CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

Lattice QCD at imaginary chemical potential in the chiral limit

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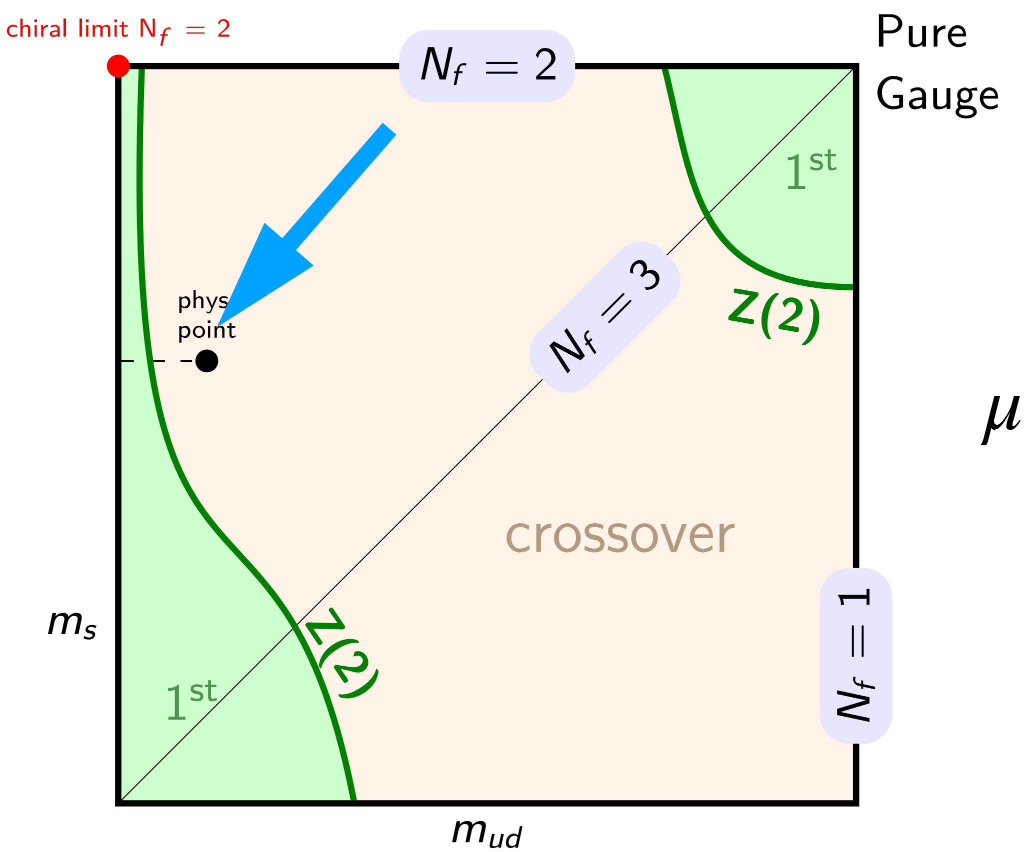
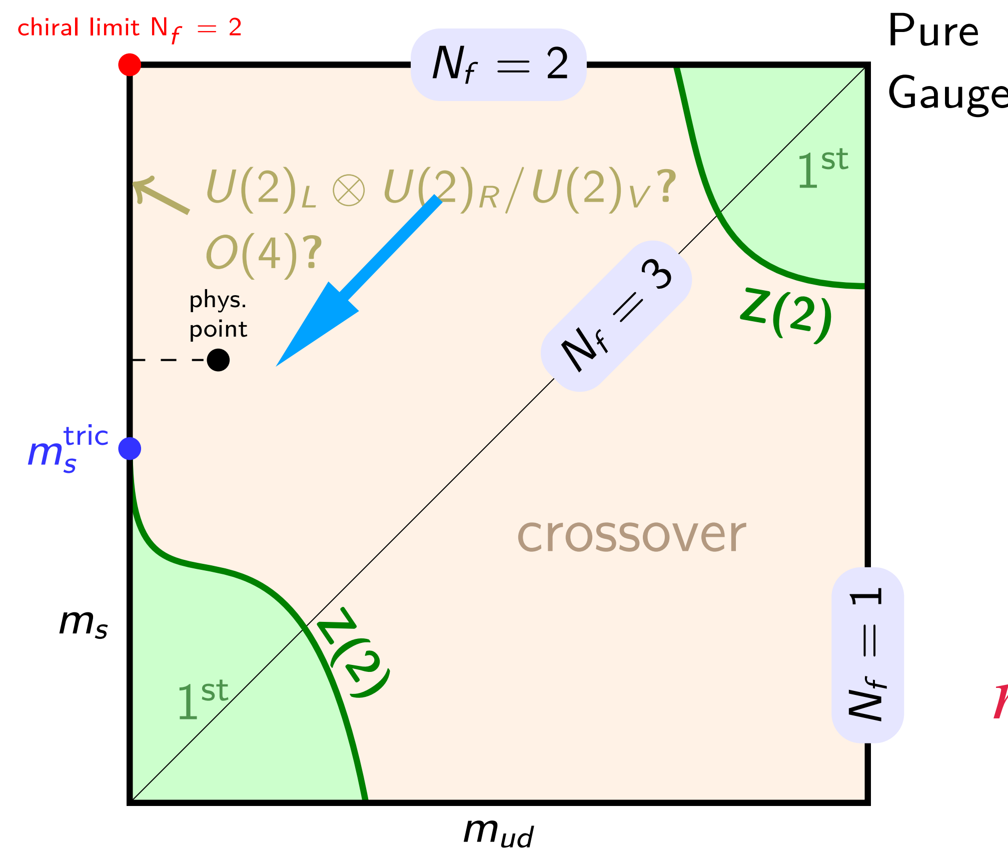
Nature of the transition in the chiral limit??



Philipsen, Pinke, PRD 93 (2016)

$\mu = 0$



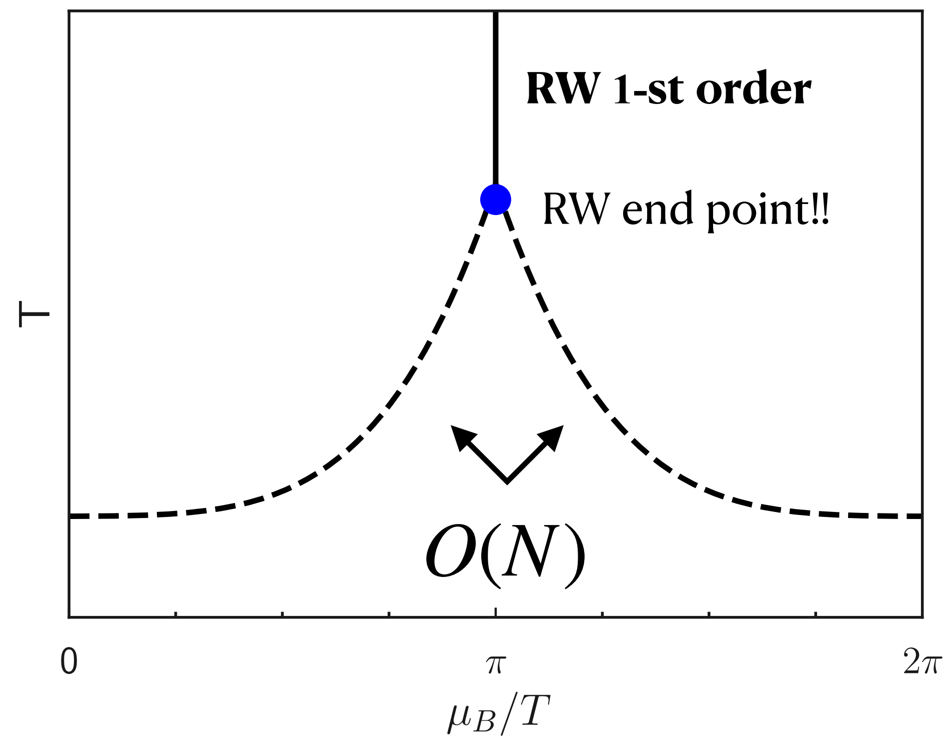
H. T. Ding et al. ,PhysRevLett.123.062002

No hint of $1st$ order for $m_\pi > 55$ MeV

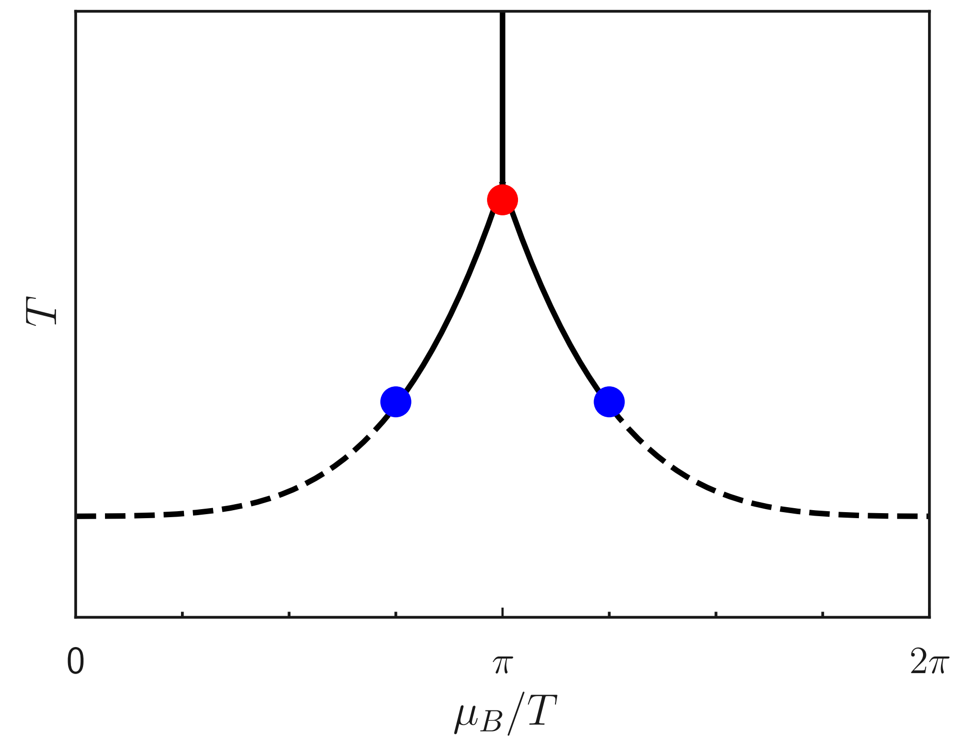
Lattice QCD study with $2+1$ flavors. Two light flavors (u, d) and one heavy flavor (s)

- **Calculations with smaller quark masses require more computer resources!!**
- **If transition is first order, a m_{crit} will exist where first order ends in a $Z(2)$. For $2nd$ order only at $m_u = m_d = 0$ it will be a $O(4)$.**
- **Calculations with imaginary chemical potential will increase the value of m_{crit} if a $1st$ order exist at $\mu = 0$.** Philipsen, Pinke, PRD 93 (2016)

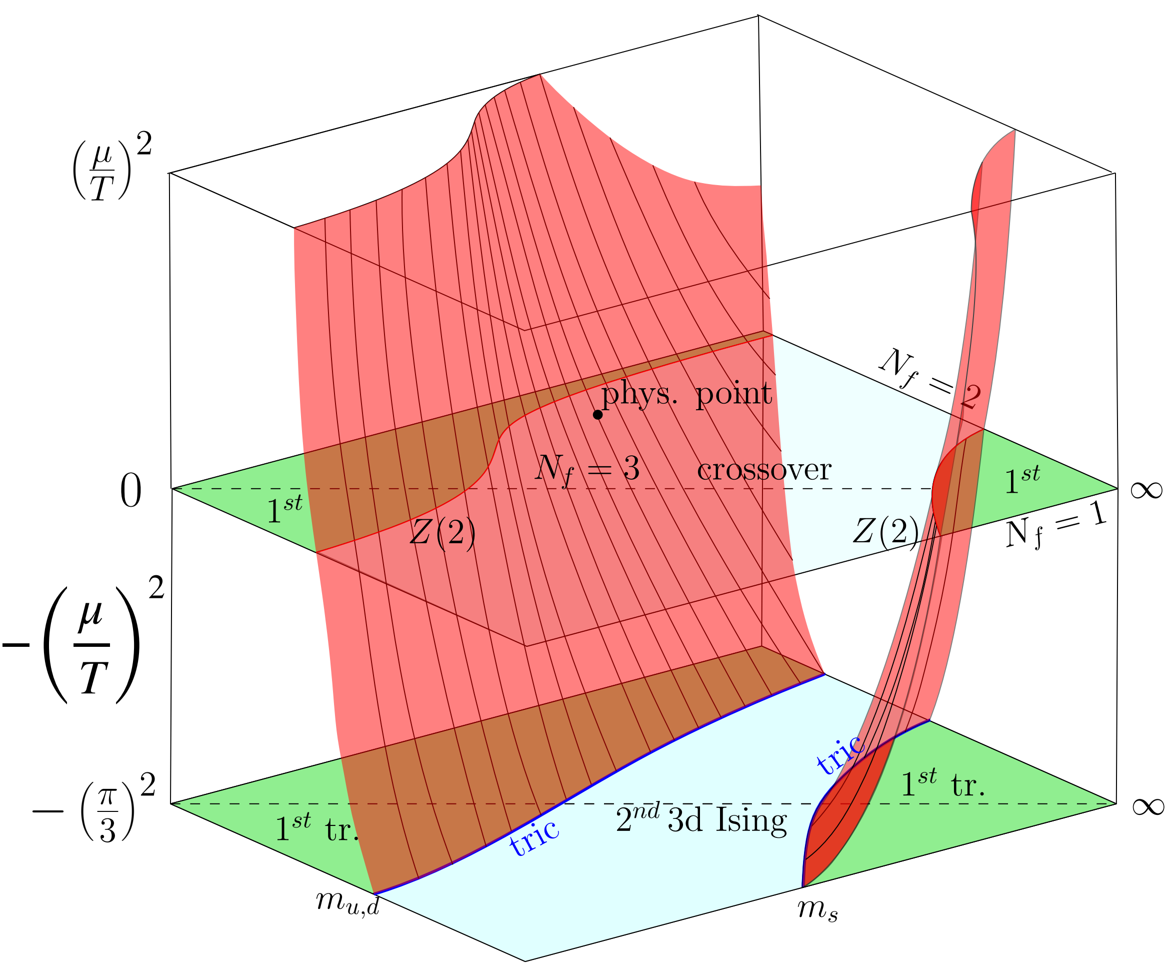
Roberge Weiss plane



$m_u, m_d \to 0$



Cuteri et. al, *PoS* LATTICE2015 (2016) 148



Partition function will be periodic under, $\mu_B/T \to \mu_B/T + 2\pi i$.

This translates to $\mu/T = 2\pi i/3$ in terms of the quark chemical potentials.

m_{crit}, if it exists will be largest in the RW plane i.e. at $\mu/T = i\pi/3$ which we will use to constrain the m_{crit} in the vanishing chemical potential.

We have used HISQ action with 2+1 flavors for this study.

Criticality and Roberge Weiss Transition

In a finite volume the Free energy and the universal functions for second order transition close to the critical point can be written as, $[t \to 0, h \to 0, t = (T - T_c)/T_c]$

$$f = b^{-d} f_s(b^{y_t} t, b^{y_h} h, b^{-1} N_\sigma) + f_{ns}$$

$$z_f = z_0 t N_\sigma^{1/\nu}$$

$$M = \left.\frac{\partial f}{\partial h}\right|_{h \to 0} \sim N_\sigma^{-\beta/\nu} f_{G,L}(z_f)$$

$$\chi_h = \left.\frac{\partial^2 f}{\partial h^2}\right|_{h \to 0} \sim N_\sigma^{\gamma/\nu}\ f_{\chi,L}(z_f)$$

$$B_4 \sim f_{B,L}(z_f)$$

$$\chi_t = \left.\frac{\partial^2 f}{\partial t^2}\right|_{h,t \to 0} \sim N_\sigma^{\alpha/\nu}\ f''_{f,L}(z_f)$$

β, ν, α and γ *are critical exponents*

Magnetic like: Derivatives with respect to h

Energy like : Derivatives with respect to t

In addition to the (approximate) chiral symmetry,

$Z(\mu/T) = Z(\mu/T + 2k\pi i/3)$, RW symmetry for any value of the m_u, m_d for $\mu/T = i\pi/3$

$$Z(T,\mu) = \int [\mathscr{D}U] det[M_{ud}(\mu_f)]^{1/2} det[M_s(\mu_f)]^{1/4} \exp[-S_G]$$

S_G **is the gauge action.**

Under the transformation, $U_\mu \to U_\mu^\dagger$

$$O_M \to -O_M$$

$$O_E \to O_E$$

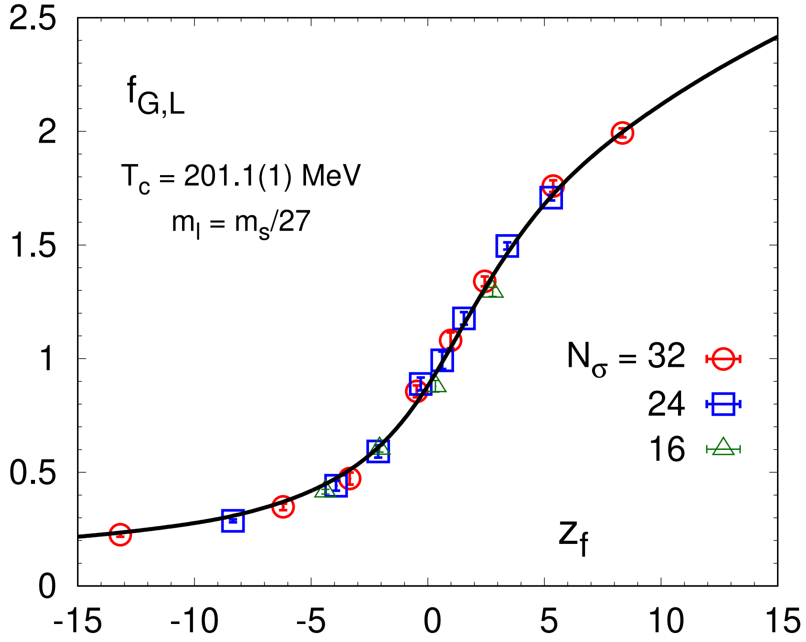
$$O_M \to Im\ L, \tan^{-1}[ImL/ReL], \mathrm{Im}[\chi_1^B]$$

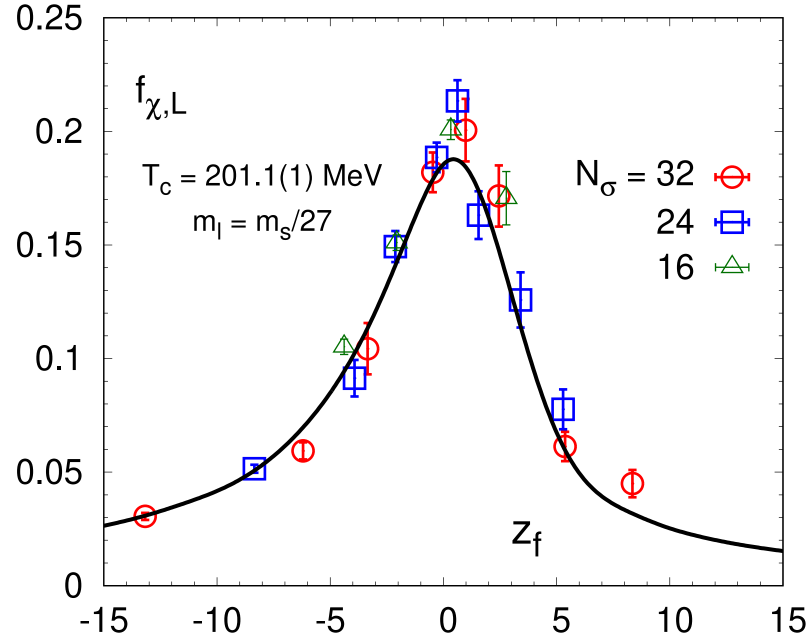
$$O_E \to Re\ L, \bar{\psi}\psi\,..$$

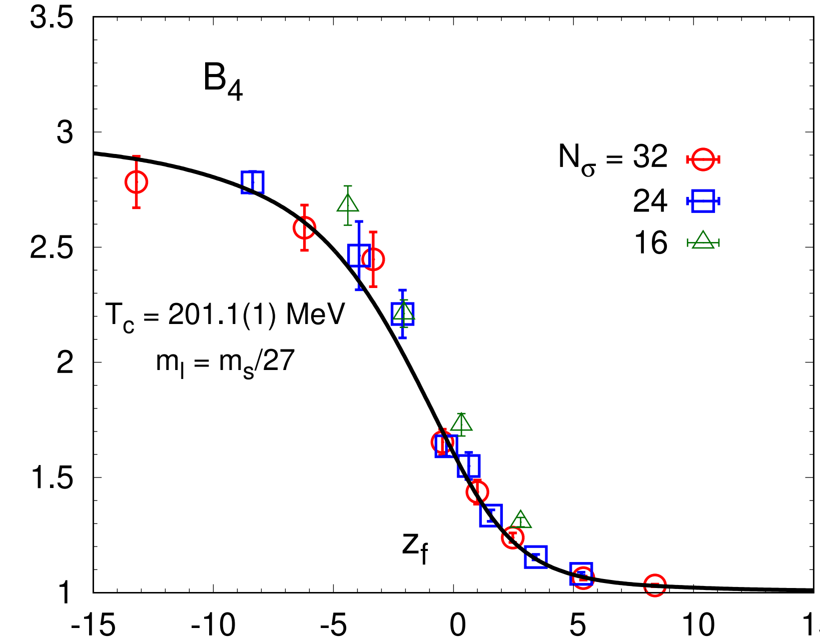
Ex. From Universality, magnetic → magnetization in ferromagnet

energy like → specific heat in ferromagnet

Finite size Scaling







$$M = A_M N_\sigma^{-\beta/\nu} f_{G,L}(z_f)$$

3-d Ising model Universal Scaling functions

Fit range, $T_c \pm 10$ MeV

$$\chi_M = A_M^2 N_\sigma^{\gamma/\nu} f_{\chi,L}(z_f) + \chi_{reg}(t)$$

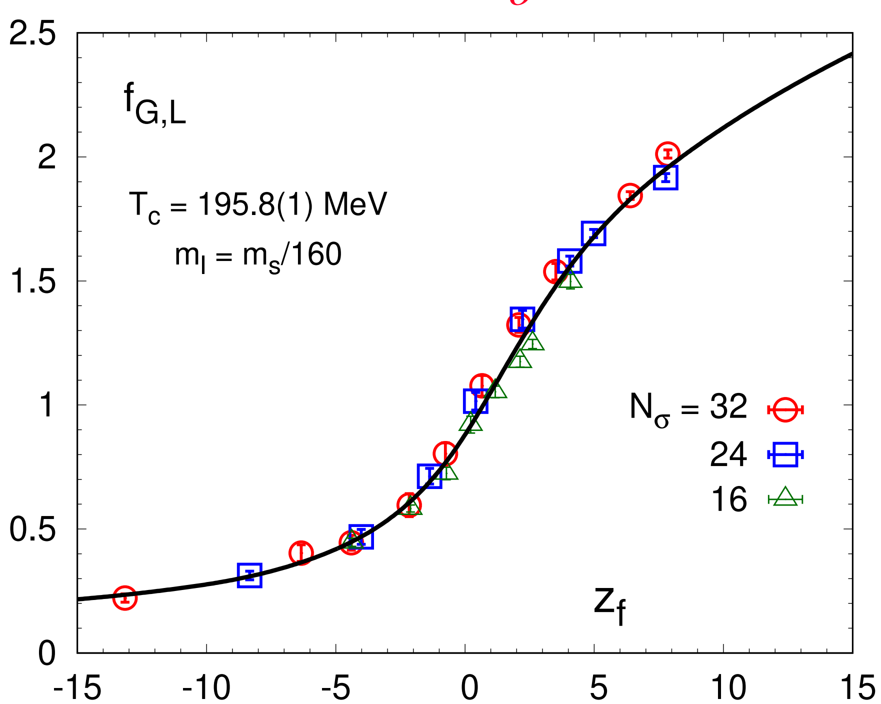
$$z_f \equiv z_0 t N_\sigma^{1/\nu},\ t = (T - T_c)/T_c$$

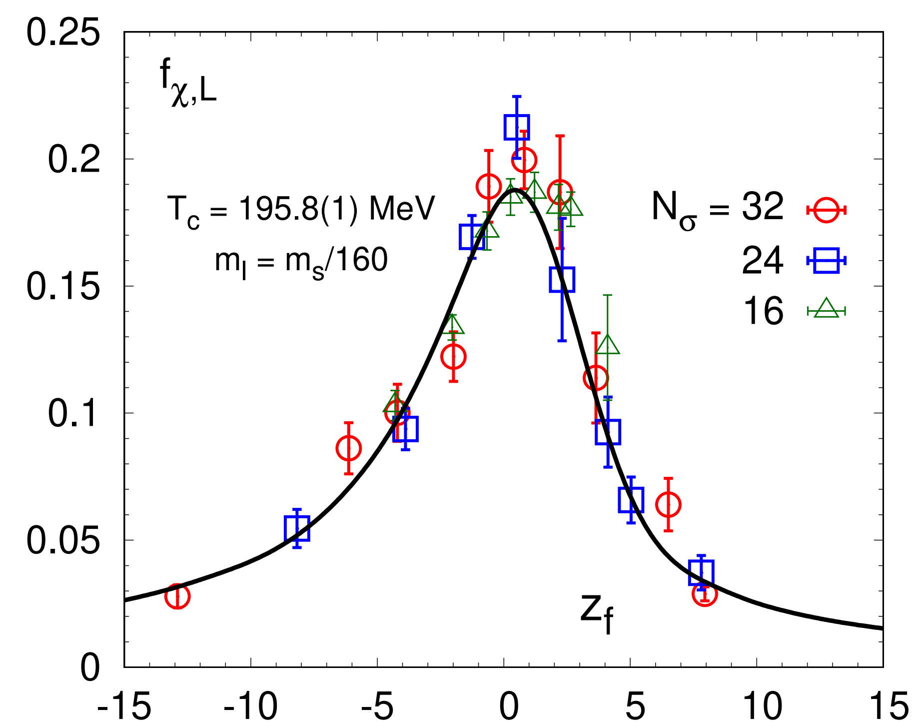
$$\chi^2/dof = 2,\ m_s/27$$
$$= 1.54,\ m_s/160$$

$$B_4 = f_{B,L}(z_f) + \frac{d}{N_\sigma^3}$$

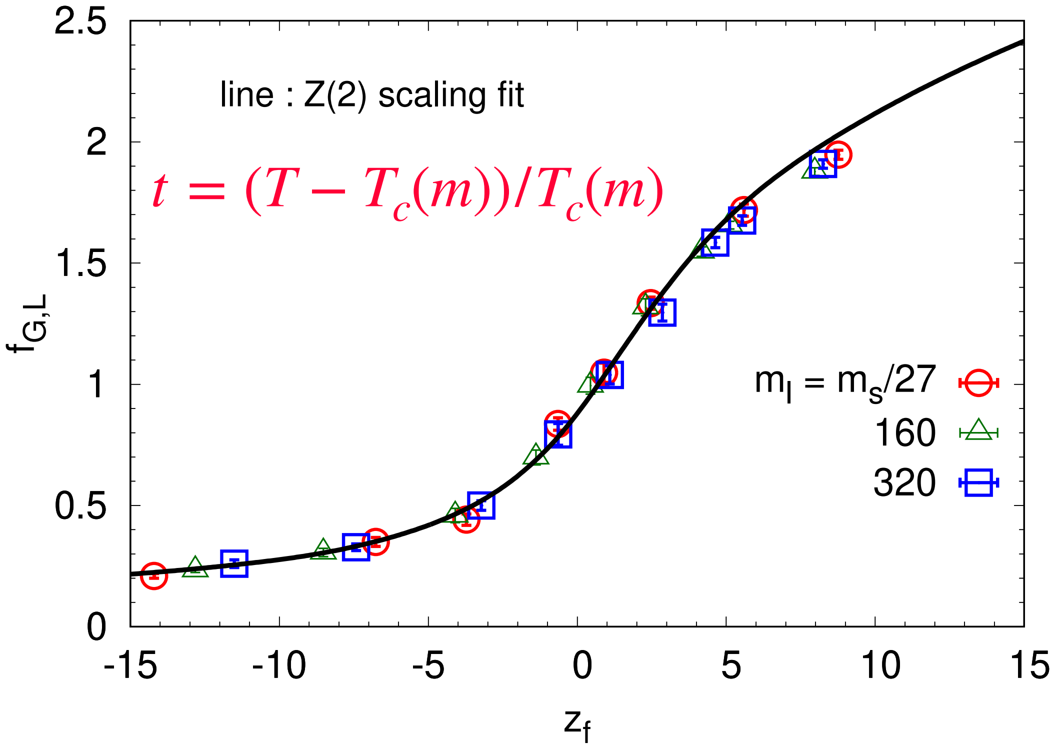
T_c, z_0, A_M **are non-universal parameters.**

Lattice Size $= N_\sigma^3 \times 4$



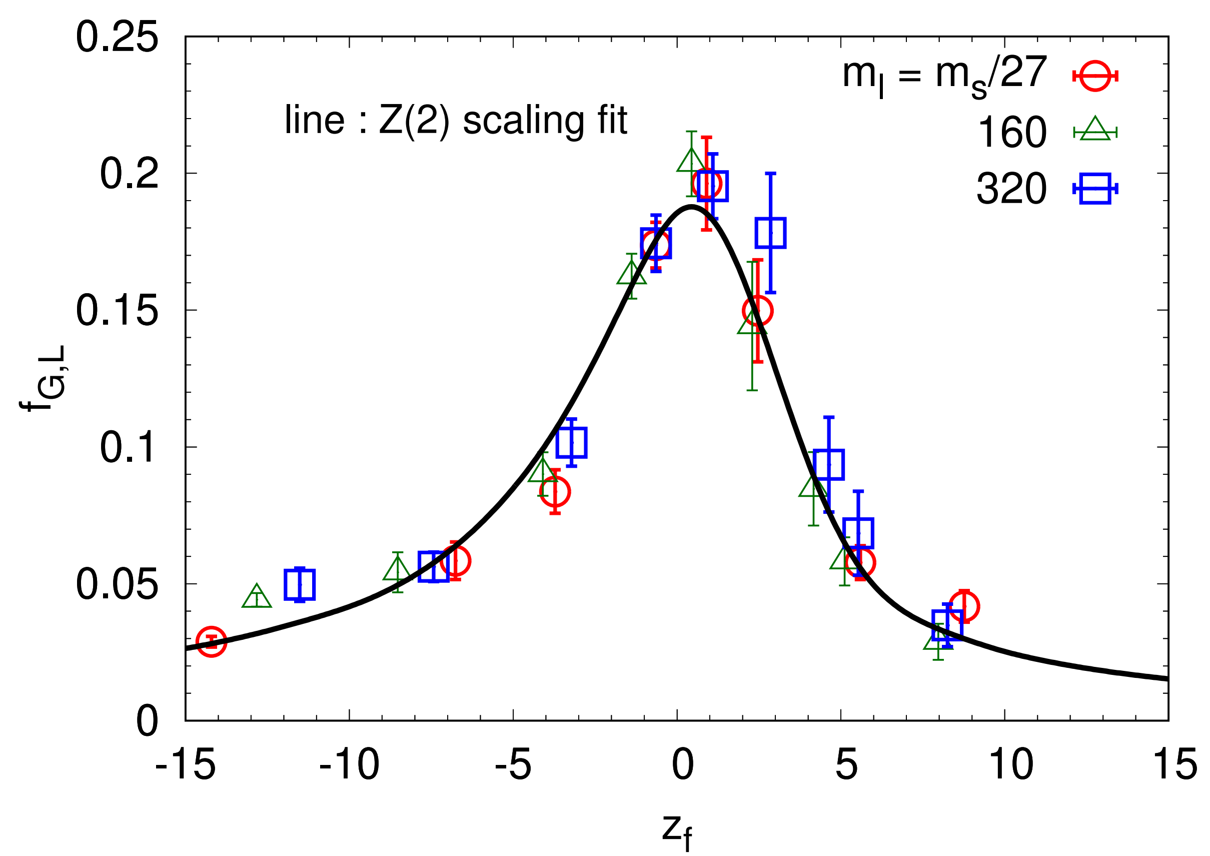


Nature of the RW end point in the chiral limit$(m_u, m_d \to 0)$

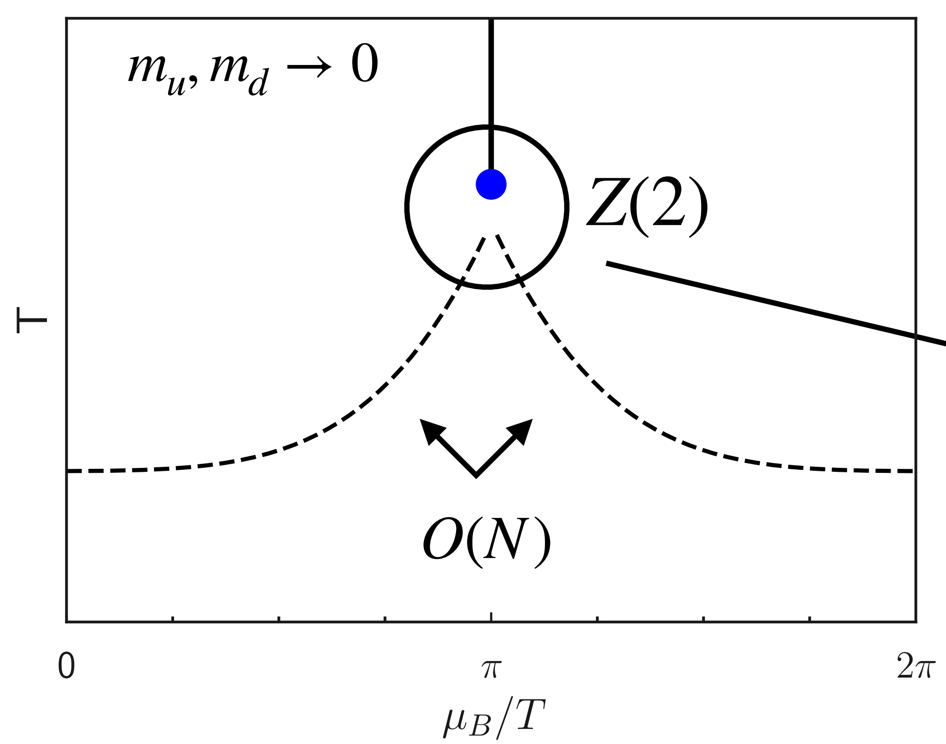


- **Order parameter and its susceptibility show good agreement with the expected finite size $Z(2)$ scaling functions.**
- **RW end point stays $Z(2)$ for $m_\pi \geq 40$ MeV, No sign of first order!!**

Consistent with: Claudio Bonati, arXiv:1807.02106 [hep-lat]



Searching for when RW end point turns to $1st$ order !!



Do RW transition and Chiral transition coincide in the chiral limit??

Chiral Observables

- The definition of order parameter of chiral transition and its susceptibility $\Delta_{ls} = \frac{2m_s}{f_k^4}(\langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s)$,

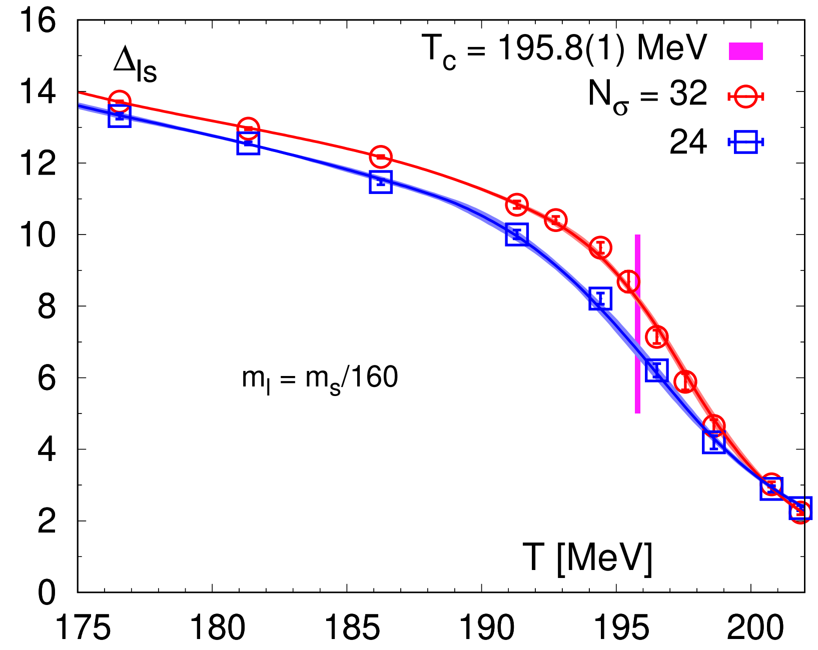
  $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2, \langle\bar{\psi}\psi\rangle_f = \frac{1}{4}\frac{1}{N_\sigma^3 N_\tau}\langle TrM_f^{-1}\rangle$

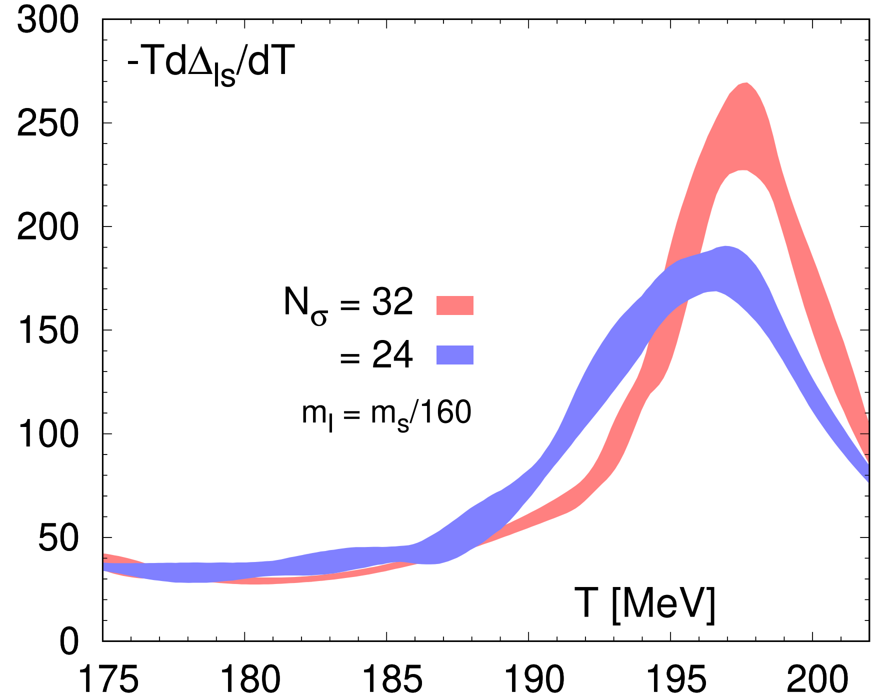
  $$\chi^{disc} = \frac{1}{4}\frac{m_s^2}{N_\sigma^3 N_\tau}\left(\langle(\mathrm{Tr}M_l^{-1})^2\rangle - \langle\mathrm{Tr}M_l^{-1}\rangle^2\right)/f_K^4,$$

  M_f is the HISQ Dirac operator for quark flavors, $f = u, d, s$.

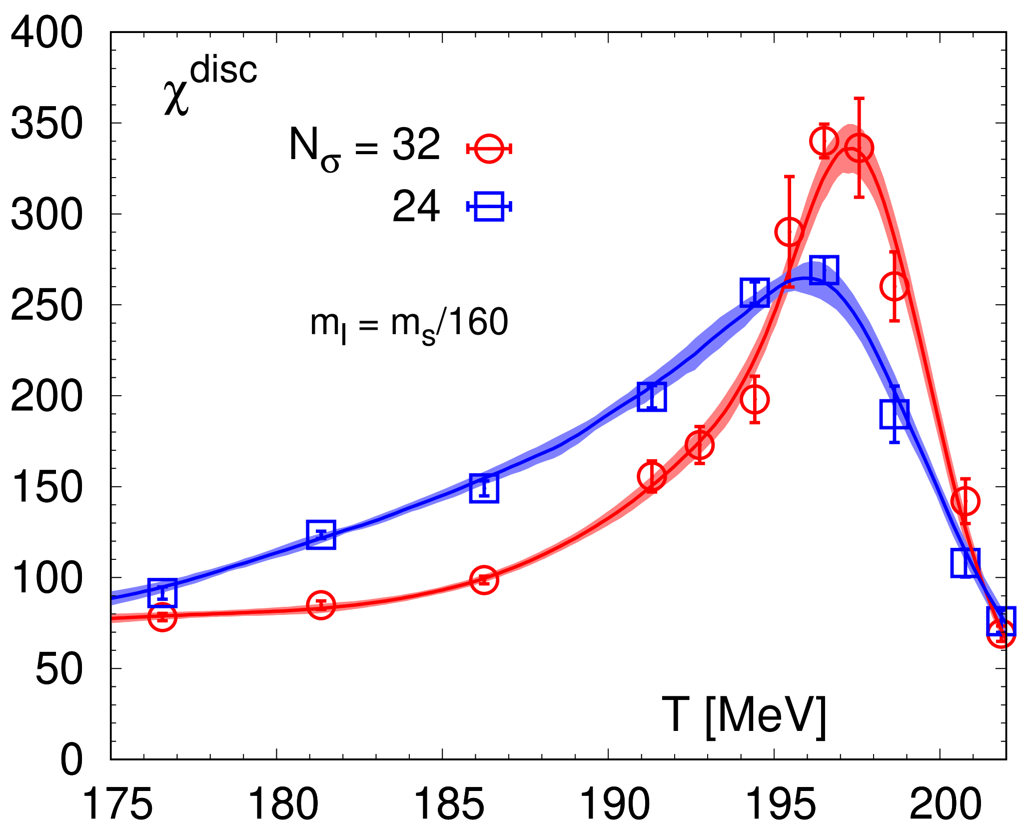
- For , $i\mu_B/T \neq \pi, m_u, m_d \to 0$, Δ_{ls} and χ^{disc} will follow the finite size behaviour of 3d, $O(N)$ model.

- For, $i\mu_B/T = \pi$, influence of additional $Z(2)$ for any value of m_u, m_d on the Δ_{ls} and χ^{disc} ??



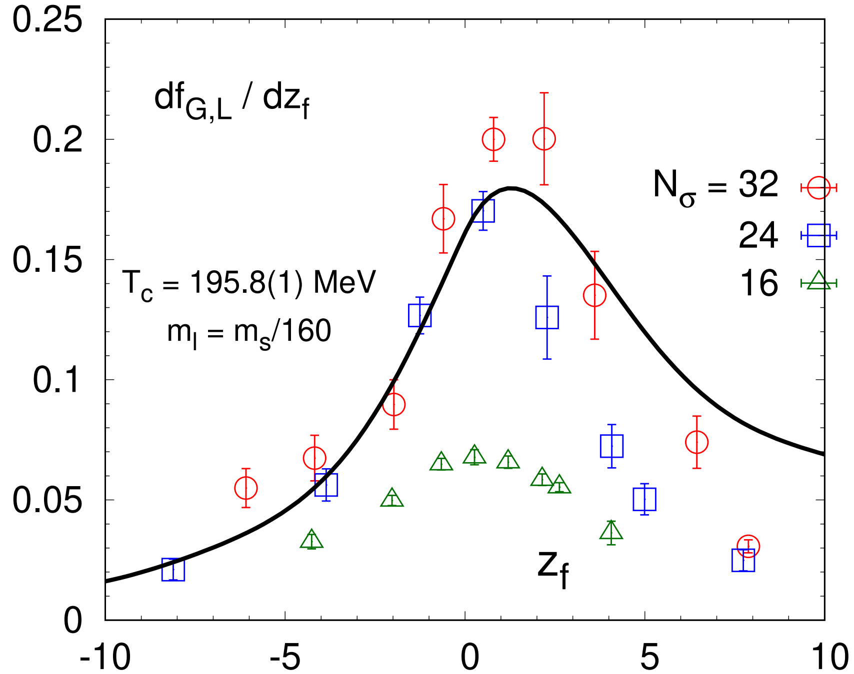


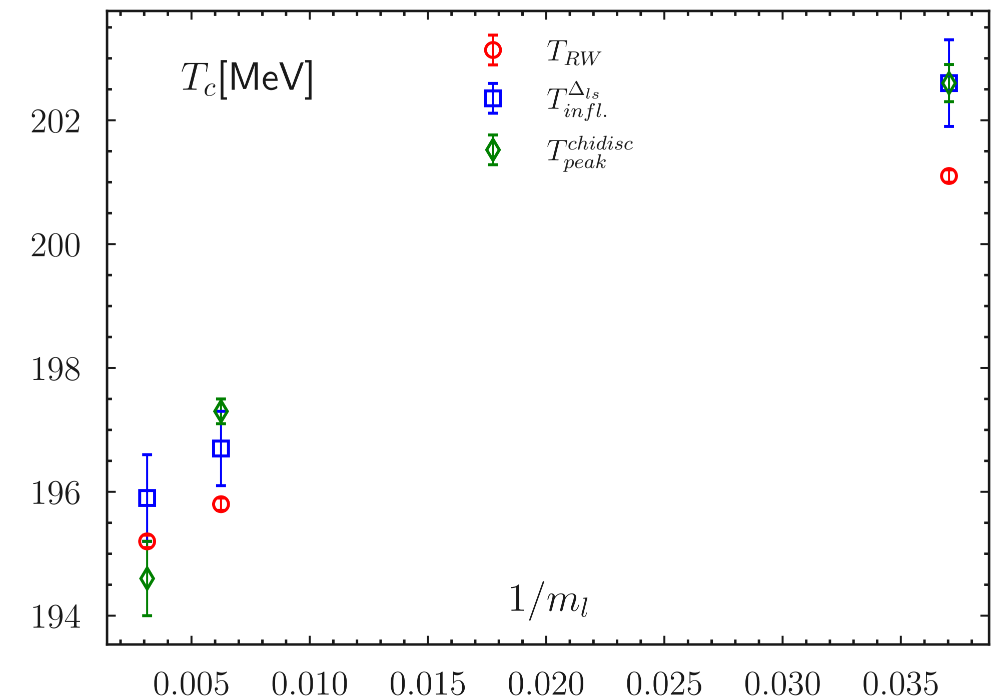
- Behaviour of Δ_{ls} looks similar to the behaviour of order parameter in $O(N)$ models.
- **But** the temperature derivative of Δ_{ls} suggests that it will have an infinite slope in the infinite volume limit.



- Finite size behaviour: χ^{disc}, $Td\Delta_{ls}/dT \sim N_\sigma^{\alpha/\nu}\ f''_{f,L}(z_f)$.
- Consistent with the specific heat of $Z(2)$ transition. In that sense Δ_{ls} could also be an energy like observable for the RW transition.
- The two transitions could coincide (happen at same temperature) in the chiral limit. The nature of the transition will differ from both $Z(2)$ and $O(4)$. Possibly a larger symmetry group/universality class will be relevant.

Outlook





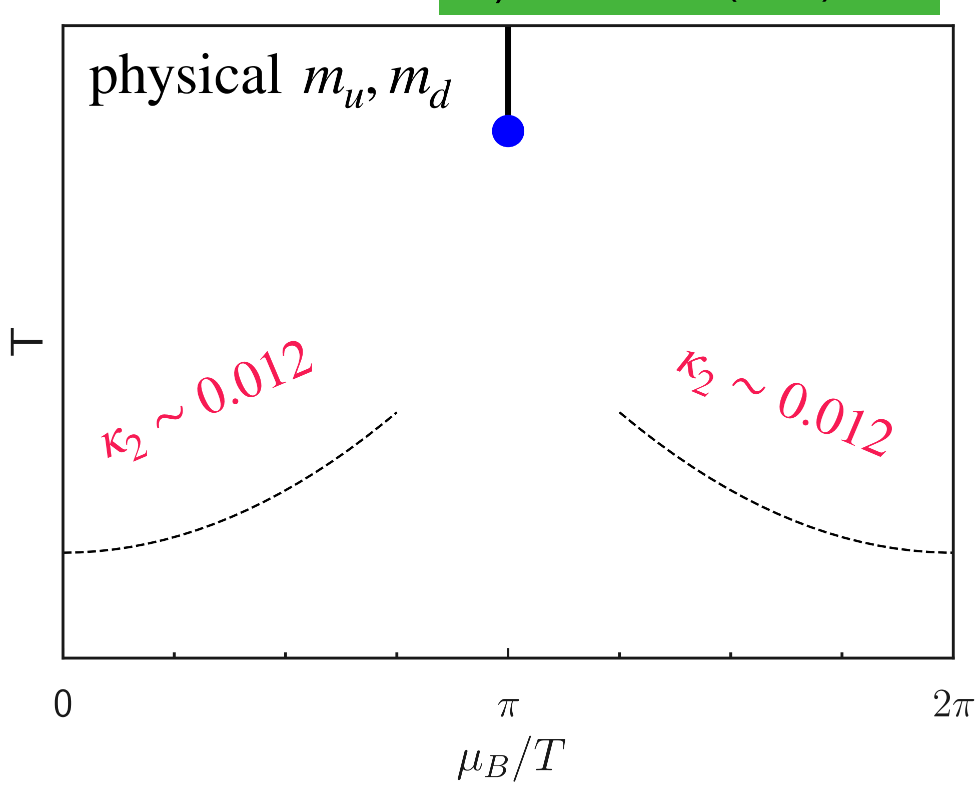
$$\chi_t = \frac{m_s}{2f_K^4}\left(\langle |\mathrm{Im}P|\,\mathrm{Tr}M_l^{-1}\rangle - \langle |\mathrm{Im}P|\rangle\langle \mathrm{Tr}M_l^{-1}\rangle\right)$$

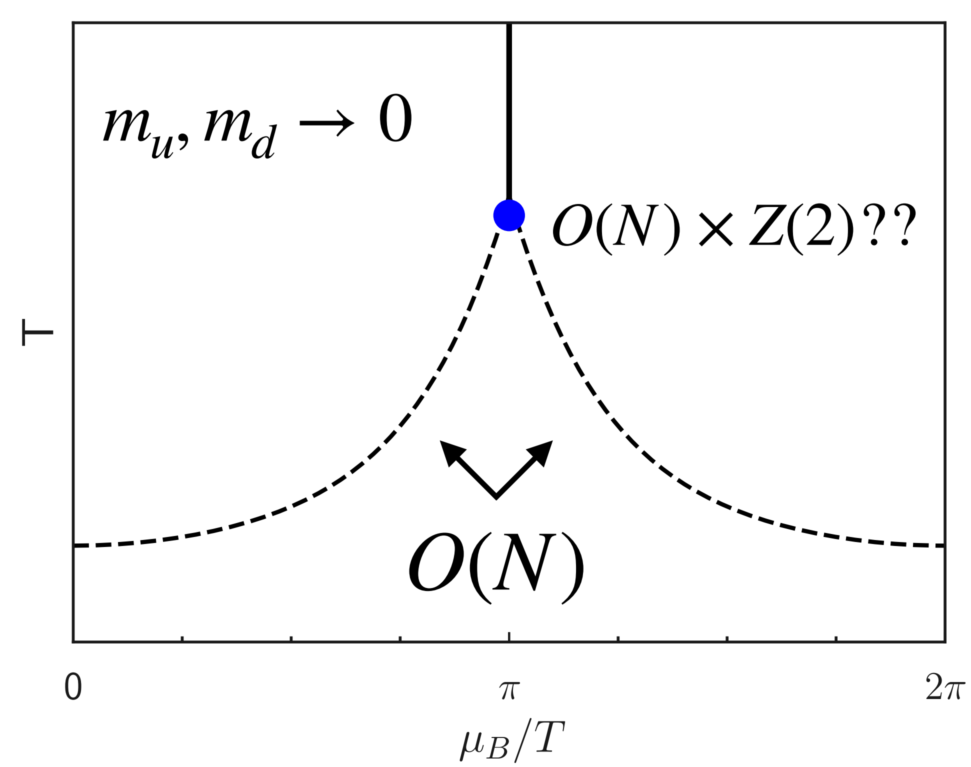
$$\chi_t(T, N_\sigma) = A_t N_\sigma^{(1-\beta)/\nu} f'_{G,L}(z_f) + regular$$

$$(1-\beta)/\nu = 1.069(5)$$

In mixed susceptibility, large finite volume effect seen for $m_l = m_s/160$.

Phys.Lett.B 795 (2019) 15-21

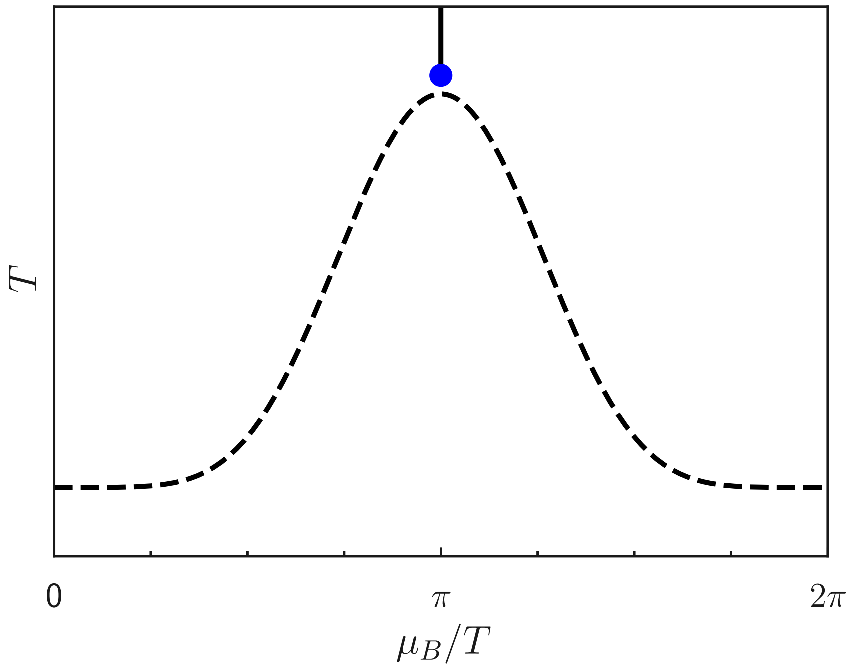




Conclusion

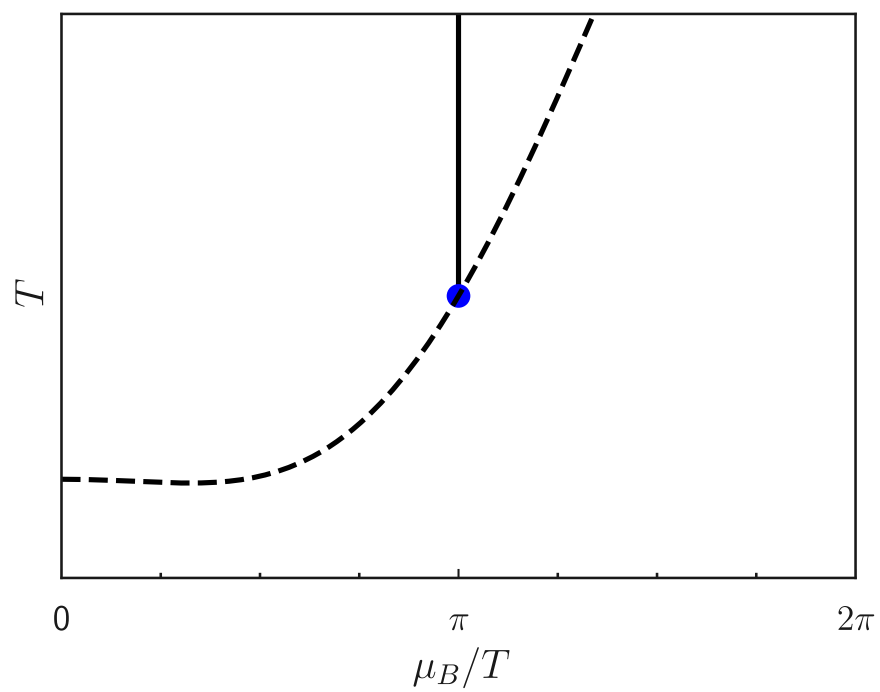
- RW end point remains Z(2) second order for $m_\pi \geq 40$ MeV. Nature of the chiral transition in the RW plane favours 2nd order O(4).
- This implies that nature of the chiral transition at vanishing chemical potential favours 2nd order O(4). Moreover, calculation at imaginary μ_B leads to more stringent limit on, $m_{crit} \leq 40$ MeV.
- Unlike vanishing chemical potential, Polyakov loop is magnetization like and chiral observables are energy like for RW transition.
- The two transitions could coincide in the chiral limit resulting in a universality class different from both $Z(2)$ and $O(4)$.

Back up



Possible parametrization!!

$$T_c(\mu_B,T) = T_c(\mu_B,T)(1+\kappa_2\ sin(\mu_B/2T-\pi)^2+\kappa_4\ sin(\mu_B/2T-\pi)^4+\kappa_6\ sin(\mu_B/2T-\pi)^6)$$
$$\sim T_c(\mu_B,T)(1-\kappa_2\ (\mu_B-\pi)^2+....)$$



$$T_c(\mu_B,T) = T_c(\mu_B,T)(1+\kappa\ cos(\mu_B/2T-\pi)+\kappa_2\ cos(\mu_B/2T-\pi)^2+\kappa_3\ cos(\mu_B/2T-\pi)^3)$$
$$\sim T_c(\mu_B,T)(1-\kappa\ (\mu_B-\pi)+\kappa_2\ (\mu_B-\pi)^2+\kappa_3\ (\mu_B-\pi)^3....)$$