Color superconductivity in a small box: a complex Langevin study

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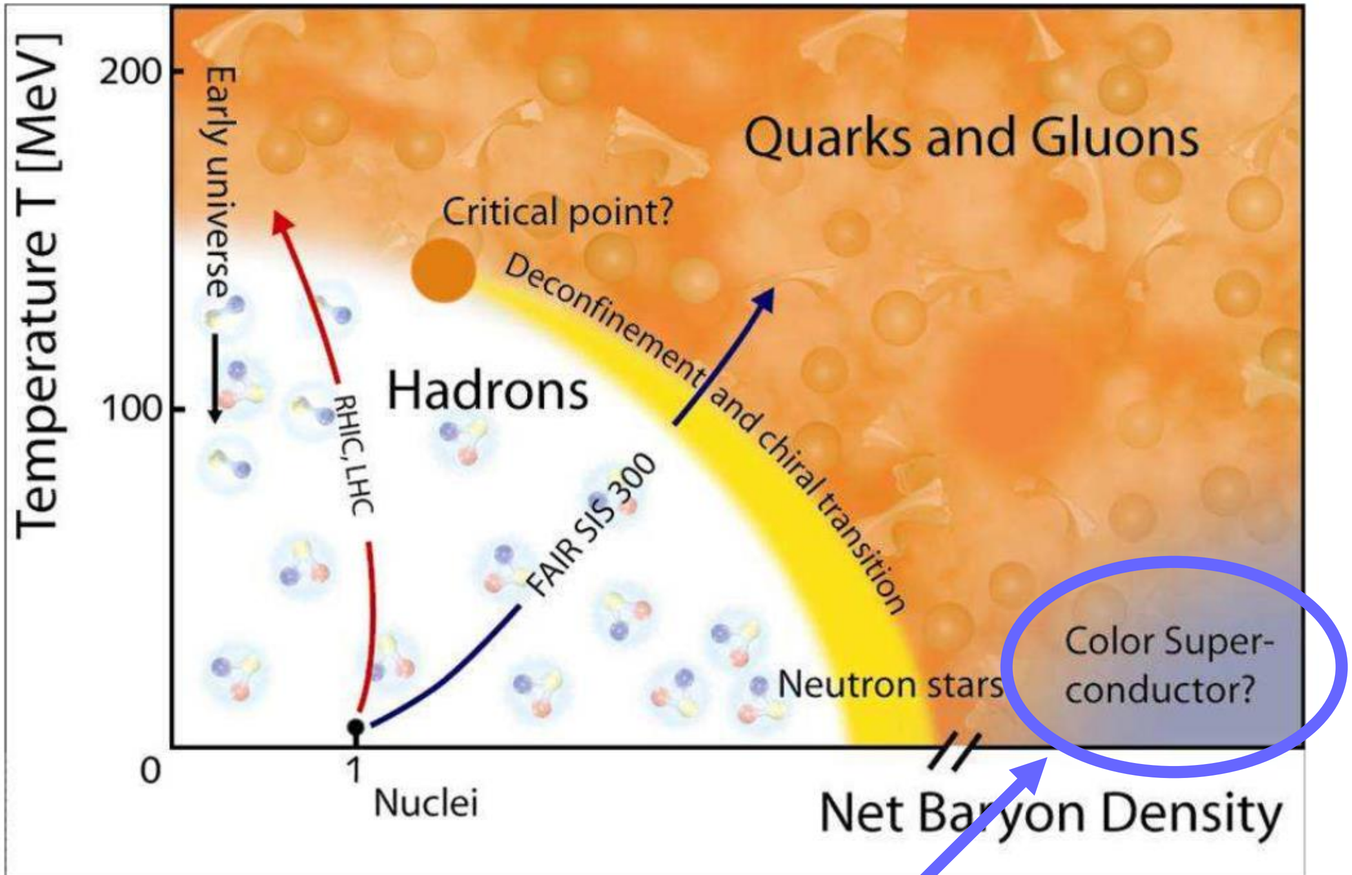
Aim of this study



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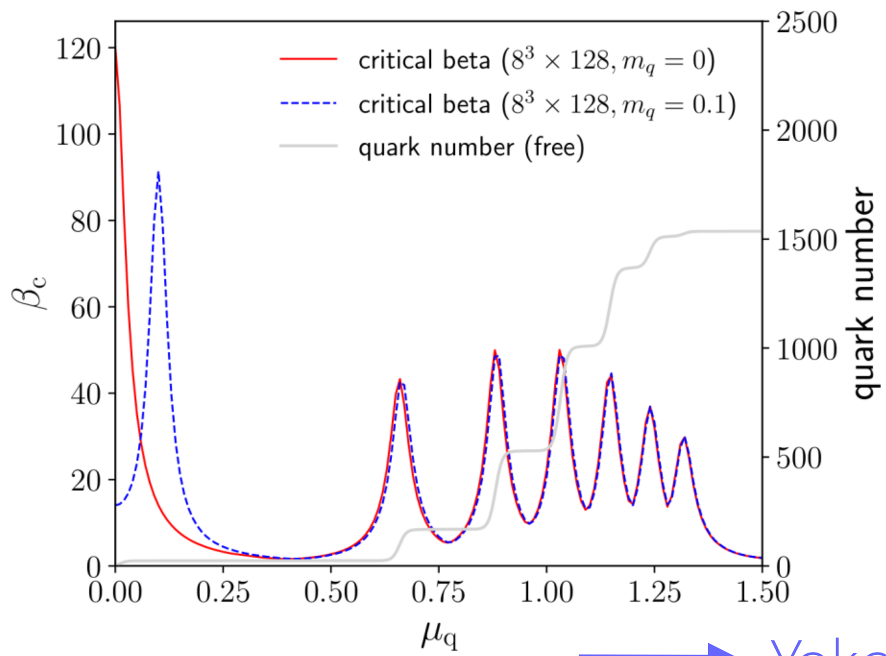
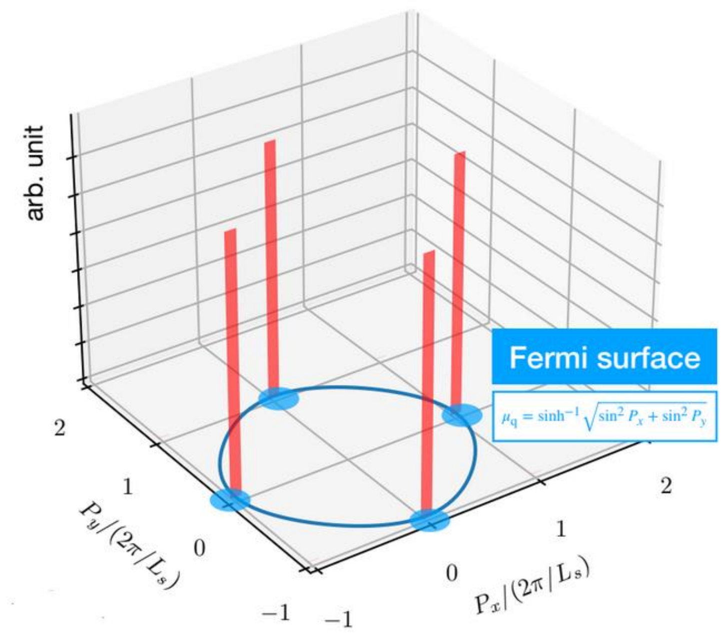
Study color superconductor (CSC) based on lattice QCD

Barrois, NPB (1977), Frautschi (1978),
Bailin, Love, Phys. Rept. (1984)
Alford, Rajagopal, Wilczek, PLB (1998),
Rapp, Schäfer, Shuryak, Velkovsky, PRL (1998)

A prediction from lattice perturbation theory

Weak coupling →

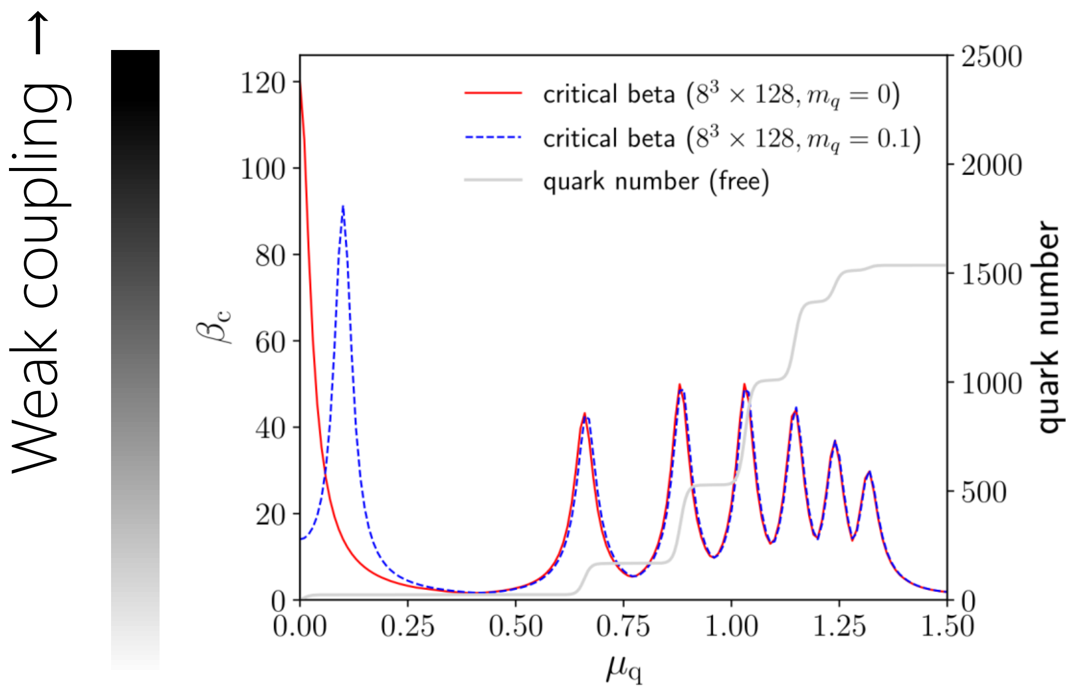


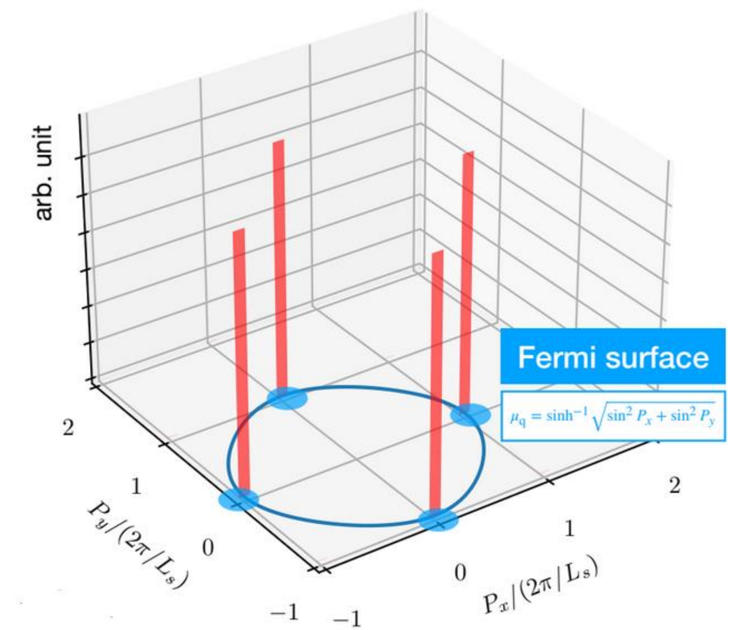


→ Yokota's talk

- CSC phase extends when μ_q=E(p) (quark energy level)
- Cooper pair is actually formed

A prediction from lattice perturbation theory





- CSC phase extends when μ_q=E(p) (quark energy level)
- Cooper pair is actually formed

→ We study these features by the complex Langevin method

Complex Langevin method (CLM)

Parisi (1983), Klauder(1984)

$$\mathcal{U}^{(\eta)}_{x,\nu}(t+\epsilon) = \exp\left[i\left(-\epsilon v_{x,\nu}(\mathcal{U}^{(\eta)}(t)) + \sqrt{\epsilon}\eta_{x,\nu}(t)\right)\right]\mathcal{U}^{(\eta)}_{x,\nu}(t)$$

$$\langle O(\mathcal{U})\rangle = \lim_{t\to\infty}\langle O(\mathcal{U}^{(\eta)}(t))\rangle_\eta$$

- We checked the validity of the CLM based on probability distributions of the drift term (*).

(*) Nagata, Nishimura, Shimasaki, PRD 92 (2015) 011501, PTEP 2016 013B01
(cf) Other criterion based on "boundary terms" is also known
Aarts, Seiler, Stamatescu, PRD 81 (2010) 054608, Aarts, James, Seiler, Stamatescu, EPJ C71 (2011) 1756

- CLM works in wide range of μ_q (e.g. $0 < \mu_q < 1.2$, for $\beta=20$)

Order parameter of CSC (O_{CSC})

O_{CSC}

$$O_{\mathrm{CSC}} = -\varphi_a^\dagger(x)\varphi_a(x)$$

Quark pair

$$\varphi_a(x) = \epsilon_{abc}\mathrm{tr}(C^{-1}\Psi_b^T(x)C\Psi_c(x))$$

4-flavor Dirac field consists of staggered fermion χ

$$\Psi(x) = \sum_{A=0,1}(\gamma_1)^{A_1}(\gamma_2)^{A_2}(\gamma_3)^{A_3}(\gamma_4)^{A_4}\chi(x+A)$$

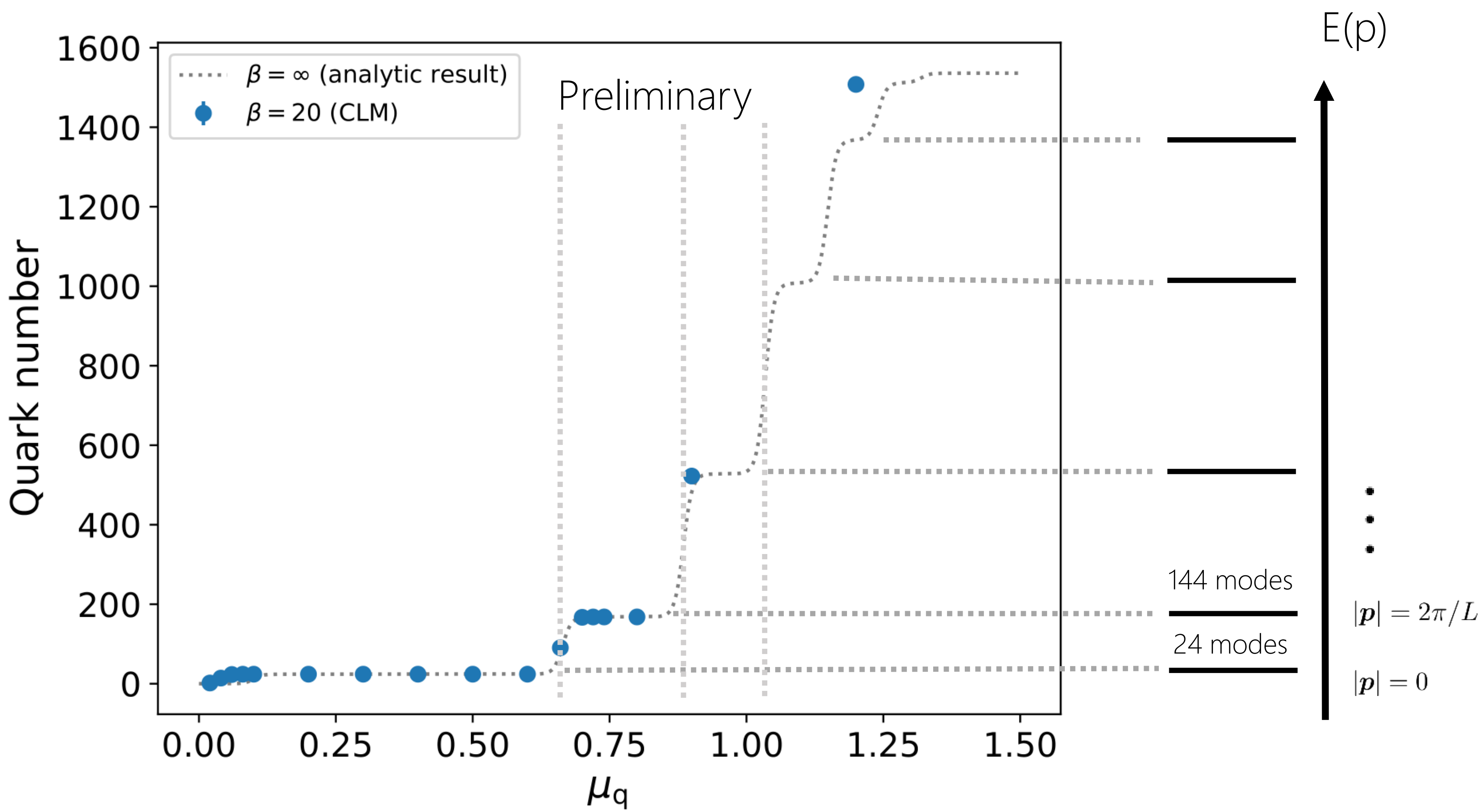
The trace is calculated by the U(1)-noisy estimator (*)

(*) Standard gaussian noisy estimator cannot be applied to calculate 4-point functions.

Numerical results

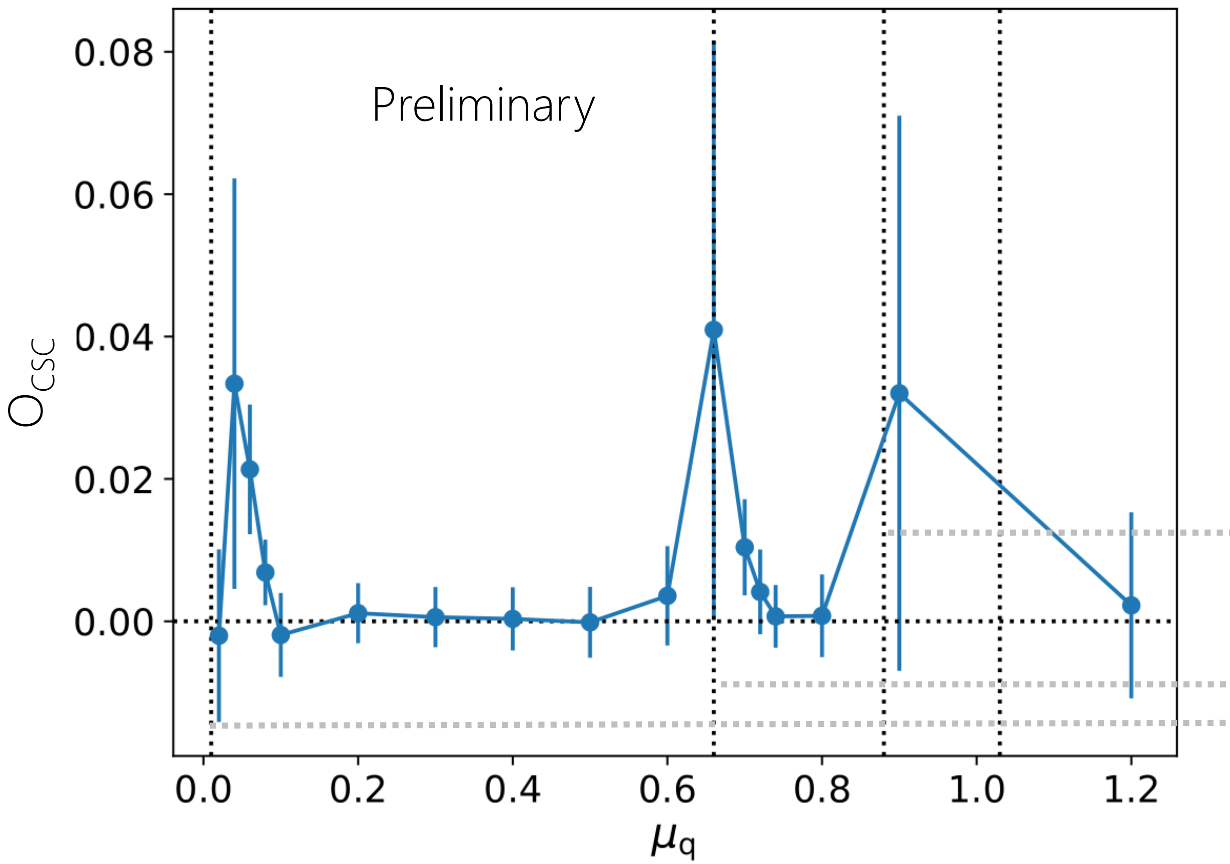
- plaquette gauge action + 4-flavor staggered fermions
- m_q = 0.01
- $8^3 \times 128$ lattice

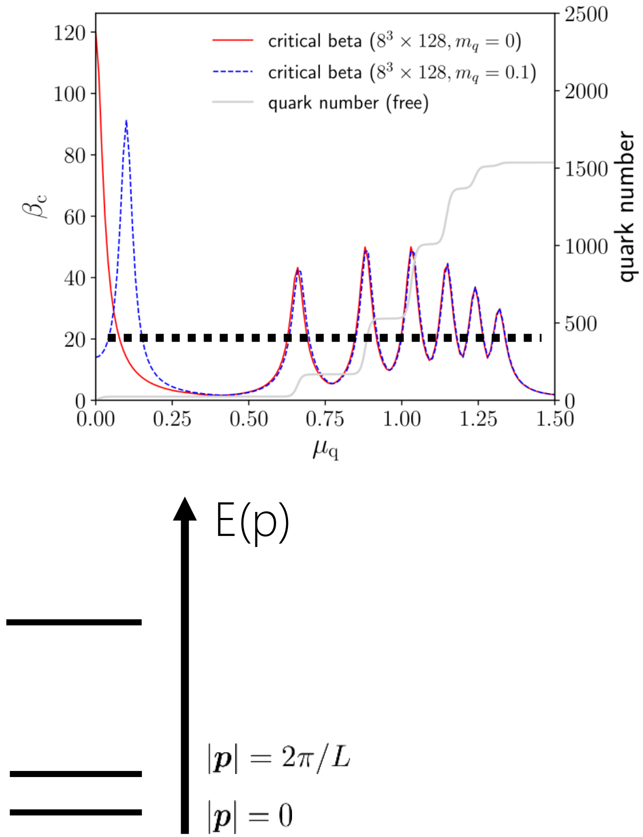
Quark production



- At β=20, quark number shows μ_q-dependence similar to that of the free case.
- Quark number as a function of μ has plateaux.
  (cf) a few plateaux were already observed in $8^3\times16$ and $8^3\times32$ lattice based on CLM in our previous study JHEP10 (2020) 144

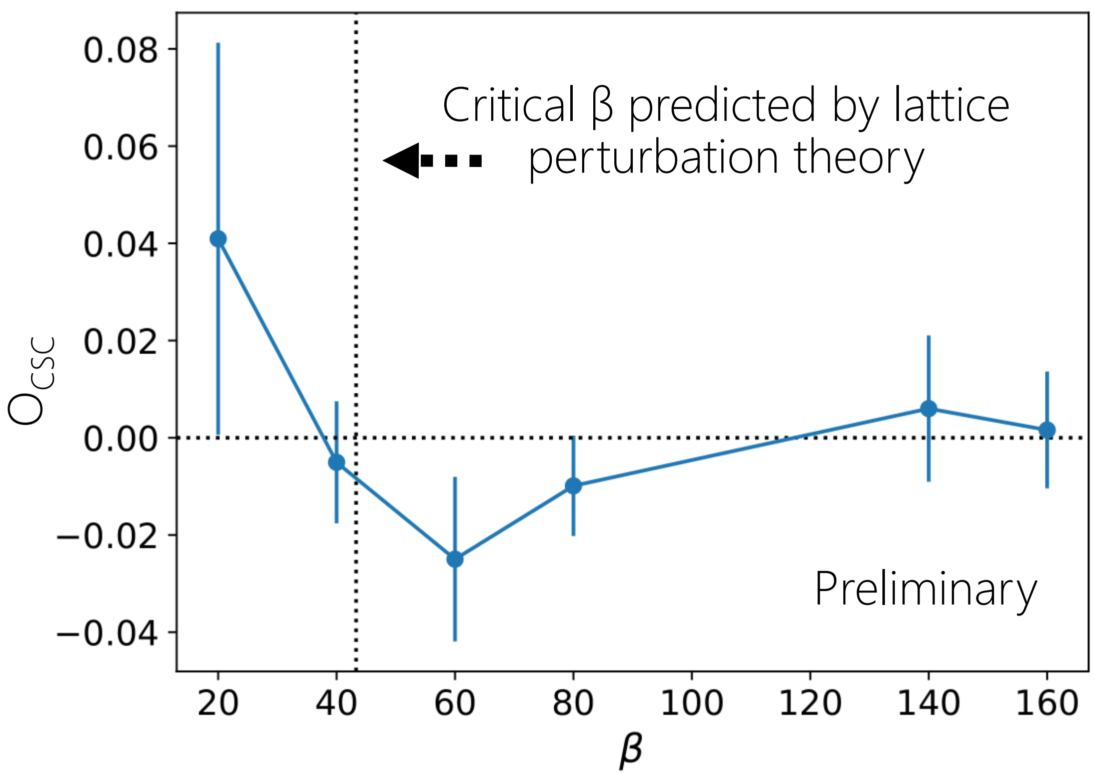
O_{CSC} at β=20

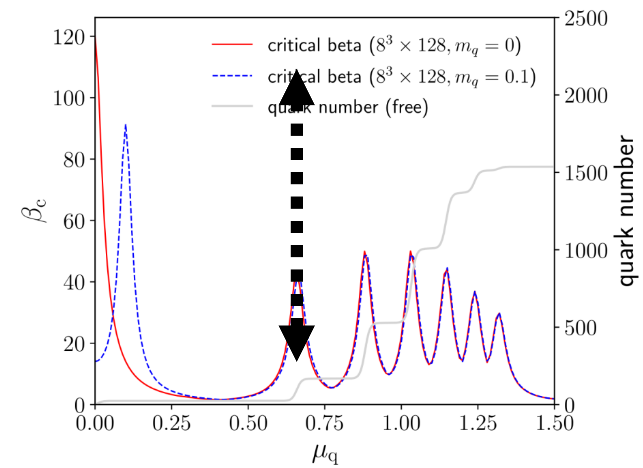




- O_{CSC} has peaks at μ_q=E(p)
- N_{noise}=20 (number of random vectors)
- Part of the error is coming from the noisy estimator (See following discussion)

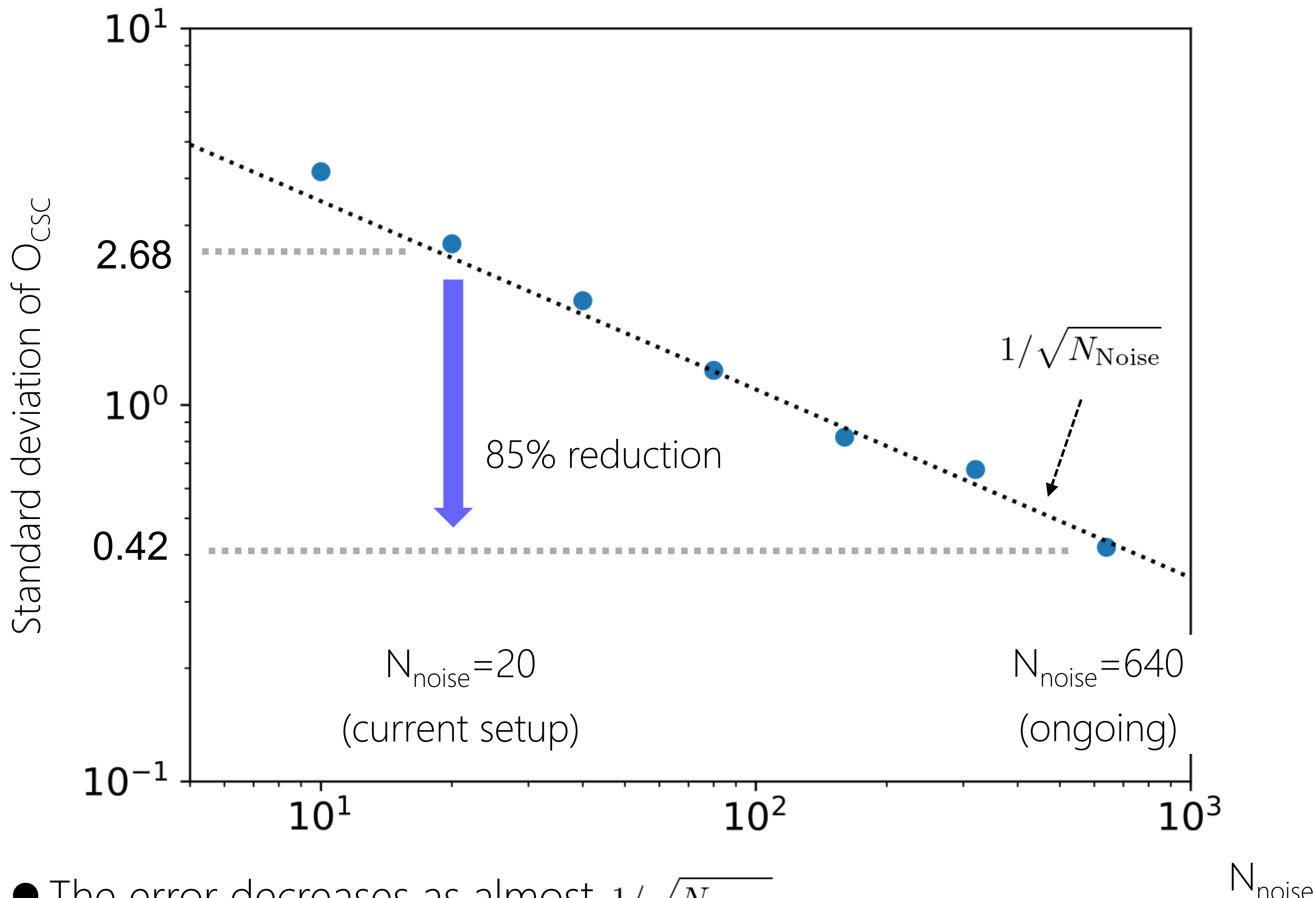
O_{CSC} at μ_q=0.66





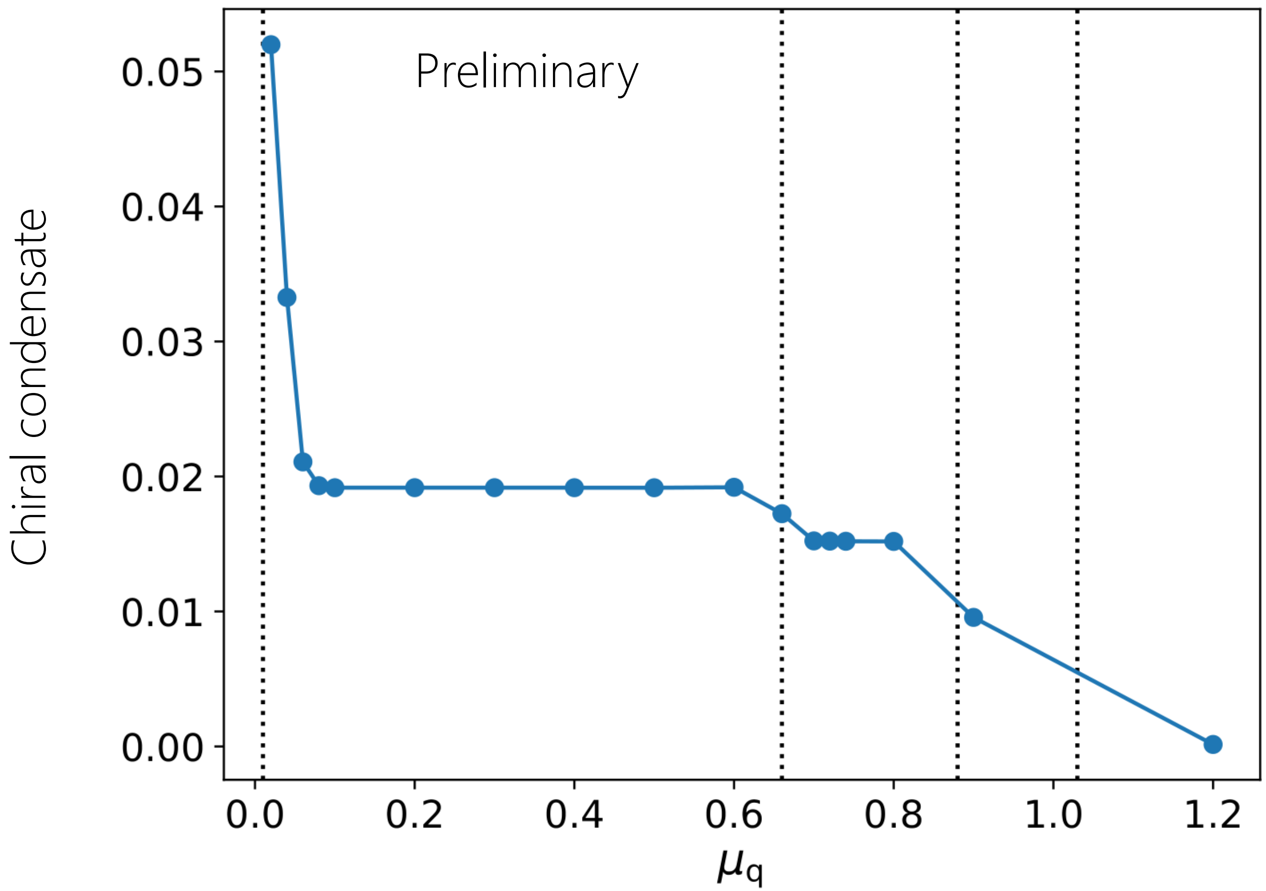
- At low β, O_{CSC} is highly fluctuating. Fluctuation is suppressed at high β.
- N_{noise}=20 (number of random vectors)
- Part of the error is coming from the noisy estimator (See following discussion)

Behavior of the error on N_{noise}



- The error decreases as almost $1/\sqrt{N_{\mathrm{Noise}}}$
- std. of O_{CSC} at N_{noise}=640 is reduced by 85% from that at N_{noise}=20

chiral condensate



- Chiral symmetry restoration due to the finite density effect

Summary

- CLM simulation is performed for 4-flavor staggered fermions, $8^3 \times 128$ lattice. CLM works in a wide range of μ_q.
- Formation of Fermi surface is confirmed up to μ_q =1.2.
- Strong fluctuation of O_{CSC} is observed when $\mu_q \sim E(p)$.
- Chiral symmetry restoration at high density is observed.

Future issues:

- Simulate on a larger lattice to observe clearer SSB.
- Simulate with Wilson fermions.