

Deep inelastic scattering off quark-gluon plasma and its photon emissivity

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Thermal QCD vector spectral functions

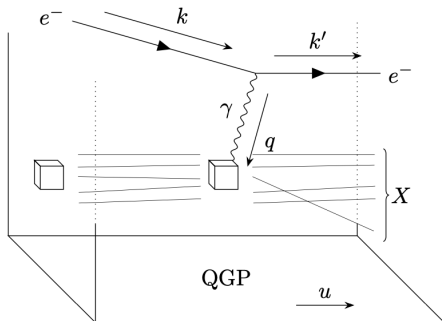
$$\rho^{\mu\nu}(q) = \int d^4x e^{iq \cdot x} \frac{1}{Z} \sum_n e^{-E_n/T} \langle n | [j^\mu(x), j^\nu(0)] | n \rangle$$

- ▶ Two kinematic variables: 'energy' $\omega \equiv q^0$ and virtuality q^2 .
- ▶ Vanishing virtuality: $\rho^{\mu\nu}$ determines the photon emissivity of the thermal medium.
- ▶ Timelike virtuality $q^2 > 0$: dilepton emission rate. [McLerran & Toimela 1985]
- ▶ Spacelike virtuality $q^2 < 0$: vector spectral functions measure the **ability of the medium to convert the energy stored in external electromagnetic fields into heat**:
 - ▶ Couple the plasma to a harmonic external vector potential $\mathbf{A}(t, \mathbf{x}) = \text{Re}(\mathbf{A}_q e^{i(\mathbf{q} \cdot \mathbf{x} - \omega t)})$, via Hamiltonian $\Delta H = -e \int d^3x \mathbf{j} \cdot \mathbf{A}$.
 - ▶ If energy of external electromagnetic fields: $E_{\text{e.m.}} = \frac{1}{2} \int d^3x (\mathbf{E}^2 + \mathbf{B}^2)$,

$$\mathbf{A}_q \perp \mathbf{q} : \quad \frac{-1}{E_{\text{e.m.}}} \frac{dE_{\text{e.m.}}}{dt} = \alpha \frac{2\pi\omega (\delta^{ij} - \hat{q}^i \hat{q}^j) \rho^{ij}(\omega, \mathbf{q})}{\omega^2 + \mathbf{q}^2},$$

$$\mathbf{A}_q \parallel \mathbf{q} : \quad \frac{-1}{E_{\text{e.m.}}} \frac{dE_{\text{e.m.}}}{dt} = \alpha \frac{4\pi \hat{q}^i \hat{q}^j \rho^{ij}(\omega, \mathbf{q})}{\omega}$$

Spacelike virtuality: possible source of external e.m. fields



Lepton scattering on quark-gluon plasma

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4 L^3}{8\pi^2 Q^4} \left(\frac{E'}{E}\right) \ell_{\mu\nu} W_{>}^{\mu\nu}(u, q),$$

$\ell_{\mu\nu}$ = leptonic tensor, $W_{>}^{\mu\nu}$ = 'hadronic' tensor.

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Structure functions of the medium

Hadronic tensor:

$$W_{>}^{\mu\nu}(u, q) = \frac{1}{4\pi Z} \sum_n e^{-\beta E_n} \int d^4x e^{iq \cdot x} \langle n | j^\mu(x) j^\nu(0) | n \rangle,$$

Kubo–Martin–Schwinger: relation to spectral function in the rest frame of the medium,

$$W_{>}^{\mu\nu}(u, q) = \frac{1}{4\pi(1 - e^{-\beta q^0})} \rho^{\mu\nu}(q^0, \mathbf{q}).$$

Tensor decomposition suitable for studying DIS limit: u = medium 4-velocity

$$\begin{aligned} W_{>}^{\mu\nu}(u, q) &= F_1(u \cdot q, Q^2) \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) \\ &+ \frac{T}{u \cdot q} F_2(u \cdot q, Q^2) \left(u^\mu - (u \cdot q) \frac{q^\mu}{q^2} \right) \left(u^\nu - (u \cdot q) \frac{q^\nu}{q^2} \right). \end{aligned}$$

F_1, F_2 = structure functions of the medium.

Deep-inelastic scattering limit

$$Q^2 \rightarrow \infty \text{ at fixed } x = \frac{Q^2}{2Tq^0}$$

There is a large cancellation in calculating Q^2 as the difference $\mathbf{q}^2 - (q^0)^2$, but Q^2 is still large compared to T^2 .

Partonic interpretation of the structure functions

Let $f_f(\xi) d\xi$ represent the number of partons of type f in the fluid cell carrying momentum $\xi T u$. This number is of order volume, L^3 .

DIS limit of the structure functions:

$$F_1(u \cdot q, Q^2) = \frac{1}{4L^3 T} \sum_f Q_f^2 f_f(x).$$

" $4 F_1 \cdot dx$ is the square-electric-charge weighted number of partons carrying a momentum xT times the fluid four-velocity u^μ per unit transverse area in a slab of fluid which in its rest frame has thickness $1/T$ in the longitudinal direction."

Moment sum rules

Using the twist expansion:

$$\int_0^\infty dx x^{n-1} [F_1(x, Q^2)]_{\text{leading-twist}} = \sum_{f,j} \frac{Q_f^2}{2} M_{fj}(Q, \tilde{\mu}) \langle O_{nj} \rangle, \quad n = 2, 4, \dots,$$

where

$$\langle O_{nj}^{\mu_1 \dots \mu_n} \rangle = T^n [u^{\mu_1} \dots u^{\mu_n} - \text{traces}] \langle O_{nj} \rangle.$$

and the $O_{nj}^{\mu_1 \dots \mu_n}$ are the usual twist-two operators of ordinary DIS (j runs over the quark flavours and the gluon operator).

NB. For n not too large, the right-hand side is computable on the lattice.

Probing the structure functions with Euclidean correlation functions

Focus here on the transverse channel:

$$H_E^T(\omega_n; Q^2) = \int d^4x e^{\sqrt{\omega_n^2 - Q^2} \hat{q} \cdot \mathbf{x} + i\omega_n x_0} \left(\frac{1}{2} (\delta_{ik} - \hat{q}_i \hat{q}_k) \langle j_i(x) j_k(0) \rangle \right).$$

Note the 'imaginary spatial momentum' required to reach the DIS regime.

Dispersive representation:

$$H_E^T(\omega_n; Q^2) - H_E^T(\omega_r; Q^2) = \int_0^\infty \frac{dx}{\pi} x \hat{\sigma}^T(x, Q^2) \frac{a_r^2 - a_n^2}{(1 + a_n^2 x^2)(1 + a_r^2 x^2)},$$

where $a_n = 2T\omega_n/Q^2$ are to be kept fixed and

$$\hat{\sigma}^T(x, Q^2) = -4\pi(1 - e^{-\beta q^0})F_1(q^0, Q^2).$$

Dispersion relation for Euclidean correlator at zero virtuality

- ▶ Let $\sigma(\omega) \equiv \rho_T(\omega, |\mathbf{q}| = \omega)$ be the relevant spectral function proportional to the photon emission rate;
- ▶ let $H_E(\omega_n) \equiv G_E(\omega_n, k = i\omega_n)$ the momentum-space Euclidean correlator with Matsubara frequency ω_n and **imaginary spatial momentum** $k = i\omega_n$;
- ▶ once-subtracted dispersion relation: ($\sigma(\omega) \sim \omega^{1/2}$ at weak coupling)

$$H_E(\omega_n) - H_E(\omega_r) = \int_0^\infty \frac{d\omega}{\pi} \omega \sigma(\omega) \left[\frac{1}{\omega^2 + \omega_n^2} - \frac{1}{\omega^2 + \omega_r^2} \right], \quad n, r \neq 0.$$

Photon emission rate per unit volume plasma:

$$\frac{d\Gamma_\gamma}{d\omega} = \frac{\alpha}{\pi} \frac{1}{e^{\beta\omega} - 1} \cdot (\omega \sigma(\omega)).$$

[HM, 1807.00781.]

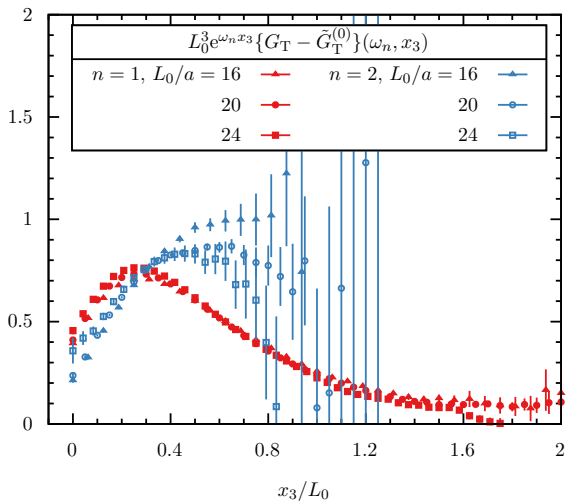
Computing $H_E(\omega_n)$ in lattice QCD

New since 1807.00781: Use the estimator

$$H_E(\omega_n) = \int_0^\beta dx_0 \int d^3x (e^{i\omega_n x_0} - e^{i\omega_n x_2}) e^{\omega_n x_3} \langle j_1(x) j_1(0) \rangle,$$

- ▶ the subtracted term ($e^{i\omega_n x_2}$) vanishes in the continuum theory, but on the lattice removes an ultraviolet divergence associated with the lack of Lorentz symmetry at finite lattice spacing.
- ▶ no cumbersome subtraction of vacuum correlators is needed!
- ▶ the above estimator of $H_E(\omega_n)$ automatically vanishes in the vacuum.

Integrand to obtain $H_E(\omega_n)$



Conclusion

- ▶ dispersion relations at fixed virtuality, rather than fixed spatial momentum, open up new perspectives on the thermal spectral functions
- ▶ certain moments of the spectrum of emitted photons can be computed in lattice QCD without solving an inverse problem
- ▶ in the same way, moments of the structure functions at fixed-virtuality can be computed in lattice QCD.

Illustration: non-interacting plasma of quarks

Explicit calculation [based on Laine 1310.0164]:

$$\lim_{Q^2 \rightarrow \infty} F_1(x, Q^2) = \frac{(\sum_f Q_f^2) N_c T^2}{4\pi^2} x \log(1 + e^{-x/2}).$$

Also: the Callan-Gross relation holds,

$$\lim_{Q^2 \rightarrow \infty} (F_2 - 2xF_1) = 0.$$

i.e. the scattering occurs on spin 1/2 constituents.

Integrand for three discretizations of the correlator

