Confinement-Deconfinement transition and Z_2 symmetry in Z_2+Higgs theory

Sabiar Shaikh

The Institute of Mathematical Sciences, Chennai.

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based on arXiv:2102.11091, in collaboration with
Minati Biswal, Sanatan Digal, Vinod Mamale

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Outline

Introduction

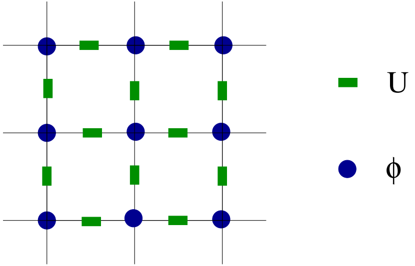
- Z_N symmetry plays an important role in the confinement-deconfinement (CD) transition in pure $SU(N)$ gauge theories. In these theories, at finite temperature, the allowed gauge transformations are classified by the centre of the gauge group, i.e Z_N.
- Previous studies of Z_N symmetry in $SU(N)$ Higgs theories have found that the Z_N symmetry is restored in the Higgs symmetric phase in the continuum limit (i.e. large number of temporal lattice points N_τ). [S. Digal et. al., Nucl.Phys.B910,30-39(2016)]
- In the Z_2 Higgs theory, the fields being Ising like, one can hope to understand better the Z_2 explicit breaking and its dependence on the coupling between the gauge and Higgs fields and N_τ.
- Our goal in this study is to investigate the strength of the explicit breaking of this symmetry by varying the parameters of the theory and N_τ.

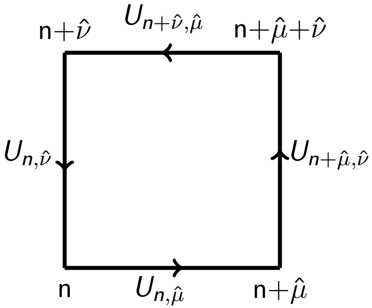
Total action in Z_2+Higgs theory

- The action for the Z_2+Higgs theory in four dimensional lattice ($N_s^3 \times N_\tau$) is given by,

$$S = -\beta_g \sum_P U_P - \kappa \sum_{n,\hat{\mu}} \Phi_{n+\hat{\mu}} U_{n,\hat{\mu}} \Phi_n. \tag{1}$$

- The plaquette $U_P = U_{n,\hat{\mu}} U_{n+\hat{\mu},\hat{\nu}} U_{n+\hat{\nu},\hat{\mu}} U_{n,\hat{\nu}}$.





- Both $U_{n,\hat{\mu}}$ and Φ_n take values ± 1.

Z_2 symmetry in pure gauge theory

- The pure gauge part of the action is invariant under the Z_2 gauge transformations,

$$U_{n,\hat{\mu}} \rightarrow V_n U_{n,\hat{\mu}} V^{-1}_{n+\hat{\mu}} \tag{2}$$

where $V_n = \pm 1 \in Z_2$. The V_n's satisfy the boundary condition,

$$V(\vec{n}, n_4 = 1) = zV(\vec{n}, n_4 = N_\tau). \tag{3}$$

$z = \pm 1 \in Z_2$. So the gauge transformations can be classified by the group Z_2 and in pure gauge theory Z_2 symmetry is always there.

- The Polyakov loop is, $L(\vec{n}) = \prod_{n_4=1}^{N_\tau} U_{(\vec{n},n_4),\hat{4}} \Rightarrow$ Order parameter

transforms non-trivially under Z_2 gauge transformations

$$L(\vec{n}) \rightarrow zL(\vec{n}). \tag{4}$$

Explicit symmetry breaking in presence of Higgs fields

- For this theory, under the Z_2 gauge transformation, Higgs field(Φ_n) in the fundamental representation transform as, $\Phi_n \to V_n \Phi_n$
- Higgs fields are periodic and satisfy the boundary condition,
$$\Phi(\vec{n}, n_4 = 1) = \Phi(\vec{n}, n_4 = N_\tau)$$
- Under Z_2 gauge transformation the Higgs fields transform as,

$$\begin{aligned} \Phi(\vec{n}, n_4 = 1) &\to V(\vec{n}, n_4 = 1)\Phi(\vec{n}, n_4 = 1) \\ &= zV(\vec{n}, n_4 = N_t)\Phi(\vec{n}, n_4 = N_\tau) \\ &= z\Phi_g(\vec{n}, n_4 = N_\tau) \end{aligned} \tag{5}$$

So the gauge transformed matter fields Φ_g satisfy the boundary condition, $\Phi_g(\vec{n}, n_4 = 1) = z\Phi_g(\vec{n}, n_4 = N_\tau)$

- Φ_g does not remain periodic when $z = -1$. Therefore, in the presence of Higgs field Φ_n the Z_2 symmetry is broken explicitly.

Symmetry in partition function

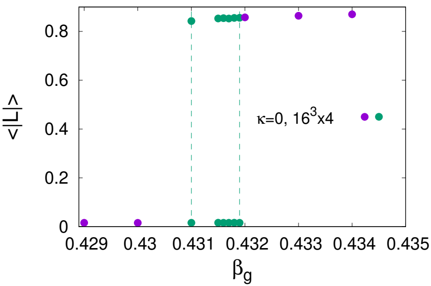
- In the partition function,

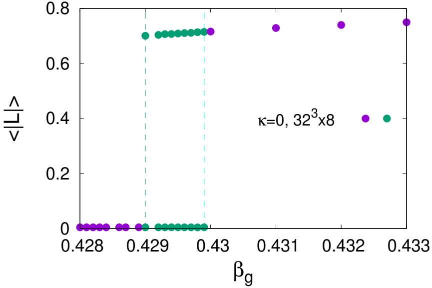
$$\mathcal{Z} = \sum_{\Phi, U} e^{-S} \tag{6}$$

- Since the action for $\kappa = 0$ is invariant under Z_2 gauge transformations, any configuration and it's gauge rotated counterpart will contribute equally to the partition function i.e there is Z_2 symmetry.
- For $\kappa \neq 0$ case given a configuration, one can define a Z_2 counterpart in which only the gauge links are Z_2 rotated i.e $(U, \Phi) \rightarrow (U_g, \Phi)$. Obviously $S(U, \Phi) \neq S(U_g, \Phi)$ and these pair of configurations will not contribute equally to the partition function.

CD transition for $N_\tau = 4, 8$ in pure gauge theory

- The average of the Polyakov loop is plotted *vs* β_g for $N_\tau = 4, 8$. There is a range in β_g for which clearly separated peaks in the distribution of the Polyakov loop has been observed.



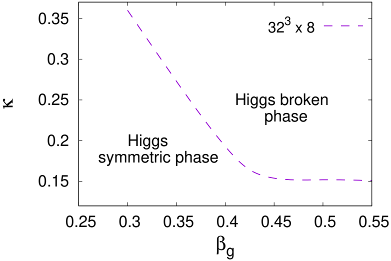


- The two peaks suggest that the transition is first order [M. Creutz et. al., Phys. Rev. Lett. 42, 1390(1979)]. For larger lattice sizes the range of β_g over which two states are observed increases.

Phase diagram

- The effect of the Φ field on the CD transition and Z_2 symmetry is expected to depend on κ. [G.A.Jongeward et. al., Phys.Rev.D21,3360(1980)]
- In the Higgs broken phase($\kappa > \kappa_c$), i.e large κ, the interaction term dominates over the entropy or DoS $\Rightarrow$ Z_2 symmetry is badly broken.

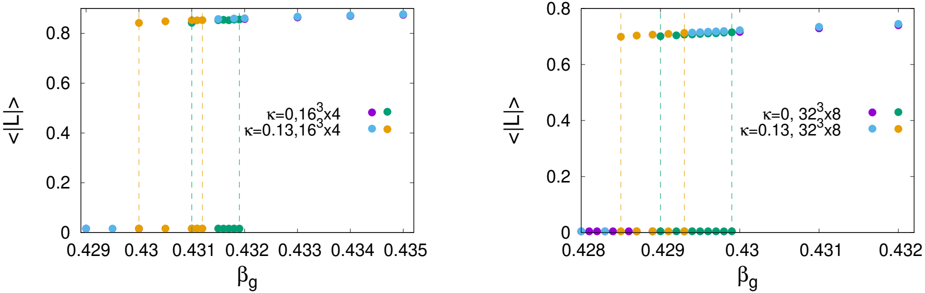
- In the Higgs symmetric phase($\kappa < \kappa_c$), it is the DoS i.e the distribution of the interaction term dominate. $\Rightarrow$ Possibility for realization of Z_2 symmetry.



- In our simulations the Higgs transition is found to be first order for intermediate range of β_g and crossover for both small and large β_g. [M. Creutz et. al., Phys. Rept. 95, 201-282 (1983)]

CD transition in presence of Higgs fields

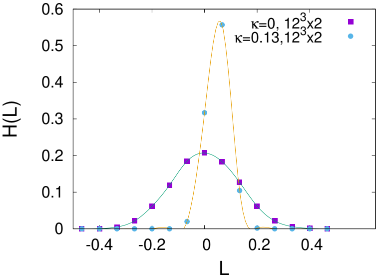
- In figure we show *CD* transition in the Higgs symmetric phase $(\kappa = 0.13)$. The *CD* transition is first order even in the presence of Φ, though the transition point shifts to lower values of β_g.

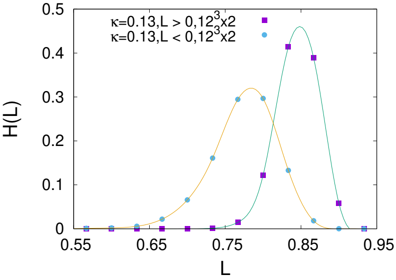


- For small but non-zero κ the CD transition is first order for $N_\tau \geq 3$.

Histogram for $N_\tau = 2$ with Higgs field

- To check the N_τ dependence of the Z_2 symmetry at $\kappa = 0.13$, the distribution of Polyakov loop is computed both in the confined and the deconfined phases
- For $N_\tau = 2$ the histograms clearly show there is no Z_2 symmetry.

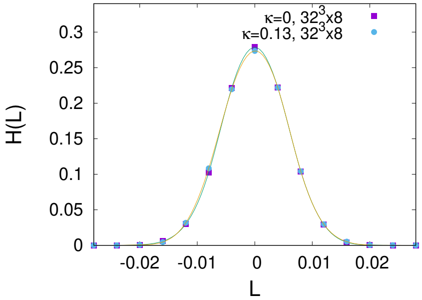


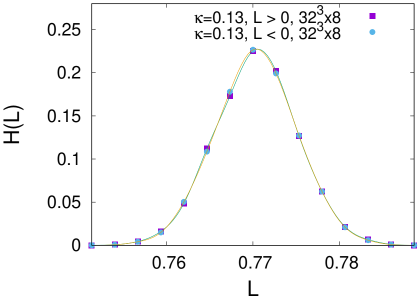


- In the deconfined phase, $L < 0$ data is Z_2 rotated and then compared with $L > 0$ data.

Histogram for $N_\tau = 8$ with Higgs field

- For $N_\tau = 8$, the histogram of Polyakov loop for two Z_2 sectors agree well with each other.

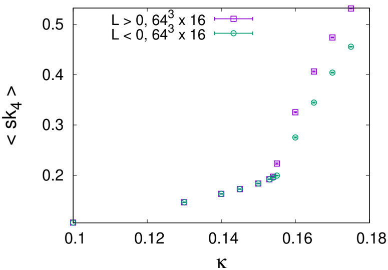


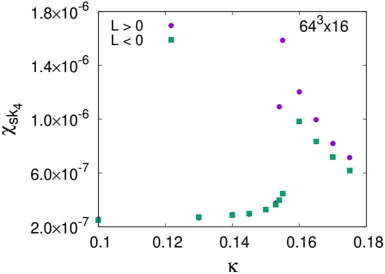


- The simulation results indicate that the Z_2 symmetry is restored at large N_τ in the presence of matter fields.

Dependence of Z_2 symmetry on the phase of Higgs

- $sk_4 = \sum_n \Phi_n U_{n,\hat{4}} \Phi^\dagger_{n+\hat{4}}$, Susceptibility $\chi_{sk_4} = \langle sk_4^2 \rangle - \langle sk_4 \rangle^2$
- Along x-axis on the left $(\kappa < 0.154)$ it is Higgs symmetric phase and on the right $(\kappa > 0.154)$ it is Higgs broken phase.





- At $\beta_g = 0.435$, for larger N_τ, the κ value at which the two polyakov loop sectors differ significantly in sk_4 and χ_{sk_4} is higher.

Role of DoS: Example in $0 + 1$D

- The temporal component of the gauge Higgs interaction corresponding to a particular spatial site can be written as,

$$S_{1D} = -\kappa sk_4, \quad sk_4 = \sum_{n=1}^{N_\tau} \Phi_n U_n \Phi_{n+1} \tag{7}$$

Φ_n satisfies the periodic boundary condition $\Phi_{N_\tau+1} = \Phi_1$.

- We set $U_i = 1$, for $i = 1, 2, ...N_\tau - 1$ and $U_{N_\tau} = L$. The partition function for $L = 1$ is nothing but that of the one dimensional Ising chain. For $L = -1$ the only difference is that the coupling between Φ_{N_τ} and Φ_1 is anti-ferromagnetic. The exact partition functions

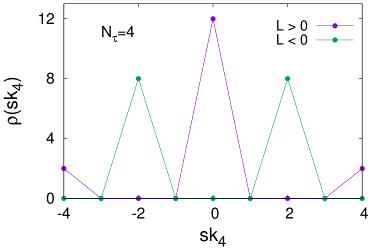
$$\mathcal{Z}(L = 1) = \lambda_1^{N_\tau} + \lambda_2^{N_\tau}, \quad \mathcal{Z}(L = -1) = \lambda_1^{N_\tau} - \lambda_2^{N_\tau} \tag{8}$$

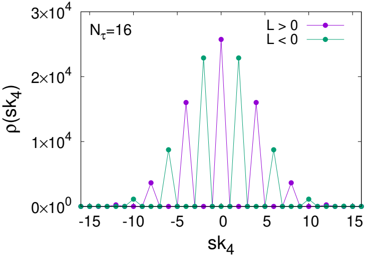
where $\lambda_1 = e^\kappa + e^{-\kappa}$ and $\lambda_2 = e^\kappa - e^{-\kappa}$. The free energies in large N_τ are, $V(L = 1) = V(L = -1) = -TN_\tau \log(\lambda_1)$

- It shows that there is Z_2 symmetry in $0 + 1$ dimensions in the limit of $N_\tau \to \infty$.

Density of states in 0 + 1D

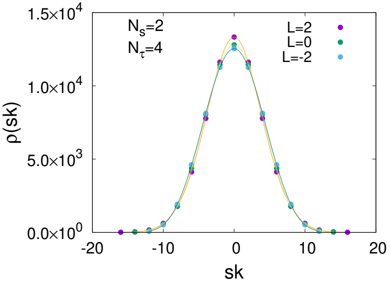
- The realisation of the Z_2 symmetry must come from the Z_2 symmetry of the entropy or the *DoS* i.e $\rho(sk_4)$.
- For small N_τ there are clear difference in $\rho(sk_4)$ for $L = \pm 1$.





- For large N_τ, $\rho(sk_4)$'s for both $L = \pm 1$ are well described by a gaussian centred at $sk_4 = 0$, with $\sqrt{N_\tau}$ as standard deviation.
- The thermodynamics in the $N_\tau \to \infty$ limit will be dominated by peak height and distribution of $\rho(sk_4)$ around the peak, which is Z_2 symmetric, for all finite κ.

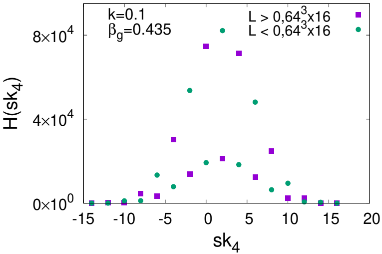
- In order to take into account the effect of nearest neighbour coupling along the spatial direction we consider $1+1$ dimensional model with $N_s = 2$ and vary N_τ. Here $sk = sk_1 + sk_4$.



- The results for the distribution of the total interaction action for $N_\tau = 4$ and $N_\tau = 16$ shows that for higher N_τ, $\rho(sk)$ around the peak $sk = 0$ do not depend on L i.e the realization of Z_2 symmetry at higher N_τ.

Histogram in $3 + 1$D

- Fig. shows the distribution $H(sk_4)$ for $N_\tau = 16$ at $\kappa = 0.1$ and $\beta_g = 0.435$. For these values of κ and β_g, the system is found to be in the deconfined and Higgs symmetric phase.
- The thermal average of the Polaykov loop for the two sectors are found to be $\langle L \rangle = 0.5896 \pm 0.002$ and -0.5897 ± 0.00199.

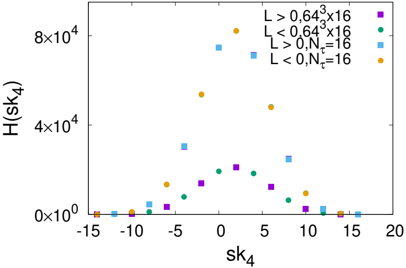


- The results clearly show that $H(sk_4)$ for both the Polyakov loop sectors can be approximately described by single function in other words the presence of Z_2 symmetry.

Comparison of $3+1$D and $0+1$D results

- We try to fit the $3+1$ dimensional simulation result with $0+1$ dimensional *DoS* by including an extra Boltzmann factor, i.e $\exp(\kappa' sk_4)$. The resulting fit agree very well with $H(sk_4)$.

- $H(sk_4) \propto \exp(\kappa' sk_4)\rho(sk_4)$
- Note here, $H(sk_4)$ values correspond to $\kappa = 0.1$, but to fit *DoS* one needs a $\kappa'(\simeq 0.106)$ value which is higher.



- This is due to the fact that in $3+1$ dimensions sk_4 at a given spatial point interacts with sk_4 at the nearest neighbour sites.

Summary

- Our results suggest that the $3+1D$ Monte Carlo simulations can be reproduced using the DoS of the $0+1D$ model.
- In presence of Higgs fields the Z_2 symmetry realization happens in the large N_τ limit in the Higgs symmetric phase.
- The realization of Z_2 symmetry is due to dominance of DoS over the Boltzman factor.
- Computing the *DoS* in $SU(N)$+Higgs theory is a difficult task as the configuration space is infinite. The Z_2+Higgs theory in four dimensions provides a suitable alternative as the field variables take values ± 1.

Thank You