Topology in high-T QCD via staggered spectral projectors

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The QCD Axion and Dark Matter

The QCD axion, being weakly coupled to the Standard Model, is also a good Dark Matter candidate.
In this context, the behavior of the axion effective potential $V_{eff}(a, T)$ at high temperatures is extremely relevant to access today axion relic abundance and mass, which are essential inputs for present and future experimental researches.

Being axion effective parameters related to QCD topological observables (χ, b_2, $\dots$), this fact constitutes a strong motivation to study QCD topology at high-T:

$$\chi = \left.\frac{\langle Q^2 \rangle}{V}\right|_{\theta=0}, \qquad b_2 = -\frac{1}{12} \left.\frac{\langle Q^4 \rangle - 3 \langle Q^2 \rangle^2}{\langle Q^2 \rangle}\right|_{\theta=0}, \dots$$

$$m_a^2 \propto \chi, \qquad \lambda_{4a} \propto b_2, \dots$$

Non-chiral fermions and would-be-zero modes

In the QCD path-integral, field configurations are weighted with the determinant of the Dirac operator:

$$\det\{\not{D} + m_q\} = \prod_{\lambda \in \mathbb{R}} (i\lambda + m_q)\,.$$

The Index Theorem relates the presence of zero-modes in the spectrum of $\not{D}$ to the topological charge of the gluon field:

$$Q = \mathrm{Index}\{\not{D}\} = \mathrm{Tr}\{\gamma_5\} = n_+ - n_-.$$

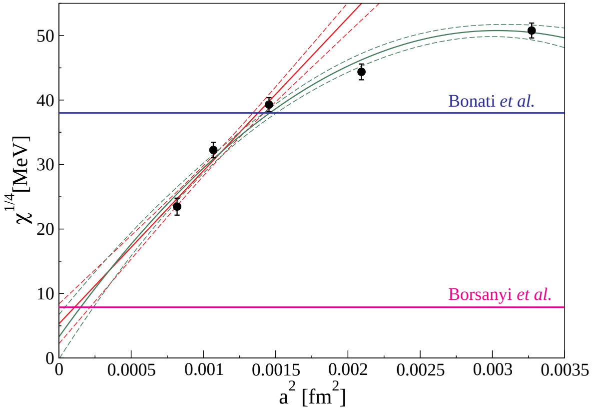
If a configuration has $Q \neq 0$, lowest eigenvalues are $\lambda_{min} = m_q$.

On the lattice, however, some fermionic discretizations (e.g., staggered) do not have exact zero-modes. $\Longrightarrow$ The determinant fails to efficiently suppress non-zero charge configurations.

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda_0, \qquad \lambda_0 \underset{a\to 0}{\to} 0.$$

Non-chiral fermions and large lattice artifacts

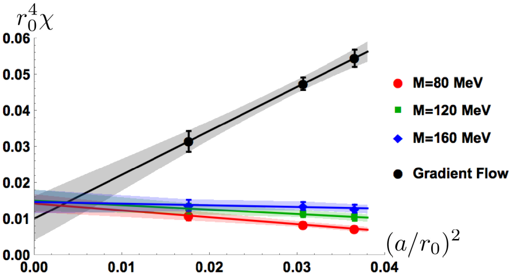
Bad suppression of non-zero charge configurations $\Longrightarrow$ large discretization corrections $\Longrightarrow$ continuum extrapolation not under control (Bonati et al., 2018):



In (Borsanyi et al., 2016) lattice artifacts affecting χ at high-T have been suppressed *a posteriori* by reweighting configurations with the corresponding continuum lowest eigenvalues of $\not{D}$.

Fermionic topological charge

Another possible solution, which does not require any ad hoc assumption, could be to switch, through the Index Theorem, to fermionic definitions of Q. Using the same "bad" operator to weight configurations and to count eigenmodes to measure Q may introduce smaller lattice artifacts.



Idea supported by results at $T = 0$ (Alexandrou et al., 2017): twisted mass Wilson fermions employed for the MC evolution and for the measure of χ through spectral projectors $\longrightarrow$ improved scaling of χ towards the continuum!

Goal: use staggered fermions spectral projectors definition (CB et al., 2019) to study χ at high-T from full QCD simulations with staggered fermions.

Spectral projectors with staggered fermions

In the continuum, only zero-modes contribute to Q. This is not true on the lattice for staggered fermions, due to the absence of exact zero-modes:

$$Q = \mathrm{Tr}\{\gamma_5\} \longrightarrow \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\},$$

$$\mathbb{P}_M = \sum_{|\lambda_k| \leq M} u_k u_k^\dagger, \qquad i\not{D}_{stag} u_k = \lambda_k u_k.$$

To avoid a mode over-counting, taste degeneration has to be considered ($n_t = 2^{d/2}$):

$$Q_{0,stag} = \frac{1}{n_t} \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M\}.$$

Lattice charge gets a renormalization $Z_Q^{stag} = \frac{Z_P}{Z_S}$, which can be derived from Ward identities for the flavor-singlet axial current:

$$Q_{stag} = \frac{Z_P}{Z_S} Q_{0,stag}, \qquad \left(\frac{Z_P}{Z_S}\right)^2 = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M\}\rangle}.$$

Choice of the cut-off mass M

The choice of the cut-off mass M is irrelevant in the continuum limit. Its renormalized value $M_R = M/Z_S$ must be kept constant as $a \to 0$ to guarantee $O(a^2)$ corrections:

$$\chi_{SP}(a, M_R) = \chi + c(M_R)a^2 + O(a^4).$$

To avoid the direct computation of Z_S for each lattice spacing, one can observe that, for staggered fermions:

$$m_{q,R} = m_q/Z_S.$$

If a Line of Constant Physics is known, it is sufficient to keep

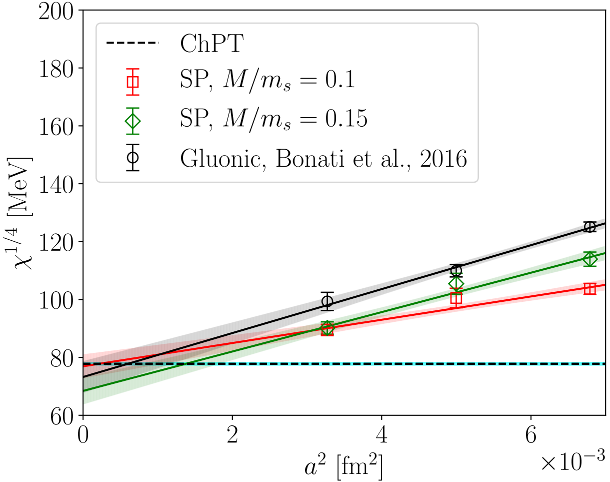
$$M/m_q = M_R/m_{q,R}$$

constant as $a \to 0$ to have M_R constant too. Since the continuum limit is independent of M/m_q, the optimal choice of M/m_s is the one that minimizes $c(M/m_q)$.

Continuum limit of $\chi^{1/4}$ $(T = 0)$

Setup: $N_f = 2 + 1$ rooted stout staggered fermions at physical point. We consider $O(a^2)$ corrections to the continuum limit:

$$\chi_{SP}^{1/4}(a, M/m_s) = \chi^{1/4} + c(M/m_s)a^2 + o(a^2).$$

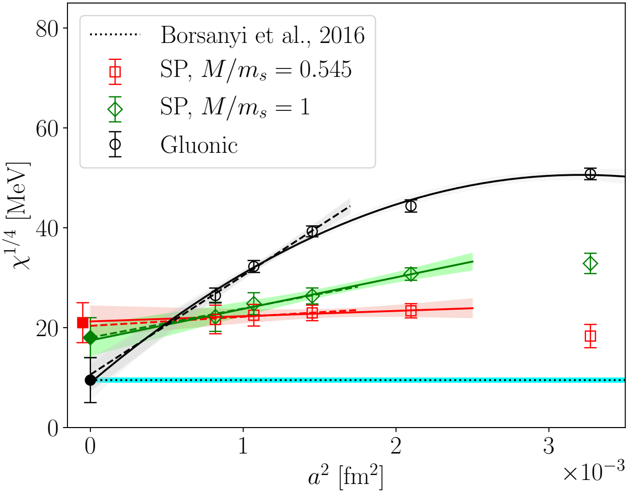


Good agreement among spectral, gluonic and leading order Chiral Perturbation Theory (ChPT) determinations. Lattice artifacts can be reduced compared to the gluonic case with a suitable choice of M: $c(0.1)/c_{gluo} \sim 0.53$, $c(0.15)/c_{gluo} \sim 0.89$.

Continuum limit of $\chi^{1/4}$ at finite T ($T = 430$ MeV)

Same lattice setup of the $T = 0$ case. Also in this case, we consider linear corrections in a^2 to the continuum limit:

$$\chi^{1/4}_{SP}(a, M/m_s) = \chi^{1/4} + c(M/m_s)a^2 + o(a^2).$$



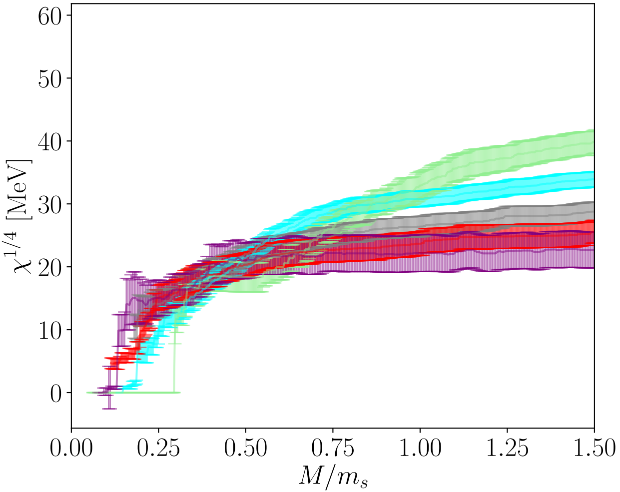
Spectral lattice artifacts are suppressed compared to the gluonic case: $c(0.2)/c_{gluo} \sim 0.04$, $c(0.3)/c_{gluo} \sim 0.3$.
This makes spectral continuum extrapolations much better controlled compared to the gluonic one.

Summary of the talk:

- spectral projectors provide a theoretically well-posed method to define the topological susceptibility,
- spectral definition of χ allows to control the magnitude of lattice artifacts through the choice of the cut-off mass M,
- systematics related to the continuum extrapolation are more under control adopting the spectral definition.
- **Future outlooks**: refine present results and explore other temperatures above the transition to study χ as a function of T and compare results with the DIGA prediction.

Thank you for your attention!

Plotting $\chi_{SP}(a, M/m_s)$ is useful to determine the best range for M/m_s. Minimal lattice artifacts are obtained when χ_{SP} mildly depends on a, i.e., where lines for different a cross each other.



Dashed = linear fit, 3 points
Solid = linear fit, 4 points
Dotted = quadratic fit, 5 points