

Heavy quark diffusion in an overoccupied gluon plasma

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Pre-equilibrium dynamics & classical field simulations

- Gluon $f \sim 1/g^2 \rightarrow$ classical \rightarrow real time lattice simulations.
- Assumptions: M large, measure force felt by a "test particle".
- Non-thermal fixed points and self-similar scaling
- Dynamically generated hierarchy of scales reminiscent of PT:
 - Coupling $g \iff$ time t
 - Temperature $T \iff$ hard scale $\Lambda \sim t^{1/7}$
 - Soft scale $gT \iff$ mass scale $\omega_{\text{pl}} \sim t^{-1/7}$
 - Ultrasoft scale $g^2 T \iff$ magnetic scale $m_{\text{mag}} \sim t^{-1/3}$

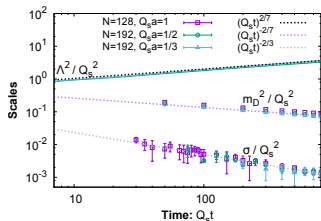


Figure: Schlichting et. al. PRD 93 (2016) 7, 074036.

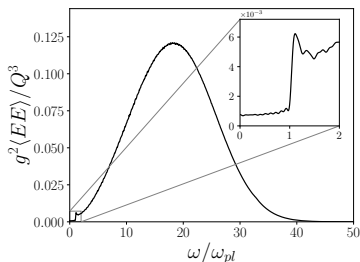
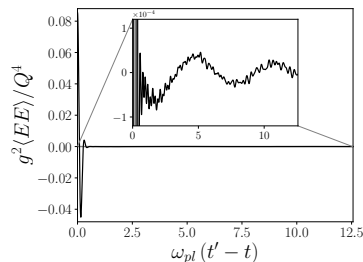
Heavy quark in a color field

Classical equation of motion

$$\dot{p}_i(t) = \mathcal{F}_i(t). \quad (1)$$

The chromoelectric force has $\langle \dot{p} \rangle = 0$ but nonzero variance

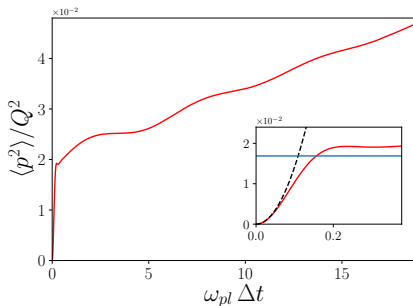
$$\langle \dot{p}_i(t) \dot{p}_i(t') \rangle = \frac{g^2}{N_c} \text{Tr} \langle E_i(t) U_0(t, t') E_i(t') U_0(t', t) \rangle = \frac{g^2}{2N_c} \langle EE \rangle(t, t'). \quad (2)$$



- RHS fig: sharp rise at $\omega = \omega_{pl}$ QP contribution. Intercept with y-axis $\approx \kappa$. Landau damping of longit. gluon fields.

Momentum broadening

$$\langle p^2(t, \Delta t) \rangle = \frac{g^2}{2N_c} \int_t^{t+\Delta t} dt' \int_t^{t+\Delta t} dt'' \langle EE \rangle(t', t''). \quad (3)$$



3 distinct features

- Rapid growth $\Delta t \approx 2\pi/\Lambda$, broad peak dominates the integration
- Damped oscillations $\Delta t \approx 2\pi/\omega_{pl}$, new feature! Transverse QP excitations.
- Linear growth in Δt , $1/\Lambda \ll \Delta t \ll t$, consistent with Langevin description.

Heavy quark diffusion vs. momentum broadening

Define $\kappa(t, \Delta t)$ as

$$3\kappa(t, \Delta t) = \frac{d}{d\Delta t} \langle p^2(t, \Delta t) \rangle \quad (4)$$

As in thermal equilibrium, diffusion coefficient $\kappa(t)$ defined at $\Delta t \rightarrow \infty$ limit

$$\frac{g^2}{2N_c} \langle EE \rangle(t, \omega = 0) = 3\kappa_\infty(t). \quad (5)$$

On the lattice $\kappa(t, \Delta t)$ given by

$$\kappa(t, \Delta t) \approx \frac{g^2}{3N_c} \int_t^{t+\Delta t} dt' \int \frac{d^3x}{V} \langle E_i^a(t, \mathbf{x}) E_i^a(t', \mathbf{x}) \rangle, \quad (6)$$

To identify $\kappa(t, \Delta t) = \kappa(t)$ we want to have $t \gg \Delta t \gg 1/\gamma_{pl}$, where γ_{pl} is the largest inverse lifetime of QP excitations.

Understanding the Δt dependence in $\kappa(t, \Delta t)$

$\kappa(t, \Delta t)$ oscillates in Δt with the frequency ω_{pl}

$$\omega_{pl}^2 = \frac{4}{3} N_c \int \frac{d^3 p}{(2\pi)^3} \frac{g^2 f(t, p)}{\omega(p)} \sim t^{-2/7} \quad (7)$$

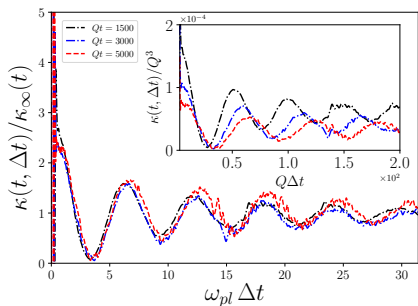


Figure: Lattice extraction of $\kappa(t, \Delta t)$

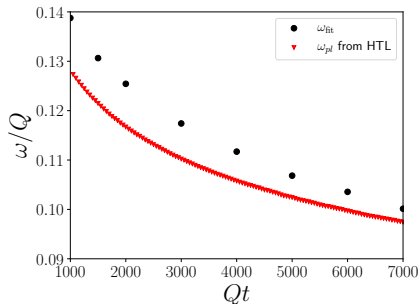


Figure: Frequency extracted from $\kappa(t, \Delta t)$ and using (7)

Spectral Reconstruction (SR) method

$$3\kappa(t, \Delta t) = \frac{g^2}{N_c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \int \frac{d^3p}{(2\pi)^3} \langle EE \rangle(t, \omega, p) \quad (8)$$

Generalized FDR (PRD 98 (2018) 1, 014006):

$$\langle EE \rangle_{T,L}(t, \omega, p) \approx \dot{\rho}(t, \omega, p) \langle EE \rangle_{T,L}(t, t, p) \quad (9)$$

$$\dot{\rho}(\omega, p) = 2\mathcal{I}m G_R^{\text{HTL}}(\omega, p) \quad (10)$$

$\langle EE \rangle$: HTL \neq far-from-equilibrium

- HTL: IR is thermal $\langle EE \rangle_T \approx T_*$ \rightarrow

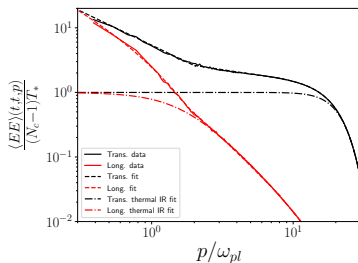
$$f(p) \approx \frac{T_*}{\sqrt{m^2 + p^2}}$$

- Far-from-equilibrium: IR enhanced compared to thermal $\langle EE \rangle_T \gg T_*$

$\dot{\rho}$ from HTL, use our data for QP properties:

- Dispersion $\omega(p)$
- Damping rate $\gamma(p)$.

Spectral Reconstruction (SR) method cont.



■ Gauge choice: $A^0 = 0 \forall t$. At readout times $\nabla \cdot \mathbf{A}(t) = 0$.
 At the $\Delta t \rightarrow \infty$ limit only longitudinal Landau damping contributes. Using the thermal IR assumption

$$\kappa_{\infty, LL}^{\text{SR}}(t) \approx \frac{N_c^2 - 1}{12\pi N_c} m_D^2(t) g^2 T_*(t) \log\left(\frac{\Lambda(t)}{m_D(t)}\right) \sim (Qt)^{-5/7} \log(Qt) \quad (11)$$

This will be our expectation for the time dependence of $\kappa(t)$.

Understanding the oscillations in $\kappa(t, \Delta t)$

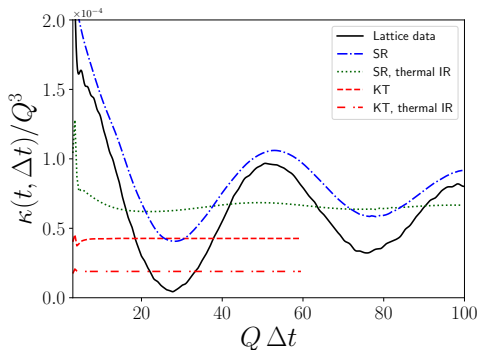
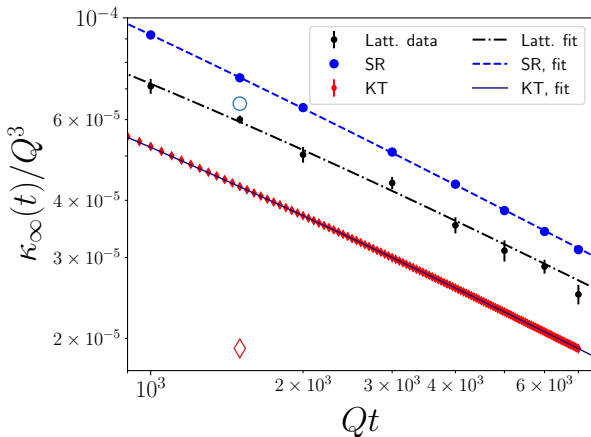


Figure: $\kappa(Q t = 1500, \Delta t)$.

- IR enhancement \rightarrow similar oscillations as on the lattice. Frequency: Trans. QP modes, Offset: Long. Landau damping.
- KT (backup): no oscillations. Trivialized frequency structure.
- Lattice vs. IR enhanced SR model \rightarrow IR enhancement exists ($\kappa(t, \Delta t)$ is gauge invariant).

$\kappa(t)$ - time-evolution

- Expectation as (11) $\kappa(t) \approx At^{-5/7}(\log(t) + B)$, use as a fit.
- Closed symbols: IR enhancement, open symbols: Thermal IR.



Conclusions

We have

- Extracted heavy quark momentum diffusion coefficient κ , t-evol consistent with $\kappa \sim t^{-5/7} \log(t)$, predicted by HTL and self-similarity.
- Observed IR enhancement (relative to thermal case) in equal time correlation functions, \rightarrow interesting oscillations in $\langle p^2 \rangle(\Delta t)$ and $\kappa(\Delta t)$ with frequency ω_{pl} .
- Heavy quarks, quarkonia, jets may reveal IR dynamics of non-equilibrium QGP.

Future plans

- Extract κ during the bottom-up thermalization scenario using EKT, and lattice extraction of \hat{q} .

Kinetic theory (generalizing PRC 71 (2005) 064904)

In the kinetic theory framework κ is given by ($gq \rightarrow gq$, t-channel gluon exchange, Compton amplitude suppressed)

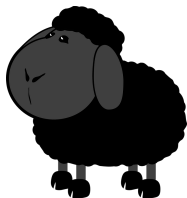
$$\kappa(\omega) = \frac{\langle \Delta k^2 \rangle}{\Delta t} = \frac{1}{6M} \int \frac{d^3\mathbf{k} d^3\mathbf{q}}{(2\pi)^6 8|\mathbf{k}||\mathbf{k}+\mathbf{q}|M} 2\pi\delta(|\mathbf{k}+\mathbf{q}|-|\mathbf{k}|-\omega) \times \mathbf{q}^2 |\mathcal{M}|_{\text{gluon}}^2(\omega) f(k) f(|\mathbf{k}+\mathbf{q}|) \quad (12)$$

k and k' gluon momenta, $q = k - k'$, p and p' incoming and outgoing heavy quark momenta.

$$|\mathcal{M}(\omega)|_{\text{gluon}}^2 = [N_c C_H g^4] \frac{4M^2(k_0 + k'_0)^2 (1 + \cos^2 \theta_{kk'})}{(q^2 - \omega^2 + m_D^2)^2}, \quad (13)$$

with $k'_0 = k_0 + \omega$. The limit $\Delta t \rightarrow \infty$ obtained at $\omega \rightarrow 0$ limit.

Real time lattice simulations



Real time lattice simulations:

- Heavy-ions: pre-equilibrium dynamics, cosmology: Higgs field after inflation (tachyonic preheating).
- $\mathcal{S} \rightarrow$ EOMS \rightarrow t-evol \rightarrow classical average
- Sign problem \rightarrow need classical (large) fields



Imaginary time lattice simulations:

- QGP in equilibrium
- $\mathcal{S} \rightarrow Z \rightarrow$
$$\langle \mathcal{O} \rangle_{\text{thermal}} = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{O} e^{-S}$$
- Quantized, wide range of applications.

Breaking down the different HTL contributions

- Transverse QP contributions \rightarrow oscillations at ω_{pl} .
- Longitudinal Landau cut $\rightarrow \kappa_{\infty}$

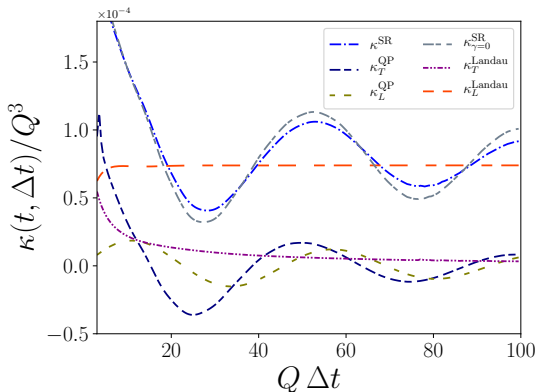


Figure: Different HTL contributions

Comparison with thermal case

Compare far-from-equilibrium to equilibrium by defining an effective coupling via scale separation:

$$\epsilon \sim T^4, \quad m_D^2 \sim g^2 T^2 \quad \rightarrow \quad \frac{m_D^2}{\sqrt{\epsilon}} \sim g_\epsilon^2 \quad (14)$$

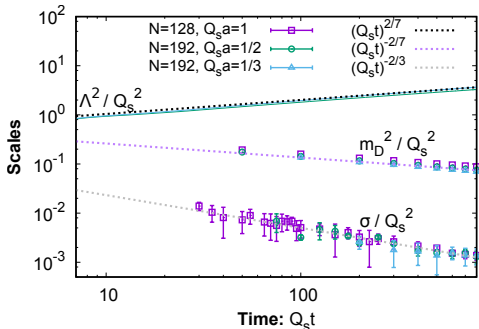


Figure: Schlichting et. al. Phys.Rev.D 93 (2016) 7, 074036. Time evolution in self-sim. regime \longleftrightarrow decreasing g in pert. theory

Comparison with thermal case

Expressing everything in terms of g :

$$\kappa^{\text{therm}} \approx 0.016 \left(\ln \frac{1}{g} + 0.25 \right) g^4 \varepsilon^{3/4} \quad (15)$$

$$\kappa_{\infty}(t) \approx 0.0050 \left(\ln \frac{g}{\tilde{g}_{\varepsilon}^2} + 0.148 \right) \tilde{g}_{\varepsilon}^5 g^{-1} \varepsilon^{3/4} \quad (16)$$

$$\kappa_{\infty}(t) \approx 0.0047 \left(\ln \frac{1}{\tilde{g}_{\Lambda}} + 0.177 \right) \tilde{g}_{\Lambda}^{5/2} g^{3/2} \varepsilon^{3/4}. \quad (17)$$

- $g_{\varepsilon}, g_{\Lambda} \gg g$ for overoccupied system. Extrapolate to $f \sim 1$ to compare to thermal.
- Coefficient of κ^{therm} larger than far-from equilibrium: IR enhancement increases m_D more than it increases κ .

SR method and generalized Fluctuation Dissipation Relation

Generalized FDR: connecting spectrum and statistical properties
(Boguslavski, Kurkela, Lappi, JP: Phys.Rev.D 98 (2018) 1, 014006):

$\langle EE \rangle_{T,L}(t, \omega, p) \approx \overbrace{\langle [\hat{E}, \hat{A}] \rangle(t, \omega, p)}^{\dot{\rho}(t, \omega, p)} \langle EE \rangle(t, t, p)$. The spectral function $\dot{\rho}$ is defined as

$$\dot{\rho}(\omega, p) = 2\Im m G_R^{HTL}(\omega, p), \quad (18)$$

with $G_R^T = \frac{-1}{\omega^2 - p^2 - \Pi_T(\omega, p)}$ and $G_R^L = \frac{p^2}{\omega^2} \frac{-1}{p^2 - \Pi_L(\omega, p)}$

$$\begin{aligned} \Pi_T(x) &= m^2 x (x + (1 - x^2) Q_0(x)) \\ \Pi_L(x) &= -2m^2 (1 - x Q_0(x)), \end{aligned} \quad (19)$$

and ($x = \omega/p$)

$$Q_0(x) = \frac{1}{2} \ln \frac{x+1}{x-1} = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| - \frac{i\pi}{2} \theta(1-x^2). \quad (20)$$

Spectral reconstruction method, quasiparticles and Landau damping

2 types of contributions:

- Particle-like excitations, at LO proportional to $\delta(\omega - \omega_{T,L})$.
- Landau damping contributions, nonzero only for $\theta(1 - \omega^2/p^2)$, damping of space charge waves in plasma.

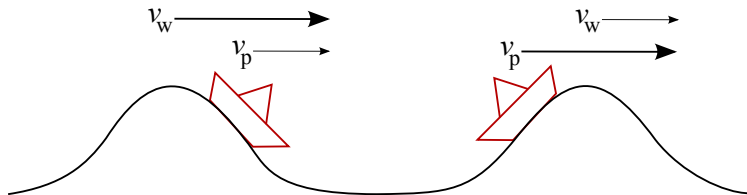


Figure: (From Wikipedia) Illustration of the Landau damping

$\Delta t \rightarrow \infty$ limit in the SR framework

At the $\Delta t \rightarrow \infty$ limit only longitudinal Landau damping contributes. Using the thermal IR assumption

$$\langle EE \rangle_L^{LL}(t, t, p) = (N_c^2 - 1) T_* \frac{m_D^2}{p^2 + m_D^2} \theta(\Lambda - p). \quad (21)$$

We get

$$\kappa_{\infty, LL}^{\text{SR}}(t) \approx \frac{N_c^2 - 1}{12\pi N_c} m_D^2(t) g^2 T_*(t) \log\left(\frac{\Lambda(t)}{m_D(t)}\right) \sim (Qt)^{-5/7} \log(Qt) \quad (22)$$

This will be our expectation for the time dependence of $\kappa(t)$.

Heavy quarks & Transport far-from equilibrium

- Heavy quarks probe the entire history of the event, $M \gg$ other scales in the medium.

Recent interest in transport out-of-equilibrium:

- Fokker-Planck evolution of heavy quarks in a non-equilibrium gluon plasma:
 - Carrington, Czajka, Mrowczynski NPA 1001 (2020) 121914
 - Mrowczynski, EPJA 54 (2018) 3, 43
- Pre-equilibrium heavy quark transport coefficients:
 - R_{AA} and v_2 : Das, Ruggieri, Scardina, Plumari, Greco J. Phys. G44(2017) no. 9 095102
 - Charm transport coefficients: Song, Moreau, Aichelin, Bratkovskaya PRC 101 (2020) 4, 044901
- Anisotropic momentum broadening in the 2+1D Glasma
 - Ipp, Müller, Schuh PLB 810 (2020) 135810

This talk: Heavy quark diffusion in pre-equilibrium QGP