Localisation of Dirac modes in finite-temperature $\mathbb{Z}_2$ gauge theory on the lattice

György Baranka

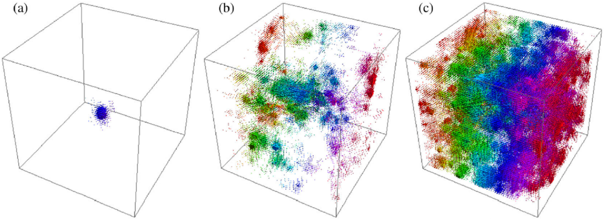
Eötvös Loránd University
Budapest

July 26, 2021
The 38th International Symposium on Lattice Field Theory

Based on arXiv:2104.03779
Work done in collaboration with Matteo Giordano

Introduction I

- Localisation of low modes of the Dirac operator was observed in QCD and other gauge theories above the deconfinement transition [Garcia-Garcia and Osborn, 2007, Ujfalusi et al., 2015]



Localisation and delocalisation from Ref. [Ujfalusi et al., 2015]

Introduction II

- Sea/island picture $\rightarrow$ Ordered Polyakov loops in deconfined phase. In this ordered "sea" modes are localised on the fluctuations of Polyakov loops [Bruckmann et al., 2011]
- To push the connection of these properties to its limit $\rightarrow \mathbb{Z}_2$ gauge theory in 2+1 dimensions and study the spectrum of the staggered Dirac operator, link variables: $U_\mu(n) = \pm 1$

Localisation of eigenmodes of the Dirac operator

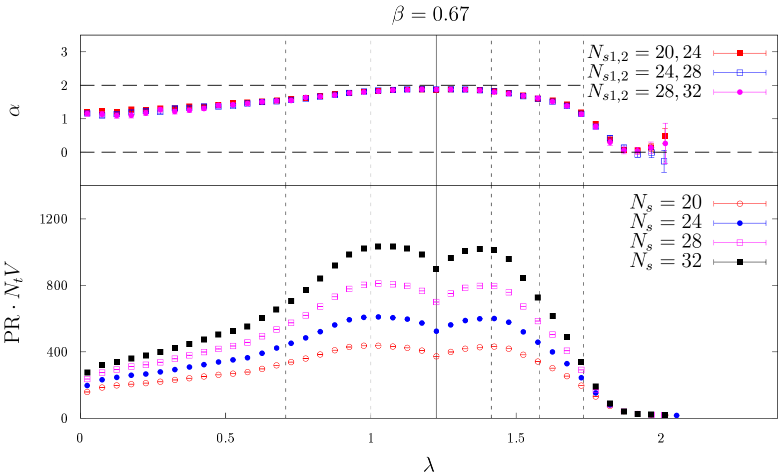
- $\mathrm{IPR}_l = \sum_n |\psi_l(n)|^4$
- $\mathrm{PR}_l = \mathrm{IPR}_l^{-1}(N_t V)^{-1}$
- The scaling of the modes can be determined by the fractal dimension:

$$\mathrm{PR}(\lambda, N_s) \approx c(\lambda) N_s^{\alpha(\lambda)-2}$$

Localised mode $\rightarrow \alpha = 0$
Delocalised mode $\rightarrow \alpha = 2$

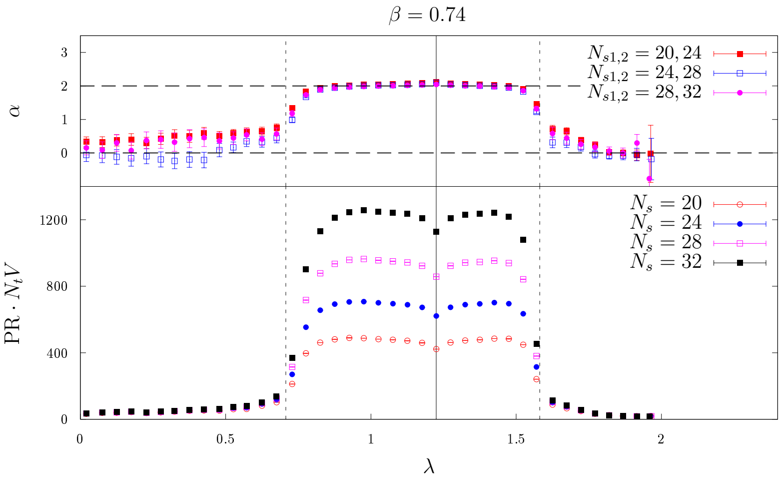
$$\alpha(\lambda) = 2 + \log\left(\frac{\mathrm{PR}(\lambda, N_{s_1})}{\mathrm{PR}(\lambda, N_{s_2})}\right) \Big/ \log\left(\frac{N_{s_1}}{N_{s_2}}\right)$$

Confined phase



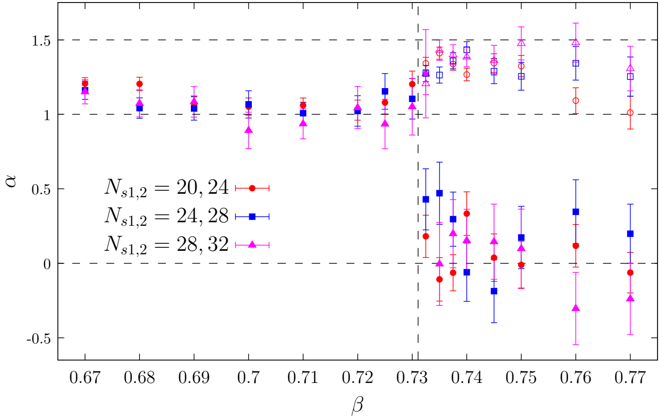
Low modes and bulk modes are not localised, high modes are localised

Deconfined phase ($\overline{P} > 0$)



Both low and high modes are localised, bulk modes are delocalised

Fractal dimension of near zero modes



The fractal dimension drops to zero at the deconfinement transition($\beta_c(N_t = 4) = 0.73107(2)$ [Caselle and Hasenbusch, 1996])

Sea/island picture of localisation

- How much of the wave function is localised on negative Polyakov loops?

$$\mathscr{P} = \sum_{x,t} P(x)|\psi(x,t)|^2$$

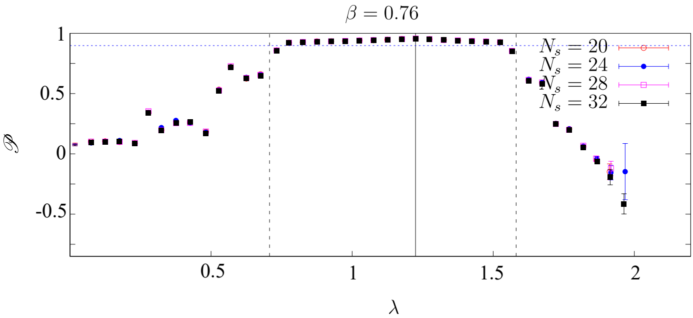
- Delocalised modes:

$$\mathscr{P} \approx \frac{1}{VN_t}\sum_{x,t} P(x) = \frac{1}{V}\sum_{x} P(x) = \overline{P}$$

- Localised modes:

$$\mathscr{P} \approx \sum_{(x,t)\in V_0} P(x)|\psi(x,t)|^2 \approx \overline{P}_{V_0}$$

Sea/island picture of localisation, deconfined phase



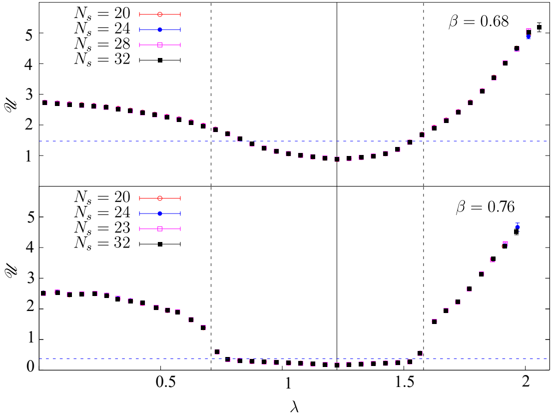
For delocalised modes $\mathscr{P}$ takes the value of the average Polyakov loops. However, for localised modes $\mathscr{P}$ drops significantly

Localisation and negative plaquettes

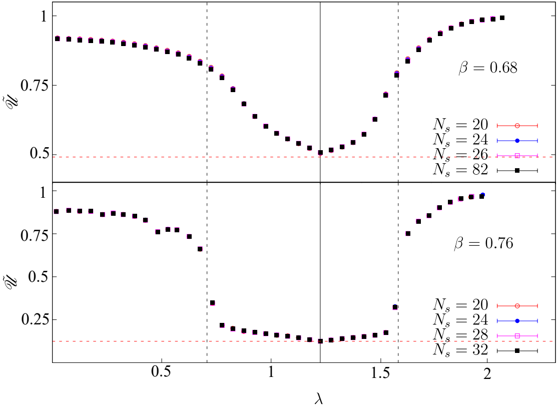
Plaquettes encode dynamics. How do localised modes correlate with negative plaquettes?

$$A(n) = \frac{1}{2} \sum_{\substack{\mu,\nu=1 \\ \mu<\nu}}^{3} [4 - U_{\mu\nu}(n) - U_{\mu\nu}(n-\hat{\mu}) - U_{\mu\nu}(n-\hat{\nu}) - U_{\mu\nu}(n-\hat{\mu}-\hat{\nu})]$$

- $\mathscr{U} = \sum_n A(n)|\psi(n)|^2$ measures the average number of negative plaquettes touched by the modes
- $\widetilde{\mathscr{U}} = \sum_{A(n)>0,n} |\psi(n)|^2$ measures how much of the modes lives on sites touched by negative plaquettes



High localised modes prefer the clusters of negative plaquettes more than low localised modes



For localised modes most part of the modes live on sites that are touched by at least one negative plaquette

Conclusion

- Localisation of low modes is present in QCD and many gauge theories, even in $\mathbb{Z}_2$ gauge theory

Conclusion

- Localisation of low modes is present in QCD and many gauge theories, even in $\mathbb{Z}_2$ gauge theory
- Numerical results confirm the predictions of the sea/island picture of localisation

Conclusion

- Localisation of low modes is present in QCD and many gauge theories, even in $\mathbb{Z}_2$ gauge theory
- Numerical results confirm the predictions of the sea/island picture of localisation
- A novel result is that the very high modes are localized in both phases of the theory

Conclusion

- Localisation of low modes is present in QCD and many gauge theories, even in $\mathbb{Z}_2$ gauge theory
- Numerical results confirm the predictions of the sea/island picture of localisation
- A novel result is that the very high modes are localized in both phases of the theory
- Localized modes display a strong correlation with the position of negative plaquettes in both phases of the theory

References I

[Bruckmann et al., 2011] Bruckmann, F., Kovács, T. G., and Schierenberg, S. (2011).
Anderson localization through polyakov loops: lattice evidence and random matrix model.
Physical Review D, 84(3):034505.

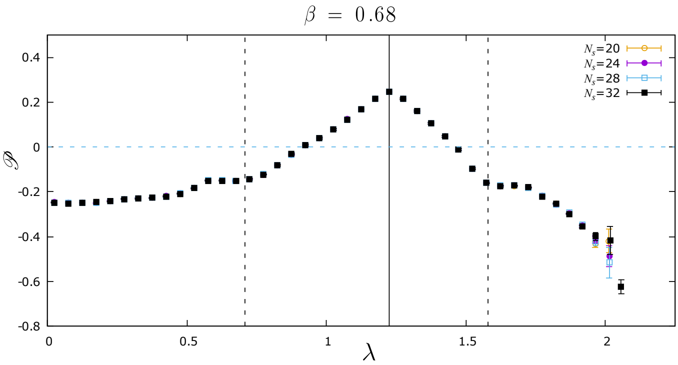
[Caselle and Hasenbusch, 1996] Caselle, M. and Hasenbusch, M. (1996).
Deconfinement transition and dimensional cross-over in the 3d gauge ising model.
Nuclear Physics B, 470(3):435–453.

[Garcia-Garcia and Osborn, 2007] Garcia-Garcia, A. M. and Osborn, J. C. (2007).
Chiral phase transition in lattice qcd as a metal-insulator transition.
Physical Review D, 75(3):034503.

References II

[Ujfalusi et al., 2015] Ujfalusi, L., Giordano, M., Pittler, F., Kovács, T. G., and Varga, I. (2015).
Anderson transition and multifractals in the spectrum of the Dirac operator of Quantum Chromodynamics at high temperature.
Phys. Rev. D, 92(9):094513.

Sea/island picture of localisation, confined phase



For localised high modes $\mathscr{P}$ becomes much lower, while for delocalised modes $\mathscr{P}$ is closer to $\overline{P}$