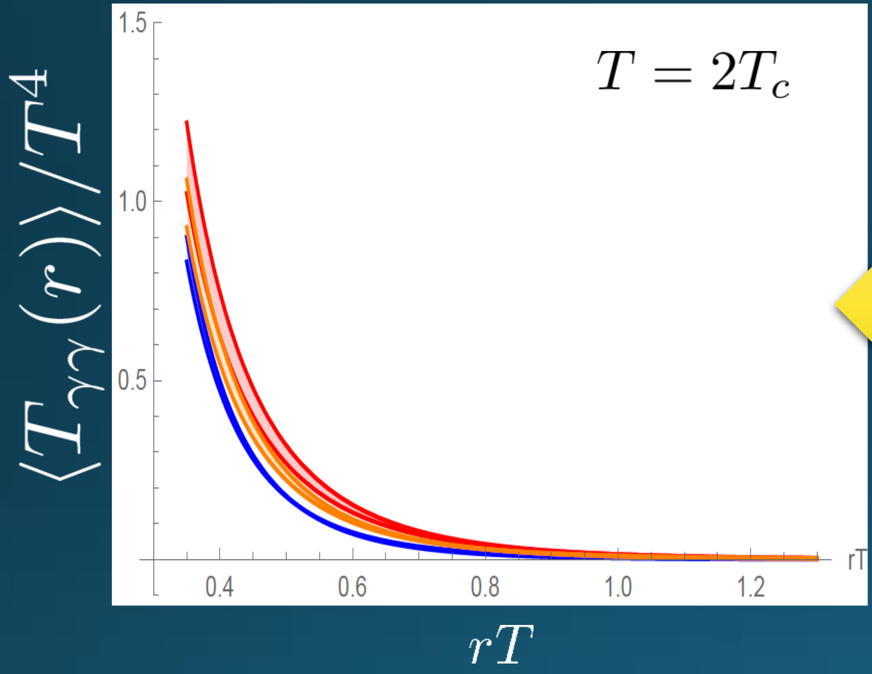
- Suppression a
- Separation of
- $|T_{44}| > |T_{rr}| \sim$
- Clearer separation for lower T

pulling pushing

Perturbative Analysis

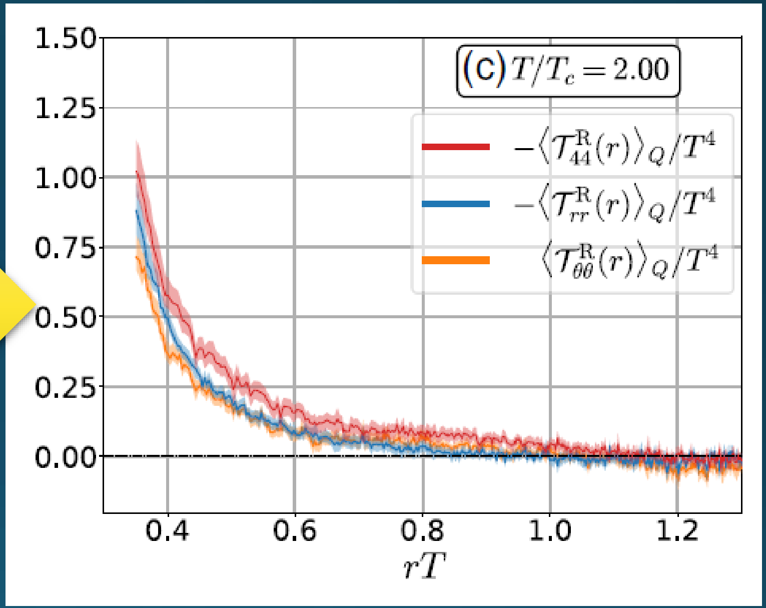
M. Berwein, private comm.

Perturbation



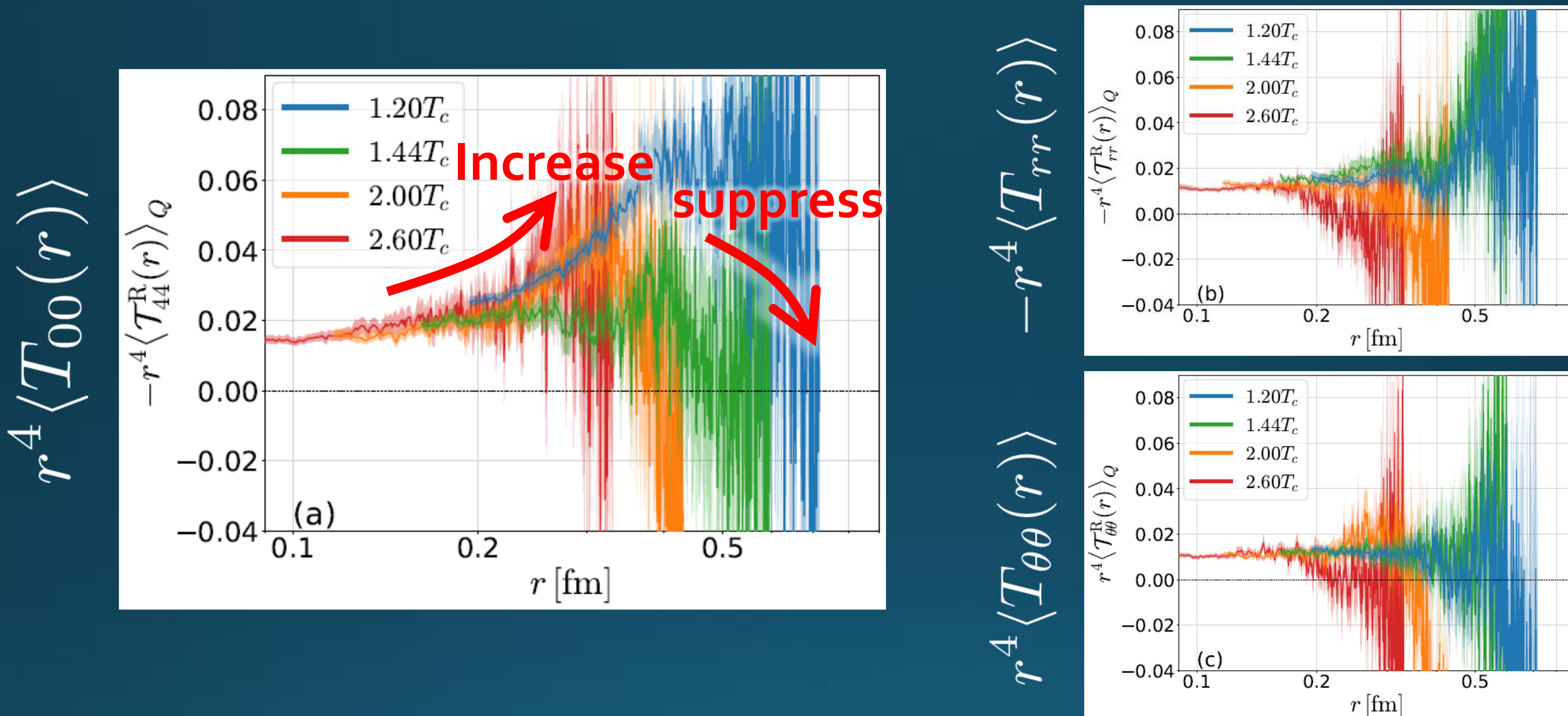
Perturbation:
Combination of
NLO pert. + NLO EQCD

Lattice



- $|T_{44}| > |T_{rr}|$ is reproduced by perturbation.
- Hierarchy of T_{rr} , $T_{\theta\theta}$ does not match?

r Dependence



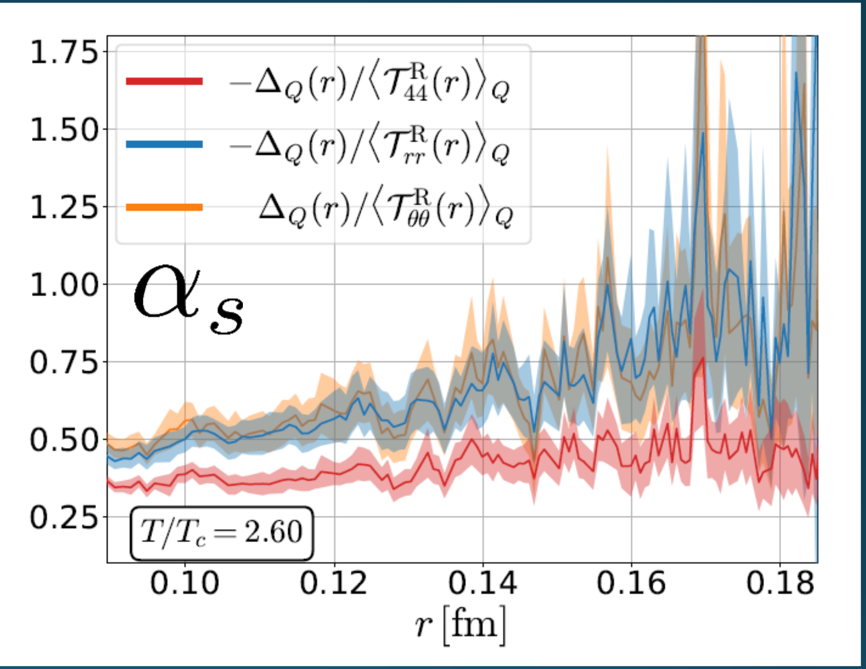
- Increase at short r / suppression at larger r
- T dependence is suppressed at $r < 1/T$
- Too noisy at large r for extracting screening mass $m_{\rm D}$

Running Coupling

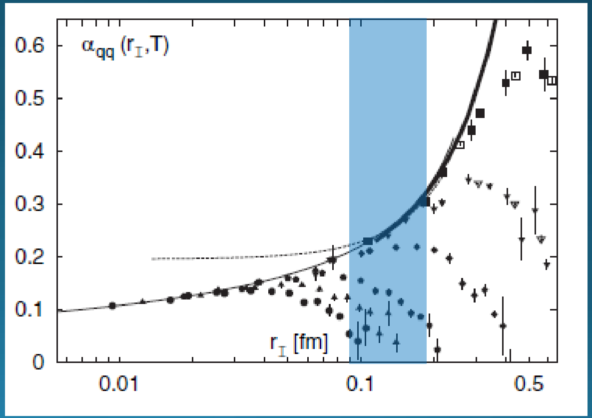
- **Estimate of α_s**

$$\left|\frac{\langle T_{\mu\mu}\rangle}{\langle \mathcal{T}_{44,rr,\theta\theta}(r)\rangle}\right| = \frac{11}{2\pi}\alpha_s + \mathcal{O}(g^3),$$

  - at the leading-order perturbation theory
  - channel dependent



- All results are approximately consistent with the estimate from $Q\bar{Q}$ potential



Kaczmarek, Karsch, Zantow, 2004

Summary

- First measurement of EMT distribution around a static quark in the deconfined phase.
- Gradient-flow (SF*t*X) method plays a crucial role to realize the measurement of EMT distribution.
- Numerical results show separation of T_{44} , T_{rr} , $T_{\theta\theta}$. Qualitative agreement with the perturbative calculation.

- **Many future studies**
  - **Single Q in full QCD** @ $T<T_c$ = heavy-light meson
  - QQQ, QQ, etc. / T dependence
  - EMT distribution inside hadrons

backup

Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

$$T_{\mu\nu}(t) = c_1(t)U_{\mu\nu}(t) + \delta_{\mu\nu}c_2(t)E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018)

Iritani, MK, Suzuki, Takaura, 2019

- **Choice of the scale of g^2**

$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E}t}$

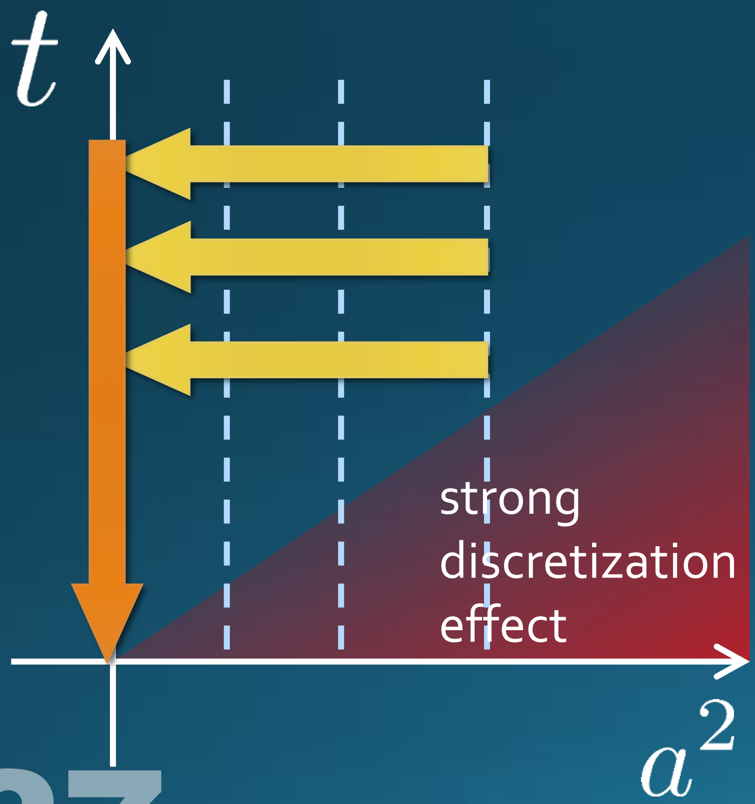
Harlander+ (2018)

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu} t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE   lattice discretization



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

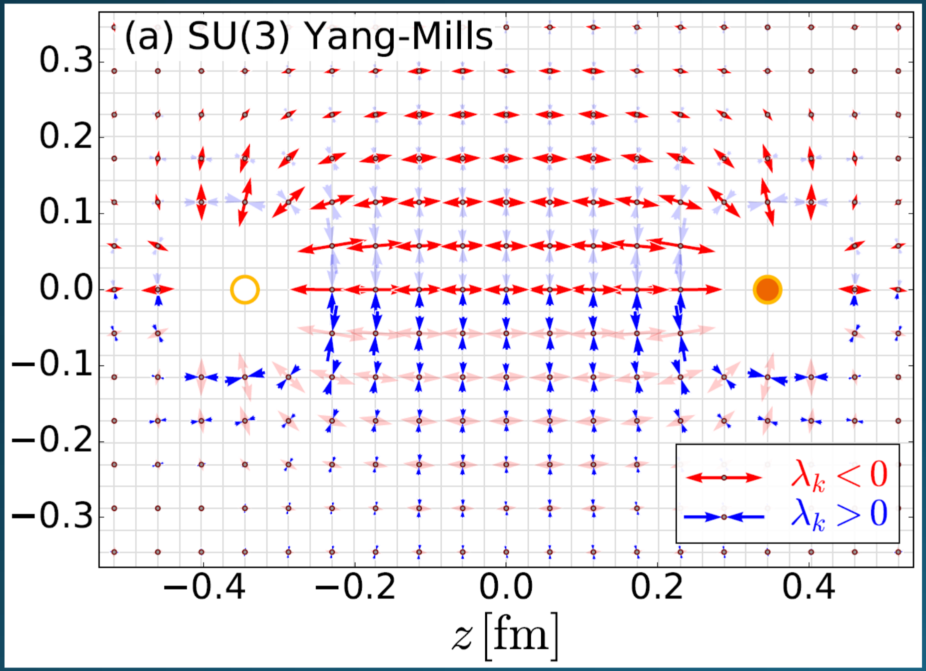
Small t extrapolation

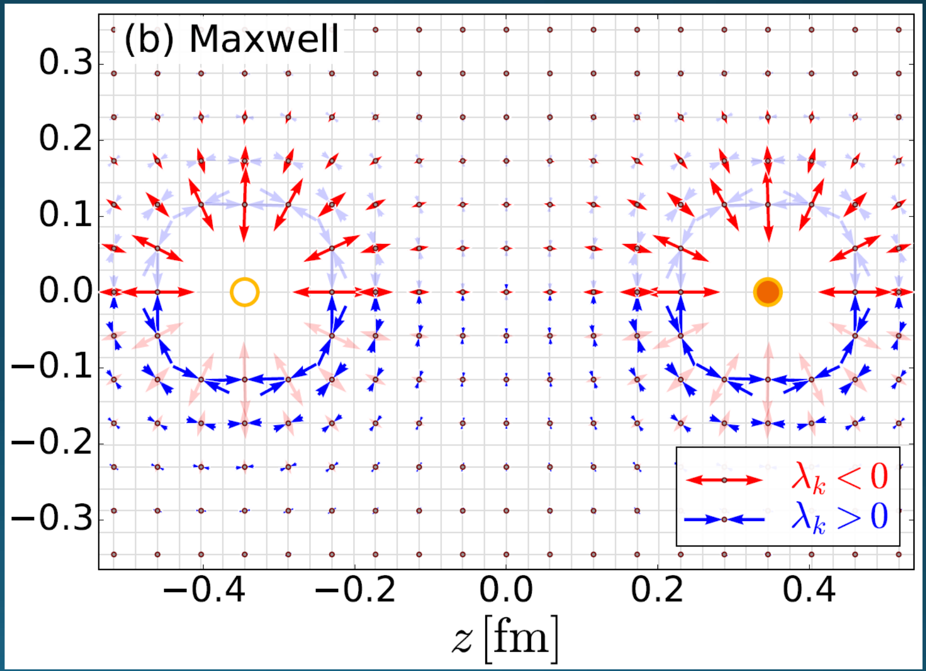
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C' t$$

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)

Maxwell
(classical)





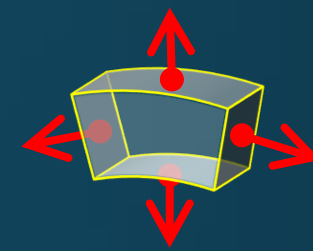
Propagation of the force is clearly different in YM and Maxwell theories!

Momentum Conservation

Yanagihara, MK, PTEP2019

- In cylindrical coordinats,

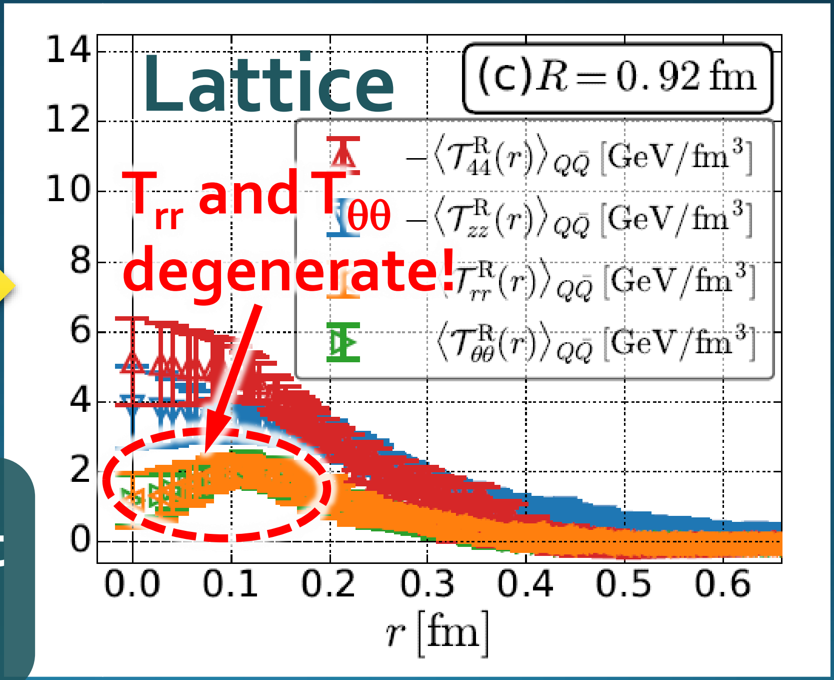
$$\partial_i T_{ij} = 0 \Rightarrow \partial_r (r T_{rr}) = T_{\theta\theta} - r \partial_z T_{rz}$$



- For infinitely-long flux tube

$$\partial_r (r T_{rr}) = T_{\theta\theta}$$

⇒ **T_{rr} and $T_{\theta\theta}$ must separate!**
$T_{\theta\theta}$ must change sign! ⇔



Effect of boundaries is important for the flux tube at R=0.92fm

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Determination of Zs are necessary.
- Non-pert. Determination of Zs
  - Shifted-boundary method
  - Full QCD with fermions

Giusti, Pepe, 2014~; Borsanyi+, 2018
Brida, Giusti, Pepe, 2020

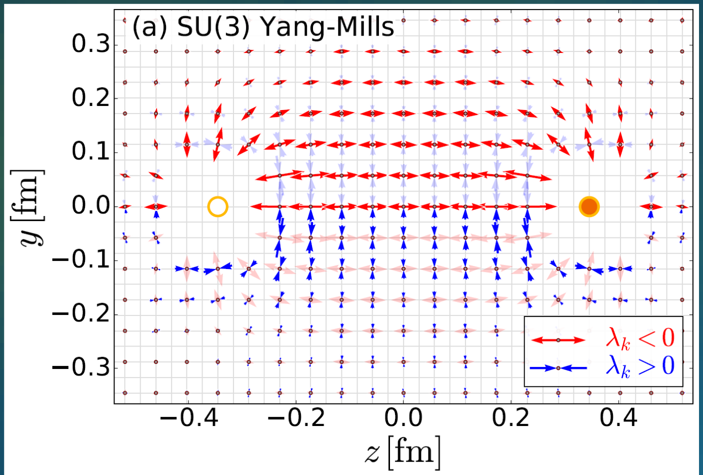
Static Charge

Fundamental tool to study field theories

This study

□ $Q\bar{Q}$

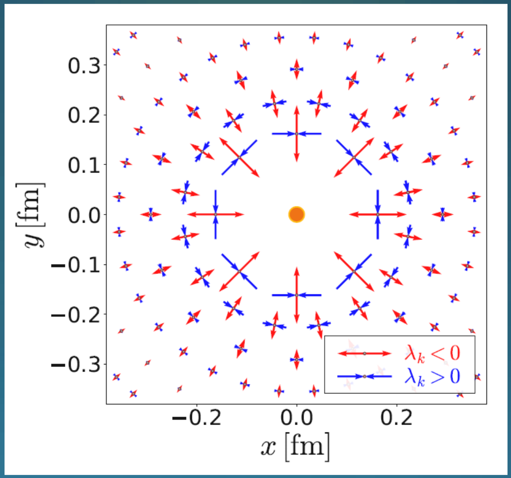
- Flux tube formation



FlowQCD, PLB **789**, 210 (2019)

□ Single Q

- Heavy-light meson @ $T<T_c$
- Debye screening @ $T>T_c$



FlowQCD, PRD **102**, 114522 (2020)