A novel approach to lattice QCD at finite baryon density

Attila Pásztor

Eötvös University, Budapest

in Collaboration with:
Sz. Borsanyi, Z. Fodor, M. Giordano, K. Kapas, S. Katz, D. Nogradi, C.H. Wong

partially based on JHEP 05 (2020) 088; Giordano, Kapas, Katz, Nogradi, Pasztor

Approaches to finite density lattice QCD

In addition to the sign problem, known approaches to finite density QCD suffer from additional serious problems. E.g.

- Taylor and imaginary μ: **analytic continuation problem**
- Reweighting: **overlap problem**
- Complex Langevin: **convergence issues**
- ...

This talk:

$\rightarrow$ a method where the only problem is the sign problem

If the sign problem is dealt with by sufficient statistics, the results are reliable, and errors (on a fixed lattice) are statistical only.

Reweighting in general

Target theory: Z_w Simulated theory: Z_r

$$Z_w = \int \mathcal{D}U \; w(U) \qquad w(U) = detM[U,\mu] e^{-S_g[U]} \in \mathbb{C}$$

$$Z_r = \int \mathcal{D}U \; r(U) \qquad r(U) > 0$$

$$\langle O \rangle_w = \frac{\int \mathcal{D}U \; w(U) O(U)}{\int \mathcal{D}U \; w(U)} = \frac{\int \mathcal{D}U \; r(U) \frac{w(U)}{r(U)} O(U)}{\int \mathcal{D}U \; r(U) \frac{w(U)}{r(U)}} = \frac{\left\langle \frac{w}{r} O \right\rangle_r}{\left\langle \frac{w}{r} \right\rangle_r}$$

Two problems that are exponentially hard in the volume:

- $\frac{w}{r} \in \mathbb{C} \rightarrow$ **sign problem**
- Tails of $\rho(\frac{w}{r})$ long $\rightarrow$ **overlap problem** $\leftarrow$ The first bottleneck when reweighting from $\mu = 0$ (see talk by Kornél Kapás)

Sign reweighting

$$Z = \int \mathcal{D}U e^{-S_g} \det M = \int \mathcal{D}U e^{-S_g} \mathrm{Re} \det \mathrm{M}$$

- Beware: the substitution $\det M \to \mathrm{Re} \det M$ can be done in Z but not in generic expectation values.
- Can calculate e.g. $\frac{\partial^n \log Z}{\partial \mu_{ud}^n}$, $\frac{\partial^n \log Z}{\partial m_{ud}^n}$ and $\frac{\partial^n \log Z}{\partial \beta^n}$

A new choice of a theory to reweight to and from:

$$\begin{array}{l} w = e^{-S_g} \mathrm{Re} \det \mathrm{M} \\ r = e^{-S_g} \left| \mathrm{Re} \det \mathrm{M} \right| \end{array} \quad \Rightarrow \quad \frac{w}{r} \equiv \epsilon = \pm 1$$

- The weights are $\epsilon = \pm 1 \to$ No tail, **no overlap problem**
- $\langle \pm \rangle_r$ measures the strength of the **sign problem**

Early mentions of the idea:
de Forcrand, Kim, Takaishi: hep-lat/0209126; Nucl.Ph.B Proc.S. 119 (2003)
Alexandru, Faber, Horvath, Liu: hep-lat/0507020; PRD72 114513

Simulation algorithm

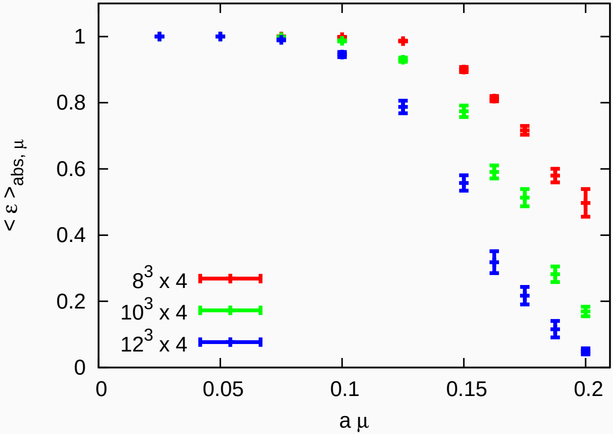
- We modify a standard RHMC at $\mu = 0$ by including an extra term in the accept-reject step.
- At the end of the trajectory $\Delta H_{\mathrm{total}} = \Delta H_\mu + \Delta H_0$ is calculated, where

$$\Delta H_\mu = \Delta \left(\log \left| \mathrm{Re} \det{}^{1/2} M_{ud} \left(\mu_{ud} \right) \right| - \log \det{}^{1/2} M_{ud}(0) \right)$$

  calculated with the reduced matrix [Hasenfratz, Toussaint; '92].
- Frequent tunnelings between the $\pm$ sectors
- Acceptance $> 50\%$

First test of the new method - unimproved staggered $N_\tau = 4$

Strength of the sign problem at $T_c(\mu)$

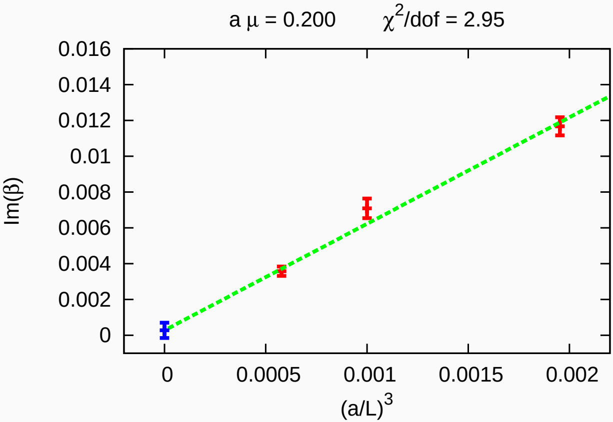


Giordano, Kapas, Katz, Nogradi, Pasztor; JHEP 05 (2020) 088

For simplicity we take $\mu_s = 0$ and $\mu_u = \mu_d = \mu_B/3$

First test of the new method - unimproved staggered $N_\tau = 4$

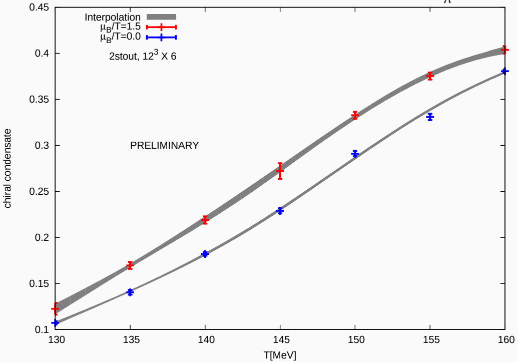
Finite volume scaling at $\mu_B/T = 2.4$



Consistent with Fodor, Katz; JHEP 04 (2004) 050. BUT: to start being relevant for phenomenology, a much better lattice action has to be used

Second test - 2stout $N_\tau = 6$ (PRELIMINARY)

$$\langle\bar{\psi}\psi\rangle_R = (\langle\bar{\psi}\psi\rangle_{0,0} - \langle\bar{\psi}\psi\rangle_{T,\mu})\frac{m_{ud}}{f_\pi^4}$$



At $T = 130$MeV and $\mu_B/T = 1.5$ the strength of the sign problem is:

$12^3 \times 6 \quad \rightarrow \quad \langle\epsilon\rangle = 0.95 \pm 0.01 \qquad 16^3 \times 6 \quad \rightarrow \quad \langle\epsilon\rangle = 0.70 \pm 0.03$

$\rightarrow$ can go further both in μ_B and in lattice size

Summary

- Current methods to study finite density QCD are typically not bottlenecked by the sign problem itself
- In particular reweighting from $\mu = 0$ is bottlenecked by the overlap problem
- We proposed a new reweighting method that is free from the overlap problem in the weights and is therefore only bottlenecked by the sign problem itself
- First test: CEP for unimproved staggered at $N_\tau = 4$, expected to be dominated by cut-off effects
- Second test: 2stout at $N_\tau = 6$; preliminary; width of transition at $\mu_B / T = 1.5 \approx$ width at 0