Roberge-Weiss transitions at imaginary isospin chemical potential

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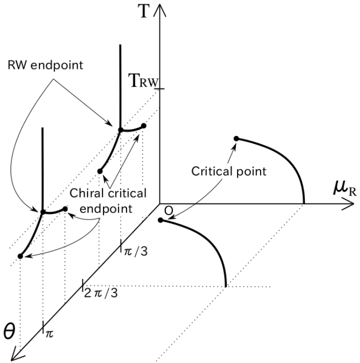
July 28, 2021

Motivation



- Why considering complex chemical potential? (Sign Problem)
- Analytic Continuation
- Our approach: Effective potential for the Polyakov loop in the presence of imaginary isospin chemical potential.
- Leads to a phase diagram

Figure from: [Kouji Kashiwa, Akira Ohnishi Phys. Lett. B (2015)]

Roberge-Weiss Periodicity

- Functional integral representation of the partition function ($\theta \equiv \mu_{\mathrm{I}}/T$):

$$\mathcal{Z}(\theta) = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \exp\left(-\int \mathrm{d}^4x \left[\bar{\psi}(\gamma D - m)\psi - \frac{1}{4}F^2 - \frac{i\theta}{\beta}\psi^\dagger\psi\right]\right) \quad (1)$$

- Perform: $\psi(\vec{\mathbf{x}},\tau) \to e^{i\tau\theta/\beta}\psi(\vec{\mathbf{x}},\tau)$

$$\mathcal{Z}(\theta) = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \exp\left(-\int \mathrm{d}^4x \left[\bar{\psi}(\gamma D - m)\psi - \frac{1}{4}F^2\right]\right) \quad (2)$$

- Roberge-Weiss-Periodicity:

$$\boxed{\mathcal{Z}(T,\theta) = \mathcal{Z}(T,\theta + 2\pi k/N)} \quad (3)$$

[André Roberge, Nathan Weiss Nucl. Phys. B (1986)]

Polyakov-Loop

- Order Parameter:

$$L(\vec{\mathbf{x}}) = \frac{1}{N} \operatorname{Tr} \mathcal{P} \exp \left\{ ig \int_0^\beta \mathcal{A}_0(\vec{\mathbf{x}}, t) \mathrm{d}t \right\} \tag{4}$$

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$$\langle L \rangle = \operatorname{Tr} e^{i\beta\phi}, \quad \phi = g A_0 = \begin{pmatrix} \phi_i & & \\ & \ddots & \\ & & \phi_N \end{pmatrix} \tag{5}$$

- Connection to the free energy:

$$\langle L \rangle = \frac{\mathcal{Z}_{\text{Quark}}}{\mathcal{Z}} = \frac{e^{-\beta F_{\text{Quark}}}}{e^{-\beta F}} = e^{-\beta \Delta F} \tag{6}$$

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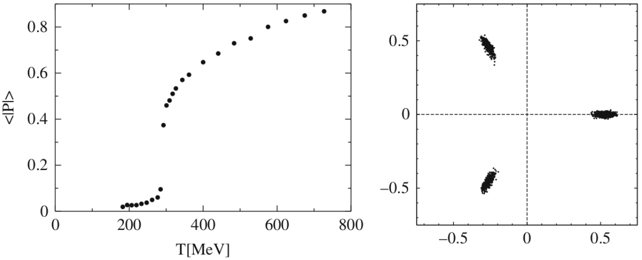
$$\langle L \rangle = \begin{cases} 0 & , \Delta F \to \infty \quad \text{confinement} \\ \neq 0 & , \Delta F \to 0 \quad \text{deconfinement} \end{cases}$$

[Alexander M. Polyakov, Phys. Lett. B (1978)]

Polyakov-Loop Phases



[Figure from: Christof Gattringer, Lect.Notes Phys. 788 (2010)]

Key Result: Effective Potential for SU(N)

Effective Potential for SU(N):

- Gluoonic:

$$V_{\text{eff}}^{\text{glu}}(\phi_i,\ldots,\phi_N)=\frac{\pi T^4}{24}\sum_{j=1}^{N}\sum_{k=1}^{N}\left\{1-\left[\left[\frac{\beta\phi_j}{\pi}-\frac{\beta\phi_k}{\pi}\right]_{\text{mod }2}-1\right]^2\right\}^2 \quad (7)$$

- Fermionic:

$$V_{\text{eff}}^{\text{ferm}}(\phi_i,\ldots,\phi_N)=-\frac{\pi T^4}{12}\sum_{j=1}^{N}\left\{1-\left[\left[\frac{\beta\phi_j}{\pi}+1\right]_{\text{mod }2}-1\right]^2\right\}^2 \quad (8)$$

- The total effective Potential is:

$$V_{\text{eff}}(\phi_i,\ldots,\phi_N)=V_{\text{eff}}^{\text{glu}}(\phi_i,\ldots,\phi_N)+V_{\text{eff}}^{\text{ferm}}(\phi_i,\ldots,\phi_N) \quad (9)$$

[Nathan Weiss, Phys. Rev. D.24 (1981)]
[Nathan Weiss, Phys. Rev D.25 (1982)]

Effective Potential

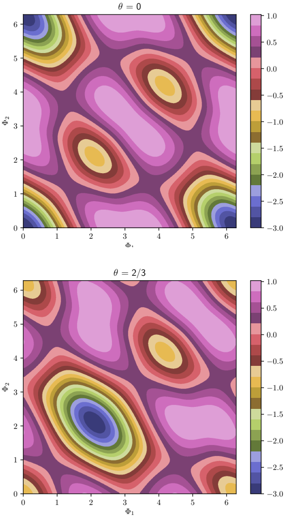
- According to stat. physics $\mathcal{H} \rightarrow \mathcal{H} - i\theta\mathcal{N}$
- If we put the Hamiltonian in the QCD-Lagrangian, we remark:

$$A_0 \rightarrow A_0 + \frac{\theta}{g\beta} \cdot \mathbb{I} \qquad \phi_i \rightarrow \phi_i + \frac{\theta}{\beta}$$

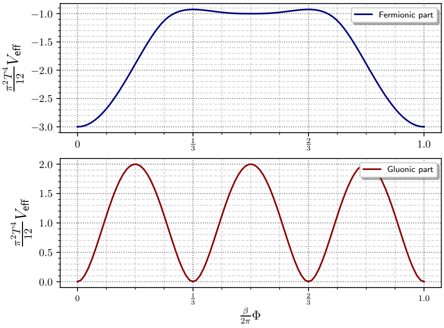
- Use this result for the effective potential (9):

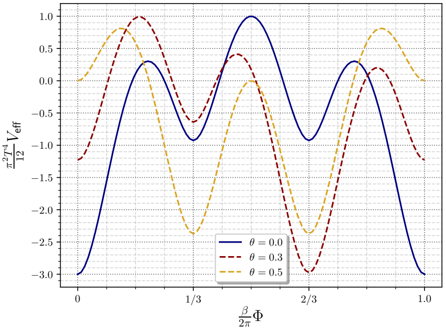
$$V_{\text{eff}}(\phi_i, \ldots, \phi_N) = V_{\text{eff}}^{\text{glu}}(\phi_i, \ldots, \phi_N) + V_{\text{eff}}^{\text{ferm}}\left(\phi_i + \frac{\theta}{\beta}, \ldots, \phi_N + \frac{\theta}{\beta}\right). \quad (10)$$

Behavior of the Potential [1]



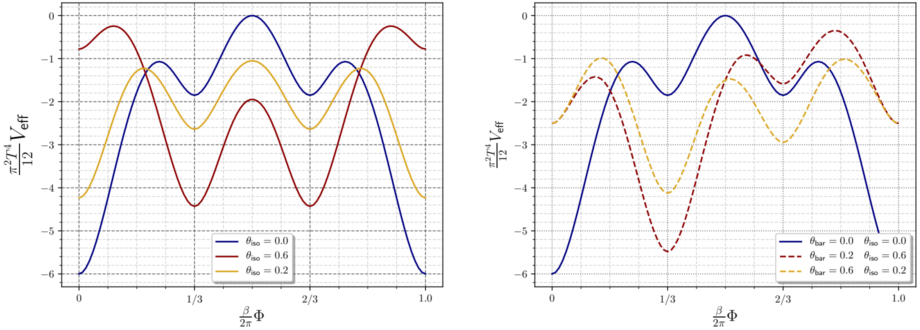
Behavior of the Potential [2]



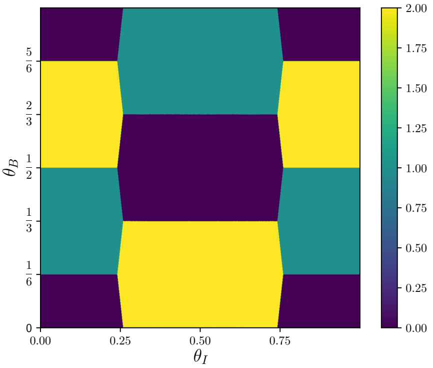


Adding isospin

- Isospin: $\theta_{\text{iso}} \equiv (\theta_u - \theta_d)/2$

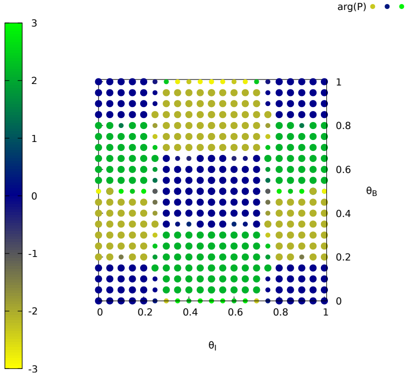


Carpet Plot



- Different colors corresponds to the phases of the Polyakov-Loop.
- Chemical potential breaks the symmetry
- Special points: Coexistence of different phases $\rightarrow$ spontaneously symmetry breaking.
- Point: $(0, 1/6)$ $Z(2)$-symmetric point or *twofold degeneracy*.

Lattice Simulation



- Setup: $8^3 \times 4$ lattices, $m = 0.025$, $\beta = 5.2$, unimproved action, 2 rooted staggered flavour
- Lattice data confirms the analytical approach.
- Passing the first order transition, problem to find the right minimum (smaller points on the plane).

Summary

- Imaginary isospin and baryonic chemical potential $\rightarrow$ interesting phase structure
- Leads to the carpet plot
- Good understanding of analytical and lattice simulations