Inhomogeneous phases in 1+1D Gross-Neveu models

at finite N_f on the lattice (handout version)

Michael Mandl

with

J. Lenz, and A. Wipf

Theoretisch-Physikalisches Institut, FSU Jena

July 29th, 2021

Handout version*

This handout is a slightly modified version of the talk given at Lattice21.
Some additional comments were added in order to give context to the figures shown.
Slides marked by an asterisk (*) were not part of the original talk.

Gross-Neveu (GN) models

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{g^2}{2N_f}\left((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right) + i\mu\bar{\psi}\gamma_0\psi$$

- chiral Gross-Neveu model
- N_f flavors
- chemical potential μ
- no bare mass term

Gross-Neveu (GN) models*

Not considering the chemical potential term, the model on the previous slide is commonly referred to as the chiral Gross-Neveu model, as opposed to the model without the $(\bar{\psi} i\gamma_5 \psi)^2$ term, which we refer to as the (conventional) Gross-Neveu model.

Motivation & Goals

Why GN models?

- **Toy models for QCD**
  Asymptotic freedom, (spontaneous breaking of) chiral symmetry, . . .
- **Solid State Physics**
  Graphene, high-T_c superconductors, polymers, . . .
- . . .

In this talk

- Study thermodynamics of GN models using Lattice Field Theory.
- Particular emphasis on inhomogeneous phases.
- 1+1 dimensions.

Chiral Gross-Neveu (χGN) model

$$\mathcal{L} = i\bar{\psi}\left(\partial\!\!\!/ + \mu\gamma_0\right)\psi + \frac{g^2}{2N_f}\left((\bar{\psi}\psi)^2 + (i\bar{\psi}\gamma_5\psi)^2\right)$$

or, equivalently

$$\mathcal{L} = i\bar{\psi}\left(\partial\!\!\!/ + \mu\gamma_0 + \sigma - i\pi\gamma_5\right)\psi + \frac{N_f}{2g^2}\left(\sigma^2 + \pi^2\right)$$

Ward identities

$$\langle\bar{\psi}\psi\rangle = \frac{iN_f}{g^2}\langle\sigma\rangle$$

$$\langle\bar{\psi}\gamma_5\psi\rangle = -\frac{N_f}{g^2}\langle\pi\rangle$$

continuous chiral symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

$$\begin{pmatrix}\sigma \\ \pi\end{pmatrix} \to \begin{pmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\sigma \\ \pi\end{pmatrix}$$

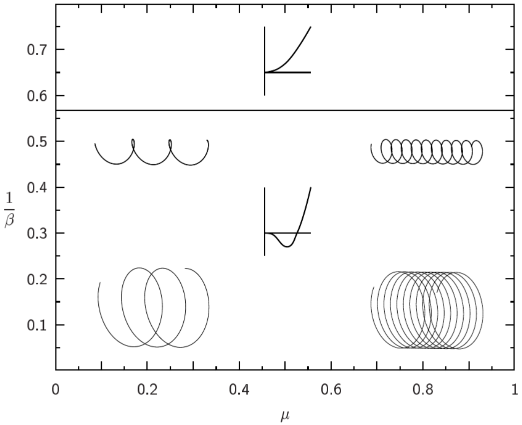
Chiral Gross-Neveu (χGN) model*

One commonly gets rid of the $(\bar{\psi}\psi)^2$ and $(\bar{\psi}i\gamma_5\psi)^2$ terms by introducing the auxiliary fields σ and π into the Lagrangian (for the conventional GN model there is only σ). These two Lagrangians are actually equivalent, as can be seen by using the equations of motion for the auxiliary fields. Notice that the χGN model has a continous $U(1)$ symmetry, whereas the GN model only has a discrete $\mathbb{Z}_2$ symmetry.

What we know*

The next few slides give an overview of known results, starting with an analytic result for the phase diagram in the temperature-chemical-potential plane of the χGN model obtained in the limit $N_f \to \infty$ in which mean field approaches become exact.

χGN, $N_f \to \infty$: chiral spiral

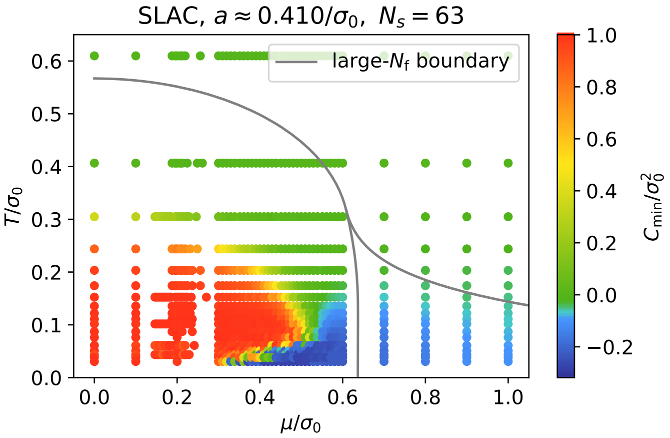


[Schön, Thies 2001]

χGN, $N_f \to \infty$: chiral spiral*

In this limit one finds two phases separated by a horizontal transition line; above this line the condensates vanish everywhere, indicating a restored chiral symmetry. Below the line chiral symmetry is spontaneously broken, with the condensates exhibiting non-trivial spatial dependence (sometimes referred to as ”chiral spiral”).
The spirals shown in the plot are $\langle\bar{\psi}\psi\rangle \propto \cos(kx)$ and $\langle\bar{\psi}\gamma_5\psi\rangle \propto \sin(kx)$ as functions of x, where k is proportional to μ.

$\overline{\text{GN}}$, finite N_f lattice results



[Lenz, Pannullo, Wagner, Wellegehausen, Wipf, 2020]

$\overline{\text{GN}}$, finite N_f lattice results*

The plot on the previous slide shows finite N_f lattice results (together with the corresponding reference) for the (conventional) GN model, where a very similar scenario holds: The $N_f \to \infty$ limit predicts the existence of inhomogenous phases (the bottom right section of the phase diagram) whose existence at finite N_f (the blue region) has been proven by the study mentioned.

This shows that inhomogeneous phases are not a large-N_f artifact and motivates the present work. The question we want to address is whether one also finds such phases in the χGN model at finite N_f.

Lattice setup

- Standard RHMC algorithm.
- SLAC fermions (exactly respecting continous chiral symmetry).
- Use spatial correlators to probe inhomogeneous phases:

$$C_{\sigma\sigma}(x) = \frac{1}{N_t N_s} \sum_{t,y} \langle \sigma(t, y + x) \sigma(t, y) \rangle$$

$$C_{\sigma\pi}(x) = \frac{1}{N_t N_s} \sum_{t,y} \langle \sigma(t, y + x) \pi(t, y) \rangle$$

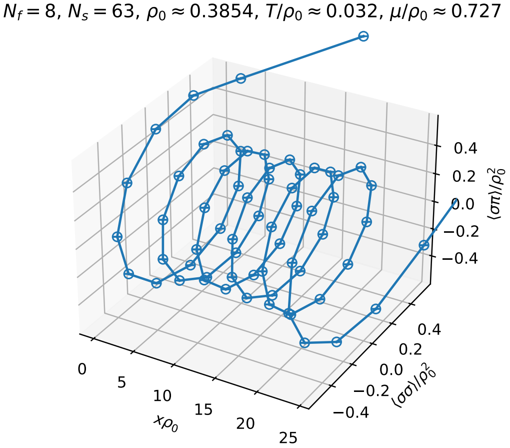
Lattice setup*

The previous study for the GN model suggests that the non-local SLAC derivative is very well suited for studying such models, which is why we use it in the present study as well.
Since attempting to measure an inhomogeneous condensate on the lattice would lead to cancellations we instead measure the spatial correlators that have proven successful for studying inhomogeneous phases in the previous study.

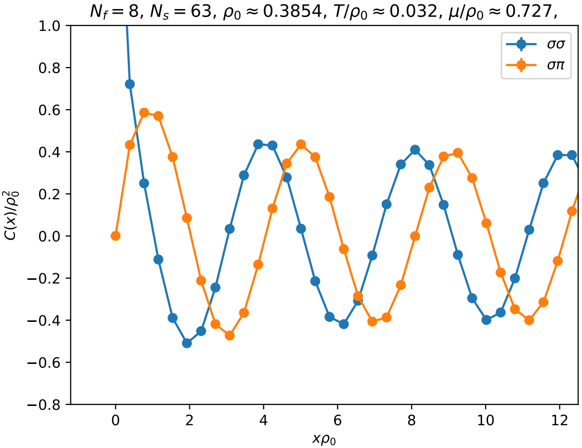
χGN, finite N_f lattice results*

The next few slides are a presentation of some of our results. We first plot the spatial correlators as a function of space in a 3D plot nicely resembling the large-N_f chiral spiral. The subsequent plot shows the same results, but in 2D. Finally we show the dependence of the wavenumber of the inhomogeneous phases as a function of the chemical potential and compare with the $N_f \to \infty$ results. We see a good agreement with large-N_f already for $N_f = 8$, similar to the previous study.

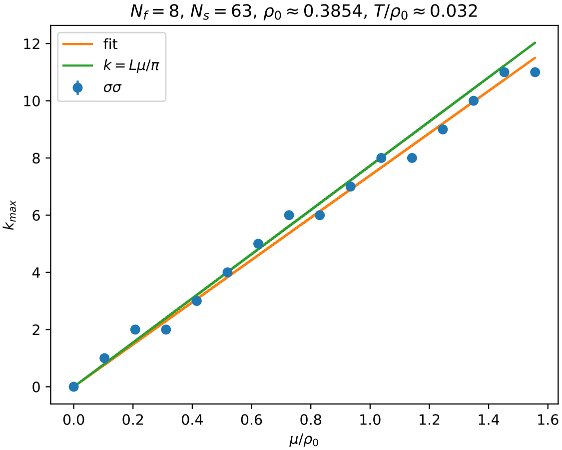
χGN, finite N_f lattice results



χGN, finite N_f lattice results



χGN, finite N_f lattice results



Discussion

- No-go theorems forbid spontaneous breaking of continuous symmetries in 2D.

  [Mermin, Wagner 1966, Coleman 1973]

- ”Almost long-range order” (BKT phase)?

  [Berezinskiĭ 1970, 1971, Kosterlitz, Thouless , 1973, Witten 1978]

- Viscous fluid instead of crystal?

- . . .?

Discussion*

The Coleman-Mermin-Wagner theorem forbids spontaneous breaking of continous symmetries in 2 dimensions (this does not hold for the $N_f \to \infty$ limit). Instead of speaking of ”spontaneous breaking” one should rather use expressions like ”structural order”.

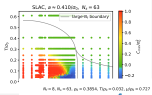
One of the most plausible explanations is that we do not see ”perfect long range order” (like one would for a spontaneous breaking) but instead ”almost long range order” which is practically indistinguishable from the former but is not at odds with the CMW theorem. In some ways the scenario resembles that of a viscous fluid.

At the present point we do not have an entirely conclusive answer as to how to precisely classify these findings.

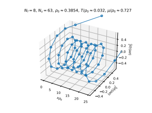
Summary & Outlook

Summary

- Inhomogeneous phases at $N_f < \infty$ in
  - **GN model**



  - **χGN model**



- No conflict with CMW theorem.
- (c.f. talk by C. Nonaka, Monday)

Outlook

- Map out phase diagram at finite N_f.
- Higher dimensions, more realistic models.
- External fields, e.g. magnetic.

Contact*

For questions/discussion please do not hesitate to contact the author of this talk via
michael.mandl@uni-jena.de