Infrared physics of the $\mathrm{SU}(2)$ Georgi-Glashow phase transition

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What was known

- The Georgi-Glashow model Higgses $\mathrm{SU}(2) \to \mathrm{U}(1)$, yielding 't Hooft-Polyakov magnetic monopoles.
- The photon-like excitation is massive due to monopole condensation.
- On the lattice, the number density of monopoles is ultraviolet divergent.

What this work adds

- We study the monopole gas, renormalizing the measurement of monopole density using gradient flow. The renormalized quantity is shown to have a well-defined continuum limit.
- We see the expected proportionality between monopole density and photon mass, even in the nonperturbative crossover region.

Continuum theory

- The Georgi-Glashow model consists of $\mathrm{SU}(2)$ gauge fields and a Higgs field ϕ in the adjoint representation. By dimensional reduction, its infrared behavior at high temperature is well described by a 3D theory with temperature dependent parameters:

$$S = \int d^3x \left\{ \frac{1}{2} \mathrm{Tr}\, F_{ij}F_{ij} + \mathrm{Tr}[D_i, \phi]^2 + m_3^2(T)\, \mathrm{Tr}\, \phi^2 + \lambda_3(T)(\mathrm{Tr}\, \phi^2)^2 \right\}.$$

  The same 3D action arises as the high-temperature limit of two-color QCD, and of beyond the Standard Model theories involving electroweak triplet scalars.
- The phase structure depends on two dimensionless ratios,

$$x = \frac{\lambda_3}{g_3^2} \quad \text{and} \quad y = \frac{m_3^2}{g_3^4}, \qquad \text{where } g_3 \text{ is the gauge coupling in 3D.}$$

  We focus on $x = 0.35$, for which the confinement-Higgs transition is of the crossover type [1].
- The monopoles give a mass to the photon-like excitation. Semi-classically [2]

$$M_\gamma^2 \sim \frac{n}{\pi g_3^2}, \qquad n \sim \frac{m_W^{7/2}}{g_3} \exp\left[-\frac{4\pi m_W}{g_3^2} f(\lambda_3/g_3^2) \right]$$

  where n is the monopole number density that counts only widely separated monopoles, and $f(z)$ an $\mathcal{O}(1)$ function.

Lattice formulation

- Writing the $\mathrm{SU}(2)$ plaquette as U_{ij}, our lattice action reads

$$S = \beta \sum_{x,i<j} \left(1 - \frac{1}{2} U_{ij}(x)\right) + 2a \sum_{x,i} \left(\mathrm{Tr}\, \phi(x)^2 - \mathrm{Tr}\, \phi(x) U_i(x) \phi(x+i) U_i^\dagger(x) \right)$$
$$+ a^3 \sum_x \left(m_L^2 \,\mathrm{Tr}\, \phi^2 + \lambda_3 (\mathrm{Tr}\, \phi^2)^2 \right), \qquad \text{where } \beta = \frac{4}{a g_3^2}.$$

- Letting $\Pi_+ = \frac{1}{2}(\mathbb{1} + \phi/\sqrt{\phi^2})$, a $\mathrm{U}(1)$ link variable can be projected out as

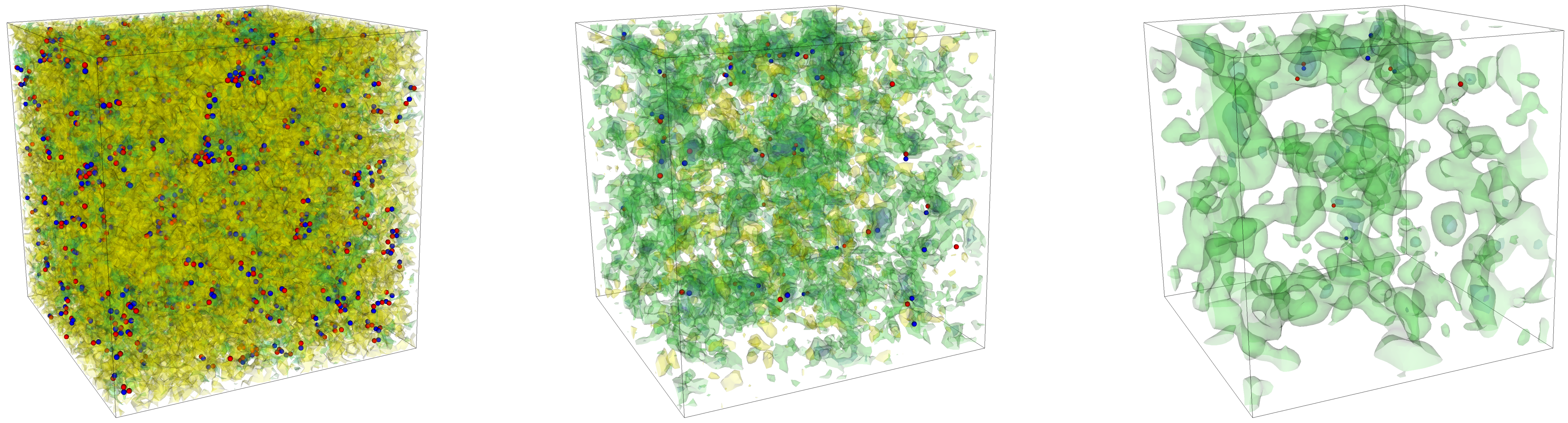
$$u_i(x) = \Pi_+(x) U_i(x) \Pi_+(x+i).$$

  The associated field-strength tensor provides a meaningful definition of the magnetic field B_i on the lattice [3].
- The magnetic monopole number density, as measured in e.g. [4], is

$$n = \frac{1}{V} \frac{g_3 a^{\frac{1}{2}}}{4\pi} \sum_{x,i} \left| B(x+i) - B_i(x) \right|.$$

  However, it is ultraviolet divergent due to short-lived monopole-antimonopole pairs. We renormalize it with gradient flow of the fields.
- The gradient flow transforms fields towards saddle point configurations of the action, removing ultraviolet fluctuations through smoothing [5]. The smoothing radius ξ in 3D is related to the flow time t by $\xi = \sqrt{6t}$.

Snapshots of the system at different stages of gradient flow

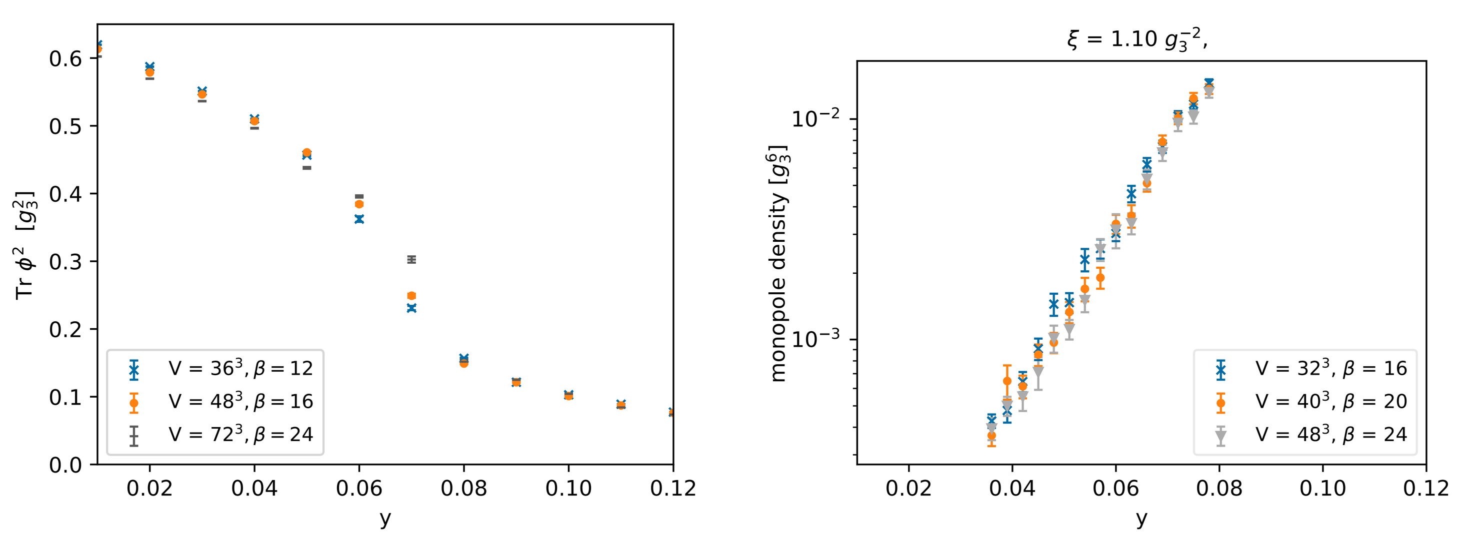


$\xi = 0.69a$ $\qquad$ $\xi = 1.55a$ $\qquad$ $\xi = 3.46a$

- The smoothing leaves only widely separated monopole-antimonopole pairs (red and blue dots). Higgs field isosurfaces are shown in green.
- Full movie available at `https://www2.helsinki.fi/fi/unitube/video/50be6eb8-3266-432d-a8db-3f7f6d47de4e`.

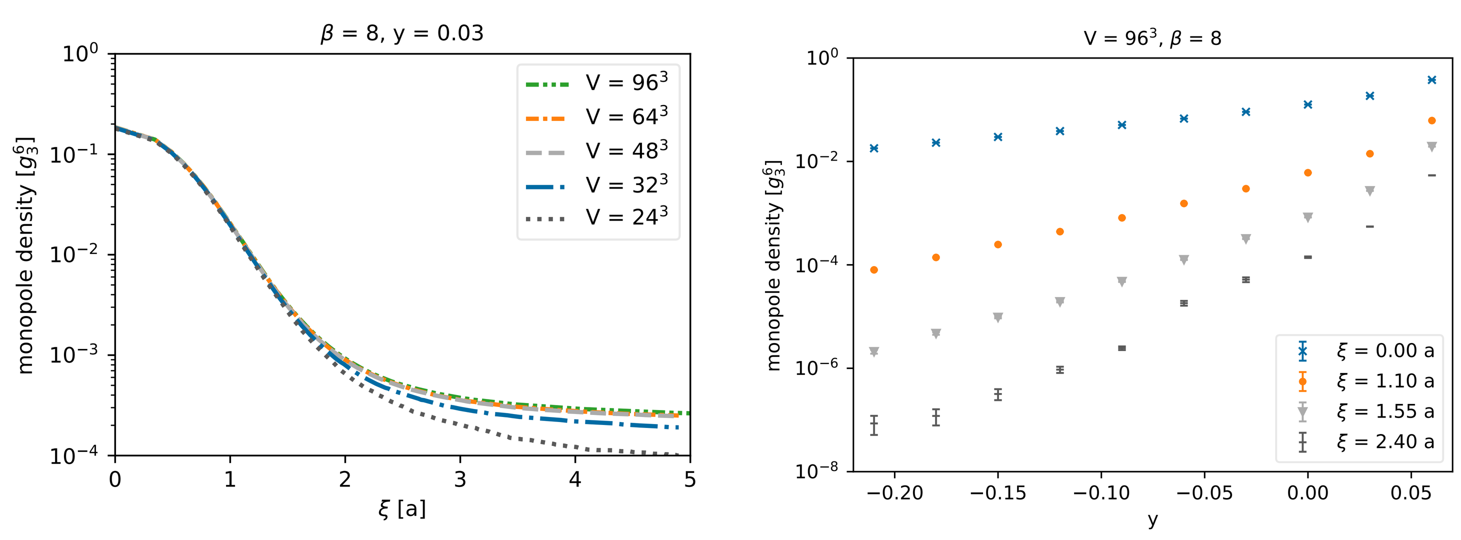
Number density of monopole gas from gradient flow

- Monopoles become heavier as the Higgs condensate grows, and the monopole density drops rapidly.



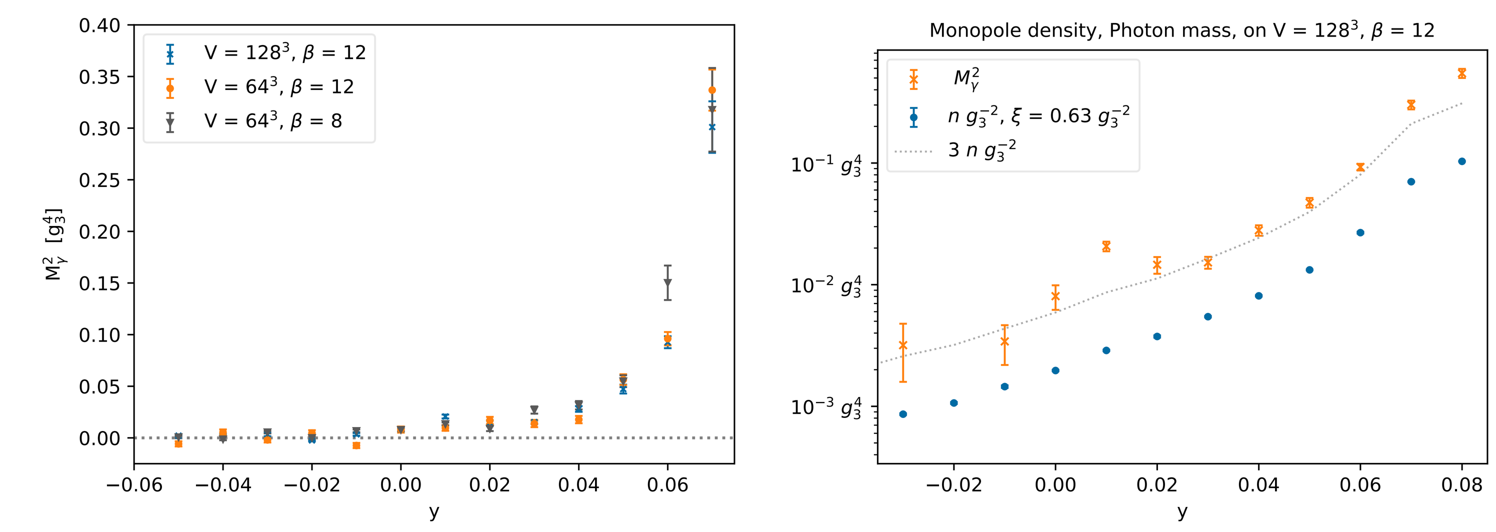
Left: Higgs field expectation value, converted to continuum $\overline{\mathrm{MS}}$.
Right: renormalized monopole density at different spacings, suggesting well-behaved $a \to 0$ limit.

- The monopoles are screened from each other at long distances. The lattice needs to be relatively big to capture this effect.



Key results: The monopole density renormalized with gradient flow has a finite continuum limit and is correlated as expected with the Higgs condensate.

Mass of photon-like excitation



- We measure the photon mass using a blocked correlator at non-vanishing momentum. The photon is almost massless deep in the Higgs regime, where the monopole gas is dilute.

Key results: The photon mass squared is proportional to the monopole density, in accordance with semi-classical expectations. Furthermore, this relationship holds in the crossover region where perturbation theory cannot be relied upon. The proportionality constant depends on how much gradient flow cooling is applied.

References

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