Coarse Graining in Effective Theories of Lattice QCD in Low Dimensions

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Lattice 2021

CRC–TR 211
Strong-interaction matter under extreme conditions

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Contents

On the Effective Theory

Mother theory: d+1 dimensional lattice QCD with

- Wilson gauge action
- Wilson-Dirac operator

Derivation

- Integrate out fermion fields
- Combined strong coupling and hopping parameter expansion
- Neglect spatial plaquettes
- Integrate out spatial link variables

[Langelage et. al, 2014; Neumann, 2015; Glesaaen, 2016; Schoen, 2018]

On the Effective Theory

Resulting theory

- Dimensionally reduced $d+1 \to d$
- Completely expressed in terms of the Polyakov loop L_x
- Effective description of LQCD at non-zero temperature and chemical potential
- Partition function given by

$$Z = \int [\mathrm{d}U] e^{-S_{\text{kin}}} \prod_x \det Q^{\text{loc}}_{\text{stat}} \prod_{\langle x,y \rangle} \left(1 + \lambda_f (L_x L_y^* + L_x^* L_y)\right)$$

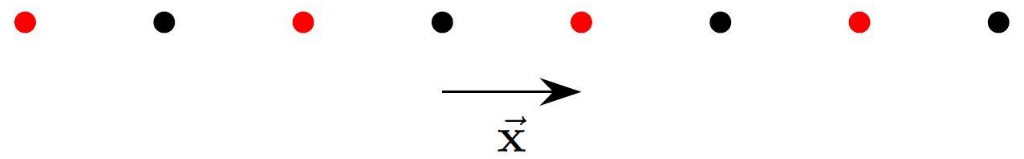
Goal: Test applicability of coarse graining in low dimensions for future use in $d = 3$

[Langelage et. al, 2014; Neumann, 2015; Glesaaen, 2016; Schoen, 2018]

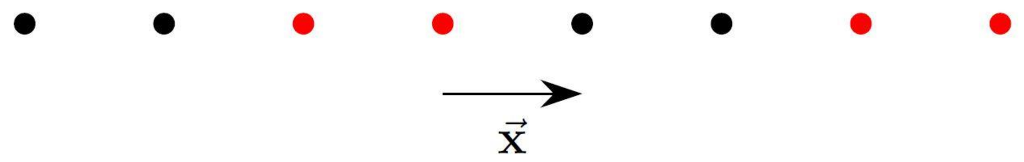
1+1D: Coarse Graining in the Effective Theory

Interpretation as spin model suggests similar renormalization scheme to Ising model

- Nearest neighbor interactions: integrate out every second site (red)



- Next to nearest neighbor interactions: integrate out every second pair of lattice sites (red)



1+1D: Evaluation of the Renormalization Scheme

Running couplings to all orders have a similar form

$$\boldsymbol{h}^{(n+1)} = \boldsymbol{h}^{(n)} \boldsymbol{g} \boldsymbol{h}^{(n)}$$

Boundary conditions e.g. in pure gauge + static quark limit

$$h_{ij}^{(0)} = \lambda_{r_i} \delta_{r_i r_j} \text{ and } g_{ij} = \int \mathrm{d}U \det Q_{\text{stat}}^{\text{loc}}(U) \chi_{r_i}(U^\dagger) \chi_{r_j}(U)$$

Analytical solution to recursion relation and partition function

$$\boldsymbol{h}^{(n)} = \left(\boldsymbol{h}^{(0)} \boldsymbol{g}\right)^{2^n - 1} \boldsymbol{h}^{(0)} \qquad Z = c_0^{N_x N_\tau} \sum_{ij} h_{ij}^{(n_x)} g_{ji} = c_0^{N_x N_\tau} \operatorname{Tr}\left[\left(\boldsymbol{h}^{(0)} \boldsymbol{g}\right)^{N_x}\right]$$

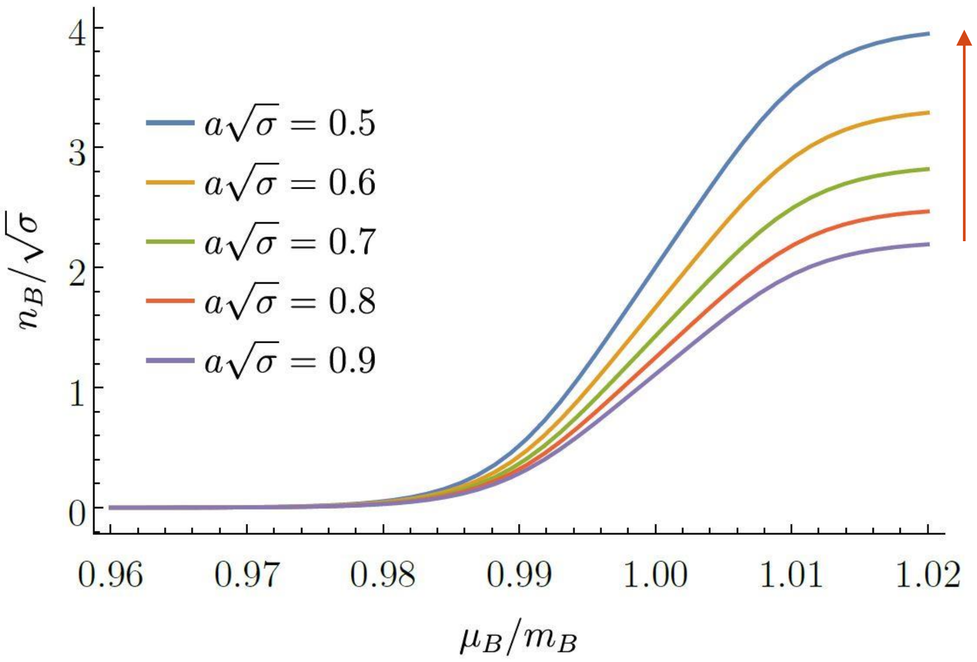
1+1D: Evaluation of the Renormalization Scheme

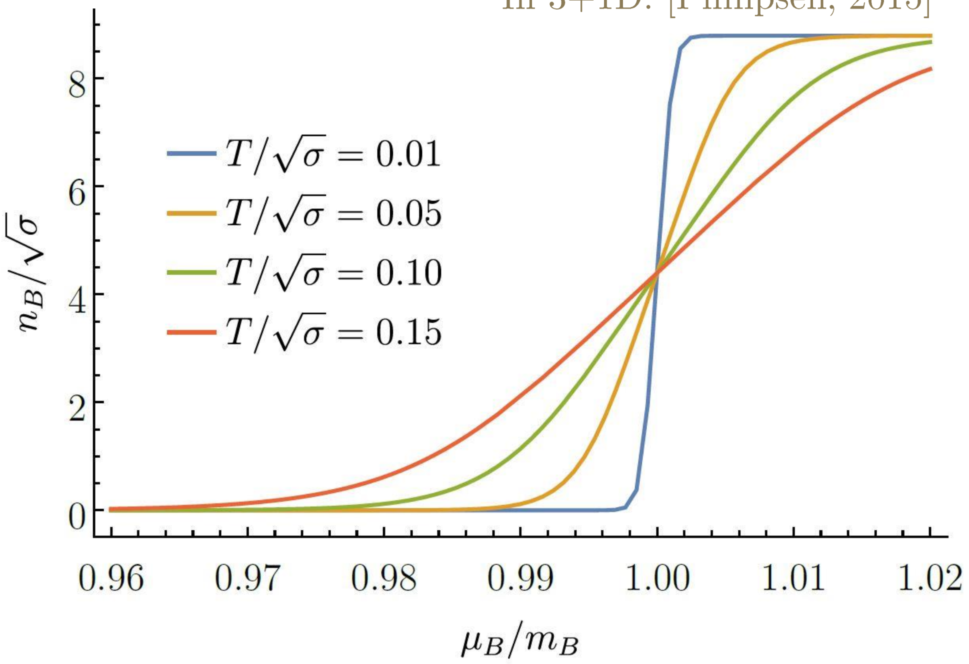
Baryon density in the static quark limit for varying lattice spacing (left) and after continuum extrapolation (right)

Scale setting with string tension: [Huang et. al, 1988]

$$N_f = 1, N_c = 3, \beta/6 \in [0, 3] \text{ and } m_B/\sqrt{\sigma} = 25$$

In 3+1D: [Philipsen, 2015]

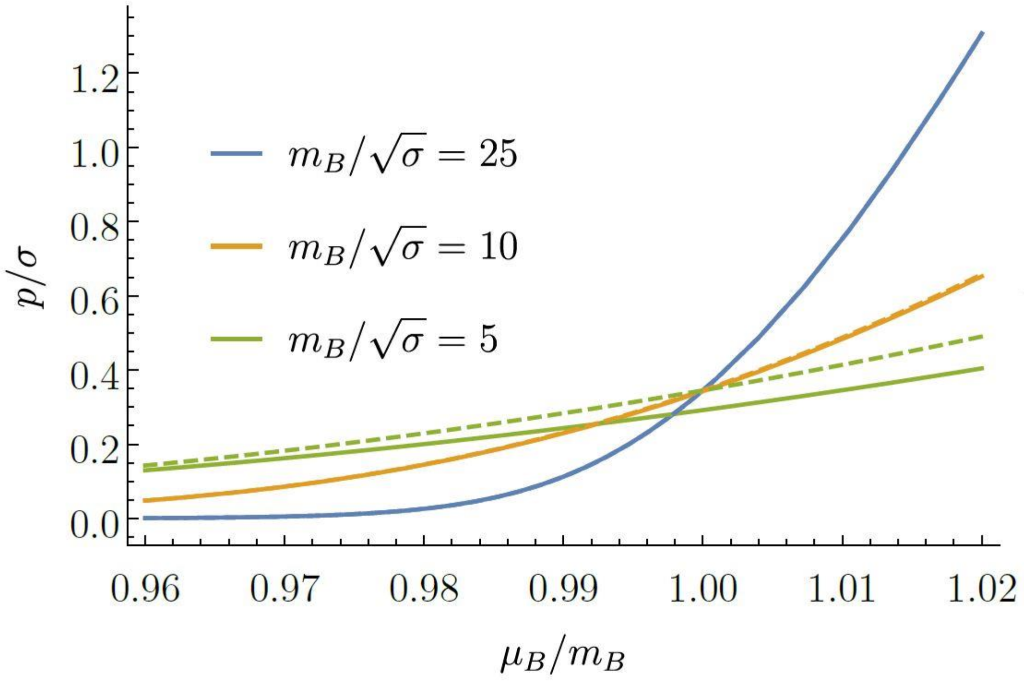




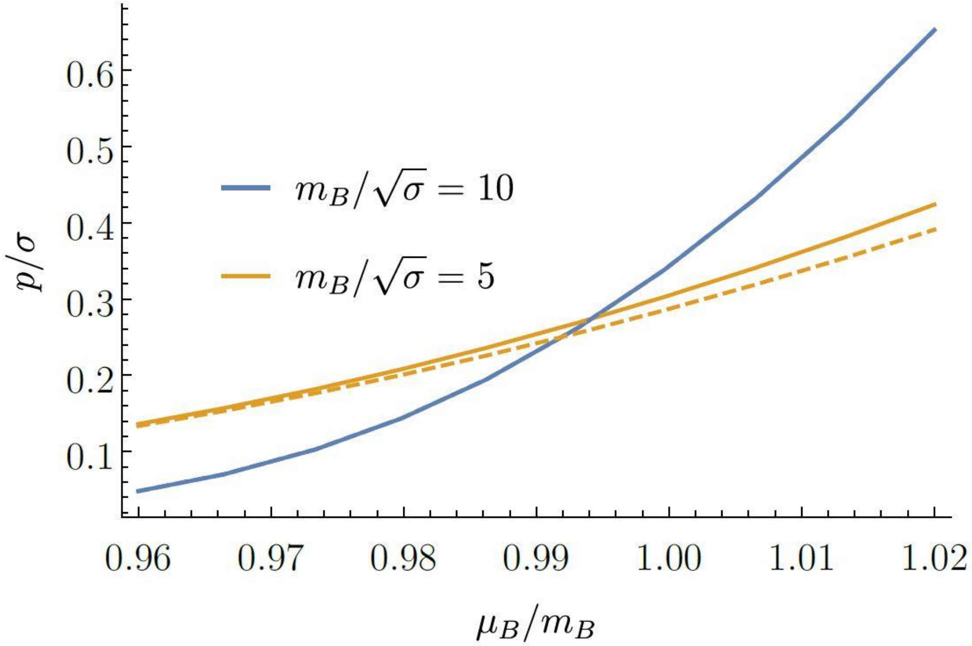
1+1D: Evaluation of the Renormalization Scheme

Comparison of the pressure for $N_f = 1, N_c = 3, T/\sqrt{\sigma} = 0.15$

Static quarks (solid lines) vs. LO corrections (dashed lines)



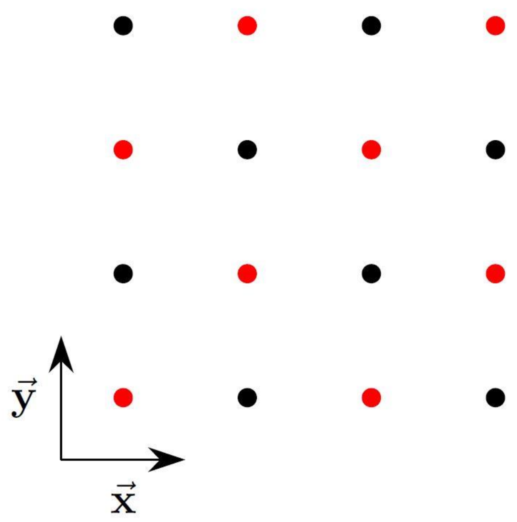
LO corrections (solid lines) vs. NLO (dashed lines)



2+1D: More Coarse Graining

Consider effective theory in the pure gauge and static quark limit

Analogous to 2D Ising model: integrate over lattice sites in a checkerboard pattern



2+1D: More Coarse Graining

Only integrals over powers of Polyakov loops with static determinant occur

$$\int \mathrm{d}U \det Q^{\mathrm{loc}}_{\mathrm{stat}} L^j L^{\dagger k} =: o(j,k)$$

Neglect terms beyond $\mathcal{O}(\lambda_f^v \kappa^{w N_\tau})$ with $v + w = 2$

Fixed point of RG transformation found at $\lambda_f = \dfrac{o(0,0)}{3o(1,1)}$

2+1D: The Deconfinement Transition

Critical couplings in the pure gauge limit for SU(3)

N_τ	β_c	$\beta_{c,\text{lit}}$	$\lvert\Delta\beta_c\rvert/\beta_{c,\text{lit}}$ in %
2	8.9267	8.1489	9.5449
3	12.6488	11.3711	11.2365
4	16.3067	14.7170	10.8021
5	19.9532	18.1310	10.0501

$N_\tau = 2, \ldots, 5$

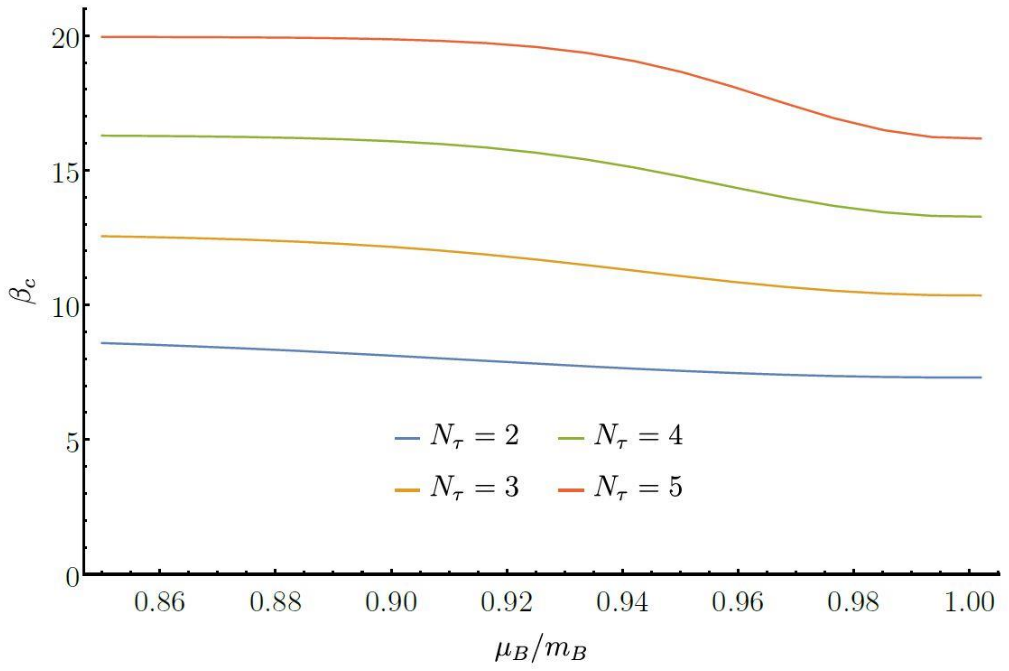
Relative deviation of <12%

Comparison to simulation
of 2+1D Yang-Mills theory

Simulation results: [Liddle et. al, 2008]

2+1D: The Deconfinement Transition

Critical couplings in the static quark limit for SU(3) and $\kappa = 0.01$



Conclusion and Outlook

Coarse Graining provided a powerful tool to evaluate the effective theory

- Obtained the transfer matrix of the effective theory in 1+1D
- Found analytical expressions for critical couplings in 2+1D
- Critical couplings agree within $<12\%$ with simulation results

Open questions

- Include parts of the kinetic quark determinant in 2+1D?
- Application in 3+1D?

Thank You!