Corrections to the hadron resonance gas from lattice QCD and their effect on fluctuation-ratios at finite density

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in collaboration with

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[2102.06625]

Hadron Resonance Gas (HRG) model

Importance:

- parameters of chemical freeze-out in experiments,
- non-critical baseline.

HRG model: interacting gas of hadrons ≈ non-interacting gas of hadrons *and* resonances.

$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{T^4}\sum_{\mathrm{h}} p_{\mathrm{h}} = \frac{1}{VT^3}\sum_{\mathrm{h}} \log \mathcal{Z}_{\mathrm{h}}\left(T, \boldsymbol{\mu} = (\mu_B, \mu_Q, \mu_S)\right)$$

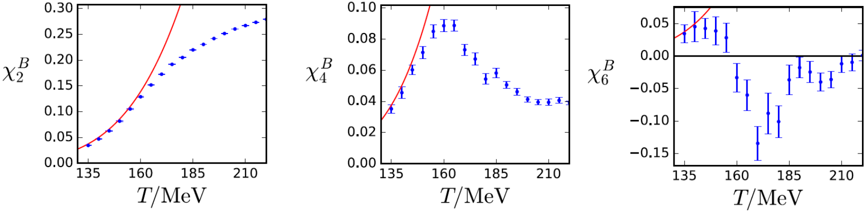
with

$$\begin{aligned}\log \mathcal{Z}_{\mathrm{h}} &= \mp \frac{V d_{\mathrm{h}}}{2\pi^2 T^3} \int_0^\infty \mathrm{d}p\, p^2 \log\left[1 \mp z_{\mathrm{h}} e^{-\beta\sqrt{m_{\mathrm{h}}^2 + p^2}}\right] \\ &= \frac{VTm_{\mathrm{h}}^2 d_{\mathrm{h}}}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\pm 1)^{n+1}}{n^2} z_{\mathrm{h}}^n K_2\left(\frac{n m_{\mathrm{h}}}{T}\right)\end{aligned}$$

and fugacity factors: $z_{\mathrm{h}} = \exp[\beta(B_{\mathrm{h}}\mu_B + Q_{\mathrm{h}}\mu_Q + S_{\mathrm{h}}\mu_S)]$.

Discrepancies of HRG and the lattice

Mostly good match for $\boldsymbol{\mu} = (0,0,0)$ in the hadronic phase:



[WB: 1805.04445]
[Pisa: 1611.08285]
[HotQCD: 2001.08530]

Possible reasons for the deviation:

- incomplete resonance list,
- finite widths are missing,
- HRG lacks non-resonant and repulsive interactions.

Fugacity expansion of the free energy

Fugacity expansion:

$$\frac{p(T,\hat{\mu}_B,\hat{\mu}_S)}{T^4} = \sum_{j,k} P^{BS}_{jk}(T)\cosh(j\hat{\mu}_B - k\hat{\mu}_S)$$

with $\hat{\mu}_{B,S} = \mu_{B,S}/T$ and $\mu_Q = 0$.

Sector coefficients P^{BS}_{jk} $\rightarrow$ contributions from Hilbert subspaces with fixed $B = j, S = k$.

E.g.

(B,S)	(0,0)	(1,0)	(0,1)	(1,1)	(1,2)	(2,0)	(0,2)	(2,1)
hadrons	π,η,ρ	p,Δ	K	Λ,Σ	Ξ	p-p	K-K	p-Λ, p-p-K

$\sim$ (main) contributions to ideal-HRG: $P^{BS}_{00}, P^{BS}_{01}, P^{BS}_{10}, P^{BS}_{11}, P^{BS}_{12}, P^{BS}_{13}$.

Imaginary chemical potential

Via analytic continuation $\mu = i\mu^I$: sign problem is absent.

$$\frac{p(T,\hat{\mu}_B^I,\hat{\mu}_S^I)}{T^4} = \sum_{j,k} P_{jk}^{BS}(T)\cos(j\hat{\mu}_B^I - k\hat{\mu}_S^I) \quad \rightarrow \quad \text{Fourier series}$$

Generalised susceptibilities:

$$\chi_{lm}^{BS} = \frac{\partial^{l+m}(p/T^4)}{\partial\hat{\mu}_B^l \partial\hat{\mu}_S^m}$$

E.g.

$$\mathrm{Im}\chi_{10}^{BS} = \sum_{j,k} j P_{jk}^{BS}(T)\sin(j\hat{\mu}_B^I - k\hat{\mu}_S^I) \qquad \mathrm{Im}\chi_{01}^{BS} = \sum_{j,k} (-k) P_{jk}^{BS}(T)\sin(j\hat{\mu}_B^I - k\hat{\mu}_S^I)$$

$$\chi_{20}^{BS} = \sum_{j,k} j^2 P_{jk}^{BS}(T)\cos(j\hat{\mu}_B^I - k\hat{\mu}_S^I) \qquad \chi_{02}^{BS} = \sum_{j,k} k^2 P_{jk}^{BS}(T)\cos(j\hat{\mu}_B^I - k\hat{\mu}_S^I)$$

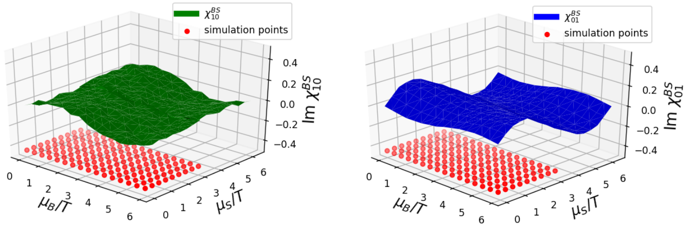
$$\chi_{11}^{BS} = \sum_{j,k} (-jk) P_{jk}^{BS}(T)\cos(j\hat{\mu}_B^I - k\hat{\mu}_S^I) \qquad \text{etc.}\ldots$$

Lattice setup

Calculation:

- 4stout-improved staggered action [WB: 1507.04627]
- 3 temporal extents: $N_\tau = 8, 10$ and 12 with $LT \approx 3$ aspect ratio.
- 4 temperatures: $T = 145$ MeV, 150 MeV, 155 MeV and 160 MeV.
- 2D scan of $(\mu_B^I, \mu_S^I) = \frac{\pi}{8}(i,j)$-plane

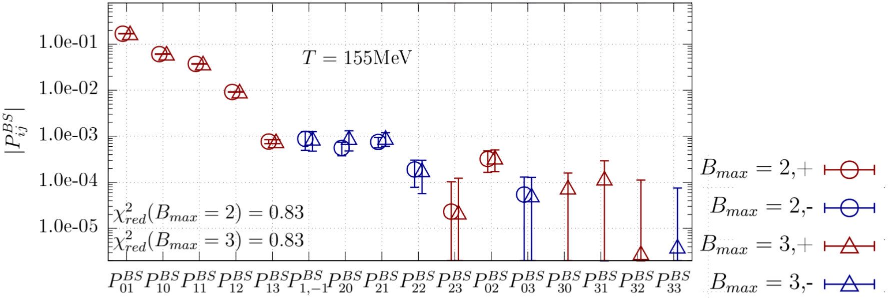
  (144 points: $i = 0, 1, \ldots, 15$ and $j = 0, 1, \ldots, 8$).



Example: $T = 150$ MeV and $N_x^3 \times N_\tau = 36^3 \times 12$.

Analysis: statistical (jackknife) + systematic errors.

Sectors

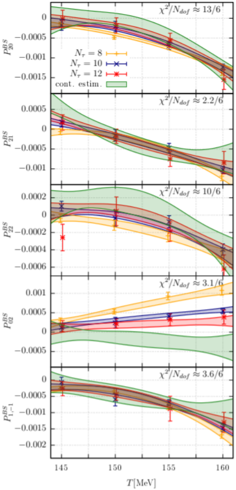


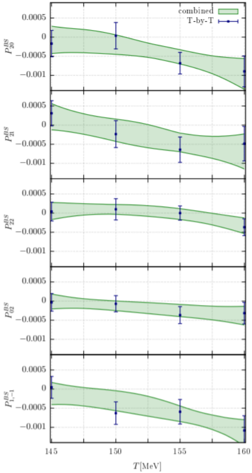
$(N_\tau = 12)$

- Hierarchy: $P_{01}^{BS} > P_{02}^{BS} > P_{03}^{BS}$; $P_{10}^{BS} > P_{20}^{BS} > P_{30}^{BS}$; $P_{10}^{BS} > P_{12}^{BS} > P_{13}^{BS}$; etc.
- $B = 2$ sector: $P_{2k}^{BS} < 0$.

Systematic error: $B_{\max} = 2$ or 3.

Sectors - continuum estimate





Ansatz:

$$f(T,N_\tau) = (a_0 + a_1 T + a_2 T^2) + (b_0 + b_1 T + b_2 T^2)/N_\tau^2$$

Systematic error: $B_{\max} = 2$ or 3 and $b_2 = 0$ or $\neq 0$.

Fluctuation ratios - at finite μ_B

Extrapolation to finite μ_B using truncated

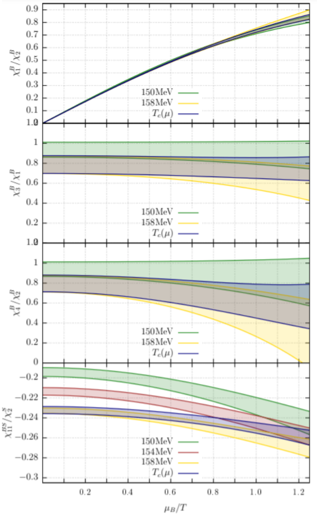
$$p/T^4 = \sum_{j,k} P_{jk}^{BS} \cosh\left[j\hat{\mu}_B - k\hat{\mu}_S(\hat{\mu}_B)\right]$$

with strangeness neutrality:
$\chi_1^S = \sum_{j,k}(-k)P_{jk}^{BS}(T)\sinh(j\hat{\mu}_B - k\hat{\mu}_S) \stackrel{!}{=} 0.$

Ratios:

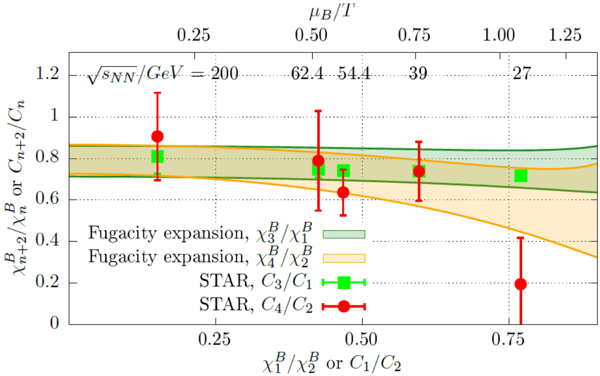
- χ_1^B/χ_2^B ~ proxy for μ_B,
- χ_3^B/χ_1^B, χ_4^B/χ_2^B ~ baryon thermometer,
- χ_{11}^{BS}/χ_2^S ~ strangeness thermometer + experimental proxy: $\sigma_\Lambda^2/(\sigma_\Lambda^2+\sigma_K^2)$.

  [WB: 1910.14592]



Crossover line: $T_c(\mu_B) \approx T_c^0(1-\kappa_2\hat{\mu}_B^2)$ [WB: 2002.02821]

Comparison with experiment



- Using C_i net-proton cumulants from STAR Experiment.
- Consistent with Taylor method: `[HotQCD: 2001.08530]`.
- Assuming that the crossover and chemical freeze-out lines are close to each other.

Summary and conclusion

- Scanning of the QCD free energy in imaginary μ_B-μ_S plane $\sim$ separation of sectors.
- Possible separation of processes like K-K or p-p scattering.
- Continuum estimates for P_{jk}^{BS} sector coefficients and fluctuation ratios.
- Clear inconsistency for $B = 2$ between the lattice and simple models:
  1. ideal-HRG ($\sim$ 5 orders of magnitude),
  2. HRG with excluded volume extension – same volume for all baryons ($\sim$ factor of 2).
- Consistency with STAR data for small μ_B.
- Our lattice results could provide help in
  1. phenomenology,
  2. construction of more realistic models.

Future:

- Examining finite volume effects due to $LT \approx 3$.
- Scanning the total μ_B-μ_Q-μ_S space $\sim$ studying electric charge fluctuations as well.

Thank you for your attention.