Deconfinement Critical Point of a Heavy Quark Effective Theory

Pham A. Quang

Joint work with O. Philipsen, J. Kim

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Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Ref.

M. Fromm, J. Langelage, S. Lottini, OP arXiv:1111.4953

J. Langelage, M. Neuman, OP arXiv:1403.4164

WHOT-QCD Collaboration: Shinji Ejiri et al. arXiv:1912.10500

J. Kim, QP, OP, Jonas Scheunert arXiv:2007.04187

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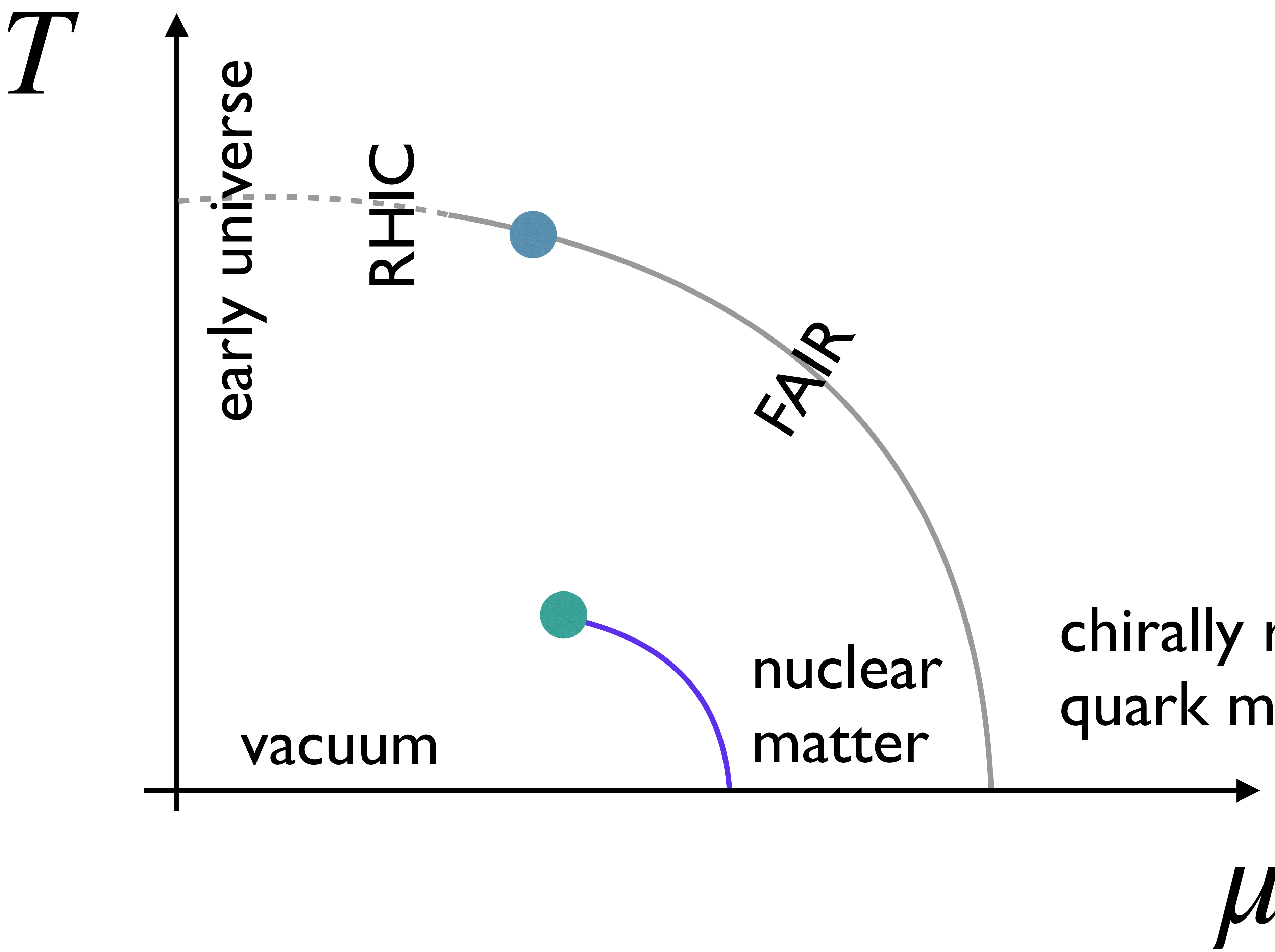
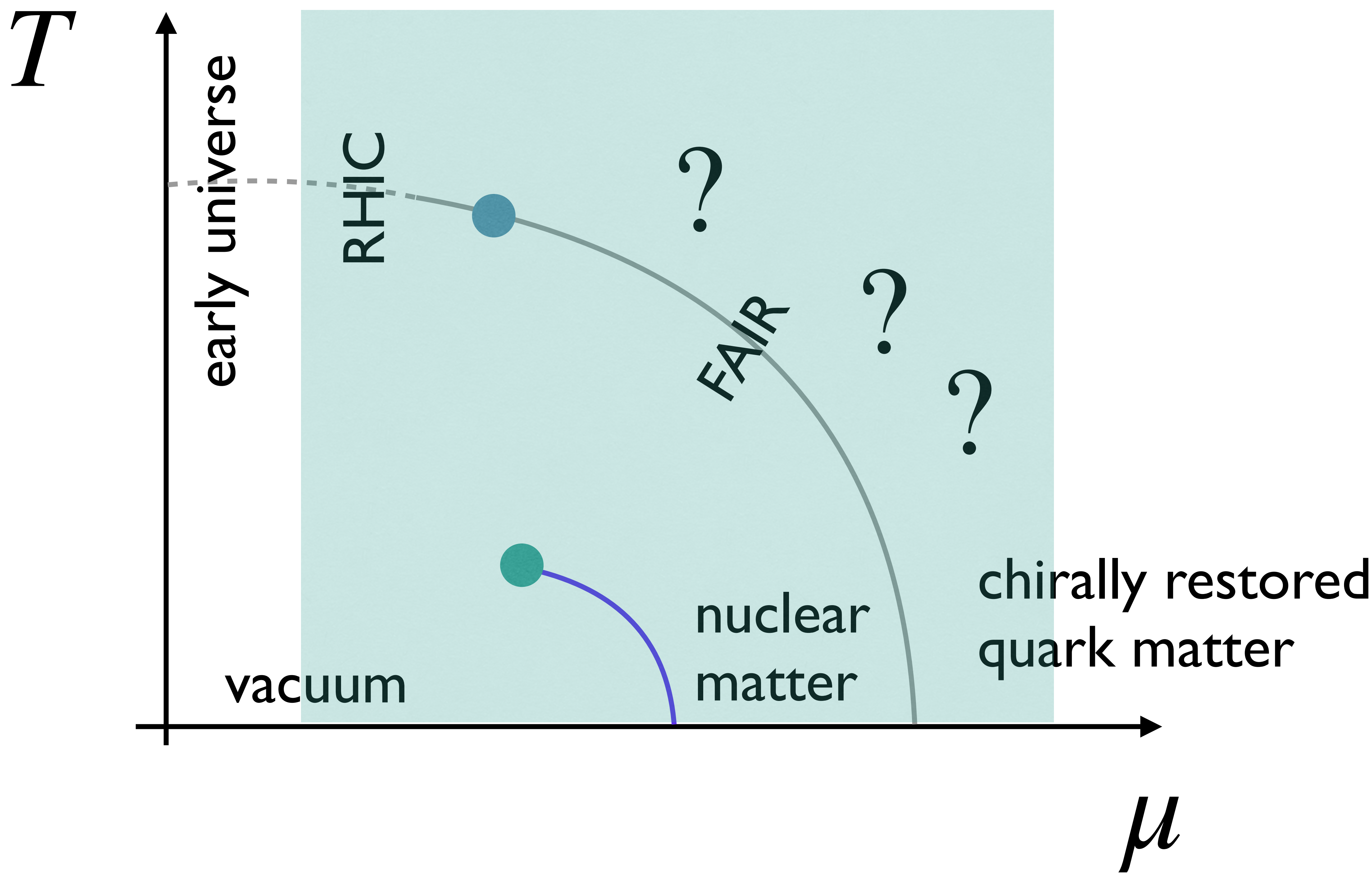
Plan

- Introduction
- Effective Yang-Mills theory
- Effective theory of heavy quark QCD
- Results
- Conclusions and Outlooks

QCD Phase Diagram



QCD Phase Diagram



Why Effective Theories?

- At finite μ, sign problem is mild enough to simulate
- Possible to carry out computations analytically e.g. Linked Cluster or High Temperature Expansion

J. Kim, QP, OP, J. Scheunert arXiv:2007.04187, 1912.01705

- Cheaper than full QCD
- ET can still provide important features which are comparable to full QCD

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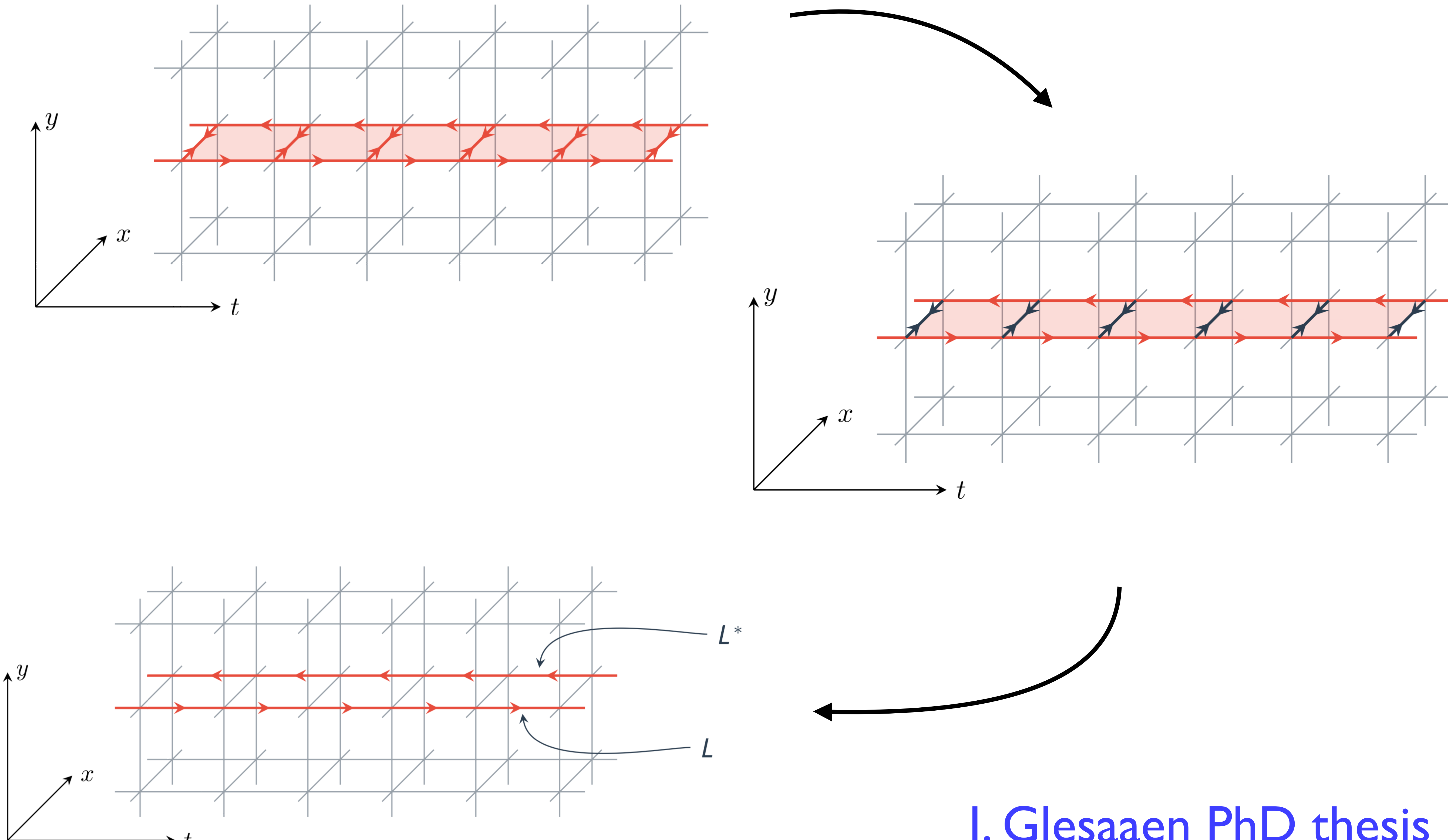
Main focus of this talk

Effective Theories

The basic idea of the effective theories is integrating out all spatial links in QCD action, then the effective theories depend only on Polyakov loops

O. Philipsen, J. Langelage, S. Lottini arXiv: 1010.0951

Eg: Yang–Mills Theory



J. Glesaaen PhD thesis

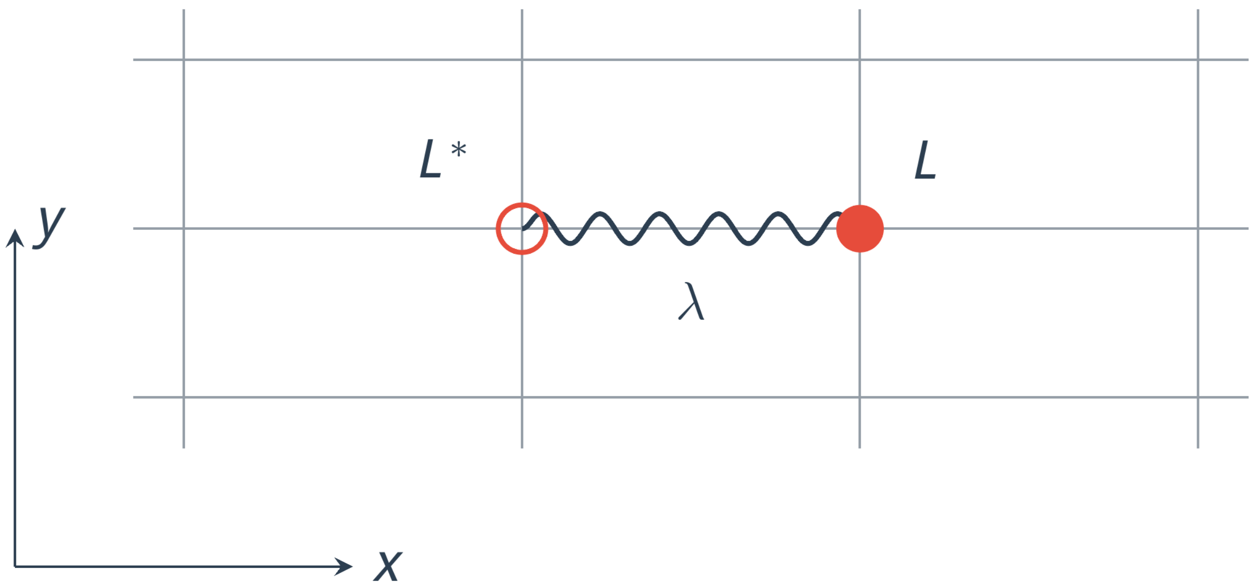
Effective Yang-Mills Theory

Partition function of ET

$$Z = \int \mathrm{D}[U_0] \prod_{\langle \vec{x}, \vec{y} \rangle} \Big(1 + \lambda (L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}}) \Big)$$

Polyakov loop

$$L_{\vec{x}} = \mathrm{tr} \prod_{t=0}^{N_t} U_0(t, \vec{x}) = \mathrm{tr} W(\vec{x})$$



Including Fermions

- Hopping matrix

$$H = T + \sum_i S_i$$

- Re-factorize the fermion determinant

$$\det Q = \det(1 - \kappa T - \kappa S) = \det(1 - \kappa T) \det\left(1 - \frac{\kappa S}{1 - \kappa T}\right)$$

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Static quark determinant

Kinetic quark determinant

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$$\kappa \ll 1$$

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Static quark determinant

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Static Quark Determinant (LO)

- Finite quark mass, but still large i.e. $\kappa \ll 1$
- The static quark determinant is derived by expanding the quark determinant in κ to leading order.

$$\det Q_{\mathrm{stat}} = \prod_{\vec{x}} (1 + h_1 L_{\vec{x}} + h_1^2 L_{\vec{x}}^* + h_1^3)^2 (1 + \bar{h}_1 L_{\vec{x}}^* + \bar{h}_1^2 L_{\vec{x}} + \bar{h}_1^3)^2$$

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$$\downarrow$$

$$h_1(\mu) = (2 e^{a\mu} \kappa)^{N_t} = \bar{h}_1(-\mu)$$

κ^2- Correction (NLO)

- κ^2 - correction corresponds to the leading order of the kinetic quark determinant.

$$S = P + M = \frac{1}{1 - \kappa T} \sum_i (S_i^+ + S_i^-)$$

$$\det Q_{\text{kin}} = \det(1 - \kappa P - \kappa M) = \exp\left(- \sum_{n=1}^{\infty} \frac{1}{n} \text{tr}(P + M)^n \right)$$

κ^2- Correction (NLO)

- κ^2 - correction corresponds to the leading order of the kinetic quark determinant.

$$S_{\kappa^2} = \frac{\kappa^2 N_t}{N_c} \sum_{\langle \vec{x},\vec{y} \rangle} \left(W_{11}(\vec{x}) - \bar{W}_{11}(\vec{x})\right) \left(W_{11}(\vec{y} - \bar{W}_{11}(\vec{y})\right) + \mathcal{O}(\kappa^4)$$

$$W_{11} = \mathrm{tr} \frac{h_1 W}{1+h_1 W}$$

Here W is called the temporal Wilson lines

κ^4- Correction (NNLO)

κ^4 - correction is very lengthy, for full expression see
J. Langenlage, M. Neuman, OP arXiv:1403.4162

$$S_{\kappa^4} = 2\frac{\kappa^4 N_t(N_t-1)}{N_c^2} \sum_{\vec{x},i,j} \mathrm{tr}\left(W_{11}(\vec{x}) - \bar{W}_{11}(\vec{x})\right) \mathrm{tr}\left(W_{11}(\vec{x}+i+j) - \bar{W}_{11}(\vec{x}+i+j)\right)$$

$$\times \mathrm{tr}\left(W_{12}(\vec{x}+i) - \bar{W}_{12}(\vec{x}+i) - \frac{2}{N_t-1}\frac{\sum_{t=1}^{N_t-1}(2\kappa^2)^{2t}}{(1+h_1 W(\vec{x}+i))(1+\bar{h}_1 W^\dagger(\vec{x}+i)}\right)$$

$$-2\frac{\kappa^4 N_t}{N_c^2}\sum_{\vec{x},i,j}\cdots$$

Observables

- We define the equilibrium Polyakov loop as

$$L = \frac{1}{V}\sum_{\vec{x}}(L_{\vec{x}} + L^{\dagger}_{\vec{x}})$$

- Then the susceptibility, skewness and kurtosis of the Polyakov loop are

$$\chi_L = \frac{\partial^2 \log Z[J]}{\partial J^2}|_{J=0} = V(\langle L^2\rangle - \langle L\rangle^2)$$

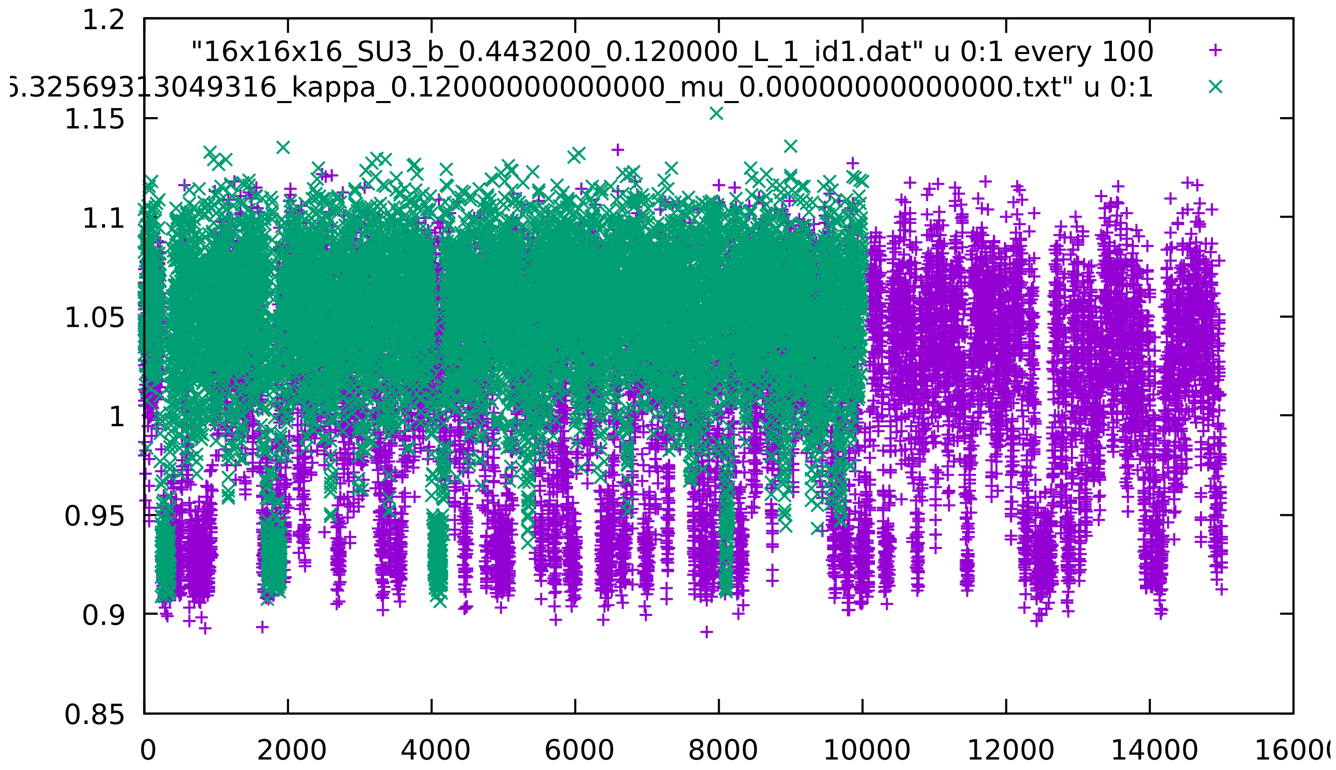
$$B_{3L} = \frac{\langle L^3\rangle - 3\langle L^2\rangle\langle L\rangle + 2\langle L\rangle^3}{(\langle L^2\rangle - \langle L\rangle^2)^{3/2}}$$

$$B_{4L} = \frac{\langle L^4\rangle - 4\langle L^3\rangle\langle L\rangle + 6\langle L^2\rangle\langle L\rangle^2 - 3\langle L\rangle^4}{(\langle L^2\rangle - \langle L\rangle^2)^2}$$

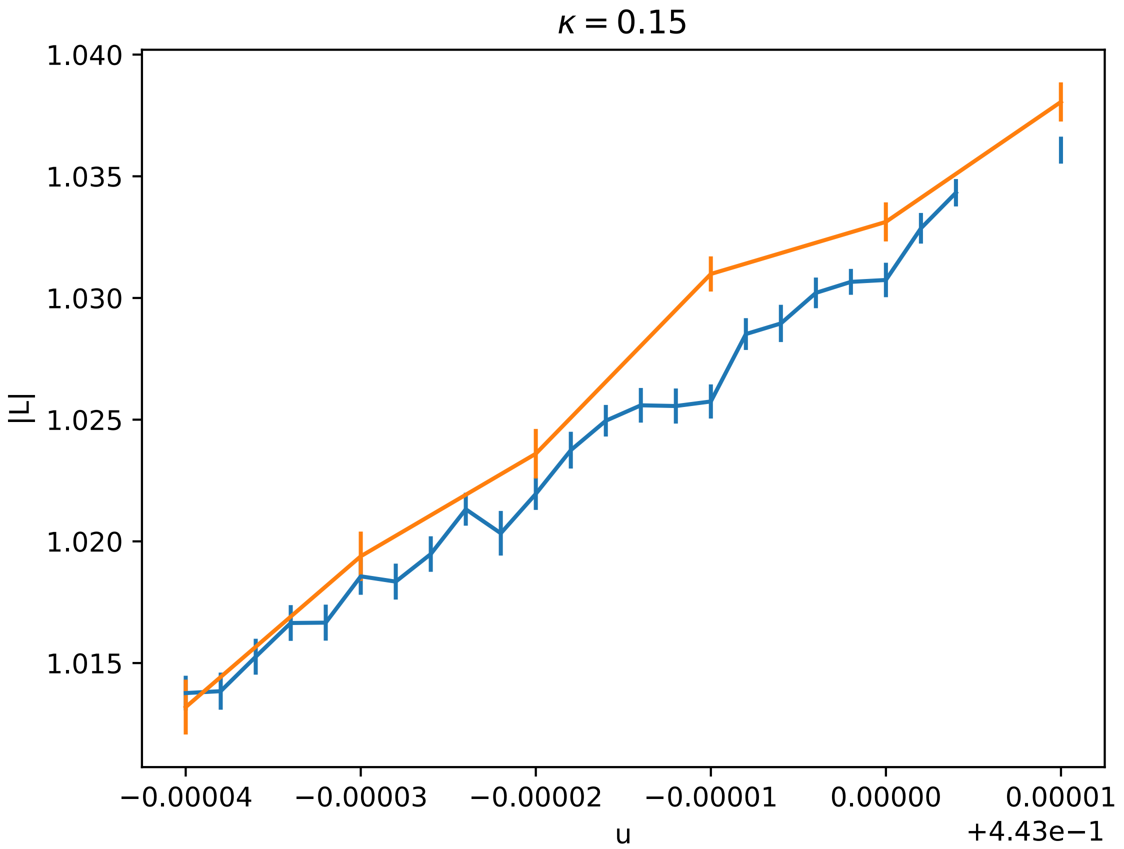
Remarks

- The standard update for our effective theory is parametrization the Polyakov with two angles θ_1, θ_2
- However, the results presented here were obtained using link update i.e. full SU(3) matrices and take the trace afterwards
- It seems more expensive but it will provide a nice solution aiming for κ^6 - correction

Update Comparison



Histogram

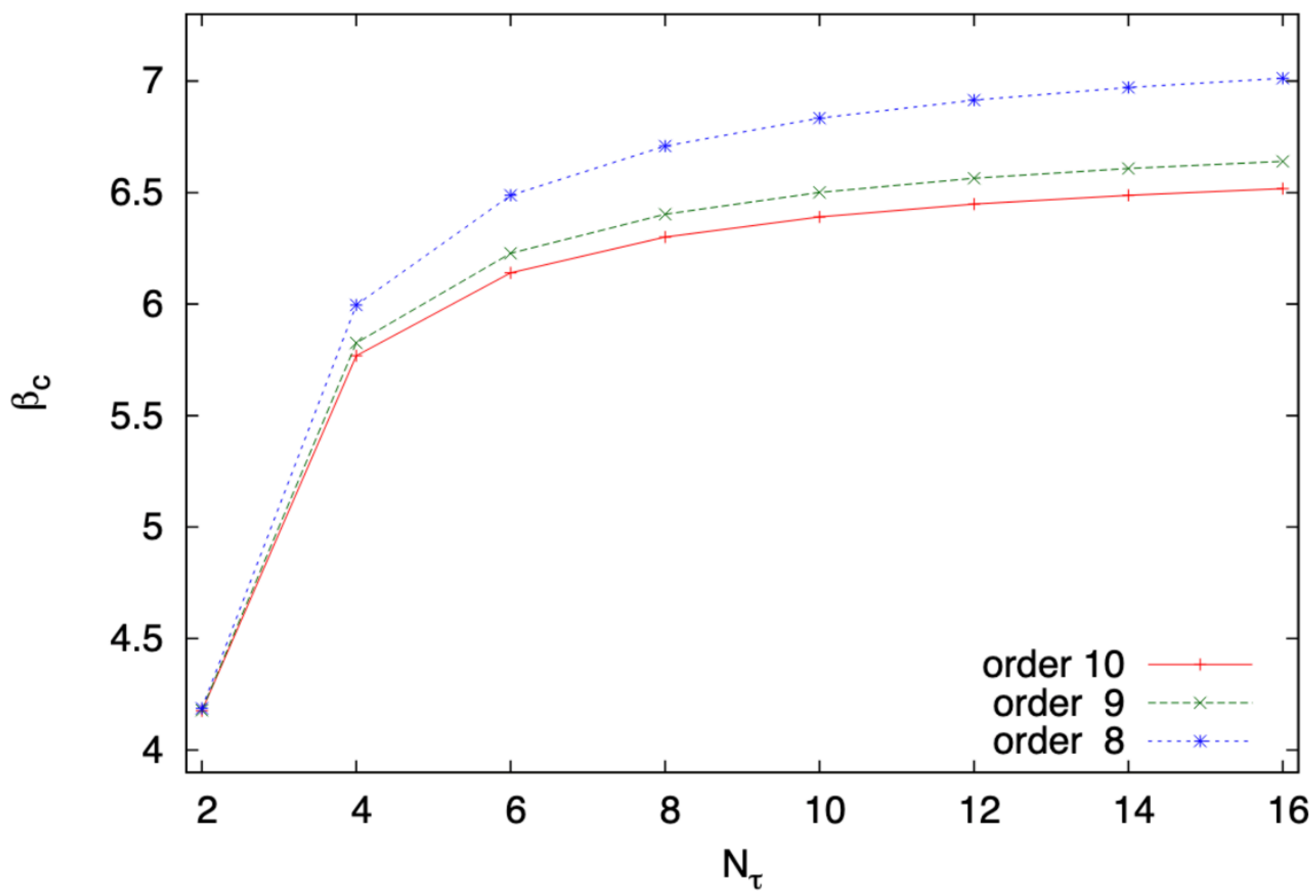


Polyakov loop

Results

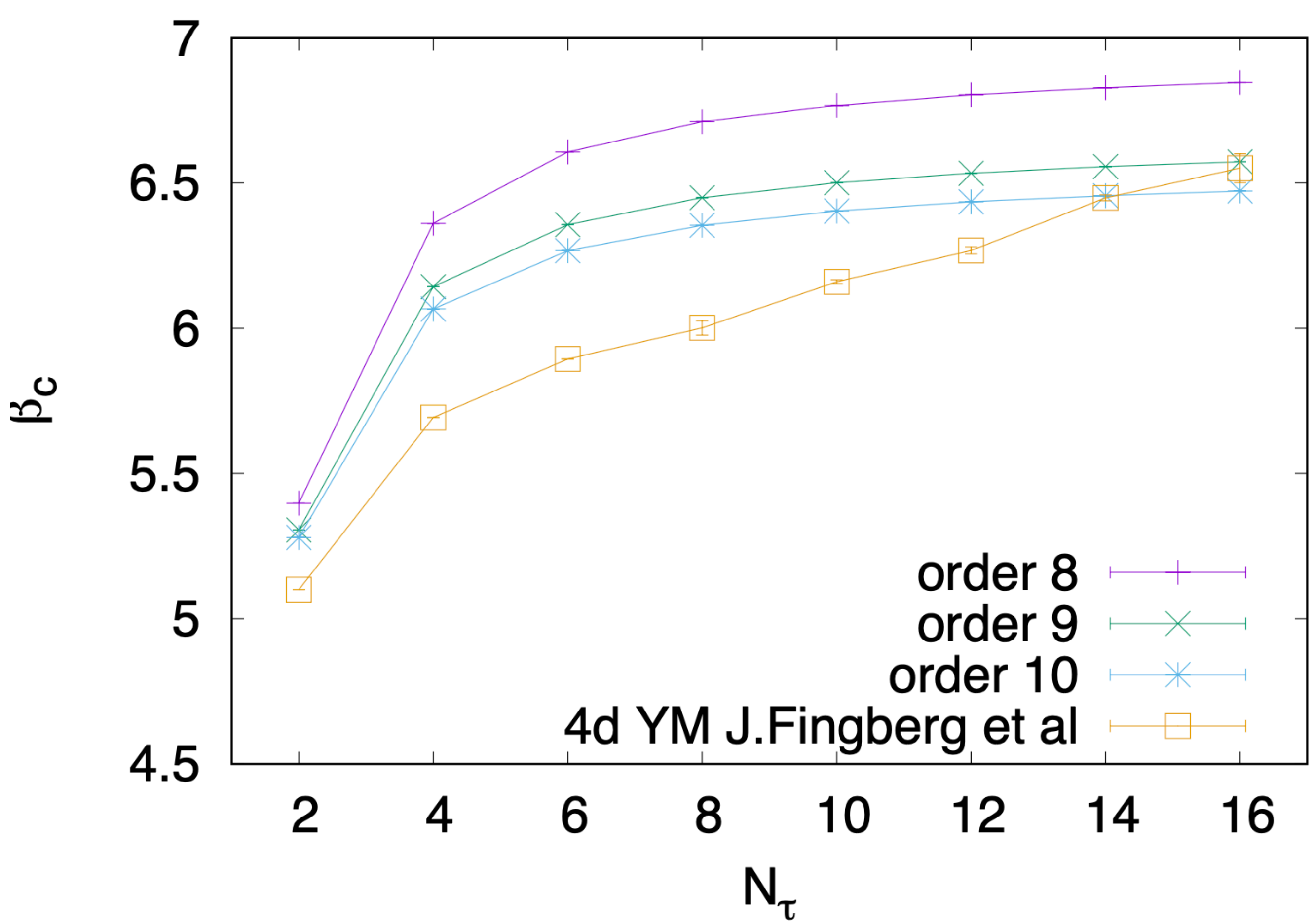
YM Effective Theory

- Effective Yang-Mills theory from simulation and analytic



Simulation results of ET

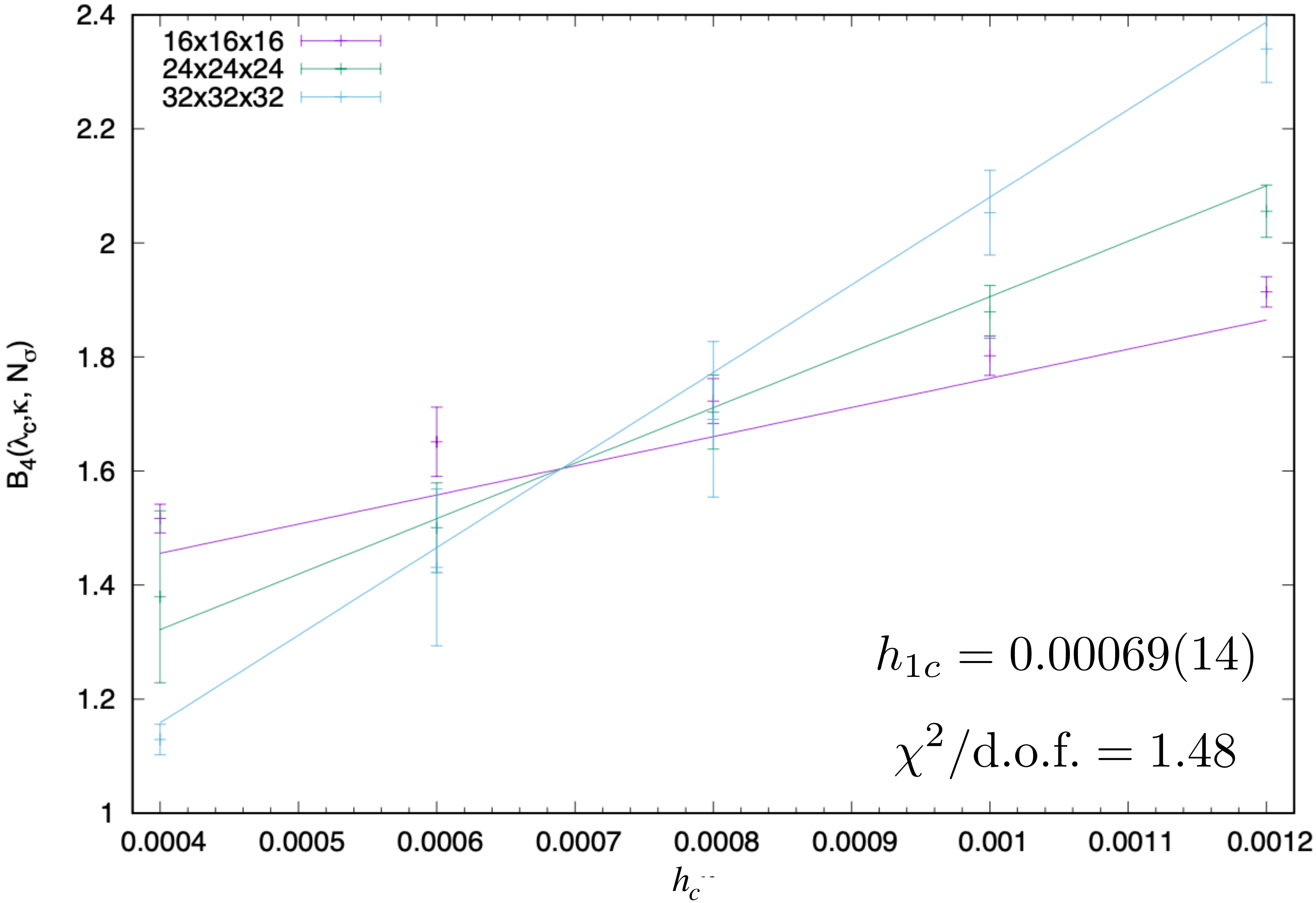
J. Langenlage, S. Lottini, O. Philipsen
arXiv:1002.1507



Analytical results of ET using High Temperature Expansion

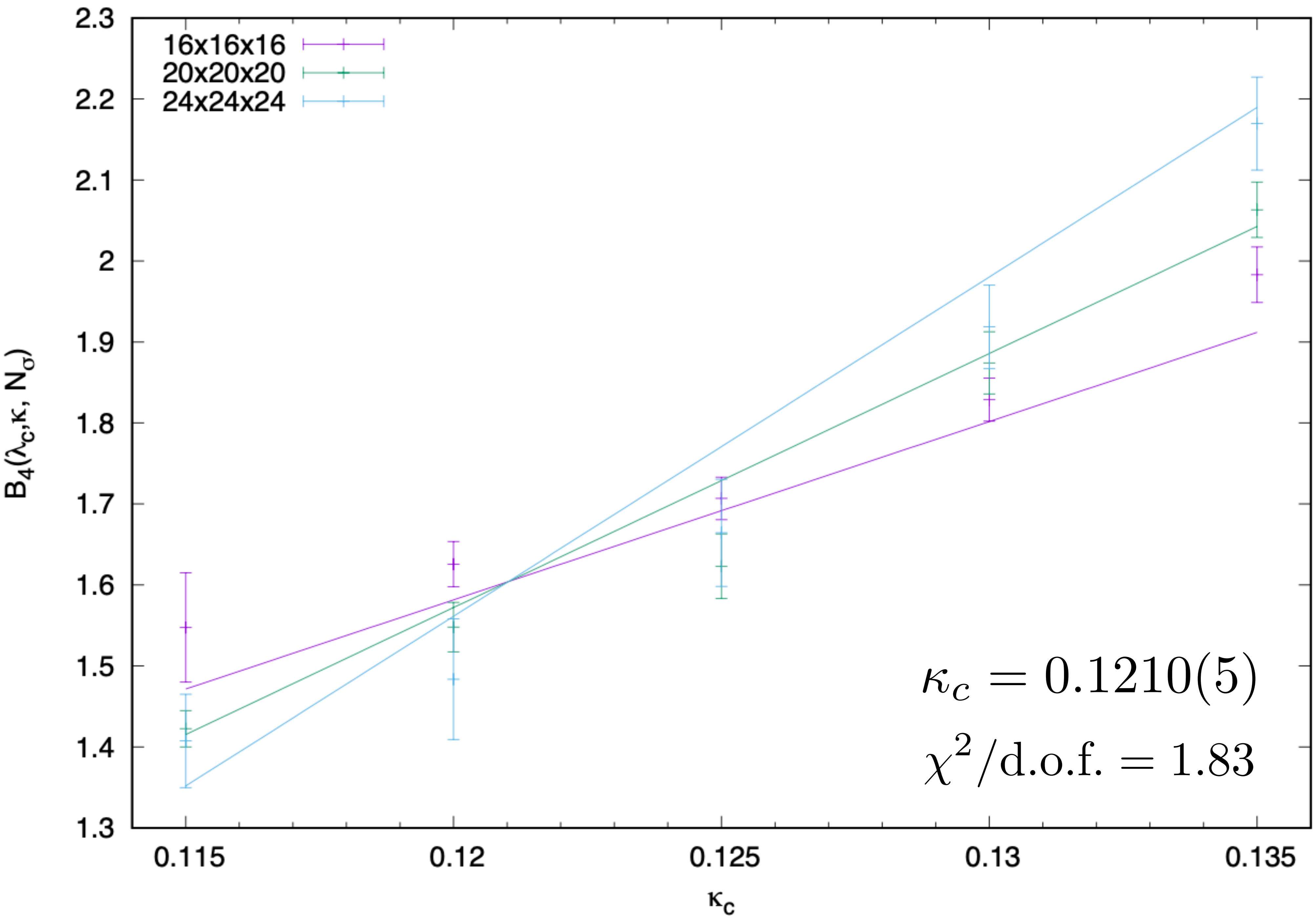
J. Kim, QP, O. Philipsen, J. Scheunert
arXiv:1912.01705

YM Effective Theory and Static Quark



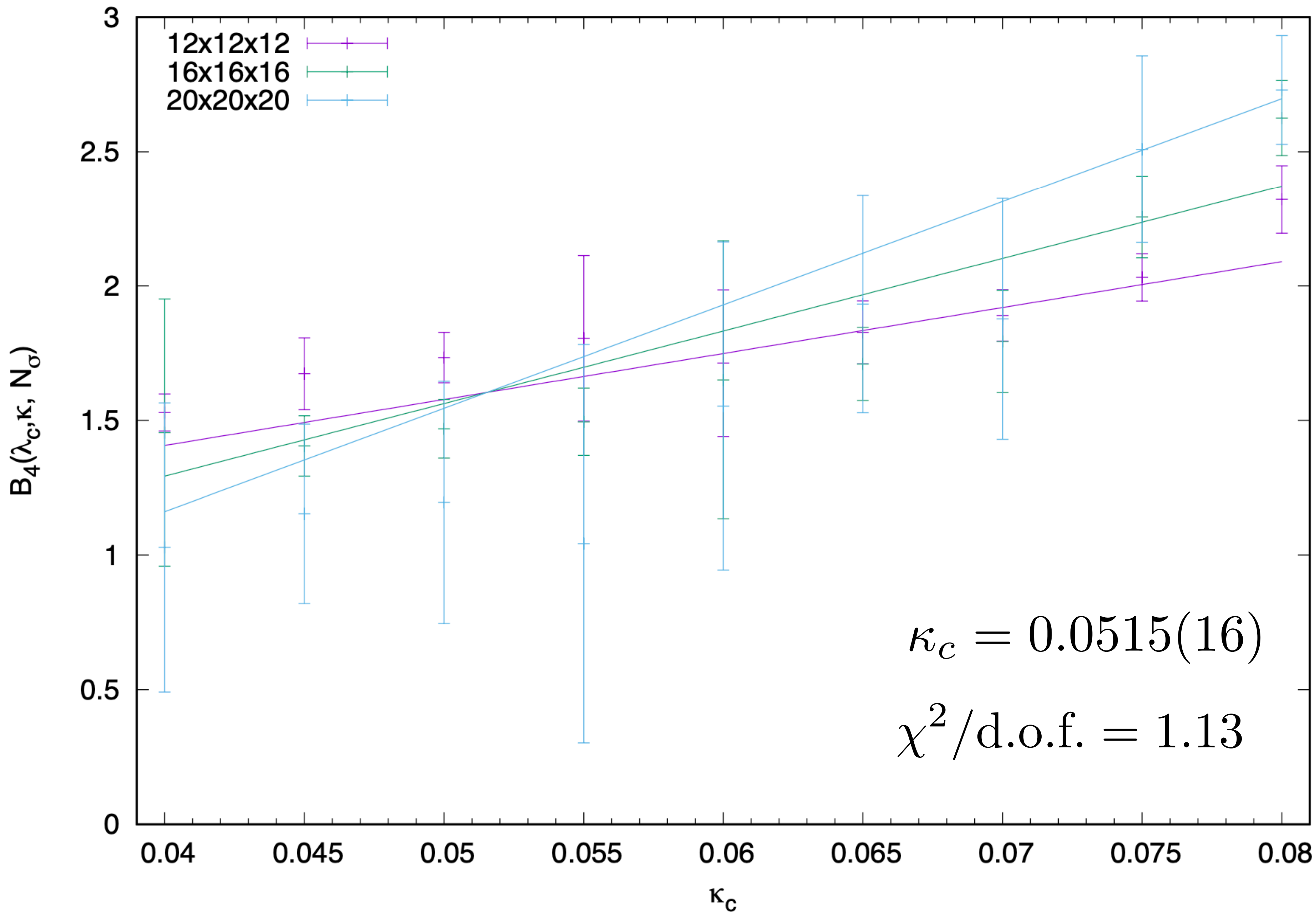
Nf = 1 and Nt = 4

YM Effective Theory, Static Quark and K2 Corrections



Nf = 2, Nt = 6

YM Effective Theory, Static Quark and K2, K4 Corrections



Nf=1, Nt =4

Compare With WHOT-QCD

Order	$N_f = 1$	$N_f = 2$
$N_t = 4$ LO	0.0810(4)	
$N_t = 4$ NLO	0.0756(6)	0.0629(4)
$N_t = 4$ NNLO	0.0515(16)	0.0443(34)
$N_t = 6$ NLO	0.1319(6)	0.1210(5)

Results from the effective theory for Nf = 1,2 and Nt = 4, 6

K_{ct}	$N_\mathrm{f} = 1$	$N_\mathrm{f} = 2$	$N_\mathrm{f} = 3$
$24^3 \times 4$, LO	0.0783(12)	0.0658(10)	0.0595(10)
$24^3 \times 4$, NLO	0.0753(11)	0.0640(10)	0.0582(9)
$24^3 \times 6$, LO	0.1525(34)	0.1359(30)	0.1270(28)
$24^3 \times 6$, NLO	0.1326(21)	0.1202(19)	0.1135(18)

Results for κ_c of WHOT collaboration for Nf=1,2,3 and Nt = 4, 6

WHOT-QCD Collaboration
Shinji Ejiri et al. arXiv:1912.10500

Conclusion

- Summarize the standard procedure for deriving ET up to κ^4 - correction
- The end point of the first-order phase transition of the heavy quark ET agrees well with those of heavy quark region in full QCD
- The end points of the NNLO decrease further which shows the consistency of our effective theory

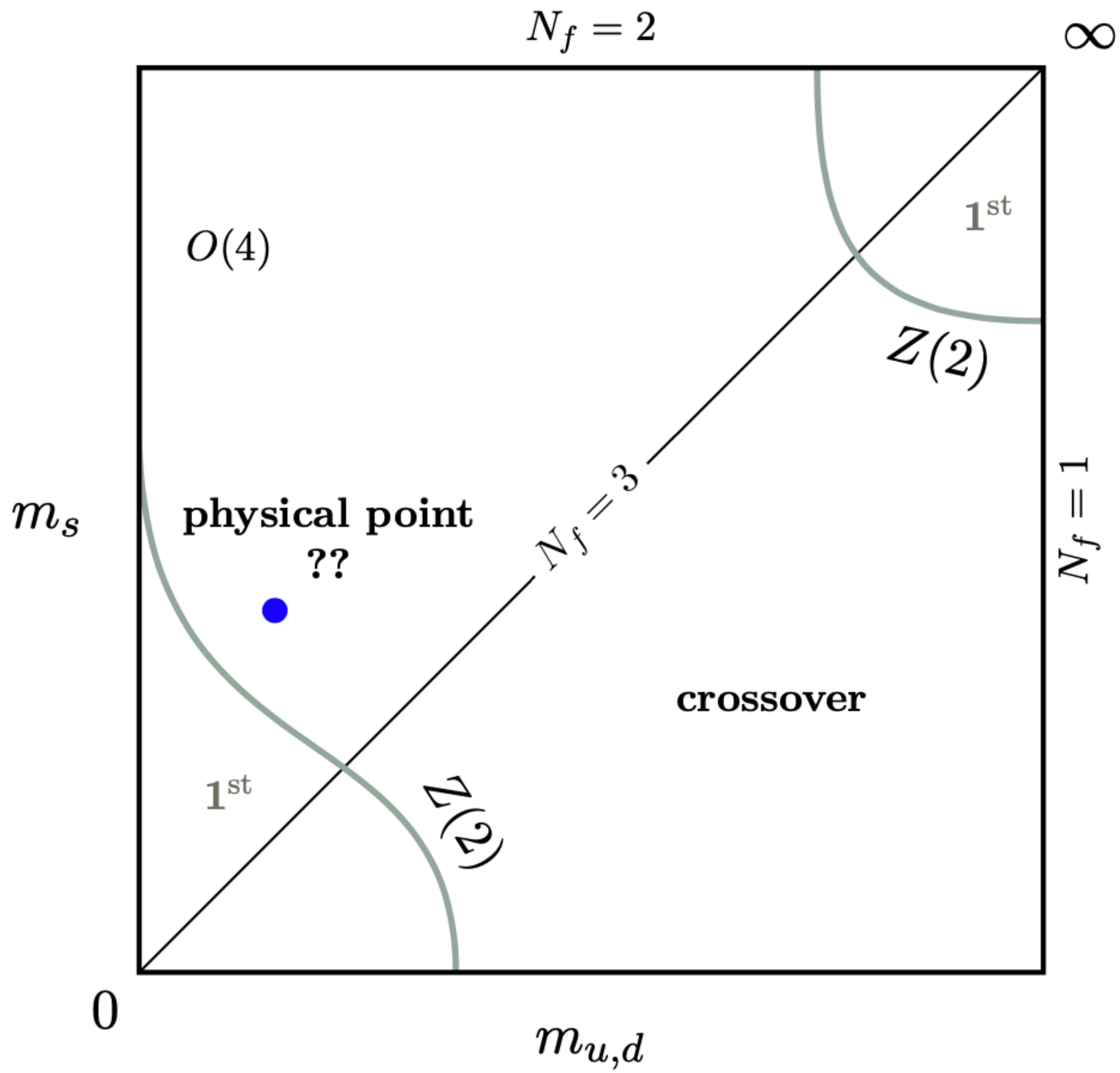
Outlook

- Including κ^6 - correction
- Explore further this region at finite chemical potential is the next step.

Thank you!

Backup slides

Columbia Plot



Milder Sign Problem

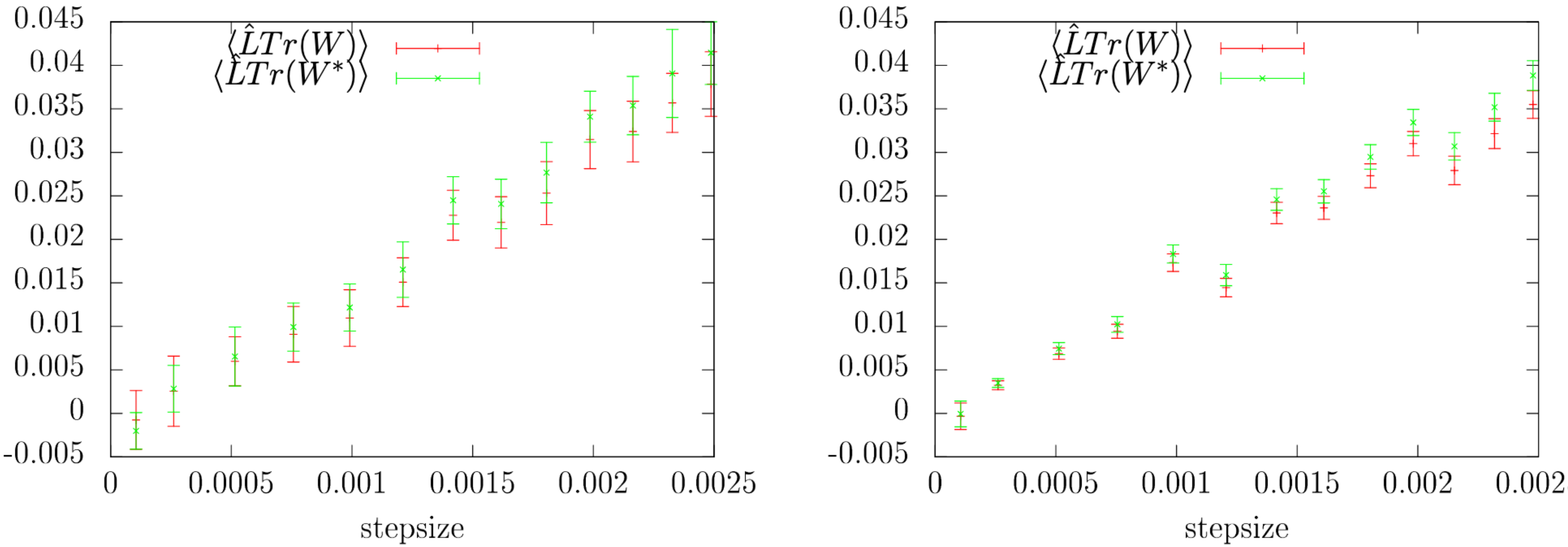


FIGURE 5.4: Applying the criteria in eq. 5.7 to $L = \text{Tr}W$ and $L^* = \text{Tr}W^\dagger$, using our effective theory to order κ^2 (left) and κ^4 (right). Parameters are $V = 10$, $\kappa = 0.0245$, $N_\tau = 50$, $a\mu_B = 9.7$ and $\beta = 6$.

M. Neuman, OP, J. Langenlage arXiv:1403.4162

Generating Functional

$$G(\alpha, \beta) = \log\det(\alpha + \beta h_1 W)$$

$$W_{nm} = \frac{(-1)^{n-1}}{(n-1)!} \frac{\partial^{n-m}}{\partial \alpha^{n-m}} \frac{\partial^m}{\partial \beta^m} G(\alpha, \beta)\bigg|_{\alpha=\beta=0}$$