Localised Dirac eigenmodes and Goldstone's theorem at finite temperature

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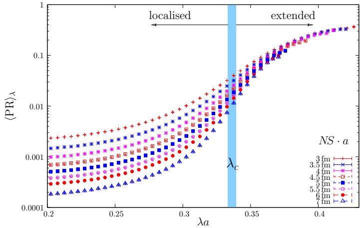
Localised Dirac eigenmodes at finite temperature

Low Dirac modes localised in high-T QCD and other gauge theories

[García-García, Osborn (2007), Kovács, Pittler (2012), . . . , Giordano, Kovács (2021)]

- delocalised mode →
  $\|\psi(x)\|^2 \sim 1/V^\alpha$
- localised mode →
  $\|\psi(x)\|^2 \sim 1/V_0 \cdot \chi_{R_0}(x)$



$$\mathrm{PR} = \frac{1}{N_t V} \left(\sum_x \|\psi(x)\|^4 \right)^{-1}$$

2+1 QCD at $T = 394\,\mathrm{MeV}$
Data from [MG *et al.* (2014)]

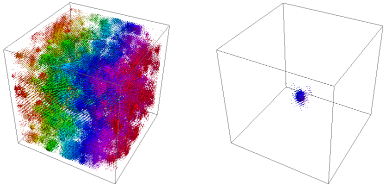


Figure from [Ujfalusi *et al.* (2015)]

- low modes localised up to λ_c ("mobility edge") in deconfined phase of gauge theories, while delocalised in confined phase
- features analogous to disordered condensed matter systems
  Anderson transition, multifractal ψ at λ_c
  [MG *et al.* (2014), Ujfalusi *et al.* (2015)]

Physical effects of localisation?

Localised modes ≈ "order parameter" for deconfinement

- localised modes appear at T_c when transition is sharp [MG *et al.* (2017), Kovács, Vig (2018, 2020), MG (2019), Bonati *et al.* (2021), Baranka, MG (2021)]
- works also in the simplest model: $\mathbb{Z}_2$ gauge theory in 2+1 dimensions

See G. Baranka's talk on Monday

More direct interpretation possible in the chiral limit where only $\lambda \approx 0$ physically relevant: **what happens if** $\rho_{\rm loc}(0) \neq 0$ **when** $m \to 0$**?**

1. 2+1 QCD: overlap on HISQ backgrounds

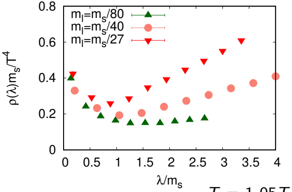
Peak of localised near-zero modes for $m_l \gtrsim m_l^{\rm phys}$ at $T \gtrsim T_c$ [Dick *et al.* (2015)]; also for $m_l < m_l^{\rm phys}$ [Kaczmarek *et al.* (2021)], loc. properties unknown

⇒ Does it survive $m_l \to 0$? Modes localised?

2. $N_f = 2$ massless adjoint fermions

Intermediate, chirally broken ($\rho(0) \neq 0$) but deconfined phase [Karsch and Lütgemeier (1999)]

⇒ Finite density of near-zero localised modes?



$T = 1.05\,T_{pc}$

Figure from [Kaczmarek *et al.* (2021)]

Goldstone modes and localisation at $T = 0$

$\rho(0) \neq 0$ at $m = 0 \Rightarrow$ SχSB $\Rightarrow$ massless bosons (Goldstone's theorem)

$$\Sigma = \frac{1}{N_f}\langle \mathcal{S}(0)\rangle \underset{m\to 0}{\to} -\pi\rho(0) \qquad \rho(\lambda) = \frac{1}{V_4}\textstyle\sum_n \langle \delta(\lambda - \lambda_n)\rangle$$

[Banks, Casher (1980)]

If near-zero modes are localised Goldstone bosons disappear

[McKane, Stone (1981), Golterman, Shamir (2003)]

$$-\langle \partial_\mu \mathcal{A}^a_\mu(x)\mathcal{P}^b(0)\rangle + 2m\langle \mathcal{P}^a(x)\mathcal{P}^b(0)\rangle = \delta^{(4)}(x)\delta^{ab}\Sigma$$

$$\underset{p\text{-space}}{\Longrightarrow} ip_\mu \mathcal{G}_{AP_\mu}(p) + 2m\mathcal{G}_{PP}(p) = \Sigma$$

$$\delta^{ab}\mathcal{G}_{\mathcal{O}_1\mathcal{O}_2}(p) = \int d^4x\, e^{ip\cdot x}\langle \mathcal{O}^a_1(x)\mathcal{O}^b_2(0)\rangle$$

- if $\mathcal{G}_{PP}(p)$ reasonable as $m \to 0$, $\Sigma \neq 0 \Rightarrow$ pole at $p = 0$ in $\mathcal{G}_{AP_\mu}(p)$
- localised modes $\Rightarrow \mathcal{G}_{PP}(p) \propto 1/m$, Goldstone bosons can disappear

$$\mathcal{G}_{AP_\mu}(p) \underset{p\to 0}{\to} -\frac{ip_\mu}{p^2}[\Sigma - \mathrm{R}] \qquad \mathrm{R} = \lim_{m\to 0} 2m\mathcal{G}_{PP}(0) \propto \rho_{\mathrm{loc}}(0)$$

Goldstone theorem **evaded**: current **not** conserved, "anomalous" $(\sim m\cdot\frac{1}{m})$
remnant R breaks symmetry explicitly also in chiral limit — **not in QCD!**

Goldstone modes and localisation at $T \neq 0$

Goldstone theorem at $T \neq 0$: spontaneous breaking of continuous symmetry implies massless quasi-particle excitations [see Strocchi (2008)]

Similar strategy: relate Euclidean Dirac spectrum and physical spectrum via axial nonsinglet Ward identity (use spectral function [see Meyer (2011)])

$$\begin{aligned}\varrho^{\mathcal{AP}}(\omega, \vec{p}=0) &= -i \int d^4x\, e^{i\omega t} \langle\langle [\hat{\mathcal{A}}_0^a(t,\vec{x}), \hat{\mathcal{P}}^b(0)] \rangle\rangle_T \\ &= -2\pi[\Sigma - \mathrm{R}]\delta(\omega) + (\text{regular at } \omega = 0)\end{aligned}$$

[MG (2020)]

$\mathrm{R} \neq 0$ if near-zero modes localised, Goldstone mode only if $\Sigma - \mathrm{R} \neq 0$

$$\Sigma \underset{m\to 0}{\to} -\pi\rho(0) \qquad \mathrm{R} = \lim_{m\to 0} 2m\mathcal{G}_{PP}(0) \propto \rho_{\mathrm{loc}}(0)$$

$$\rho(\lambda) = \tfrac{T}{V}\langle \textstyle\sum_n \delta(\lambda - \lambda_n)\rangle \qquad \rho_{\mathrm{loc}}(\lambda) = \tfrac{T}{V}\langle \textstyle\sum_{n\in\mathrm{loc}} \delta(\lambda - \lambda_n)\rangle$$

Pseudoscalar correlator

IR divergence from pseudoscalar correlator (continuum):

- expand in Dirac modes $\not{D}\psi_n = i\lambda_n\psi_n$

$$\langle \mathcal{P}_B^a(x)\mathcal{P}_B^b(0)\rangle = -\frac{\delta^{ab}}{2}\left\langle \sum_{n,n'} \frac{\mathcal{O}^{\gamma_5}_{n'n}(x)\mathcal{O}^{\gamma_5}_{nn'}(0)}{(i\lambda_n + m_B)(i\lambda_{n'} + m_B)} \right\rangle = -\delta^{ab}\Pi_B(x)$$

$$\mathcal{O}^{\Gamma}_{nn'}(x) \equiv (\psi_n(x), \Gamma\psi_{n'}(x))$$

- renormalise

$$\Pi(x) = Z_m^2[\Pi_B(x) - \Pi^{\mathrm{add.\,div.}}(x)] \qquad m = Z_m^{-1} m_B \qquad \lambda_n^R = Z_m^{-1}\lambda_n$$

$$\boxed{\lim_{m\to 0} 2m\Pi(x) = 2\lim_{m\to 0}\int_0^{\frac{\mu}{m}} dz \left(\frac{C^1(mz;m;x)}{z^2+1} + \frac{(1-z^2)C^{\gamma_5}(mz;m;x)}{(z^2+1)^2} \right)}$$

Zero modes negligible as $V \to \infty$
$|\lambda^R| > \mu$ negligible as $m \to 0$

$$C^{\Gamma}(\lambda;m;x) \equiv \left\langle \textstyle\sum_{\lambda_n^R \neq 0} \delta(\lambda - \lambda_n^R)\mathcal{O}^{\Gamma}_{nn}(x)\mathcal{O}^{\Gamma}_{nn}(0) \right\rangle$$

Pseudoscalar correlator and localised modes

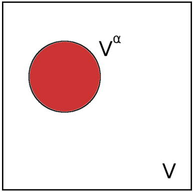
Large-V behaviour of $C_V^\Gamma = \left\langle \sum_n \delta(\lambda - \lambda_n^R) \mathcal{O}_{nn}^\Gamma(x) \mathcal{O}_{nn}^\Gamma(0) \right\rangle$

$$|\langle \mathcal{O}_{nn}^\Gamma(x)\mathcal{O}_{nn}^\Gamma(0)\rangle| \le \langle \|\psi_n(x)\|^2 \|\psi_n(0)\|^2\rangle \le \langle \|\psi_n(0)\|^4\rangle = \frac{T}{V} \langle \overbrace{\textstyle\int_T d^4x \|\psi_n(x)\|^4}^{\mathrm{IPR}_n} \rangle$$

$$\textstyle\int_T d^4x = \int_0^{\frac{1}{T}} dt \int d^3x$$

ψ_n supported in $V^\alpha \Rightarrow \|\psi_n\|^2 \sim \frac{1}{V^\alpha}$ inside R_α

$$\langle \mathrm{IPR}_n \rangle \sim \underbrace{\frac{1}{V^{2\alpha}}}_{\|\psi_n(0)\|^4} \times \underbrace{V^\alpha}_{\text{size of } R_\alpha} \sim \frac{1}{V^\alpha}$$



$$|C_V^\Gamma(\lambda; m; x)| \lesssim \frac{T}{V} \textstyle\sum_n \langle \delta(\lambda - \lambda_n^R)\, \mathrm{IPR}_n \rangle \sim \rho(\lambda) \times V^{-\alpha(\lambda)}$$

$C_V^\Gamma \to 0$ unless $\alpha = 0 \Rightarrow$ only localised modes contribute

$$\boxed{C^\Gamma = \lim_{V\to\infty} C_V^\Gamma = C_{\mathrm{loc}}^\Gamma}$$

Localised modes and Goldstone excitations

Assume near-zero localised modes in $\lambda \in [0, \lambda_c(m)] \implies C^{\Gamma}_{\rm loc}, \rho_{\rm loc} \neq 0$

$$-\mathrm{R} = \int_T d^4x \lim_{m\to 0} 2m\Pi(x) = \pi\xi\, \rho_{\rm loc}(0) \qquad \xi = \lim_{m\to 0} \frac{2}{\pi} \arctan \frac{\lambda_c(m)}{m}$$

$\lambda_c(m)/m$ is renormalisation group invariant
Localised modes $\Rightarrow$ can exchange order of limits

$$\varrho^{\mathcal{AP}}(\omega, \vec{p}=0)|_{\rm singular} = -2\pi[\Sigma - \mathrm{R}]\delta(\omega) = 2\pi^2 \rho(0) \left(1 - \xi \frac{\rho_{\rm loc}(0)}{\rho(0)}\right) \delta(\omega)$$

Localised and delocalised modes usually
do not coexist $\Rightarrow \rho_{\rm loc}(0)/\rho(0) = 0, 1$

1. no near-zero localised modes, $\rho_{\rm loc}(0) = 0$, or $\rho_{\rm loc}(0) \neq 0$ but $\xi = 0$
   $\Rightarrow$ Goldstone excitations present if $\rho(0) \neq 0$ (standard scenario)
2. near-zero localised modes, $\rho_{\rm loc}(0) \neq 0$, and $0 < \xi < 1$
   $\Rightarrow$ Goldstone excitations present (but modified "residue")
3. near-zero localised modes, $\rho_{\rm loc}(0) \neq 0$, and $\xi = 1$ (e.g., $\lambda_c(0) \neq 0$)
   $\Rightarrow$ Goldstone excitations disappear

Summary and outlook

In the presence of a finite density of near-zero **localised** Dirac modes

- pseudoscalar correlator diverges $\sim 1/m$ (unless $\frac{\lambda_c}{m} \to 0$)
- axial nonsinglet Ward identity in the chiral limit modified by "anomalous" remnant, current **not** conserved (Goldstone theorem evaded)
- spectral function modified, Goldstone excitations disappear if $\frac{\lambda_c}{m} \to \infty$ (modified residue if $\frac{\lambda_c}{m} \to$ finite)

Open issues:

- is there an explicit realisation of the non-standard scenarios?
- phenomenological consequences?

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Proof of Goldstone theorem at $T \neq 0$ using WI

$T \neq 0$ Goldstone theorem: spontaneous breaking of continuous symmetry implies massless quasi-particle excitations [see Strocchi (2008)]

- axial nonsinglet Ward identity in coordinate space holds also at $T \neq 0$, compact t-direction restricts p_4 to Matsubara frequencies $\omega_n = 2\pi n T$, $n \in \mathbb{Z}$

$$i\omega_n \mathcal{G}_{AP\,4}(\omega_n, \vec{p}) + i\vec{p} \cdot \vec{\mathcal{G}}_{AP}(\omega_n, \vec{p}) + 2m\mathcal{G}_{PP}(\omega_n, \vec{p}) = \Sigma$$

- take $\vec{p} \to 0$ and analytically continue to $\Omega \in \mathbb{C}$ (Carlson's theorem)

$$\bar{\mathrm{G}}(\Omega) \equiv \bar{\mathcal{G}}_{AP\,4}(\Omega, \vec{0}) = \tfrac{1}{i\Omega}[\Sigma - 2m\bar{\mathcal{G}}_{PP}(\Omega, \vec{0})] \equiv \tfrac{1}{i\Omega}[\Sigma - \bar{\mathrm{R}}(\Omega)]$$

$\vec{p} \cdot \vec{\mathcal{G}}_{AP}(\omega_n \neq 0, \vec{p}) \to 0$ as $\vec{p} \to 0$ due to relativistic locality

- zero-momentum A-P spectral function using KMS condition [see Meyer (2011)]

$$i\varrho^{\mathcal{AP}}(\omega, \vec{p} = 0) = \bar{\mathrm{G}}(0^+ - i\omega) - \bar{\mathrm{G}}(0^- - i\omega)$$
$$= -2\pi i[\Sigma - \bar{\mathrm{R}}(0^+)]\delta(\omega) + (\text{regular at } \omega = 0)$$

- $\mathrm{R}(0) = \bar{\mathrm{R}}(0^+)$ if transport peak $\varrho(\omega) \propto \omega\delta(\omega)$ absent

Not expected and not observed in pseudoscalar channel [Burnier *et al.* (2017)]

- chiral limit: if $\mathrm{R} \to 0$ and $\Sigma \neq 0$ (SSB) $\Rightarrow$ massless quasi-particle excitation
- if $\mathrm{R} \not\to 0$, Goldstone mode present if $\Sigma - \mathrm{R}(0) \neq 0$