NuclearScience Computing Center at CCNU

# Chiral properties of (2+1)-flavor QCD in strong magnetic fields at zero temperature

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bases on *Phys. Rev. D 104 (2021) 014505*, in collaboration with

Heng-Tong Ding, Sheng-Tai Li, Akio Tomiya, Yu Zhang

LATTICE 21

JULY 26-30 2021, ZOOM/GATHER@MIT

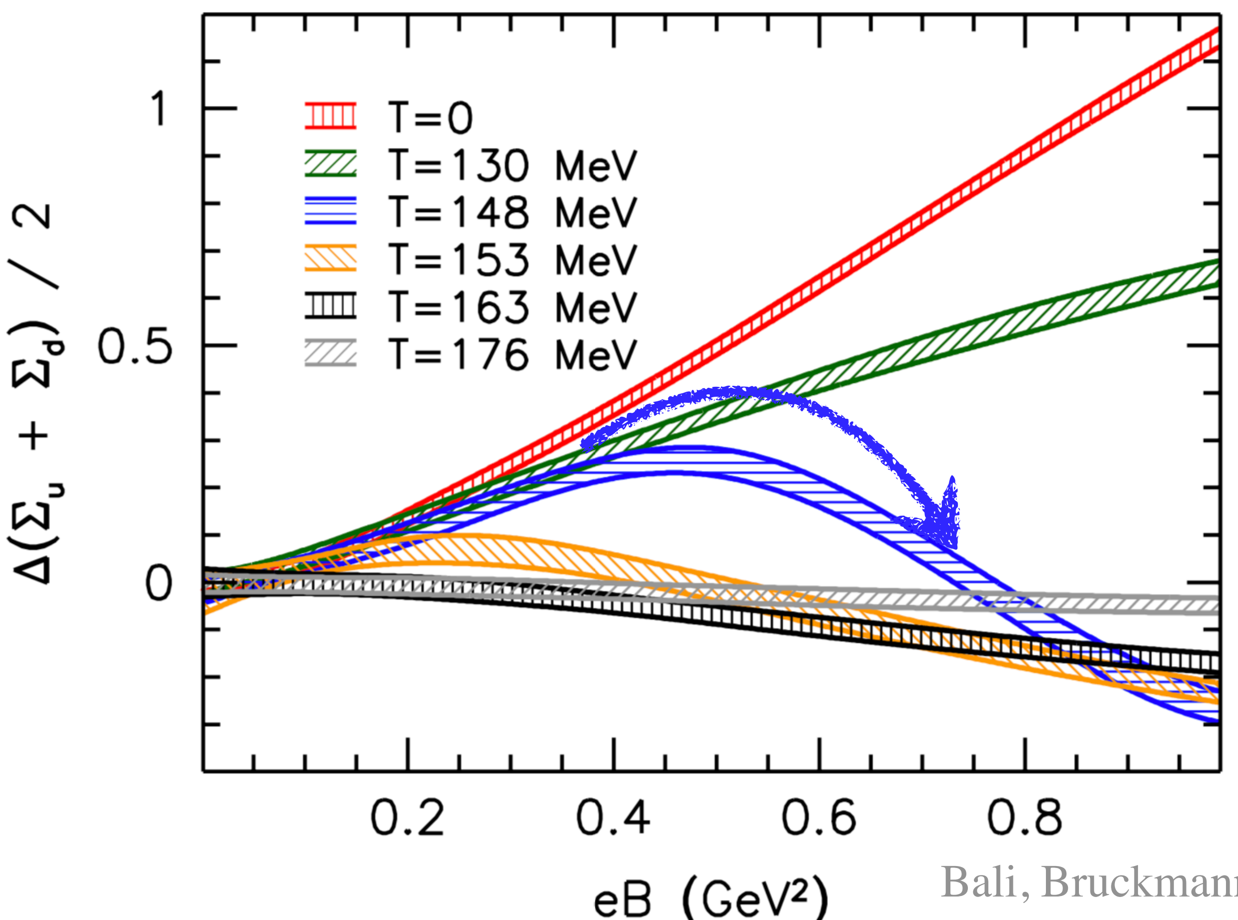
# Outline

- Motivation and Introduction
- Lattice Setup
- Results
- Summary

# Motivation

## (Inverse) Magnetic catalysis

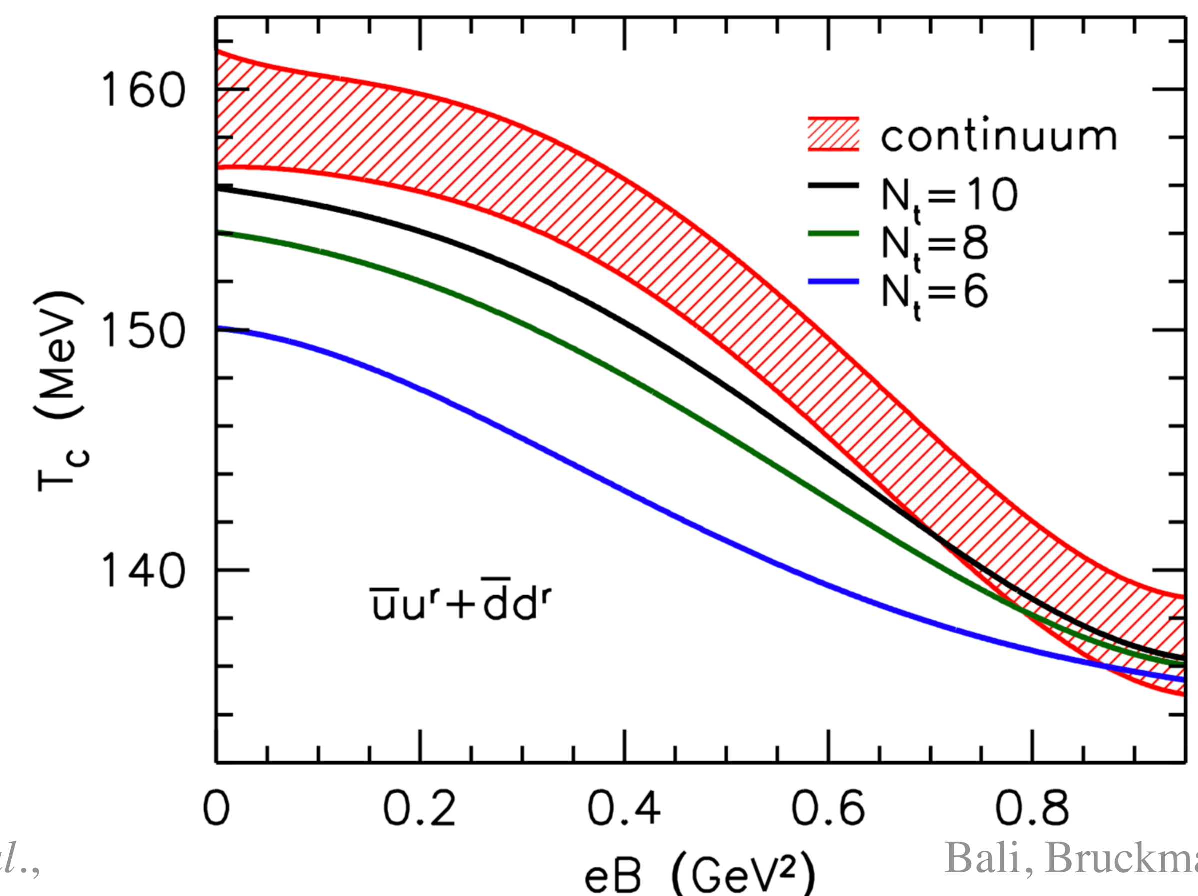
*Stout Staggered fermion (@physical quark mass)*




Bali, Bruckmann *et al.*,
Phys. Rev. D86 (2012) 071502

- T = 0: magnetic catalysis
- T ~ $T_{pc}$: inverse magnetic catalysis

## reduction of $T_{pc}$



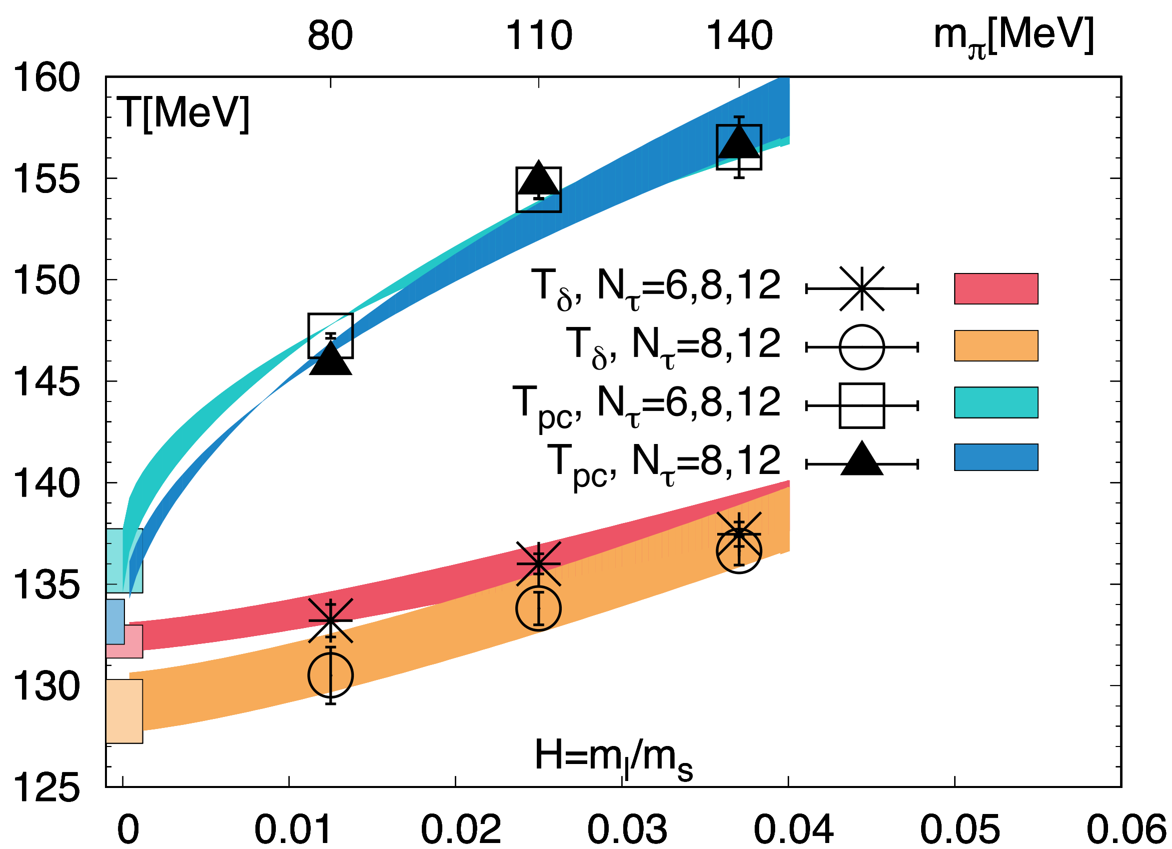

Bali, Bruckmann *et al.*,
JHEP 1202 (2012) 044

- critical temperature decreases as eB increases

☆The connection between (I)MC and the reduction of $T_{pc}$ is non-trivial.

# Motivation

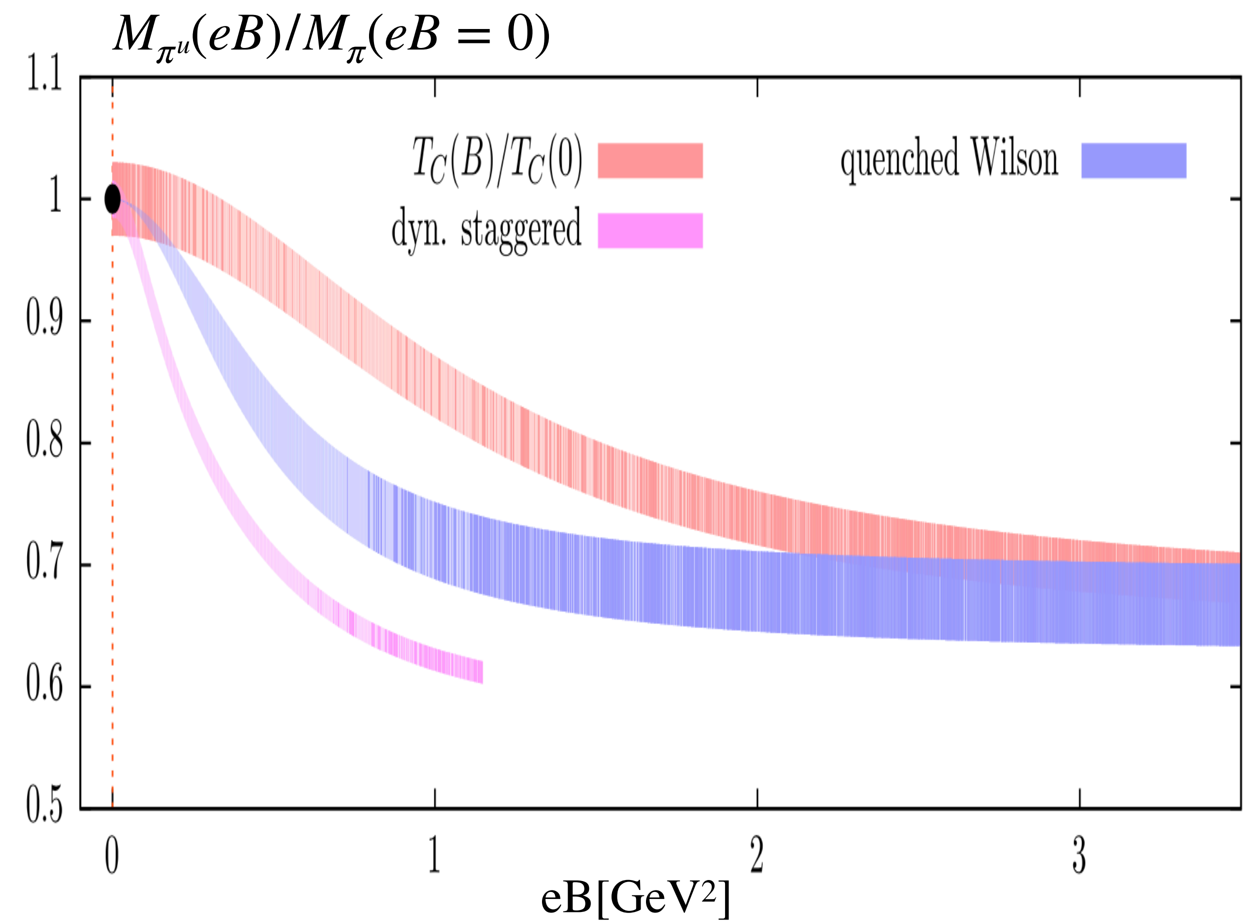
**eB = 0**




HTD, P. Hegde O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 062002

H.-T. Ding, Nuclear Physics A 1005 (2021):121940

**eB ≠ 0**




Bali *et al*., Phys. Rev. D 97, 034505

⭐Given that pion is still the lightest Goldstone boson, the mass reduction of pion explains the reduction of $T_{pc}$

**Is neutral pion still a Goldstone boson at eB≠0 ?**

# Introduction Gell-Mann-Oakes-Renner relation

- $(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2\, f_{\pi^0}^2\, M_{\pi^0}^2 (1 - \delta_{\pi^0})$

M. Gell-Mannet al, Phys. Rev. 175, 2195
Jamin et al, Phys. Lett. B 538, 71
Bordes et al, JHEP 05, 064
Bordes et al, JHEP 10, 102

- $(m_s + m_d)\left(\langle\bar{\psi}\psi\rangle_s + \langle\bar{\psi}\psi\rangle_d\right) = 2\, f_K^2\, M_K^2\, (1 - \delta_K)$

Gasser et al. Nucl. Phys. B 250, 465

**explicit symmetry breaking** **spontaneous symmetry breaking**

- GMOR relation has been confirmed on lattice in the vacuum without magnetic fields

Boucaud et al, Phys. Lett. B650, 304

- The GMOR relation for neutral pion valid in chiral limit in chiral perturbation theory in :
  - Low temperature with zero magnetic field — J. Gasser and H. Leutwyler , Phys. Lett. B184, 83
  - Weak magnetic field at zero temperature — I. A. Shushpanov and A. V. Smilga, Phys. Lett. B402, 351
  - Weak magnetic field at low temperature — N. O. Agasian and I. A. Shushpanov, JHEP 10, 006

⭐Is GMOR relation valid in the presence of strong magnetic field?

# Lattice Setup

Phys. Rev. D 104 (2021) 014505
H.-T. Ding, S.-T. Li, Akio Tomiya, X.-D.Wang, and Yu Zhang

- (2+1) flavor Dynamical HISQ fermion at T=0

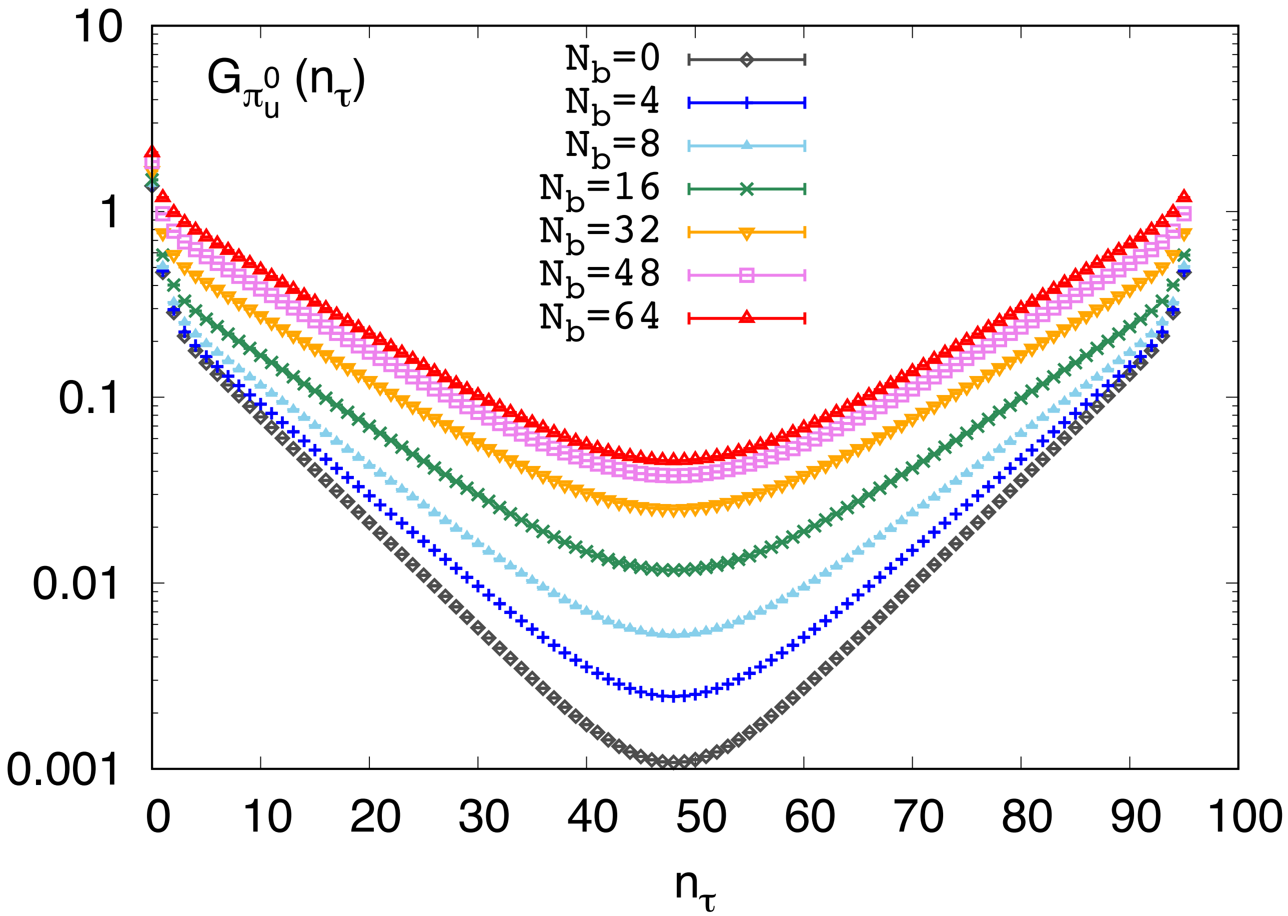
- Lattice size: $32^3 \times 96$, a = 0.117 fm

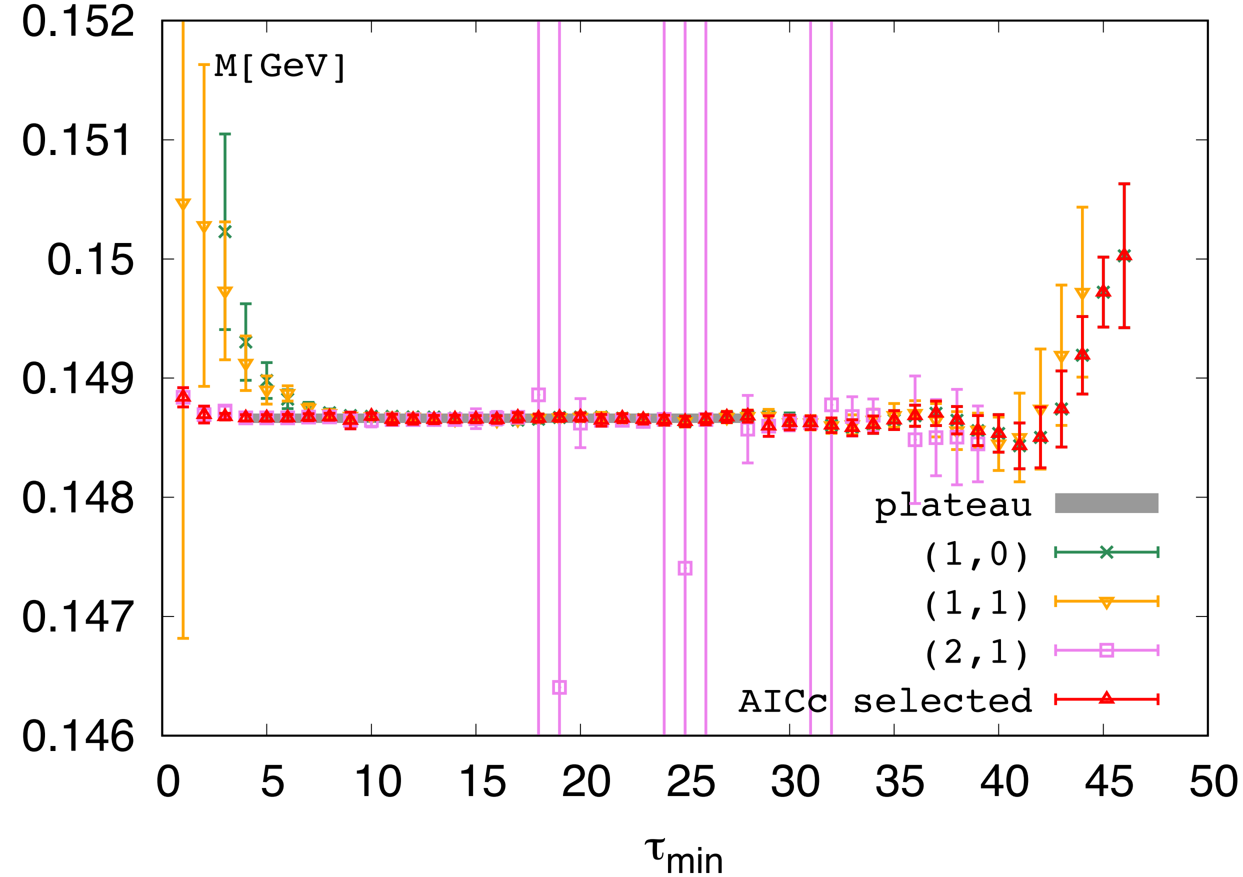
- Our simulation tuned to $M_\pi = 220$ MeV, while $f_\pi = 96.93(2)$ MeV, $f_K = 112.50(2)$ MeV, $f_K \,/\, f_\pi = 1.1606(3)$

Flag's review 2019: $f_\pi$ = 92.1(6) MeV, $f_K$ = 110.1(5) MeV, $f_K \,/\, f_\pi$ = 1.1917(37)

- Magnetic field was set along z direction and quantized as $eB = \dfrac{6\pi N_b}{N_x N_y} a^{-2}$
  - $N_b = 0, 1, 2, 3, 4, 6, 8, 10, 12, 16, 20, 24, 32, 40, 48, 64$
  - $0 < |eB| \lesssim 3.35 \text{ GeV}^2$ ( $\sim 70\, M_\pi^2$)

# Correlators and Meson mass



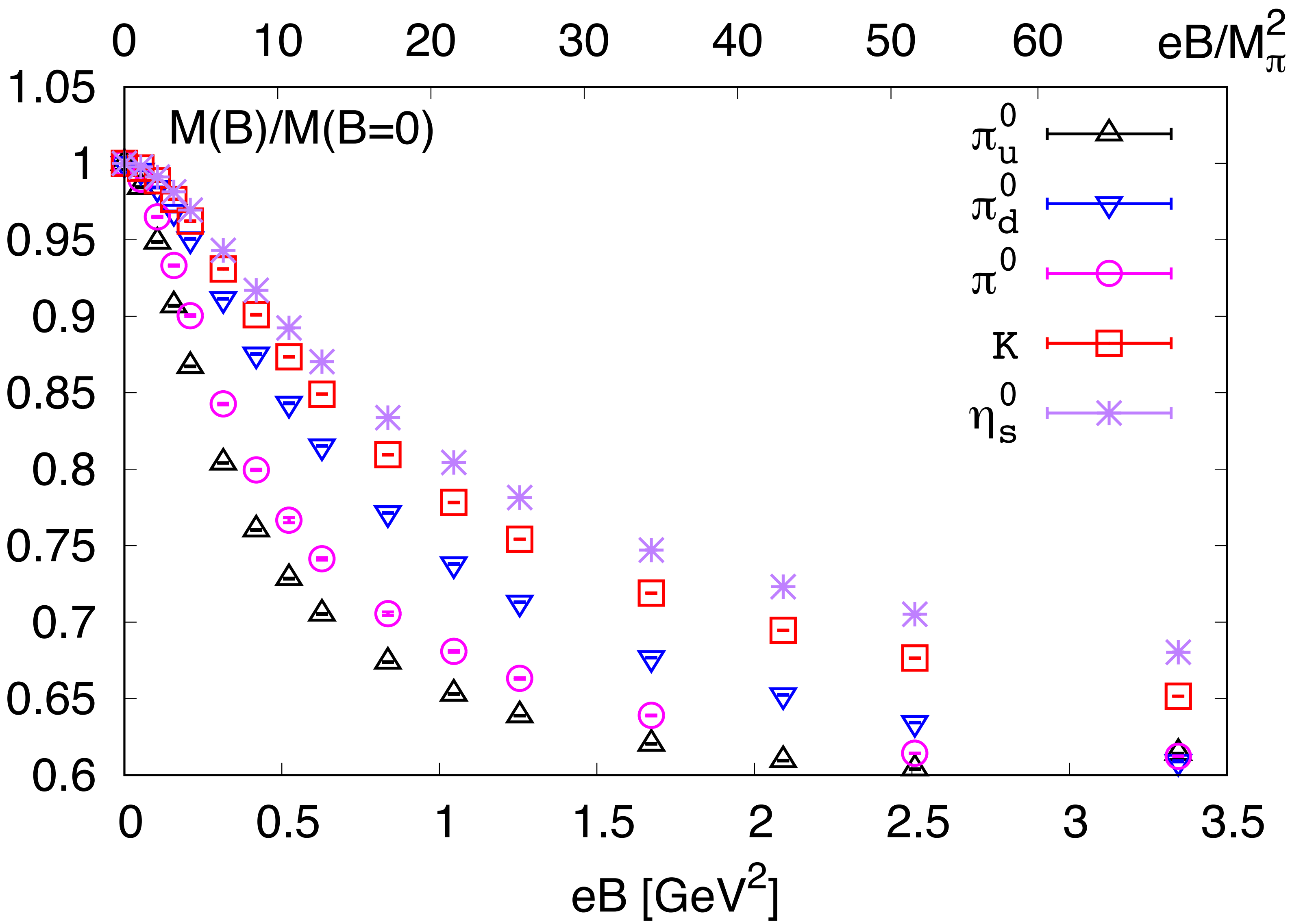

- Wall sources have been used to improve the signal

  Reduction of $\delta G/G$ : single point source $\xrightarrow{6}$ single wall source $\xrightarrow{\sqrt{\#\ of\ sources}}$ multiple wall sources

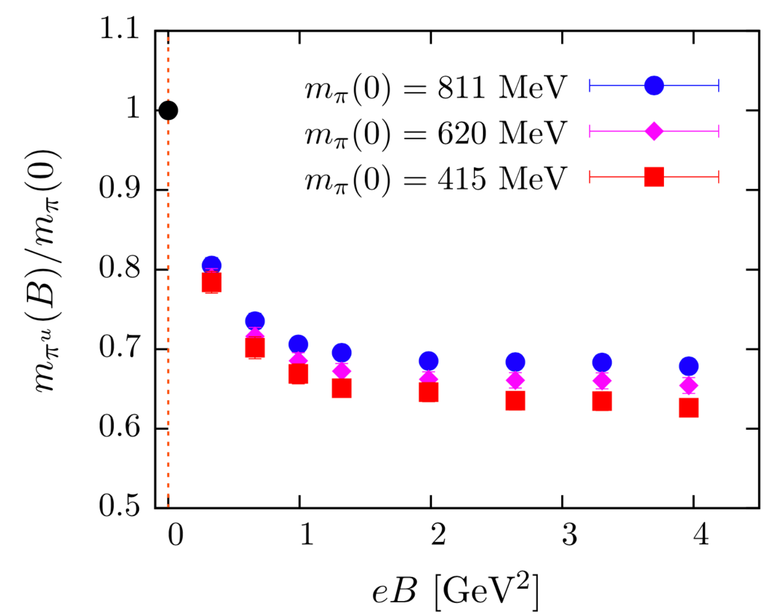
- $$G\left(n_\tau\right) = \sum_{i=1}^{N_{nosc}} A_{nosc,i} \exp\left(-M_{nosc,i}\, n_\tau\right) - (-1)^{n_\tau} \sum_{i=0}^{N_{osc}} A_{osc,i} \exp\left(-M_{osc,i}\, n_\tau\right)$$

- $$\mathrm{AICc} = 2k - \ln(\hat{L}) + \frac{2k^2 + 2k}{n - k - 1}$$ H. Akaike 1997, J. E. Cavanaugh 1997

# Mass of Neutral Pseudo-scalar Meson



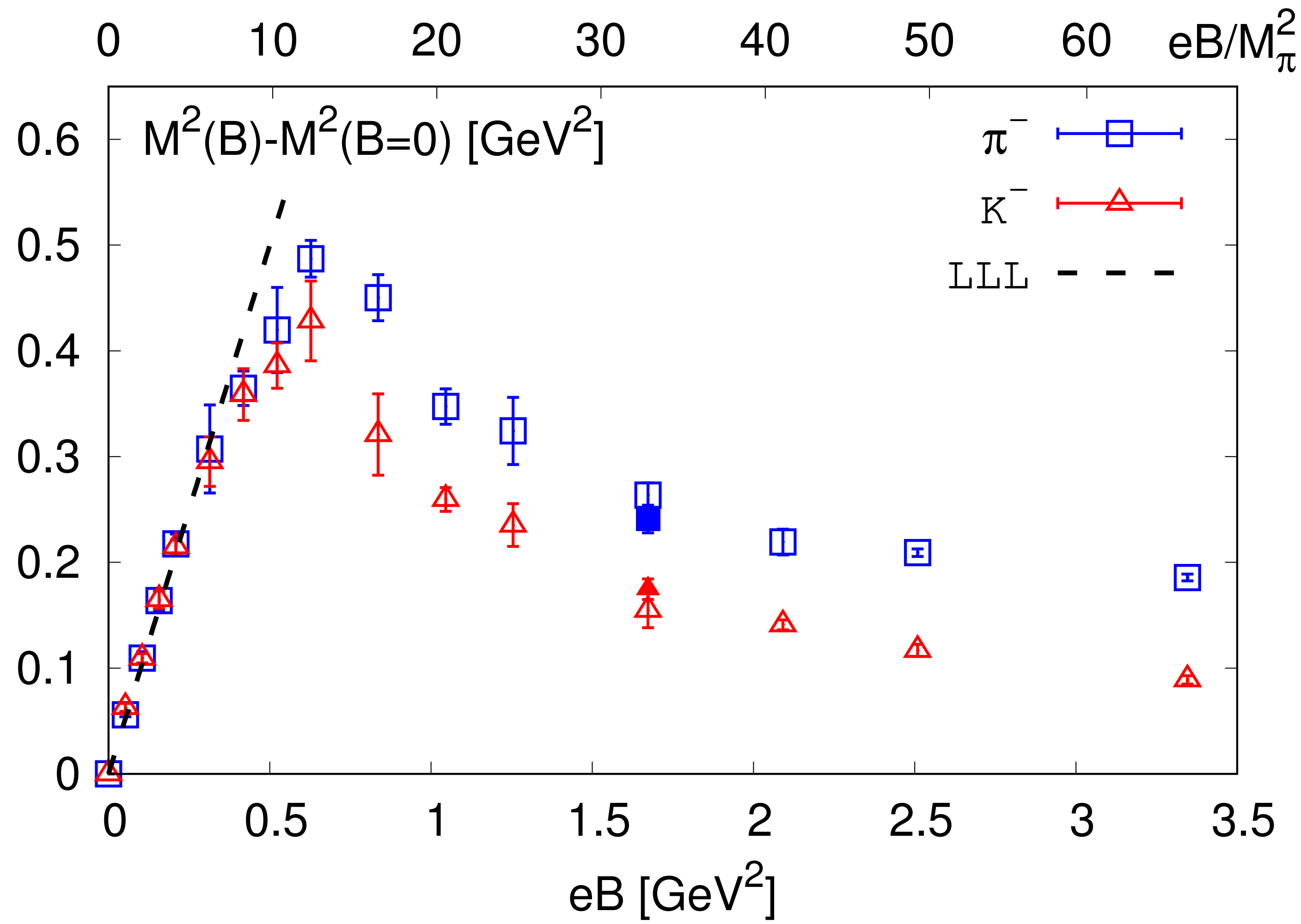

**Quenched Wilson Fermion**



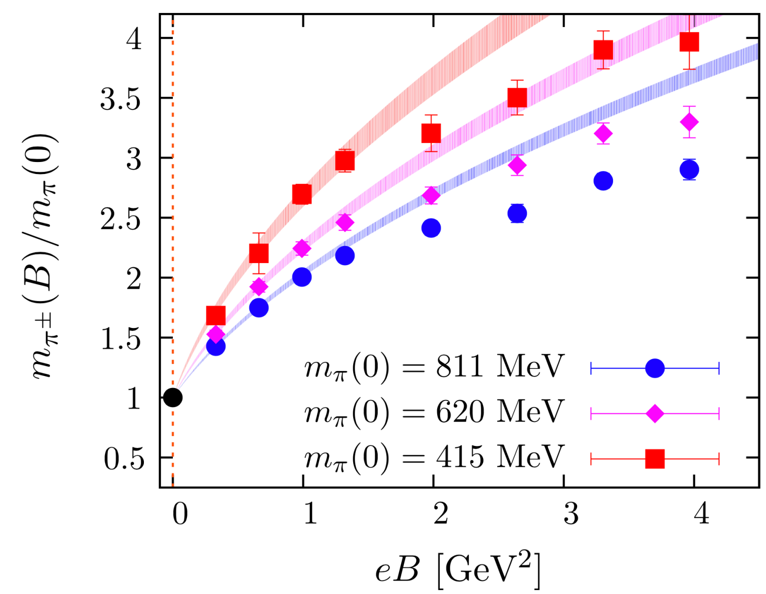

Bali, Brandt *et al.*,
Phys. Rev. D 97, 034505 (2018)

- Neutral PS mesons' masses decrease as eB grows and saturate at large eB
- Lighter mesons are more affected by magnetic field
- Neutral PS mesons have quite large (30~40%) mass reduction, CANNOT be considered as point particles anymore

# Mass of Charged Pseudo-scalar Meson




Quenched Wilson Fermion




Bali, Brandt *et al.*,
Phys. Rev. D 97, 034505 (2018)

- In general, charged PS particles' masses have non-monotonous behavior, which is different from quenched lattice result.
- At $0 < eB \lesssim 0.3$ GeV$^2$,PS particles' masses can be well described by Lowest Landau level approximation.

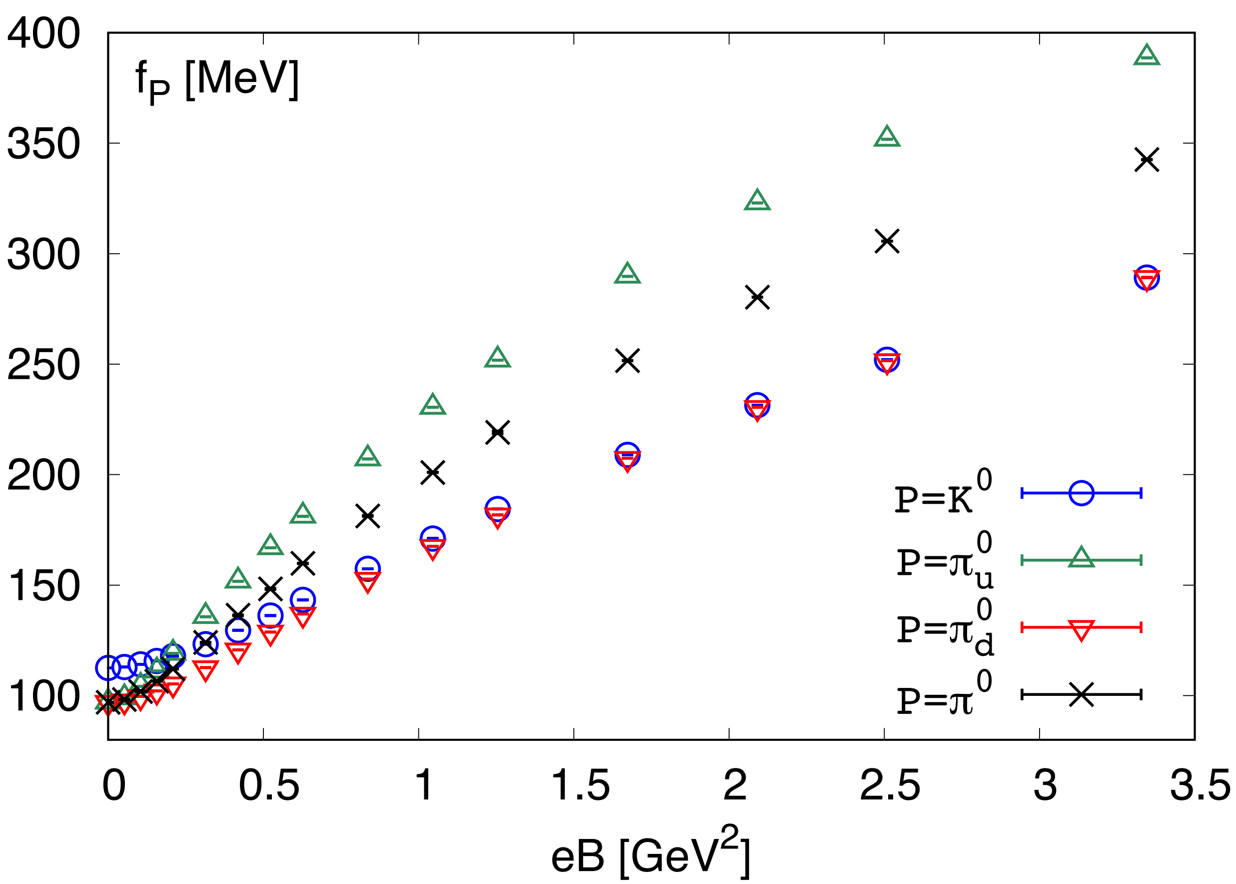
$$LLL\ approx: E^2 = p_z^2 + (2n+1)|qB| - gs_z qB + m^2 \longrightarrow \quad m_{\pi^\pm}^2(B) = m_{\pi^\pm}^2(B=0) + eB$$

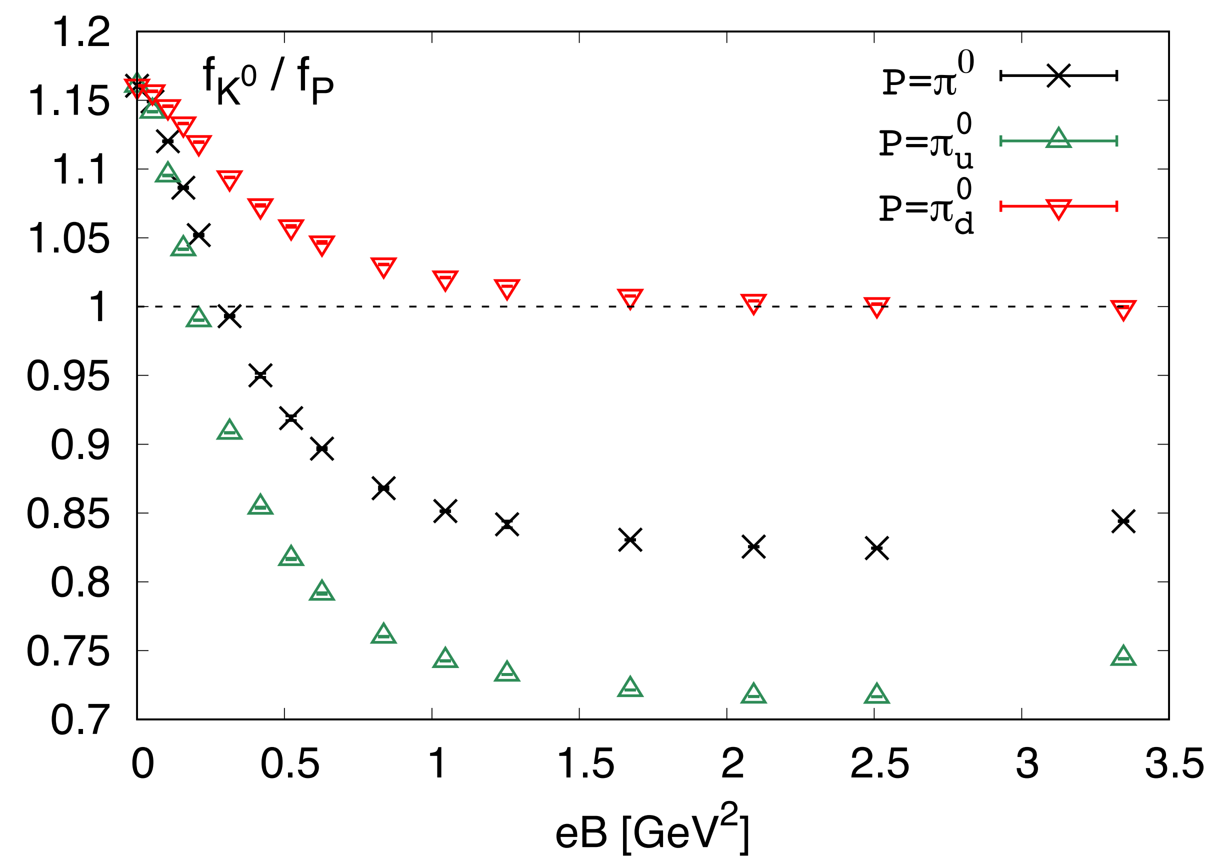
# Decay Constants

$$\sqrt{2} f_{\pi_u^0} M_{\pi_u^0}^2 = 2 m_u \left\langle 0 \left| \bar{u}\gamma_5 u \right| \pi_u^0(\boldsymbol{p}=0) \right\rangle$$

$$C_{O_s \mathrm{P}_W} = \frac{\left\langle 0 \left| O_s \right| \mathrm{P}(\vec{p}=0) \right\rangle \left\langle \mathrm{P}(\vec{p}=0) \left| \mathrm{P}_W \right| 0 \right\rangle}{2 M_\mathrm{P} V_s}$$

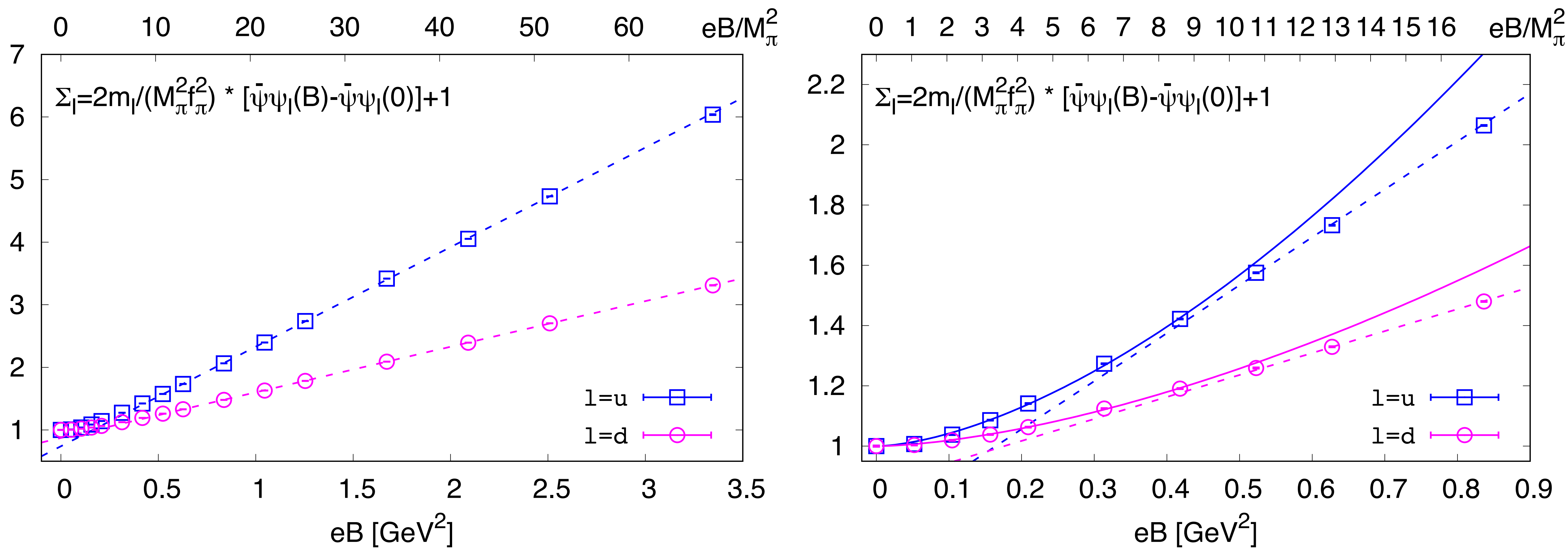
$$f_{\mathrm{P}^\pi} = 2 m_l \sqrt{\frac{V_s}{4}} \sqrt{\frac{C_{\mathrm{P}_W^\pi \mathrm{P}_W^\pi}}{M_{\mathrm{P}^\pi}^3}}$$




- Neutral pion and kaon decay constants increase as eB grows
- $f_{K^0} / f_P$ decreases as eB increases in eB ∈ [0,1.5] $\mathrm{GeV}^2$
- $f_{K^0} / f_P$ saturates in [1.5, 2.5] $\mathrm{GeV}^2$

# Chiral Condensates



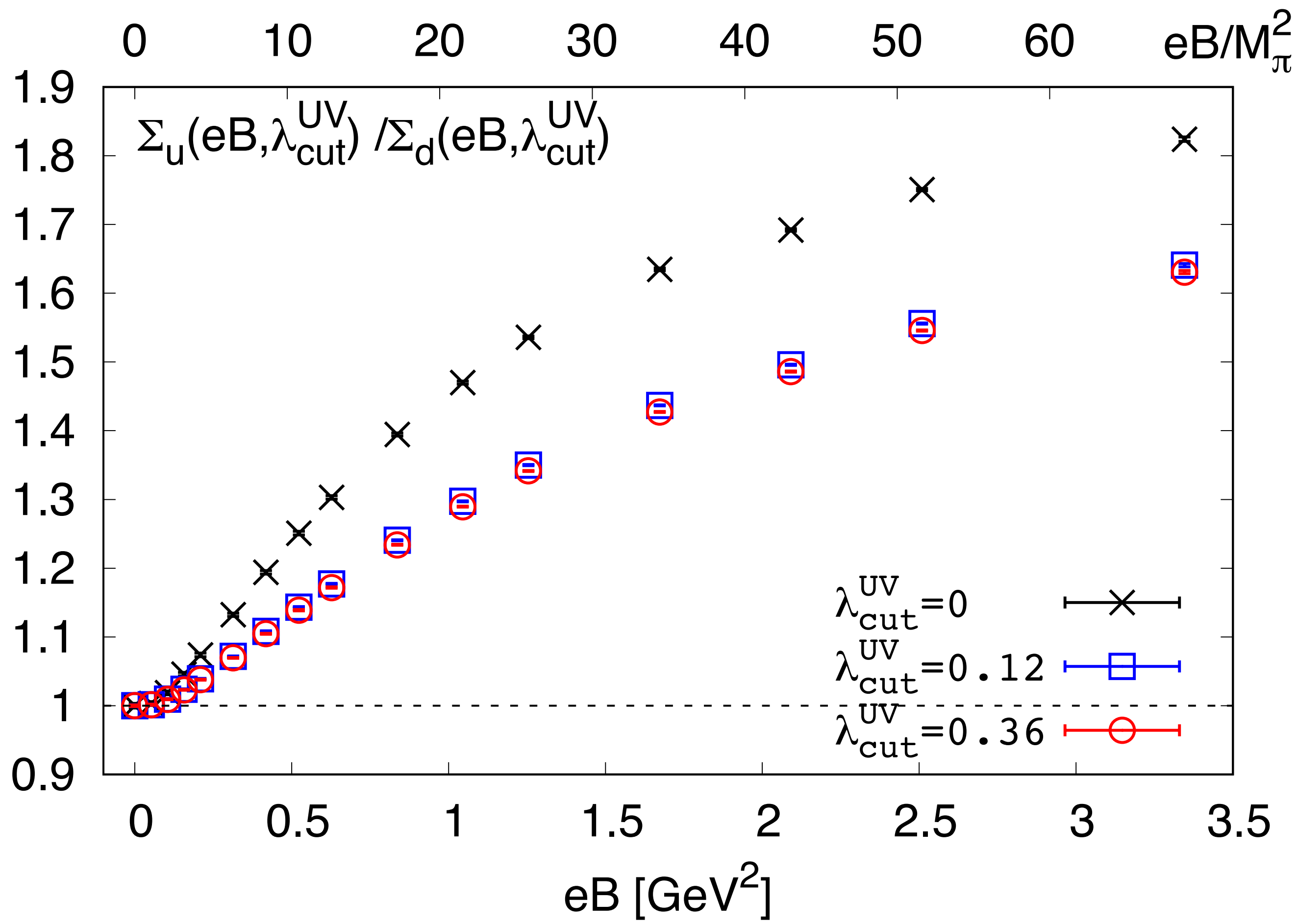

- Chiral condensates increase as eB grows
- **Two-parameter fits:**
  - In large eB $\in$ [0.5, 3.5] GeV$^2$, $\Sigma_l$ is almost linear in eB (dashed line)
  - In small eB $\in$ [0, 0.5] GeV$^2$ , $\Sigma_l$ can be well described by $h(eB)^\gamma + 1$ (solid line)

# Chiral Condensates

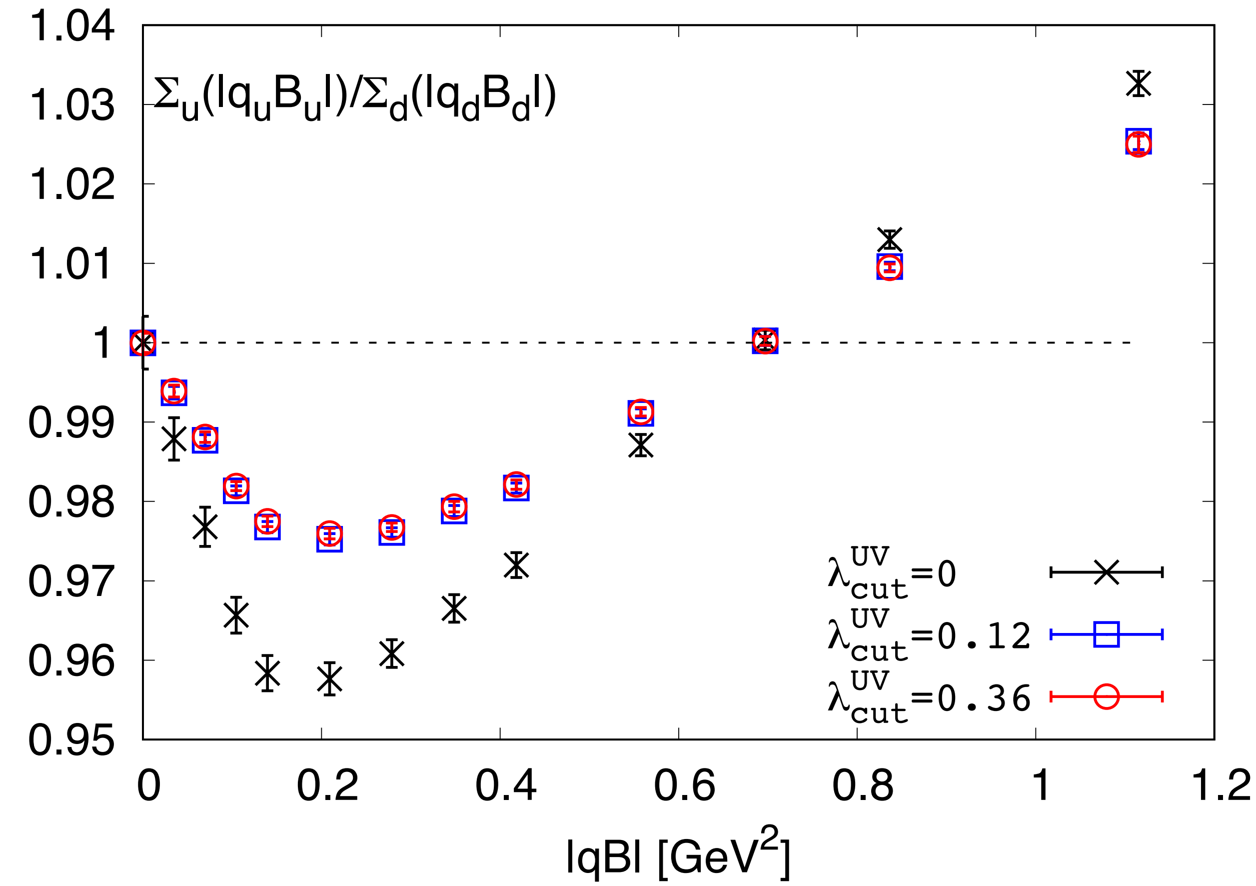
complete Dirac eigenvalue spectrum

Phys. Rev. Lett. 126, 082001(2021)

$$\Sigma_l(B, \lambda_{cut}^{\rm UV}) = \frac{2m_l}{M_\pi^2 f_\pi^2}\left(\langle\bar{\psi}\psi\rangle_l(B) - \langle\bar{\psi}\psi\rangle_l^{\rm UV}(B=0,\, \lambda_{cut}^{\rm UV})\right) + 1$$



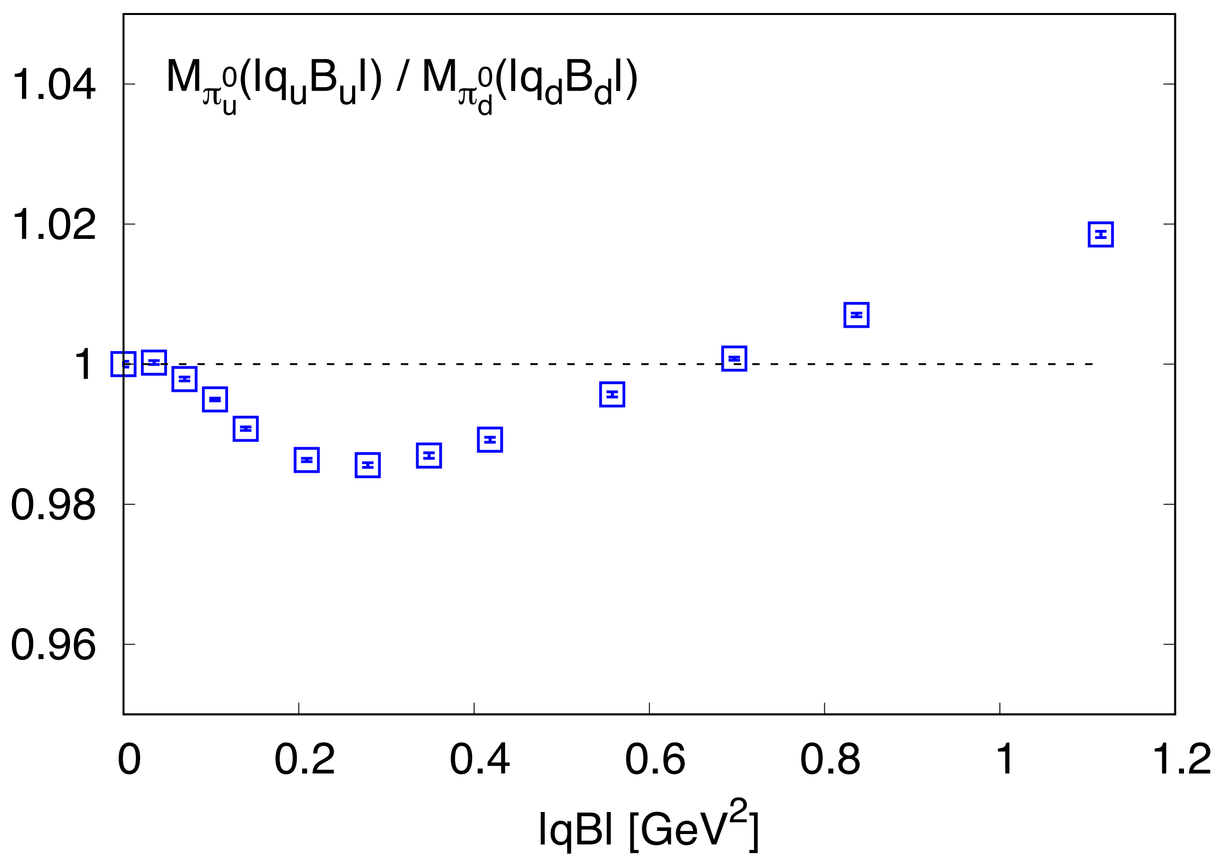

$|qB| = |q_u B_u| = |q_d B_d|$

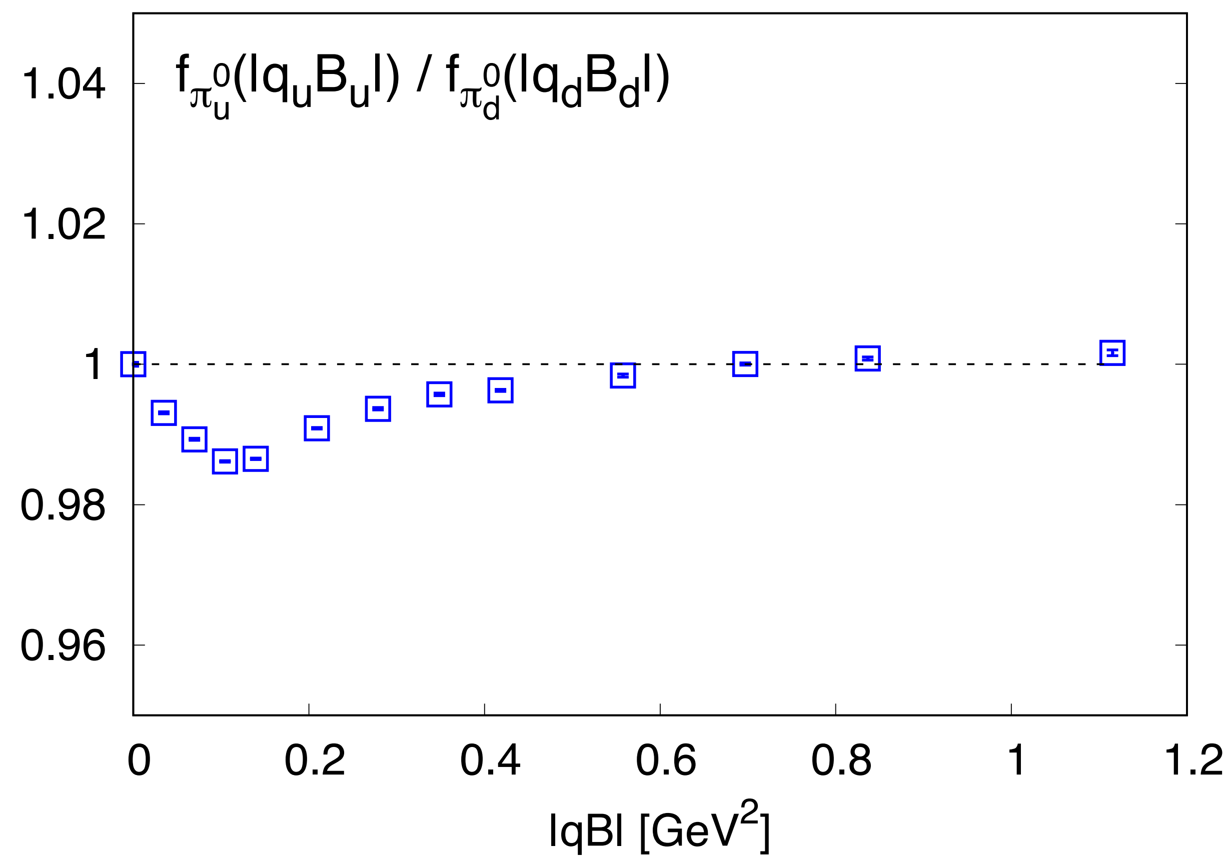



**qB scaling:**

It is the electric charge of quark multiplied by B that affects the behavior of quantities

# qB scaling



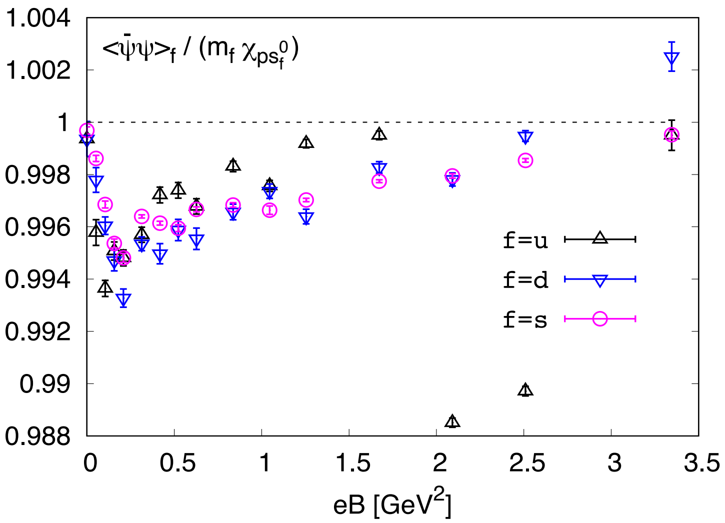




- qB scaling holds for $M_{\pi_u^0}(M_{\pi_d^0})$ , $\Sigma_u(\Sigma_d)$ , $f_{\pi_u^0}(f_{\pi_d^0})$
- qB scaling should be exact in quenched approximation

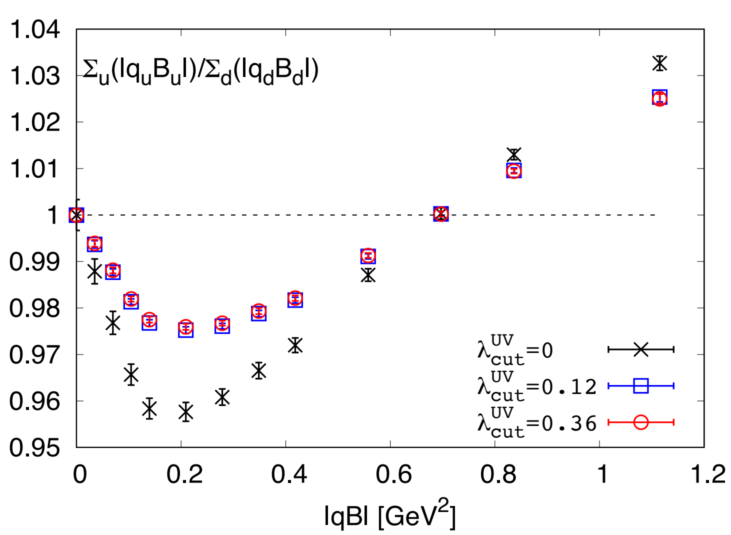
  ➡ contribution from dynamic quark is small

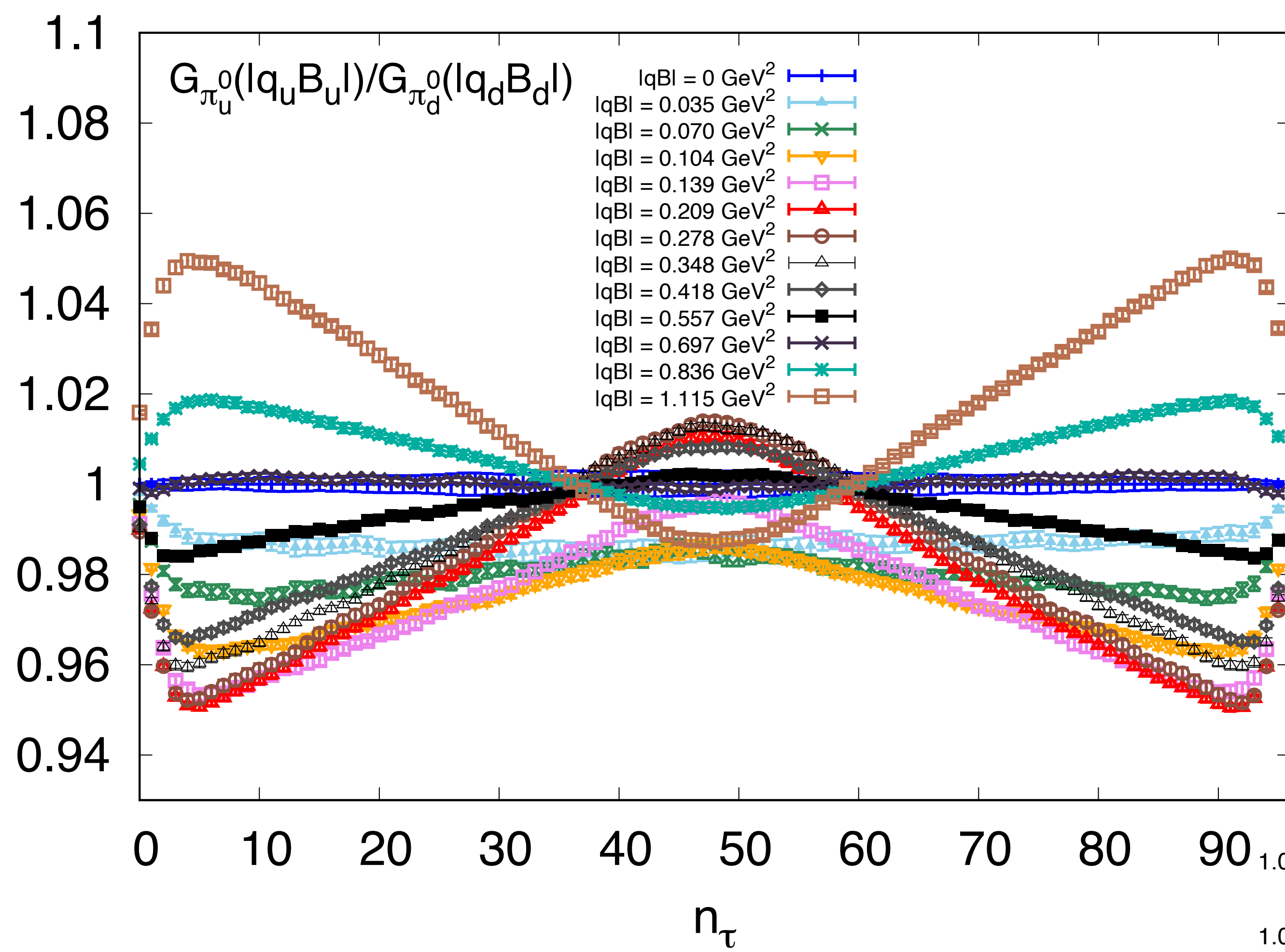
# qB scaling

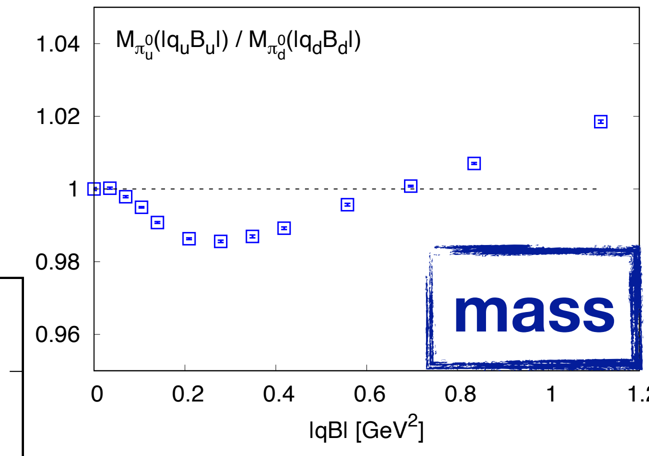
Ward Identity : $\langle \bar{\psi}\psi \rangle = m\chi$




**chiral condensate**

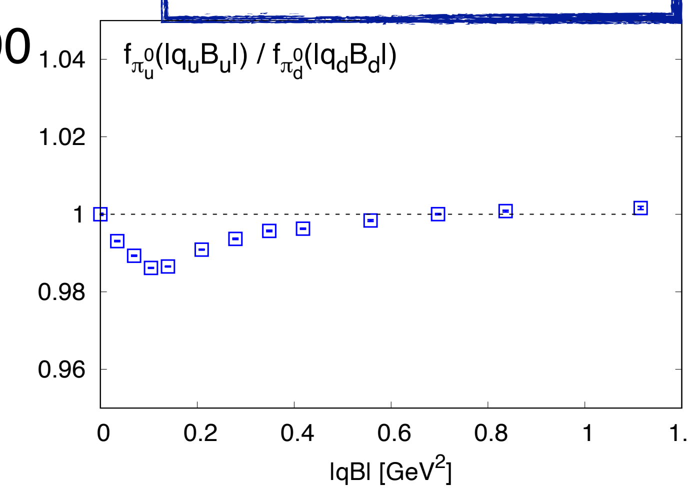



**mass**

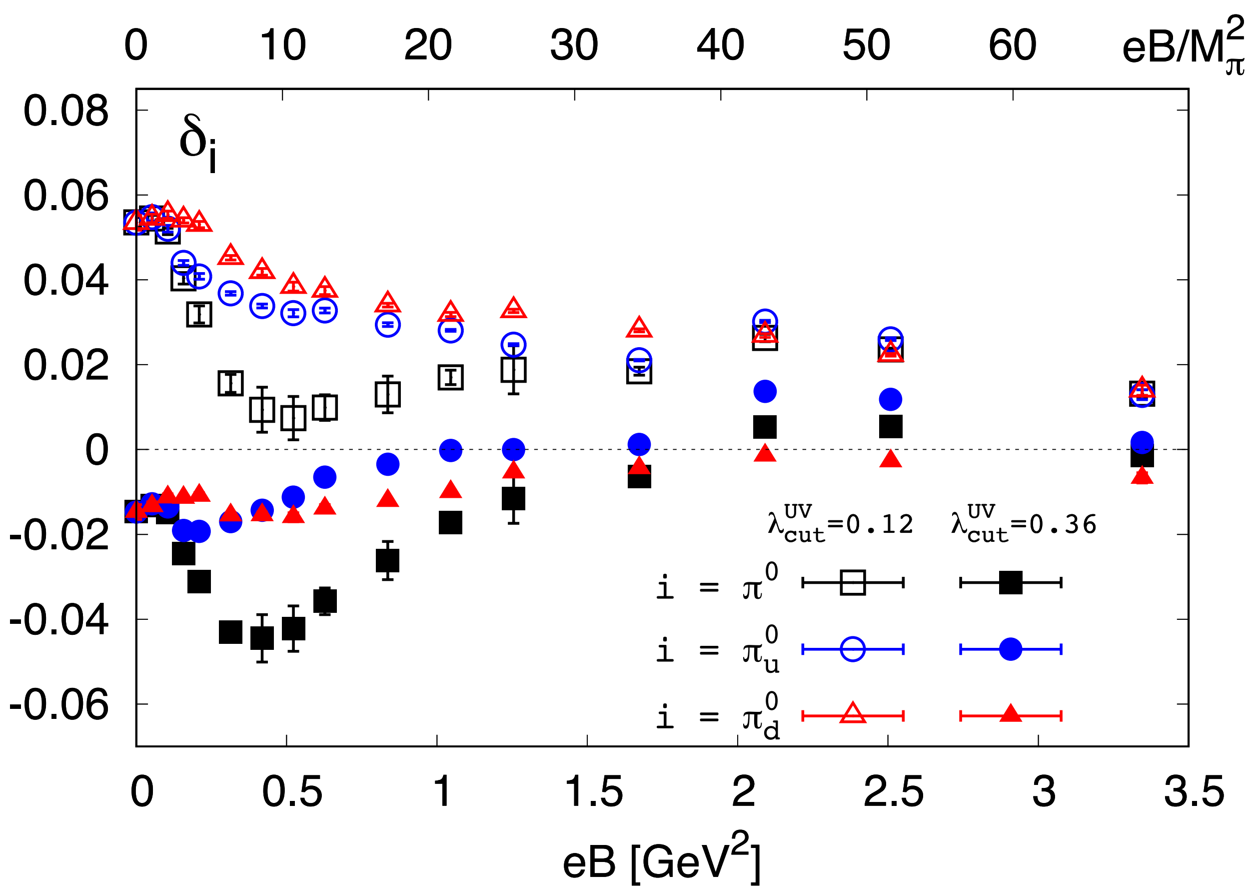
**decay constant**

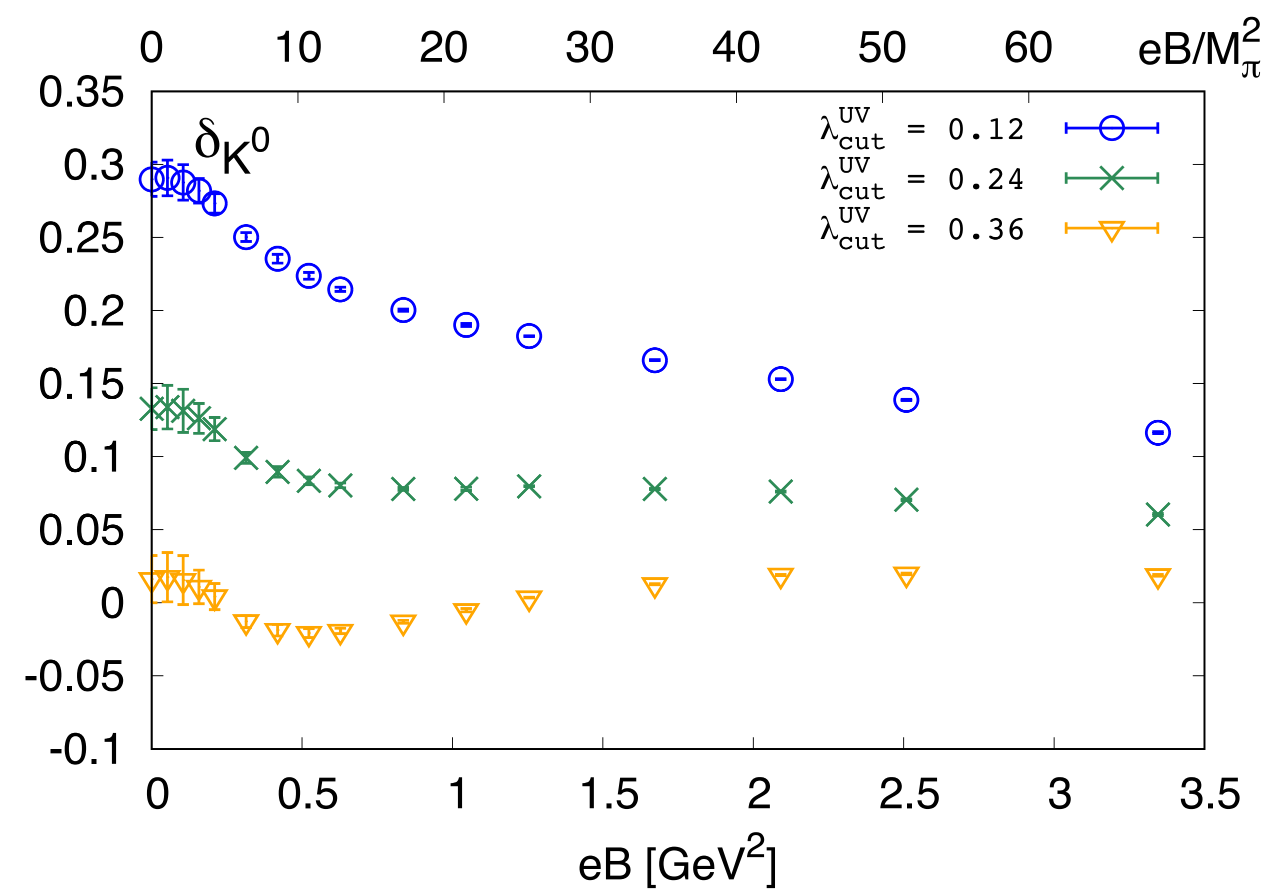



- The origin of all is the correlator, $G_{\pi_u^0}\left(\tau, q_u B_u\right)/G_{\pi_d^0}\left(\tau, q_d B_d\right)$ itself holds for qB scaling.

# GMOR relation

- $4m_u \langle\bar{\psi}\psi\rangle_u = 2f^2_{\pi^0_u} M^2_{\pi^0_u}(1-\delta_{\pi^0_u})$
- $4m_d \langle\bar{\psi}\psi\rangle_d = 2f^2_{\pi^0_d} M^2_{\pi^0_d}(1-\delta_{\pi^0_d})$
- $(m_u+m_d)\ \big(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\big) = 2\,f^2_{\pi^0}\ M^2_{\pi^0}(1-\delta_{\pi^0})$
- $(m_s+m_d)\ \big(\langle\bar{\psi}\psi\rangle_s + \langle\bar{\psi}\psi\rangle_d\big) = 2\,f^2_K\ M^2_K\ (1-\delta_K)$




$\chi_{PT}$ : $\delta_\pi = 6.2\ \pm\ 1.6\,\%$

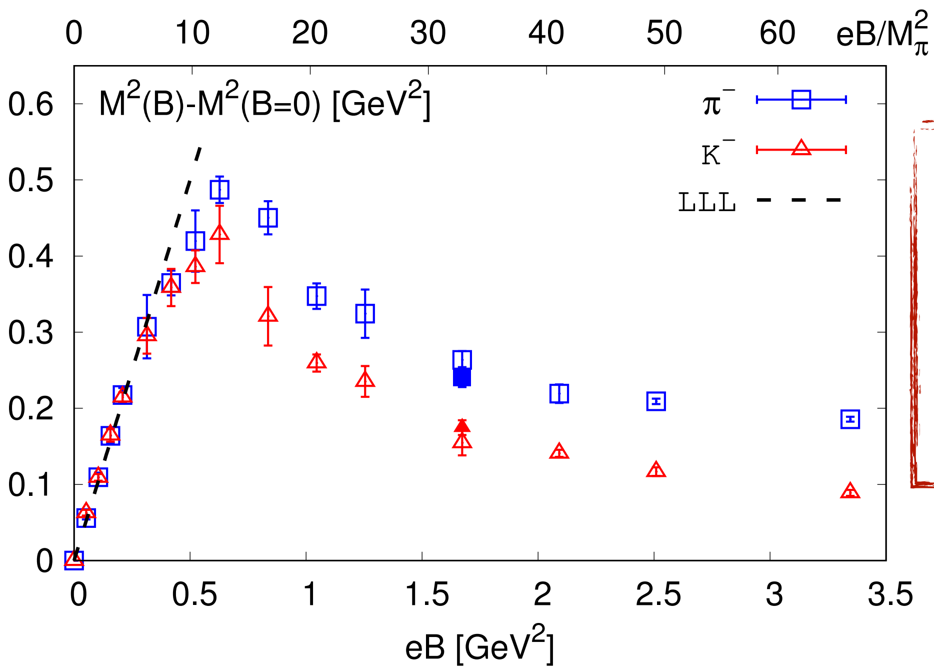
M. Jamin. Phys. Lett. B 538, 71

J. Bordes et al. JHEP 05, 064

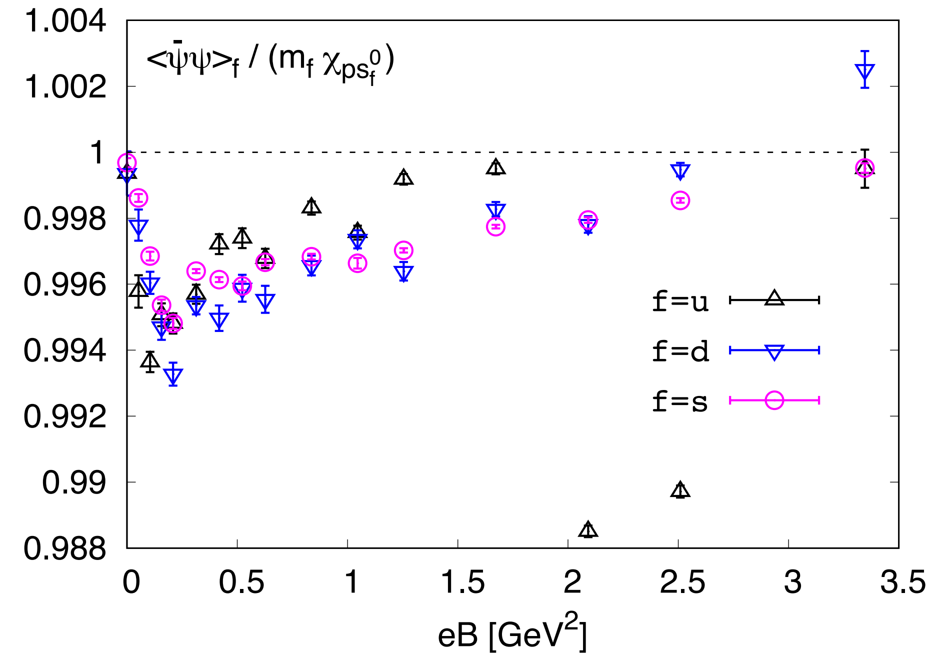
$\chi_{PT}$ : $\delta_K = 55\ \pm\ 5\,\%$

J. Bordes et al. JHEP 10, 102

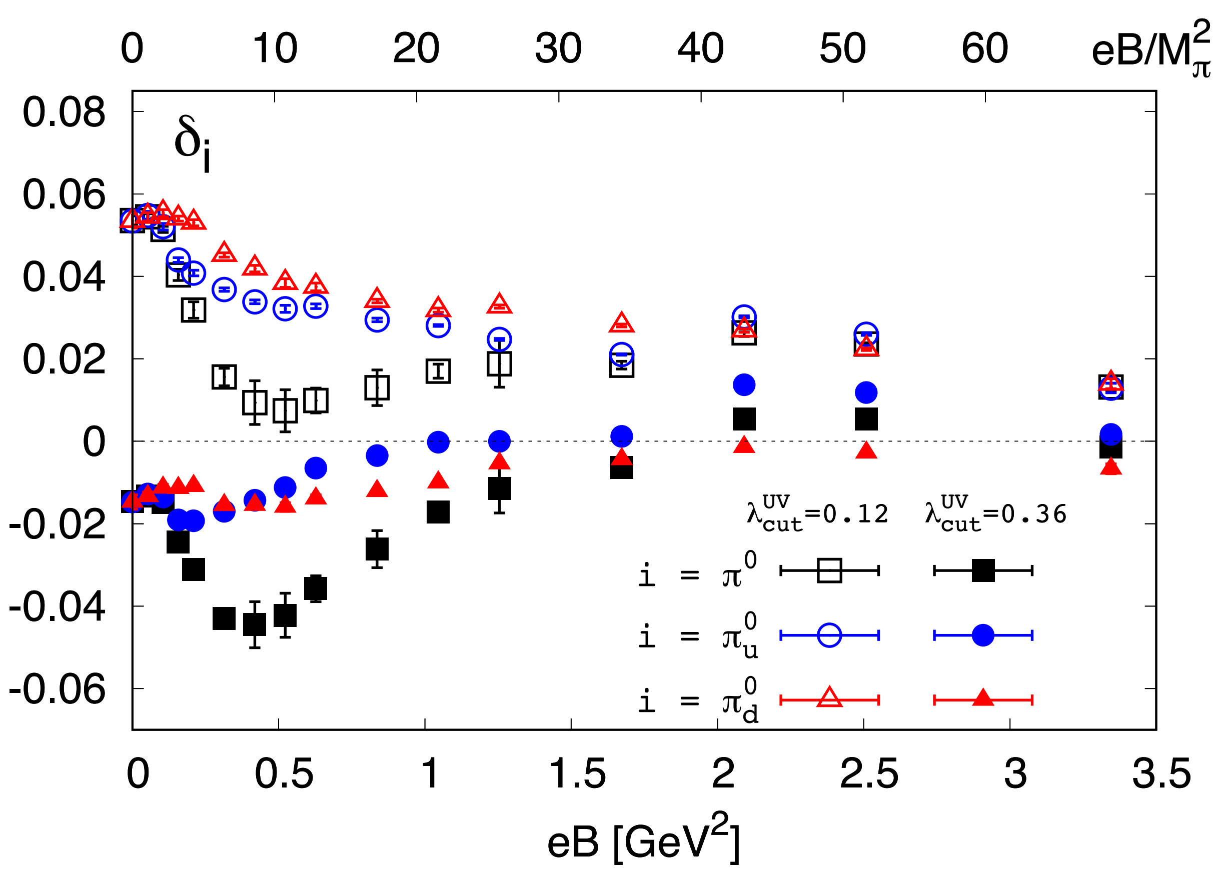
# Summary




novel non-monotonous behavior of charged Pseudo Scalar mesons




Ward Identity valid

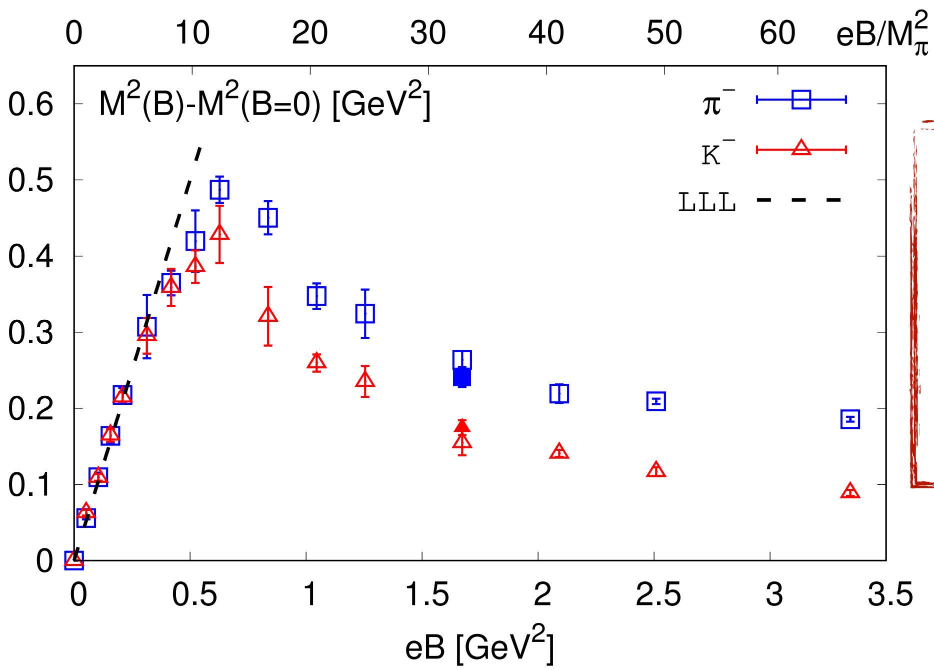



$$(m_u + m_d)\ \left(\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d\right) = 2\, f_{\pi^0}^2\ M_{\pi^0}^2 (1 - \delta_{\pi^0})$$

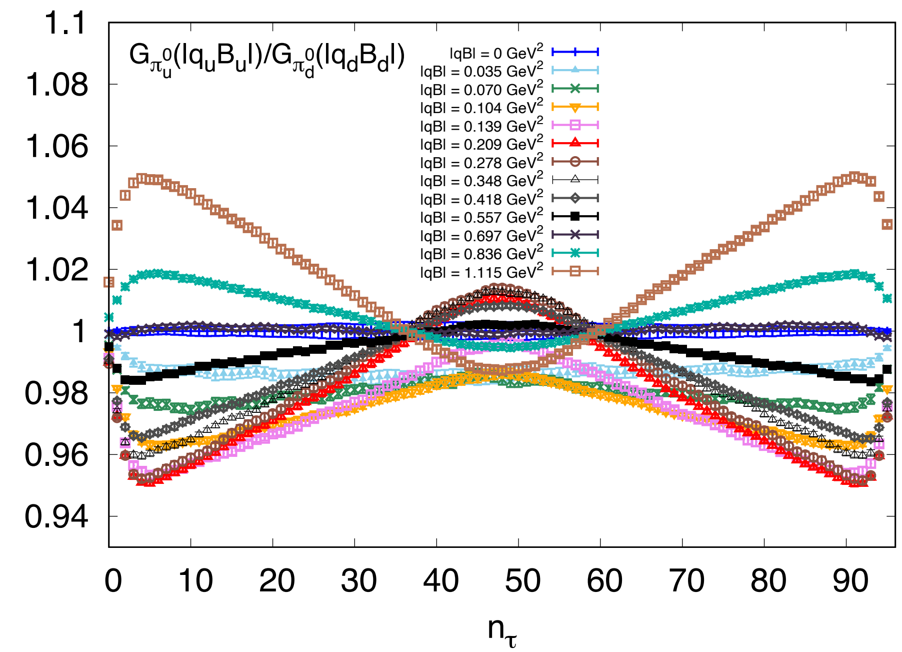
GMOR relations valid with small corrections

- ➡ $m_l$ explicitly breaks chiral symmetry
- ➡ reduction of $T_{pc}$ and neutral pion mass
- ➡ ~$T_{pc}$ , non-monotonous behavior of pion screening mass may be expected

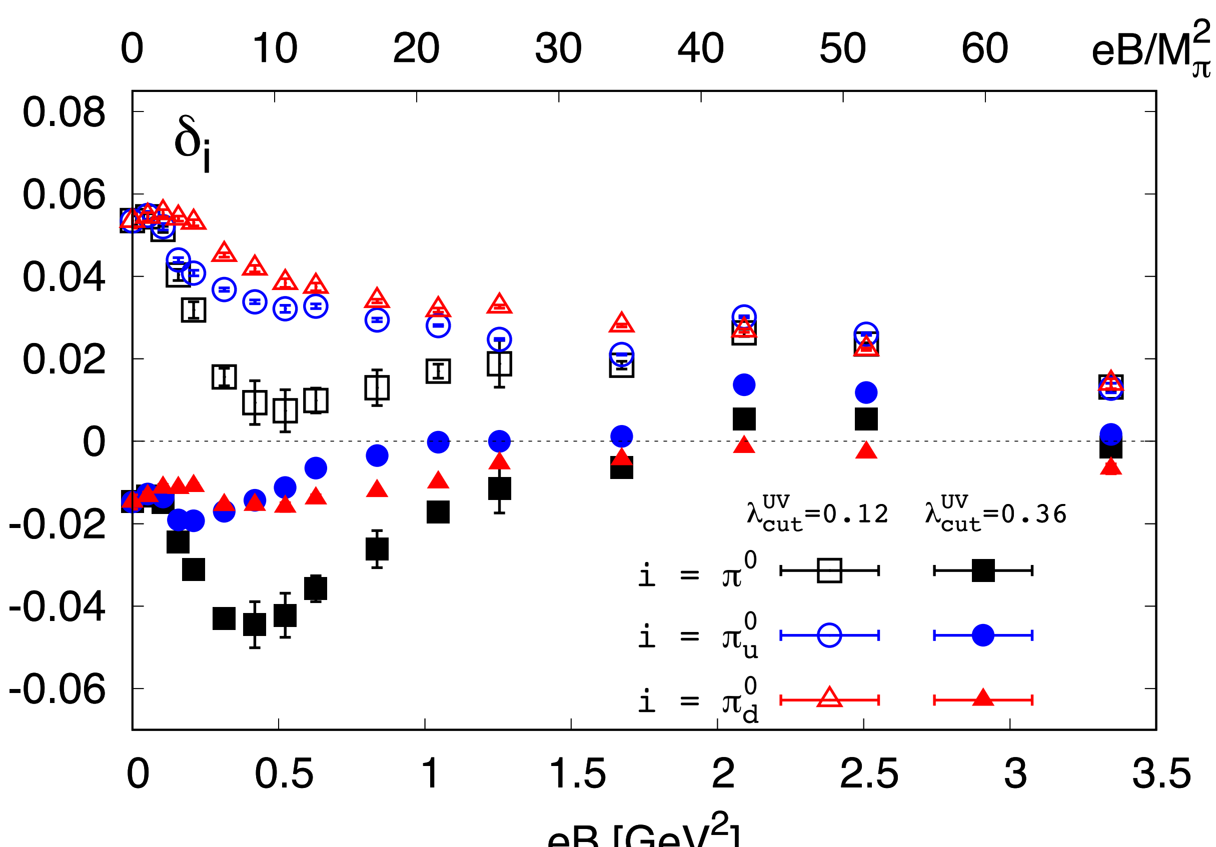
# Summary




novel non-monotonous behavior of charged Pseudo Scalar mesons




qB scaling

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- ➡ $m_l$ explicitly breaks chiral symmetry
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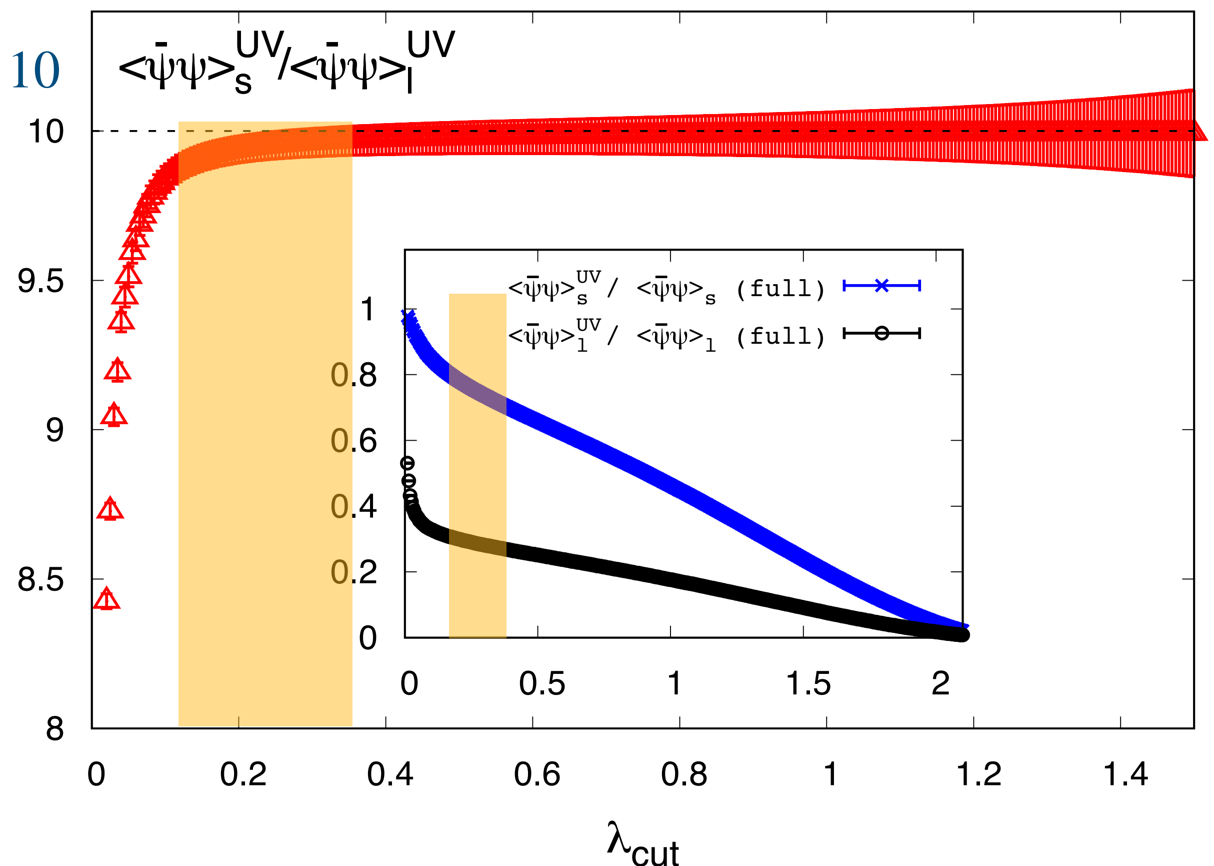
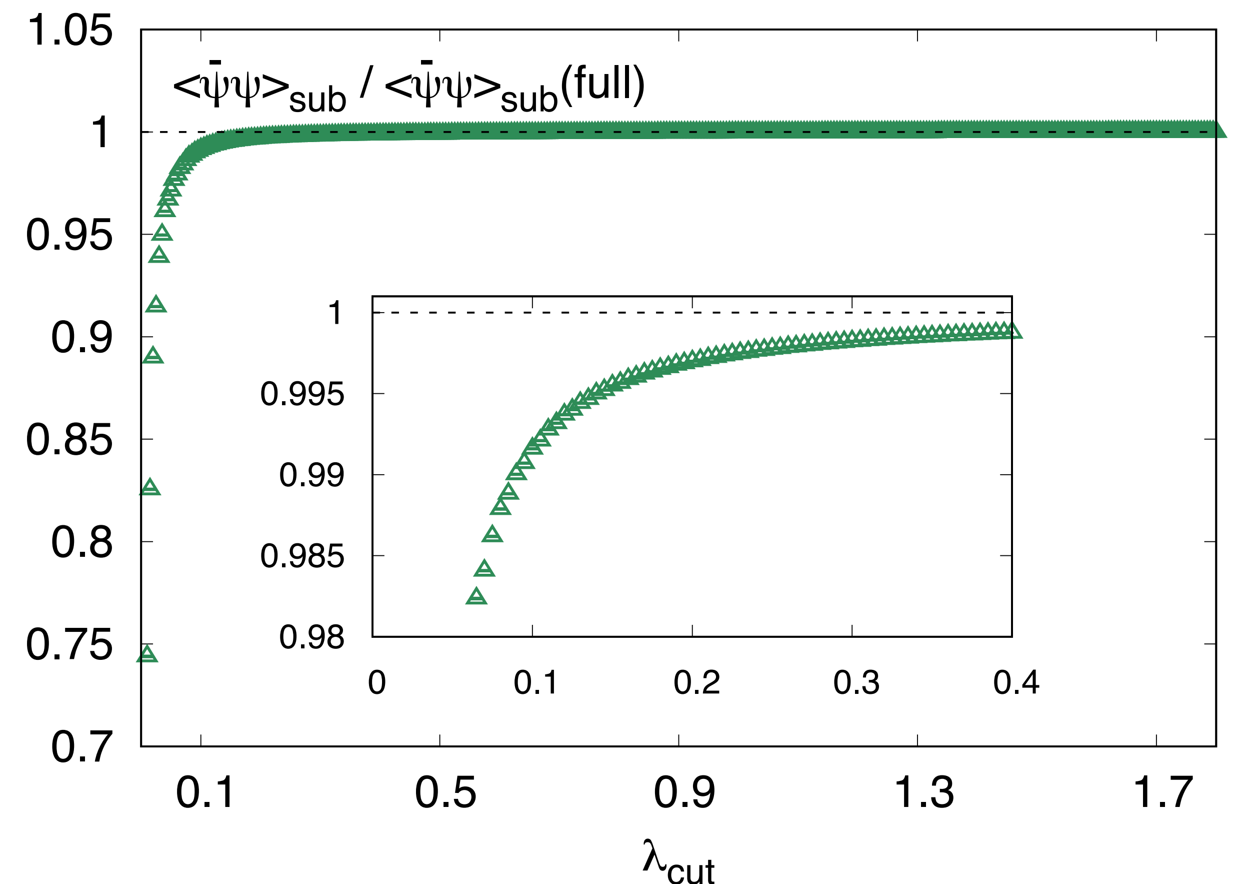
# UV-divergence of Chiral Condensates

★ $m_s/m_l = 10$




- **Complete Dirac eigenvalue spectrum was obtained and used to estimate UV-divergence part of chiral condensate.**

Yu Zhang et al, POS Lattice 2019. L. Giusti and M. Luscher ,JHEP 03, 013

G. Cossu, PTEP 2016, 093B06 . Z. Fodor, PoS LATTICE2015, 310

- $\langle\bar{\psi}\psi\rangle_{sub} \equiv \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s = \int_0^\infty \frac{2m_l(m_s^2 - m_l^2)\rho(\lambda)}{(\lambda^2+m_l^2)(\lambda^2+m_s^2)}\mathrm{d}\lambda$
- $\langle\bar{\psi}\psi\rangle_{l,s} = \int_0^\infty \frac{2\,m_{l,s}\,\rho(\lambda)}{\lambda^2+m_{l,s}^2}\mathrm{d}\lambda$ , $\langle\bar{\psi}\psi\rangle_{l,s}^{\mathrm{UV}} = \int_{\lambda_{cut}^{\mathrm{UV}}}^\infty \frac{2\,m_{l,s}\,\rho(\lambda)}{\lambda^2+m_{l,s}^2}\mathrm{d}\lambda$

| $\lambda_{cut}^{UV}$ | $\langle\bar{\psi}\psi\rangle_l^{UV}/\langle\bar{\psi}\psi\rangle_l(full)$ | $\langle\bar{\psi}\psi\rangle_s^{UV}/\langle\bar{\psi}\psi\rangle_s(full)$ |
|---|---|---|
| 0.12 | 32% | 83% |
| 0.24 | 29% | 76% |
| 0.36 | 27% | 71% |