Estimating the thermal photon production rate using lattice QCD

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in collaboration with:
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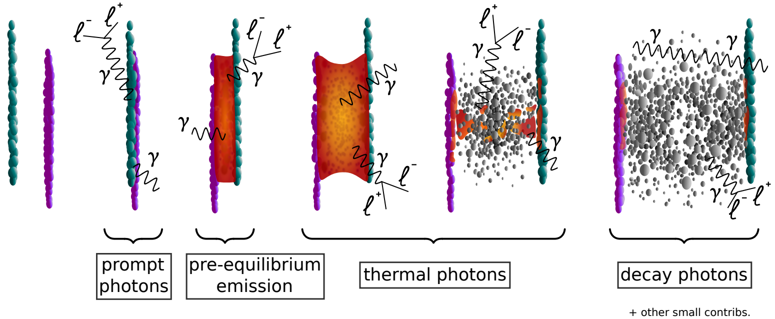
Motivation

- electromagnetic probes (photons, dileptons) interact weakly with the QGP medium $\rightarrow$ on-going experimental research (RHIC, LHC, GSI)

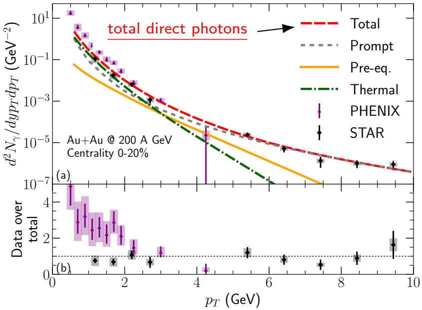


direct photons = total - [decay photons]

Motivation

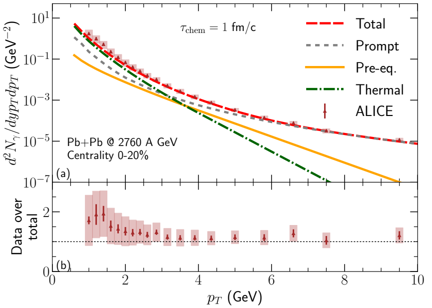
PHENIX, STAR (RHIC)

[1405.3940, 1607.01447]



ALICE (LHC)

[1509.07324]



source of figs: [2106.11216]

- discrepancy between PHENIX & STAR
- discrepancy between PHENIX & theory (/ALICE & theory)
- thermal photon yield: assuming weakly coupled plasma + hydro

Introduction

Thermal photon rate per unit volume of the QGP

$$\frac{\mathrm{d}\Gamma_\gamma(k)}{\mathrm{d}^3k} = \frac{\alpha_{\mathrm{em}}}{\pi^2}\,\frac{\rho_V(\omega = k, k)}{4k}\,\frac{1}{e^{k/T}-1} + \mathcal{O}(\alpha_{\mathrm{em}}^2)$$

The spectral function of the electromagnetic current is defined as

$$\rho_{\mu\nu}(\omega, \mathrm{k}) = \int \mathrm{d}^4x\, e^{i(\omega t - \mathrm{kx})}\langle[J_\mu^{\mathrm{em}}(x), J_\nu^{\mathrm{em}}(0)^\dagger]\rangle, \qquad \rho_V = -\rho^\mu_\mu$$

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Consider the following combination [1710.07050, 2001.03368]

$$\rho(\omega, k, \lambda) \equiv \underbrace{\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\rho_{ij}}_{2\rho_T} + \lambda \underbrace{\left(\frac{k_i k_j}{k^2}\rho_{ij} - \rho_{00}\right)}_{\rho_L}$$

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Ward-identity:

$$\omega^2 \rho_{00}(\omega, k) = k_i k_j \rho_{ij}(\omega, k) \implies$$

$\rho(\omega, k, \lambda)$ is independent of λ when $\omega = k$

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$$\lambda = 1 \qquad \lambda = -2 \qquad \lambda = 0$$

$$\boxed{V} \qquad \boxed{T - L} \qquad \boxed{T}$$

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$\lambda = 1$ $\boxed{V}$

$\lambda = -2$ $\boxed{T - L}$

- non-negative below $\omega = k$
- UV-finite
- superconvergent sum rule

$$\int_0^\infty \mathrm{d}\omega\, \omega\, \rho(\omega, k, \lambda = -2) = 0$$

$\lambda = 0$ $\boxed{T}$

- non-negative for all ω
- does not couple to the diffusion pole

Introduction

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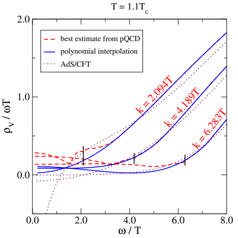
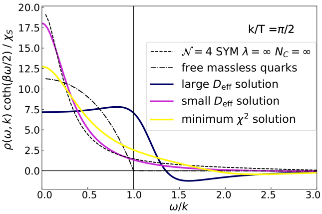


quenched SU(3): [1604.07544]



dyn. QCD, $N_f = 2$: [2001.03368]

this talk

Spectral function reconstruction

How to determine the SF using lattice QCD?

$$G_{\mu\nu}(x_0, k) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\cosh[\omega(\beta/2 - x_0)]}{\sinh(\omega\beta/2)} \rho_{\mu\nu}(\omega, k)$$

- ill-posed problem...
- method: fit + pert. theory (similar to [1604.07544])

$$\rho(\omega) = \boxed{\rho_{fit}(\omega)} (1 - \Theta(\omega, \omega_0, \Delta)) + \boxed{\rho_{pert}(\omega)} \Theta(\omega, \omega_0, \Delta)$$

$$\Theta(\omega, \omega_0, \Delta) = (1 + \tanh[(\omega - \omega_0)/\Delta])/2$$

$$\rho_{pert}(\omega) = \rho_{NLO}(\omega) + \rho_{LPM}(\omega)$$

[1910.07552, 1407.7955]

Ensembles & observables

ensembles:

- $N_f = 2$, clover-improved Wilson fermions
- $m_\pi \approx 280$ MeV
- $T \approx 250$ MeV $\approx 1.2\,T_c$
- four lat. spacings: 0.0658, ..., 0.033 fm (volume $\sim (3.1\text{ fm})^3$)
  $48^3 \times 12, \ldots, 96^3 \times 24$

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observables:

- $I = 1$ contribution:

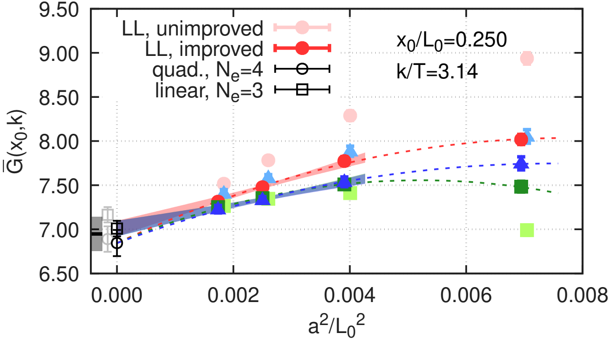
$$G_{\mu\nu}(x_0, k) = \sum_{f=u,d} Q_f^2\, G_{\mu\nu}^{conn}(x_0, k) + \Big[\mathrm{disc.}\Big]$$
$$\approx \sum_{f=u,d} Q_f^2\, 2G_{\mu\nu}^{I=1}(x_0, k) = \frac{10}{9}\, G_{\mu\nu}^{I=1}(x_0, k)$$

- different discretizations:
  local / conserved current $\rightarrow$ LL, LC, CL, CC correlators
- division by the static susc. $\Rightarrow$ no need for renormalization factors

Continuum extrapolation

Simultaneous, correlated continuum extrapolation of different discretizations:

- tree-level improvement
- linear & quadratic fits in a^2/L_0^2



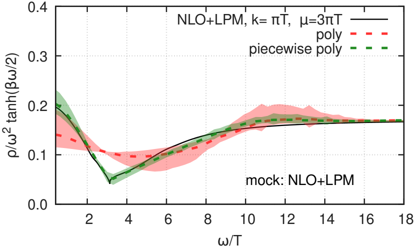
Mock analysis

$$\rho(\omega) = \rho_{fit}(\omega)\ (1 - \Theta(\omega, \omega_0, \Delta)) + \rho_{pert}(\omega)\Theta(\omega, \omega_0, \Delta)$$

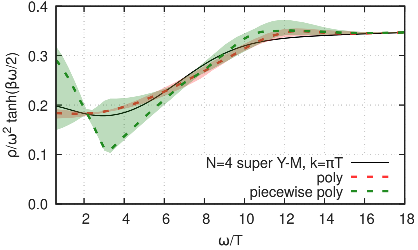
polynomial

piecewise polynomial

NLO perturbation theory + LPM resummation



strongly coupled $\mathcal{N} = 4$ super Yang-Mills theory using AdS/CFT



Results from the lattice

sources of systematic errors:

- fit ansätze: 2 or 3 fit parameters
- matching: Δ, ω_0
- perturbation theory: renorm. scale, $\chi_s^{(pert)}$ $\longrightarrow$ histogram
- lattice data: $\#$(corr. data pts), systematic error of the cont. limit values

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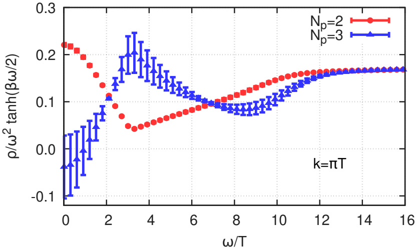
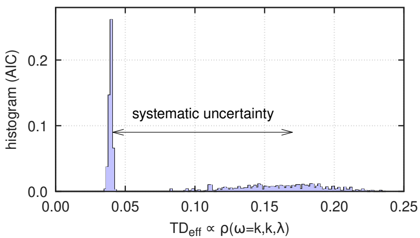
$\longrightarrow$ histogram

piecewise polynomial ansatz:





Results from the lattice

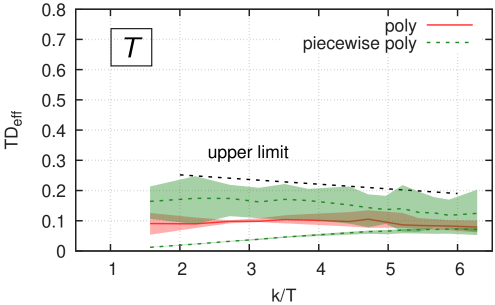
thermal photon production rate:

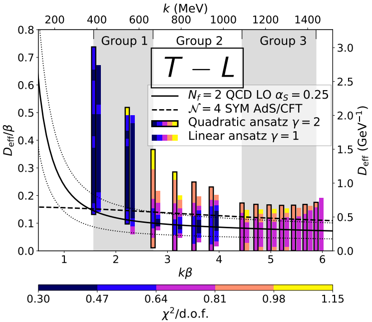
$$\frac{\mathrm{d}\Gamma_\gamma(k)}{\mathrm{d}^3 k} = \frac{\alpha_{\mathrm{em}}}{\pi^2} \frac{\rho(\omega = k, k, \lambda)}{4k} \frac{1}{e^{k/T} - 1} + \mathcal{O}(\alpha_{\mathrm{em}}^2).$$

effective diffusion constant:

$$D_{eff}(k) = \frac{\rho(\omega = k, k, \lambda)}{4k} \frac{1}{\chi_s}$$

[2001.03368]





Summary

- first analysis of transverse channel correlator at nonzero spatial momentum
- $N_f = 2$ clover-improved Wilson fermions
- simultaneous continuum extrapolation of different discretizations using four lattice spacings
- spectral reconstruction using fit $+$ pert. theory
- results for the effective diffusion coefficient: consistent with weak coupling results at $T = 250$ MeV

Thank you for your attention!

BACKUP

Fit ansätze

SF has to be odd and positive:

$$\rho(-\omega) = -\rho(\omega), \qquad \rho(\omega) > 0^*$$

* fit: within 1 std. dev.

polynomial:

$$\frac{\rho_{fit,1}(\omega)}{T^2} = \sum_{n=0}^{N_p-1} A_n \left(\frac{\omega}{\omega_0}\right)^{1+2n}$$

piecewise polynomial:

$$\frac{\rho_{fit,2}(\omega)}{T^2} = \begin{cases} A_0 \frac{\omega}{\omega_0} + A_1 \left(\frac{\omega}{\omega_0}\right)^3, & \text{if } \omega \leq k \\ B_0 \frac{\omega}{\omega_0} + B_1 \left(\frac{\omega}{\omega_0}\right)^3, & \text{if } \omega > k \end{cases}$$

$$A_1 = B_1 + (B_0 - A_0)\frac{\omega_0^2}{k^2}$$