Flavor number dependence of QCD at finite density by the complex Langevin method

Yusuke Namekawa (Yukawa Institute of Theoretical Physics)

in collaboration with

Y.Asano(Tsukuba), Y.Ito(Tokuyama Col.), T.Kaneko(KEK), H.Matsufuru(KEK), J.Nishimura(KEK), A.Tsuchiya(Shizuoka), S.Tsutsui(RIKEN), T.Yokota(RIKEN)

Contents

1 Introduction

QCD at high density has been investigated by Complex Langevin Method(CLM) Sexty(2013),...

→ It is important to find ”validity region of CLM” where the following two conditions are satisfied for correct CLM results

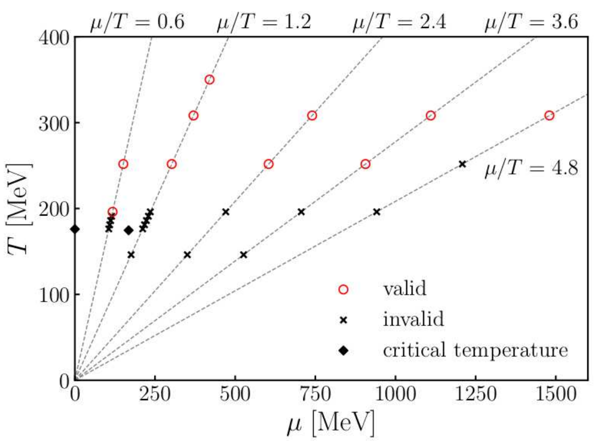
- Excursion problem must be under control Aarts et al.(2011)
  Gauge configurations must not be far away from $SU(3)$.
  - ◇ Excursion problem can be tamed in part by Gauge cooling(gauge trans. for complexified variable). It does not solve the problem but gives better control. Seiler et al.(2013)
- Singular drift must be under control Nishimura and Shimasaki(2015)
  Quark matrix can have near zero eigenvalues, which leads to singularities in the drift term and CLM converges to wrong result.
  - ◇ Histogram of drift term gives a good criterion: exp fall-off or not Nagata et al.(2016). (Consistency with the boundary term has been confirmed Scherzer et al.(2019). cf. talks by Sexty(Wed), Seiler(Wed))

We have found the validity region of CLM with $N_f = 4$ staggered quarks on $8^3 \times 16$ and $16^3 \times 32$ Ito et al.(2020), as well as on $24^3 \times 12$ Tsutsui et al.(coming soon)

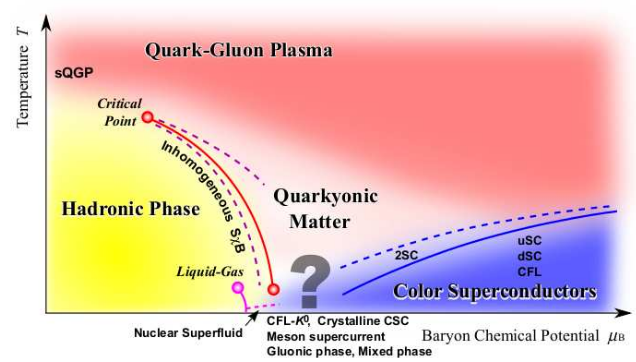
→ Next page

[Our previous works for validity region of CLM with $N_f = 4$ staggered quark]

- Validity region of CLM at $\beta = 5.7$ is found to be $\mu_{\rm quark}/T = 5.2 - 7.2$ on $8^3 \times 16$, $\mu_{\rm quark}/T = 1.6 - 9.6$ on $16^3 \times 32$ Ito et al.(2020)
    - ◇ We have successfully confirmed formation of the Fermi surface, though $L = 0.4 - 0.7$ fm
- Validity region of CLM has been identified in $\mu_{\rm quark}/T = 0.6 - 4.8$ on $24^3 \times 12$ ($L = 1.1 - 2.7$ fm) Tsutsui et al.(coming soon)
    - ◇ Our next target is QCD at low T and high $\mu_{\rm quark}$ toward non-perturbative study of color superconducting(CSC) phase



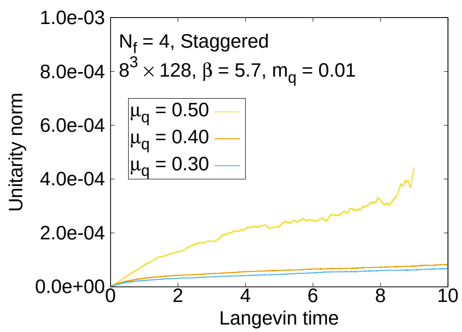
Tsutsui et al.(coming soon)

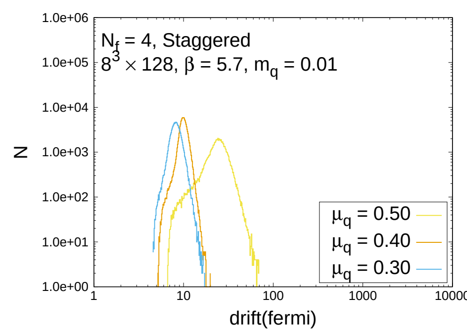


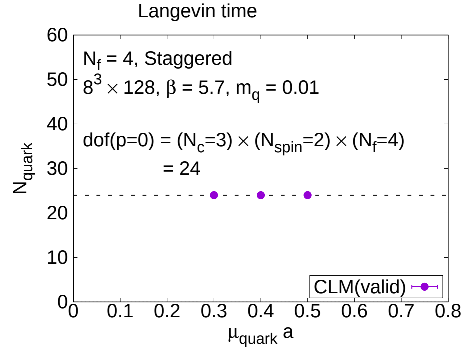
Fukushima and Hatsuda(2011)

[Validity region of CLM with $N_f = 4$ staggered quark on $8^3 \times 128$]
Is there validity region of CLM at low T and high $\mu_{\rm quark}$? Yes!

- On $8^3 \times 128$, validity region of CLM at $\beta = 5.7$ is found in $\mu_{\rm quark}/T = 38 - 64$ (CG not converged at $\mu_{\rm quark}/T = 26, 77$)
    - ◇ Control of unitarity norm and exp fall-off fermion drift histogram ensures validity of CLM
    - ◇ Quark number shows plateau behavior, expected from formation of Fermi surface Ito et al.(2020)







$$N_{\rm quark} := \frac{\partial}{\partial \mu_{\rm quark}} \log Z_{QCD}$$

[Our strategy toward non-perturbative study of color superconducting phase]

- Explore validity region of CLM by $N_f = 2, 3, 4$ Wilson quark
  $\leftarrow$ This talk

  ◇ N_f dependence is important for CSC classification:
  – Color-Flavor locked phase for $N_f = 3$
  – 2-flavor CSC for $N_f = 2$

  ◇ Wilson quark has advantage that it has exact flavor-symmetry
  (NB. chiral symmetry can be also important for CSC, requiring chirally improved quarks)

- Perturbative analysis of CSC on the lattice
  $\leftarrow$ T.Yokota's talk

- Lattice simulation using CLM with $N_f = 4$ staggered quark at low T and high μ_{quark} toward CSC
  $\leftarrow$ S.Tsutsui's talk

2 CLM with $N_f = 2, 3, 4$ Wilson quark at finite density

[Simulation setup] Preliminary

- Action: plaq gauge + $N_f = 2, 3, 4$ Wilson quark
- Lattice size: $8^3 \times 128$
- $\beta = 6.0, 8.0$ (ex. $a_{r_0}^{-1} = 2.2$ GeV for $\beta = 6.0$, $N_f = 2$)
- $\kappa_{\rm quark} = 0.12195 - 0.12497$

  ($m_{\rm quark}^{\rm bare} a \sim 0.100 - 0.001$, assuming $\kappa_c \sim 1/8$ to be determined non-perturbatively)
- $\mu_{\rm quark} = 0.10 - 0.80$ ($\mu_{\rm quark}/T = 13 - 102$)

- Algorithms: improved partial RK2 Bali et al.(2013) + adaptive step size Aarts et al.(2010) with $\epsilon_L = 10^{-5}$, nested CG, gauge cooling Seiler et al.(2013)

  ◇ NB. CLM with odd N_f can be calculated with no additional prescription, because of no use of probability in CL updates
  ← HMC needs additional prescription such as rational approx. for odd N_f part

  ◇ cf. previous works with $N_f = 2$ Wilson fermion Scherzer et al.(2020), talk by B.Jäger(Tue)

[Results of this work]

- We explore validity region of CLM on $8^3 \times 128$ lattice by $N_f = 2, 3, 4$ Wilson quark
  - ♢ Validity region of CLM is enlarged as we decrease N_f

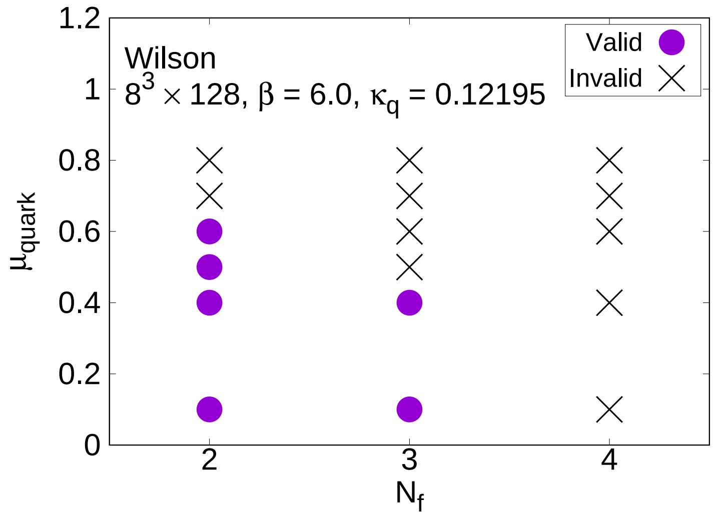
    → The trend is opposite to $N_f = 2$ staggered quark on $12^3 \times 6$ Kogut,Sinclair(2018) which utilizes rooting and has finite temperature effects
  - ♢ Validity region of CLM do not significantly depend on m_{quark}
  - ♢ Validity region of CLM is enlarged at higher β
- Quark numbers in the validity region of CLM shows expected N_f dependence in the heights of plateaus

[Validity region of CLM with $N_f = 2, 3, 4$ Wilson quark at $\beta = 6.0$ on $8^3 \times 128$]

- We investigate $N_f = 2, 3, 4$
- Validity region of CLM is enlarged as we decrease N_f

  (NB. strict comparison along the line of constant physics is desirable)

  ◇ Unitarity norm decreases and drift histogram shows exp fall-off, as we decrease N_f (See next page for details)
  $\rightarrow$ It is an encouraging result for realistic $N_f = 2+1$, in which validity region of CLM must be wider than that of $N_f = 3, 4$



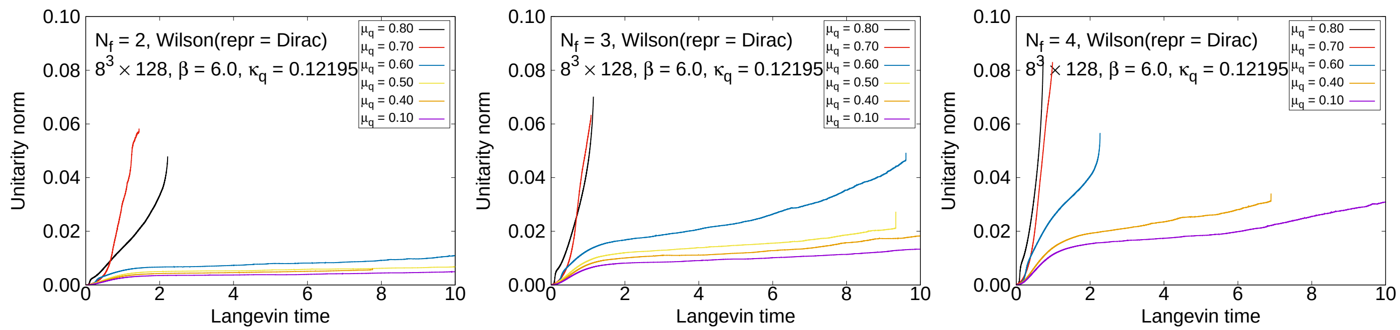
- (For staggered quark, validity region of CLM shrinks as we decrease $N_f = 4 \rightarrow 2$ on $12^3 \times 6$ Kogut,Sinclair(2018) which utilizes rooting and has finite temperature effects)

[Validity region of CLM with $N_f = 2, 3, 4$ Wilson quark at $\beta = 6.0$ on $8^3 \times 128$]

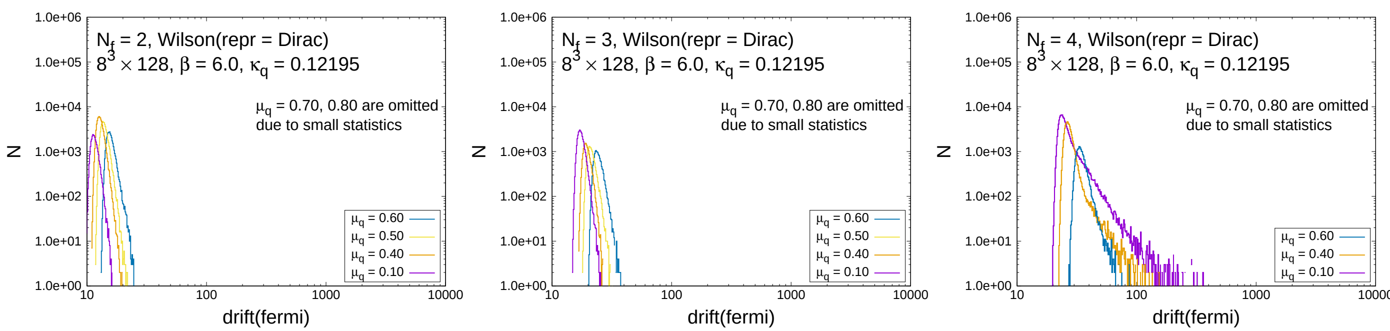
- We investigate $N_f = 2, 3, 4$
- Validity region of CLM is enlarged as we decrease N_f

(NB. strict comparison along the line of constant physics is desirable)

◇ Unitarity norm decreases as we decrease N_f

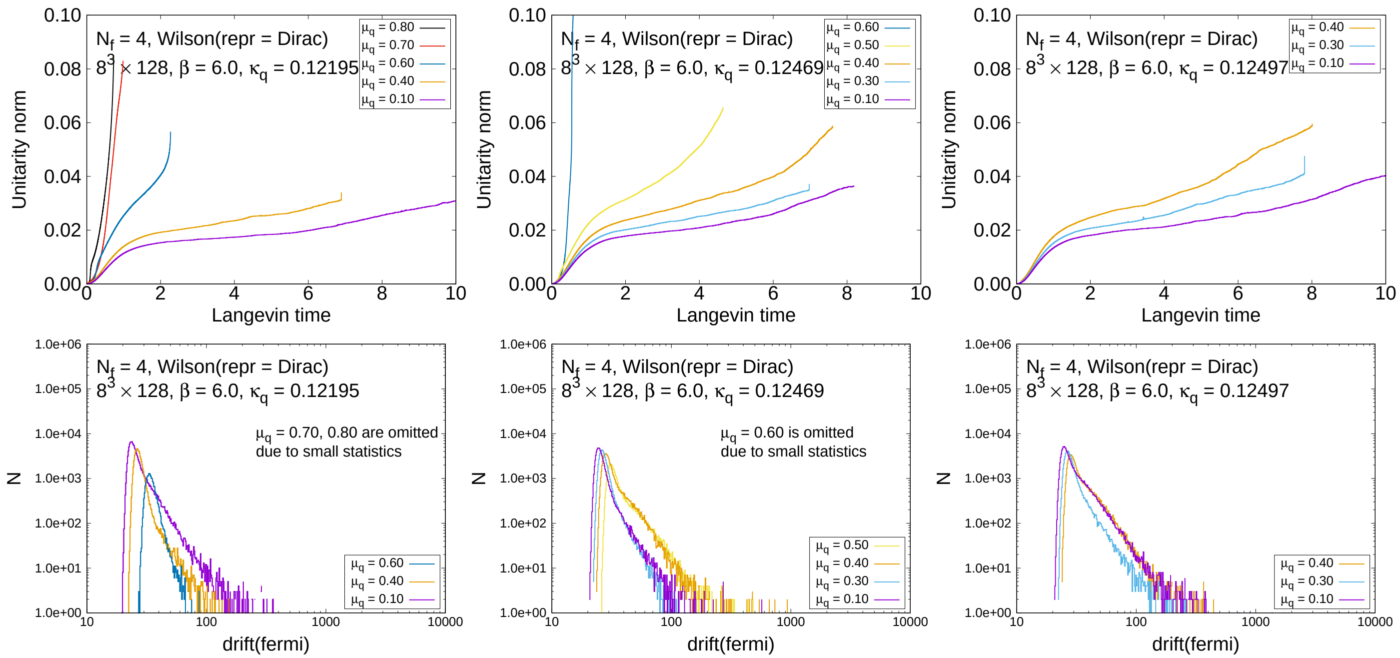


◇ Drift histogram shows exp fall-off at $\mu_{\mathrm{quark}} \leq 0.60$ in $N_f = 2, 3$, while power-law tail appears in $N_f = 4$



[m_q dependence of validity region of CLM with $N_f = 4$ Wilson quark]

- We investigate $\kappa_{\text{quark}} = 0.12195 - 0.12497$
- Validity region of CLM does not significantly depend on m_{quark}
  $\leftarrow m_{\text{quark}}$ may not be small enough (small m_{quark} causes "CG not converged")



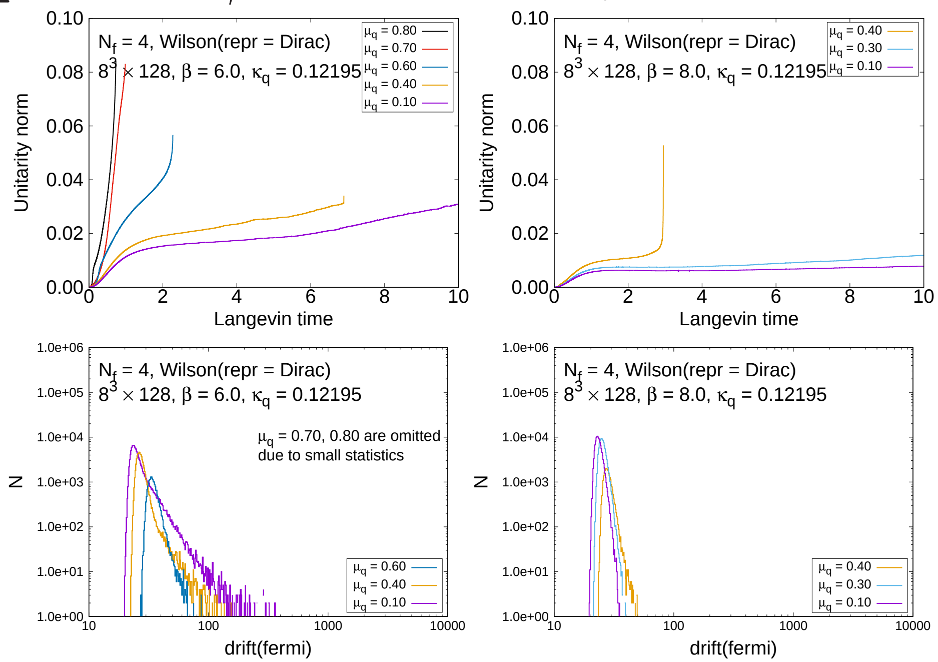
- (For staggered quark, validity region of CLM is enlarged for smaller m_{quark} Kogut,Sinclair(2019); Tsutsui et al.(coming soon))

[β dependence of validity region of CLM with $N_f = 4$ Wilson quark]

- We investigate $\beta = 6.0, 8.0$
- Validity region of CLM is enlarged for higher β

  (NB. strict comparison along the line of constant physics is desirable)

  ◇ Unitarity norm decreases and drift histogram has exp fall-off in $\mu_{\text{quark}} \leq 0.30$ at $\beta = 8.0$ → It may indicate usefulness of improved actions

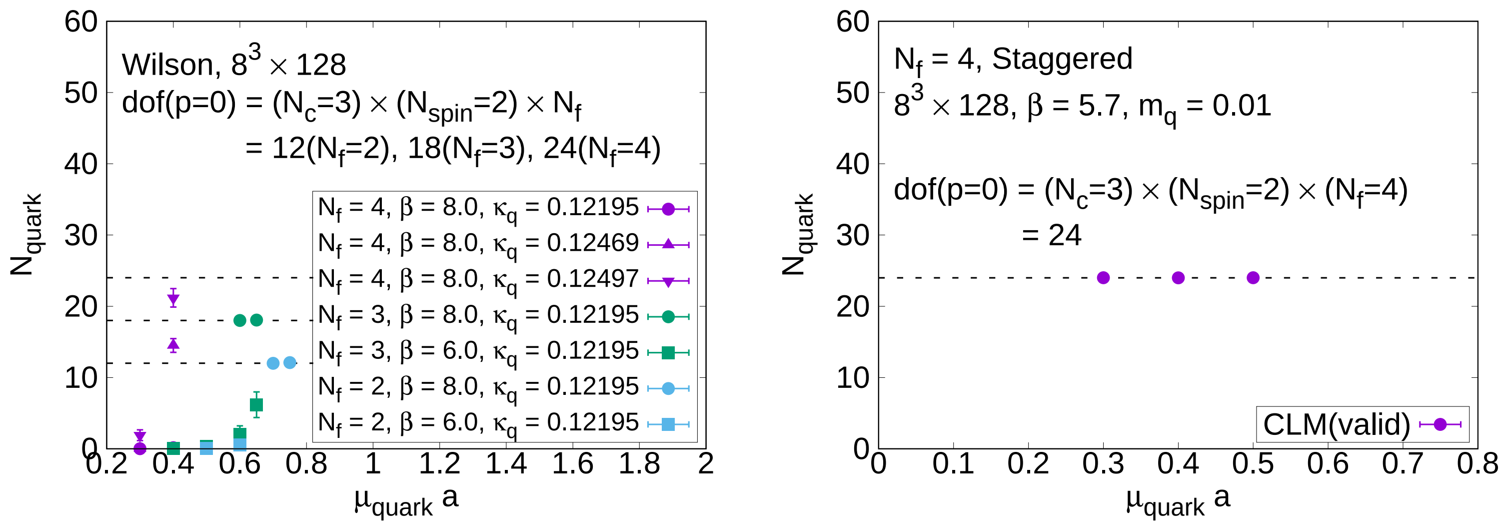


- (For staggered quark, validity region of CLM is enlarged for higher β Kogut,Sinclair(2019); Tsutsui et al.(coming soon))

[Quark number in the validity region of CLM with $N_f = 2, 3, 4$ Wilson quark]

- Quark number $N_{\rm quark}$ shows expected N_f dependence
  - ◇ Proportionality of plateau height to N_f is confirmed for $N_f = 2, 3$

    ← not yet for $N_f = 4$ due to small validity region
  - ◇ Our result indicates formation of Fermi surface in this parameter regime



3 Summary

We investigated finite temperature/density QCD by Complex Langevin Method

- We explore the validity region on $8^3 \times 128$ by $N_f = 2, 3, 4$ Wilson quark
  - ♢ Validity region of CLM is enlarged as we decrease N_f

    → The trend is opposite to $N_f = 2$ staggered quark on $12^3 \times 6$ Kogut,Sinclair(2018) which utilizes rooting and has finite temperature effects
  - ♢ Validity region of CLM does not significantly depend on m_{quark}

    → Our m_{quark} may not be small enough (small m_{quark} causes "CG not converged")
  - ♢ Validity region of CLM is enlarged for higher β

    → It may indicate usefulness of improved actions
- Quark numbers shows expected N_f dependence in the heights of plateaus, indicating formation of Fermi surface

[Future direction]

- Perform realistic simulations on large volume with improved action and new algorithms(selected inversionBloch,Schenk(2018),implicit solverAlvestad et al.(2021))