Dual Polyakov loop model at finite density: phase diagram and screening masses

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Overview

Partition function

We start with the effective 3d Polyakov loop model with an exact static determinant, describing the $(3+1)d$ theory with one flavour of fermions.

$$Z = \int \prod_x dW(x) \, \exp\left(\beta \sum_{x,n} \mathrm{Tr}\, W(x) \, \mathrm{Tr}\, W^\dagger(x+e_n)\right) \times$$
$$\times \prod_x B(m, \mu; W(x)) \, ,$$

where

$$B(m, \mu; W) = A(m) \det\left[1 + h_+ W\right] \det\left[1 + h_- W\right] \, ,$$

$$A(m) = h^{-3} \, , \; h_\pm = h e^{\pm \mu / T} \, , \; h = e^{-N_t \operatorname{arcsinh} am}$$

Dual form

One can perform integration over $W(x)$, resulting in a following dual form

$$Z = \prod_{l} \sum_{s(l)=0}^{\infty} \sum_{m(l)=0}^{\infty} \frac{\left(\frac{\beta}{2}\right)^{s(l)+m(l)}}{s(l)!m(l)!} \prod_{x} R(n(x), p(x))$$

$$n(x) = \sum_{\nu=1}^{d} \left(s_\nu(x) + m_\nu(x - e_\nu)\right) \ ,$$

$$p(x) = \sum_{\nu=1}^{d} \left(m_\nu(x) + s_\nu(x - e_\nu)\right) \ ,$$

$$R(n, p) = \int dW \, (\mathrm{Tr}\, W)^n \, (\mathrm{Tr}\, W^\dagger)^p \, B(m, \mu, W) \ .$$

Integral calculation

The group integral

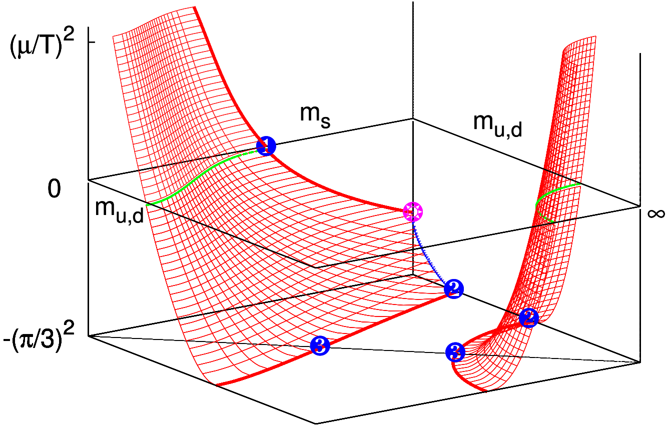
$$R(n,p) = \int dW \, (\mathrm{Tr}\, W)^n \, (\mathrm{Tr}\, W^\dagger)^p \, B(m,\mu,W) \ .$$

gives for the $SU(3)$ group

$$\begin{aligned} R(n,p) = {} & Q(n+1,p)\left(h_+ + h_-^2 + h_+ h_-^3 + h_+^3 h_-^2\right) + \\ & + Q(n,p)\left(1 + h_+^3 + h_-^3 + h_+^3 h_-^3\right) + \\ & + Q(n,p+1)\left(h_- + h_+^2 + h_+^3 h_- + h_+^2 h_-^3\right) + \\ & + Q(n+1,p+1)\left(h_+ h_- + h_+^2 h_-^2\right) + \\ & + Q(n+2,p) h_+ h_-^2 + Q_3(n,p+2) h_+^2 h_- \ . \end{aligned}$$

$$\begin{aligned} Q(n,p) &= \int dW \, (\mathrm{Tr}\, W)^n \, (\mathrm{Tr}\, W^\dagger)^p \\ &= \sum_{q==\infty}^{+\infty} \delta_{n,p+3q} \sum_{\lambda \vdash min(n,p)} d(\lambda) d(\lambda + |q|^3) \end{aligned}$$

Phase structure



P. de Forcrand, C. Bonati, M. D'Elia, O. Philipsen, F. Sanfilippo (LATTICE 2011)

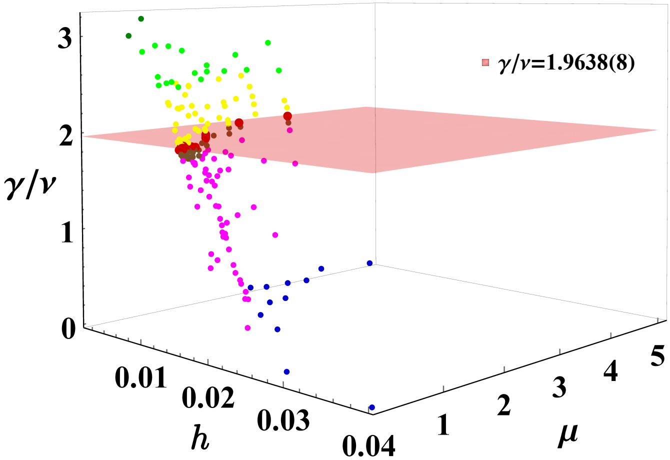
Simulation details

We perform simulations for a set of β, h and μ values on the lattices with size $L = 16, 20, 24, 32$

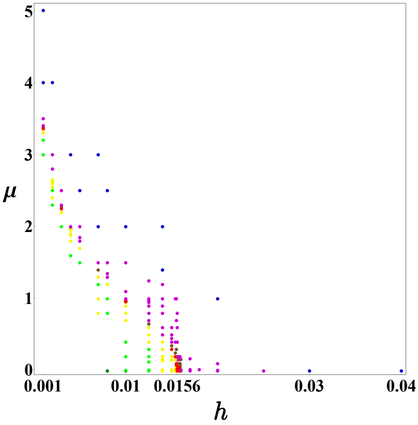
- Precomputed values of Q and R functions
- Metropolis multihit update attempting to update link variables by ± 1 or ± 3.
- For smaller values of h a skipping procedure is implemented – allowing to generate in constant time the number of failed Metropolis attempts and the final accepted update.

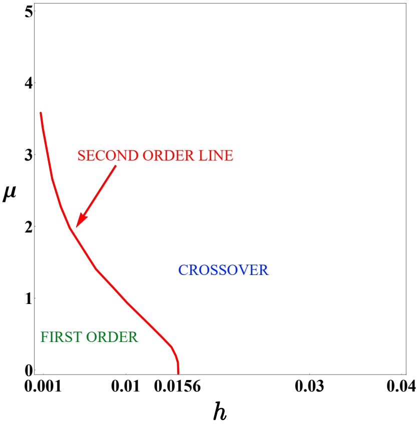
Phase structure

For each set of h, μ values we extracted the critical index ratio $\frac{\gamma}{\nu}$ from the ratios of the peak values of magnetization susceptibilities on different lattice sizes, and compared it with the one in 3d Ising model.



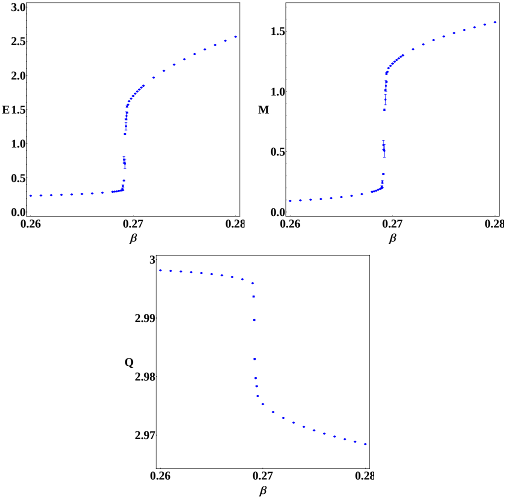
Phase structure





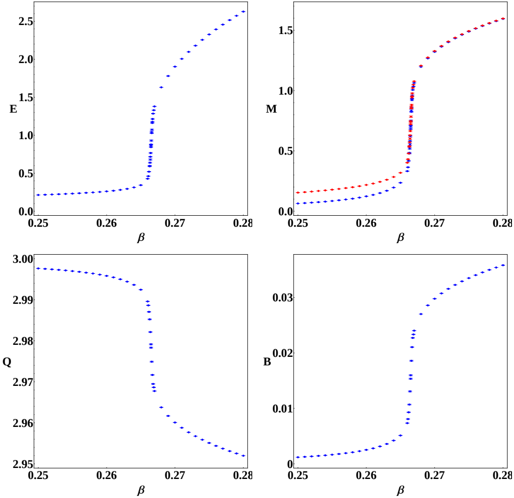
First order transition

$h = 0.01,\ \mu = 0$



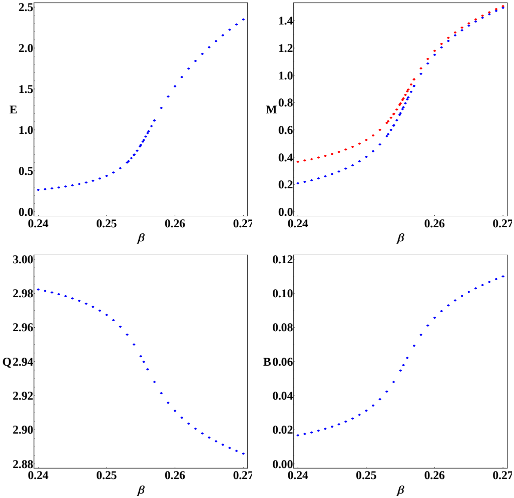
Second order transition

$h = 0.01,\ \mu = 0.9635$



Crossover

$h = 0.01,\ \mu = 2.0$



Correlation functions

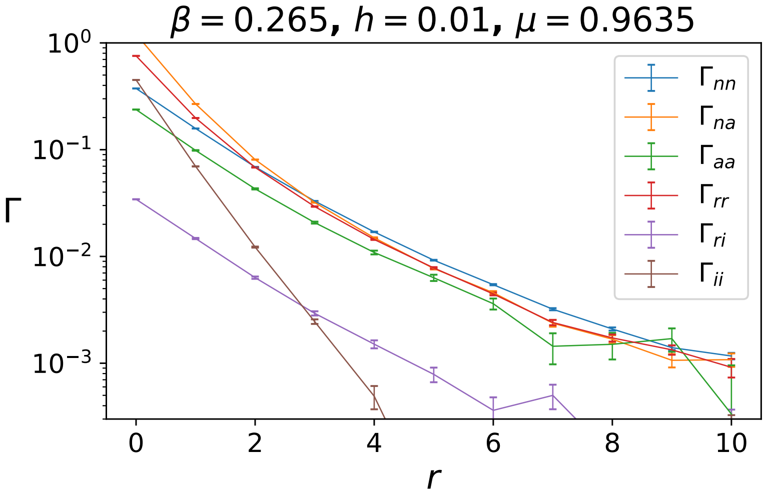
At nonzero h the correlation functions in the dual model can be expressed as ratios of the R functions.

$$\langle \mathrm{Tr}\, W(0)\, \mathrm{Tr}\, W(r) \rangle = \left\langle \frac{R(n(0)+1, p(0))}{R(n(0), p(0))} \frac{R(n(r)+1, p(r))}{R(n(r), p(r))} \right\rangle \ ,$$

$$\left\langle \mathrm{Tr}\, W(0)\, \mathrm{Tr}\, W^{\dagger}(r) \right\rangle = \left\langle \frac{R(n(0)+1, p(0))}{R(n(0), p(0))} \frac{R(n(r), p(r)+1)}{R(n(r), p(r))} \right\rangle \ ,$$

$$\left\langle \mathrm{Tr}\, W^{\dagger}(0)\, \mathrm{Tr}\, W^{\dagger}(r) \right\rangle = \left\langle \frac{R(n(0), p(0)+1)}{R(n(0), p(0))} \frac{R(n(r), p(r)+1)}{R(n(r), p(r))} \right\rangle \ .$$

Correlation functions



Screening masses

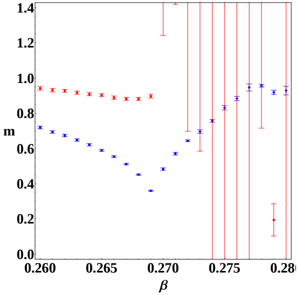
For $\mu = 0$ we can describe the decay of the connected parts of correlation functions as

$$\Gamma_{rr,conn}(r) = A_{rr}\frac{e^{-m_M r}}{r} \ ,$$
$$\Gamma_{ii,conn}(r) = A_{ii}\frac{e^{-m_E r}}{r} \ .$$

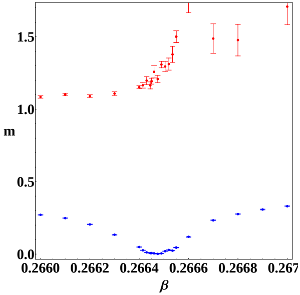
For nonzero μ both correlation functions have electric and magnetic components, but one can extract pure magnetic and pure electric correlation function by diagonalizing the correlation matrix

$$\begin{pmatrix} \Gamma_{rr,conn}(r) & \Gamma_{ri,conn}(r) \\ \Gamma_{ri,conn}(r) & \Gamma_{ii,conn}(r) \end{pmatrix}$$

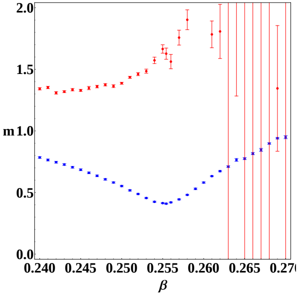
Screening masses



$h = 0.01,\ \mu = 0$



$h = 0.01,\ \mu = 0.9635$



$h = 0.01,\ \mu = 2$

Conclusions

- The dual formulation for the effective $SU(3)$ Polyakov loop model is free of sign problem, and can be effectively simulated numerically.
- At large masses (small h) the regions of first order phase transition, and crossover region are separated by the second order line belonging to the 3d Ising universality class
- The electric and magnetic screening masses were extracted for a large set of model parameters. Electric screening mass is systematically higher than the magnetic one
- Around the phase transition the magnetic mass becomes smaller (goes to zero for second order transition). The electric mass does not change much in the confined phase, and grows rapidly in the deconfined phase.