# Chiral Phase Transition Temperature in 3-Flavor QCD

Lattice 2021

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Strong-interaction matter
under extreme conditions

IISc

# Motivation

- $SU(N_f)_L \times SU(N_f)_R$

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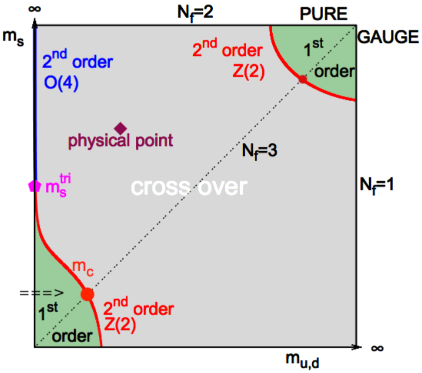


Figure: Columbia Plot
[A.Peikert, Ph.D. Thesis, 2000]

- In the chiral limit the order of the QCD phase transition depends on the number of quark flavours that become massless. For $N_f \geq 3$ massless quark flavors, this phase transition is first order.
  [Pisarski & Wilczek, 1983]

# Past work

- $N_\tau = 4$ lattices for various values of $N_\sigma$ were construced using staggered fermions. Critical parmeters turned out to be significantly cut-off dependent.

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- If $O(4)$ for $N_f = 2$ and first order for $N_f \geq 3$ in the chiral limit, then there should be a tricritical point in between. The upper limit for $N_f^{tri}$ was slightly less than 3, while lower limit was not conclusive.

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  [F. Cuteri et al., 2018]
- Wilson-Clover fermion action was used to put an upper bound on critical pion mass which was found to be equal to 110 MeV. $T_E$ was quoted to be equal to 134(3) MeV.

  [Y. Kuramashi et al., 2020]

# Universal properties near a critical point

- In the vicinity of the second order critical point, system behaves in a characteristic way independent of the details of the microscopic interactions. The singular part of the free energy density dominates.

$$f_s(t, h, ...) = b^{-d} f_s(b^{da_t} t, b^{da_h} h, ...)$$

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- t and h are the reduced scaling variables :

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c}, \quad h = \frac{H}{h_0}; \quad t_0,\ h_0 \text{ are non universal parameters.}$$

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- b can be chosen such that $f_s$ becomes function of one argument :

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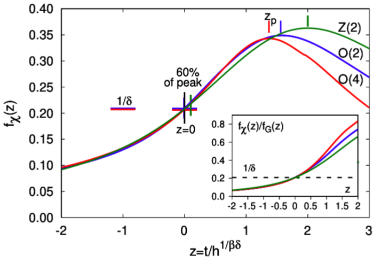
- General scaling features hold for both $O(2)$ & $Z(2)$ with different critical exponents. H will get modified for $Z(2)$ to account for a non vanishing critical mass. So far we haven't found any evidence of a first order region, therefore, only $O(2)$ scaling functions have been used in the analysis.

# Universal properties near a critical point

$$M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z)$$

$$\chi_M = \frac{\partial M}{\partial H} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z)$$

$$\frac{f_\chi(z)}{f_G(z)} = \frac{1}{\delta}\left(1 - \frac{z}{\beta}\frac{f'_G(z)}{f_G(z)}\right)$$

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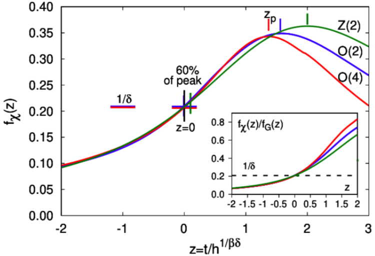


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- $f_\chi(z)$ has a maximum at $z_p$, which is a characteristic value for a given theory. This fact can be used to find $T_c$ (and critical exponent $\delta$).
- $f_\chi/f_G$ has a unique crossing point at $z = 0$. This is a second way to find $T_c$.

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- For the HISQ/tree action calculations, Bielefeld GPU code has been used.
- $N_\tau$ value is fixed to be equal to 8 and three different aspect ratios $(N_\sigma/N_\tau)$ : 5,4,3 have been explored.
- $f_K$ scale setting from 2+1 flavor has been used to fix the temperatures for various values of gauge couplings as well as quark masses $(m_\ell)$, which are relative to the LCP for 2+1 flavor, which corresponds to physical strange quark mass $(m_s)$. [A.Bazavov et al., 2012]

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- Every trajectory separated by 5 TU has been saved. 100 random vectors have been used for calculating fermionic observables.
- Number of saved trajectories:

| $m_s/m_\ell$ | 27 | 40 | 60 | 80 |
|---|---|---|---|---|
| | O(1000) | O(1000) | O(3500) | O(5000) |

# Chiral Observables

- Partition function of QCD with $N_f$ degenerate quark flavors :

$$\mathcal{Z} = \int \mathcal{D}[U](detD)^{N_f/4} e^{-S_g}$$

$$\frac{1}{\mathcal{Z}} \frac{\partial \mathcal{Z}}{\partial m} = \frac{N_f}{4} \langle tr(D^{-1}) \rangle = N_\sigma^3 N_\tau \langle \bar{\psi}\psi \rangle \rightarrow M$$

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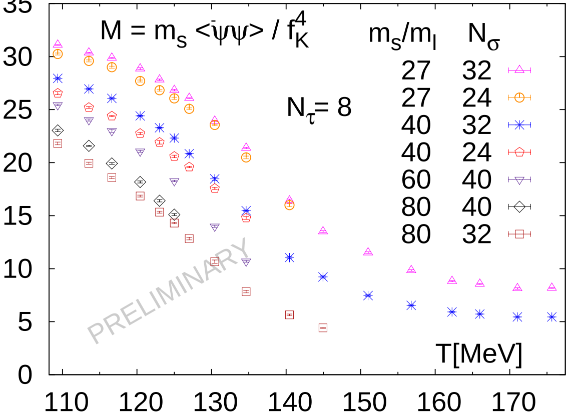
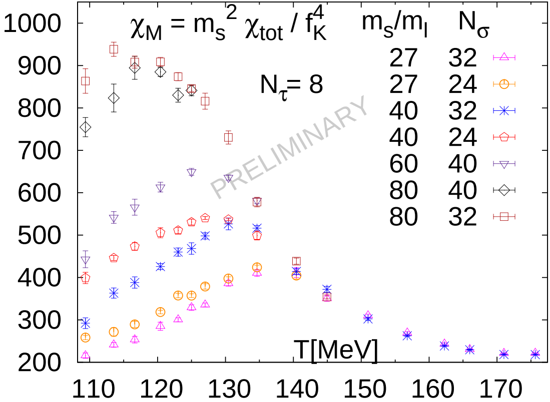
- $H \to m_\ell / m_s$ , $T \to T$

# Chiral condensate in $f_K$ normalisation




- In the lower temperature region for smaller quark masses, finite size effects increase.

# Total susceptibility in $f_K$ normalisation




- Consistent with $t_p(h) = z_p h^{1/\beta\delta}$.

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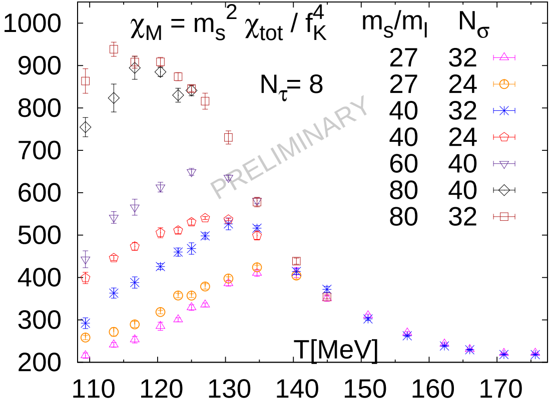


- Consistent with $t_p(h) = z_p h^{1/\beta\delta}$.
- Peak height doesn't increase with the volume. No evidence of a first order transition in the pion mass range explored.

# Locating $T_p$ using quadratic fits

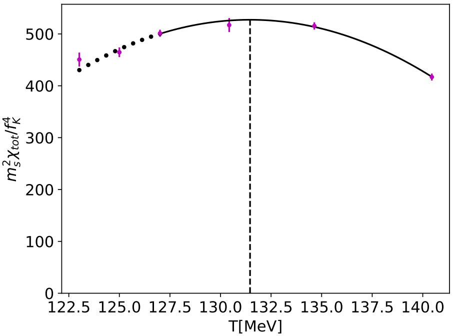


Figure: Sample quadratic fit for $m_s/40$, $N_\sigma = 32$.

- 100 Bootstrap samples were created for each mass and volume.

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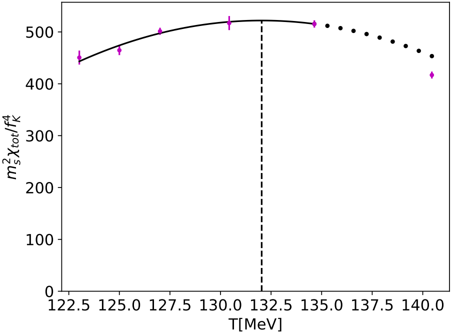
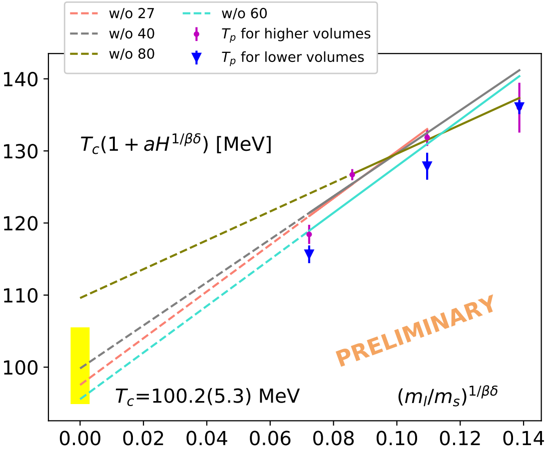



Figure: Sample quadratic fit for $m_s/40$, $N_\sigma = 32$.

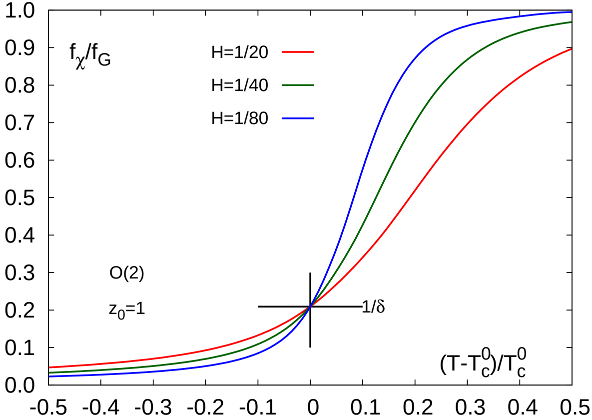
- 100 Bootstrap samples were created for each mass and volume.
- Fit interval was varied for each bootstrap sample to take care of the systematics.

# Infinite volume scaling of $T_p$




- Only highest available volumes have been considered for fitting. $\beta$ and $\delta$ are the given $O(2)$ critical exponents.

# $T_\delta$ expectation for $O(2)$ from the ratio of scaling functions




- $\dfrac{H\chi_M}{M} = \dfrac{1}{\delta}$ at $T_\delta$. This unique crossing point for different values of H can give $T_c^0 = \lim_{H\to 0} \lim_{V\to\infty} T_\delta$.

# Finite size dependence of scaling functions

- One additional relevant scaling field $\ell = L_0/L$ can be added to the singular part of the free energy density :

$$f_s(t, h, \ell, ...) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, b\ell ...)$$

[Engels & Karsch, 2014]

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- By choosing $b = h^{-1/y_h}$, $f_s$ can be brought to this form :

$$f_s = h_0 h^{1+1/\delta} f_f(z, z_L)$$

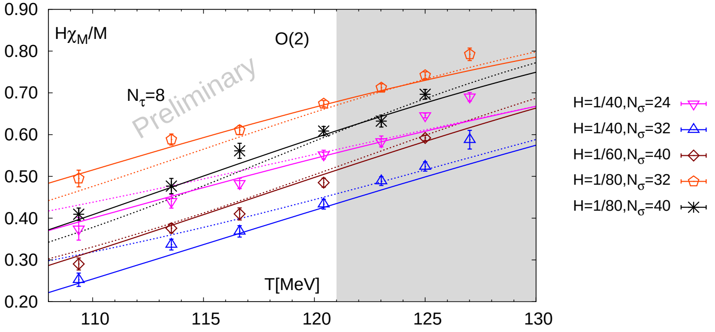
where, $z = t/h^{1/\beta\delta}$ and $z_L = \ell/h^{\nu/\beta\delta}$

$$M = -\frac{\partial f_s}{\partial H} = h^{1/\delta} f_G(z, z_L) + H \times reg$$

$$\chi_M = \frac{\partial M}{\partial H} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z, z_L) + reg$$

$$reg = a_0 + a_1 * (T/T_c^0 - 1)$$

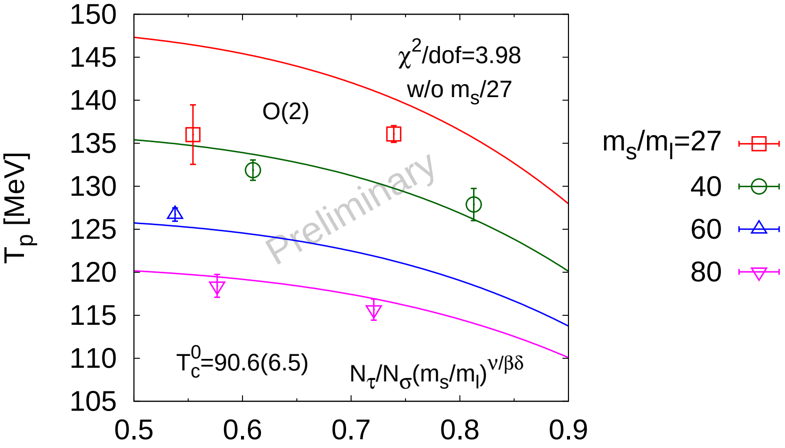
# Joint fit to the ratio (Grey region is not included in the fit)




| | w/o *reg* (dashed) | $reg = a_0 + a_1 * (T/T_c^0 - 1)$ (solid) |
|---|---|---|
| $T_c^0$ [MeV] | 103.3(1.4) | 99.8(2.2) |
| $\chi^2/dof$ | 5.30 | 2.80 |

- Ratio should have a unique crossing point in the thermodynamic limit, which in this case is below the temperature range explored and gets spoiled by the regular contribution.

# Finite size scaling of $T_p$




$$T_p(H,L) = T_c^0\left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta}\right)$$

[H.-T. Ding et al., 2019]

# Conclusions

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- In the pion mass range 80 to 140 MeV, no evidence of first order phase transition has been found.
- Chiral observables show universal finite size scaling behaviour which is not inconsistent with $O(2)$.