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The QCD chiral phase transition for different numbers of quark flavours

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CRC-TR 211 Strong-interaction matter under extreme conditions

NIC

Long history, (incomplete) key results for the following

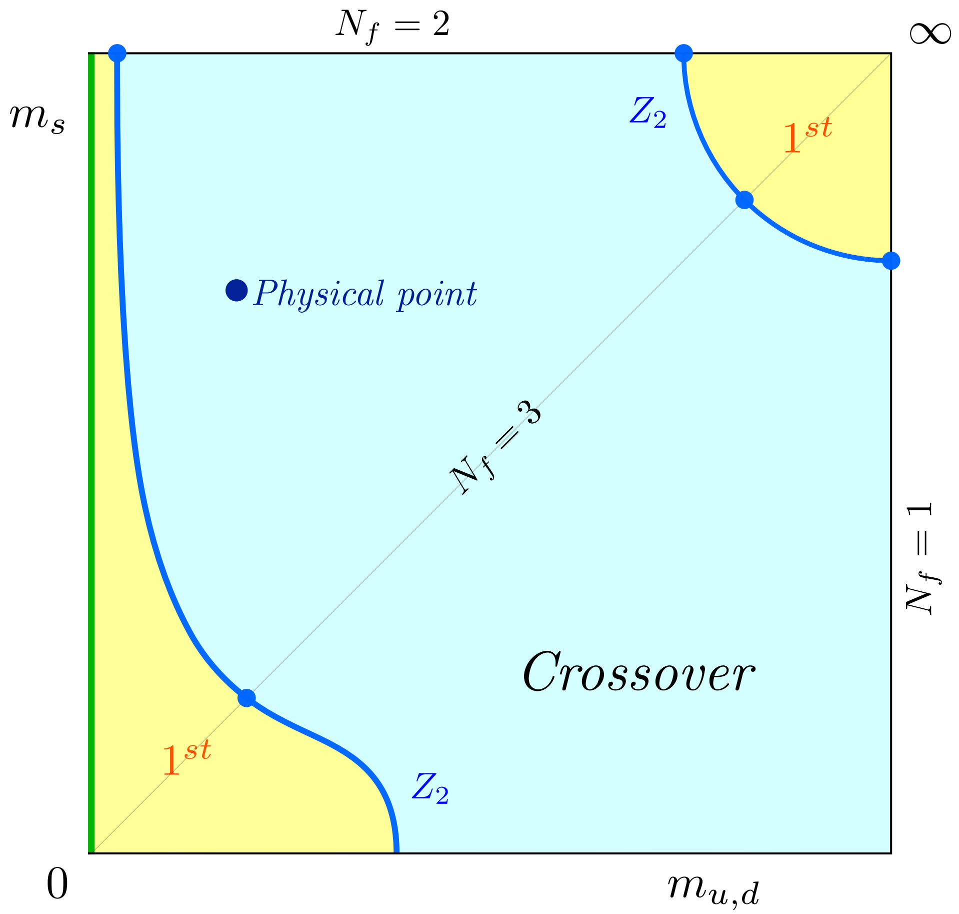
- [Pisarski, Wilczek PRD 84]
  Universality, 3D sigma models: 1st order for $N_f \geq 3$
  $N_f = 2$ depends on $U(1)_A$

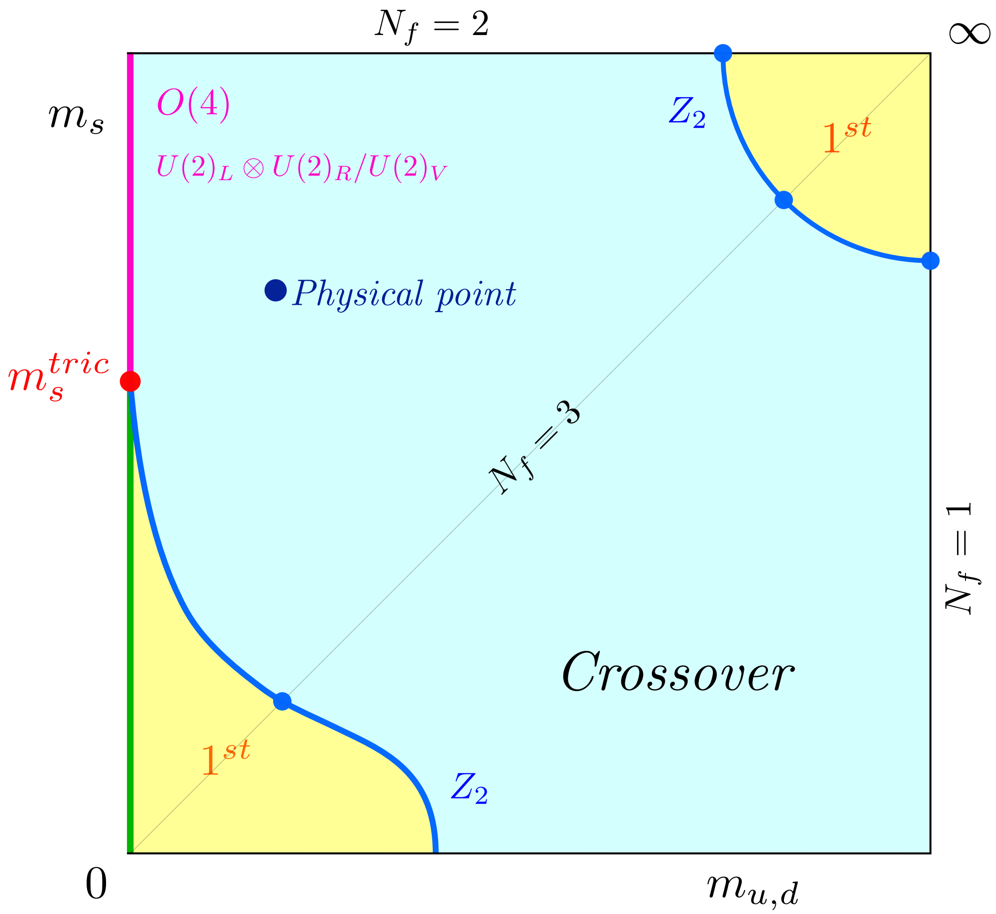
- Shrinking of 1st-order region towards continuum, will something remain?:
  Standard staggered $N_f = 2$ [Bonati et al. PRD 14, Cuteri et al. PoS LAT 18]
  $N_f = 3$ [de Forcrand, O.P. PoS LAT 07]
  $N_f = 4$ [de Forcrand, D'Elia PoS LAT 16]

  O(a)-improved Wilson $N_f = 3$ [Jin et al. PRD 15,17; Kuramashi et al. PRD 20]
  $N_f = 4$ [Ohno et al. PoS LAT 18, this conference]

- No 1st order transition seen at all:
  HISQ, $N_f = 3, m_{PS} \geq 50\ \mathrm{MeV}$ [Bazavov et al. PRD 17]

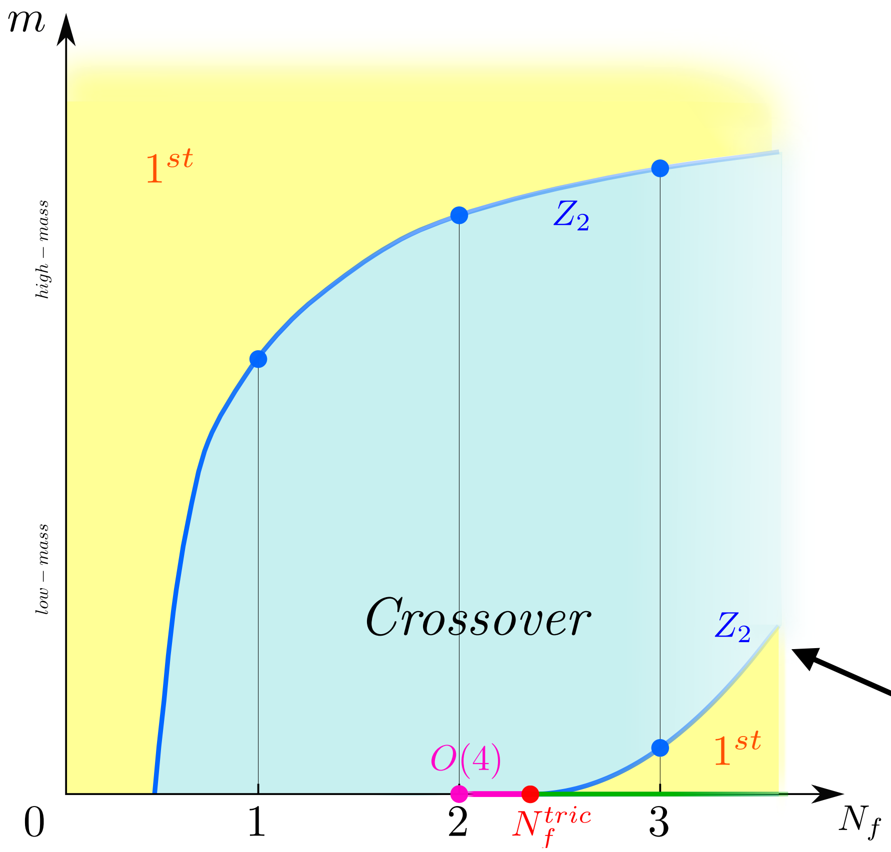
The Columbia plot





- Nature of chiral p.t. in massless limit still not settled
- Coarse lattices, unimproved actions: 1st order
- 1st order region shrinks rapidly as $a \to 0$
- Change from 1st to 2nd order in chiral limit: tricritical point

The Columbia plot for mass-degenerate quarks



$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_\mathrm{f}} \, e^{-\mathcal{S}_\mathrm{YM}[A_\mu]}$$

straightforward for staggered fermions

$$N_\mathrm{f}^c(am) = N_\mathrm{f}^\mathrm{tric} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\big((am)^{4/5}\big)$$

- Consider analytic continuation to continuous N_f
- Tricritical point guaranteed to exist if there is 1st order at any N_f
- Known exponents for critical line!
- Continuation to $a \neq 0$: Z(2) surface ends in tricritical line

[Cuteri, O.P., Sciarra PRD 18]

Methodology

Order of phase transition:

Finite size scaling of generalised cumulants

$$B_n = \frac{\langle(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle)^n\rangle}{\langle(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle)^2\rangle^{n/2}}$$

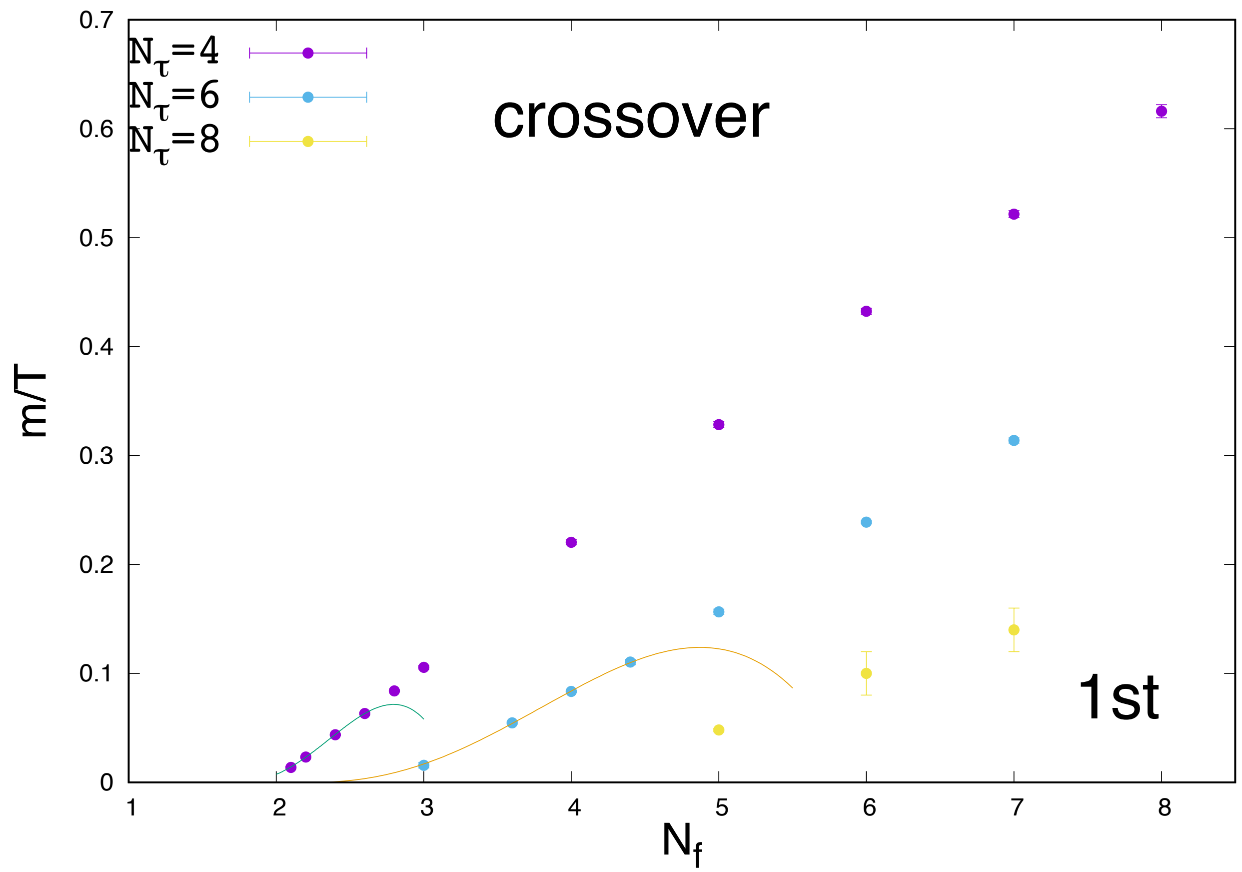
Standard staggered fermions, bare parameters: β, am, N_f, N_τ

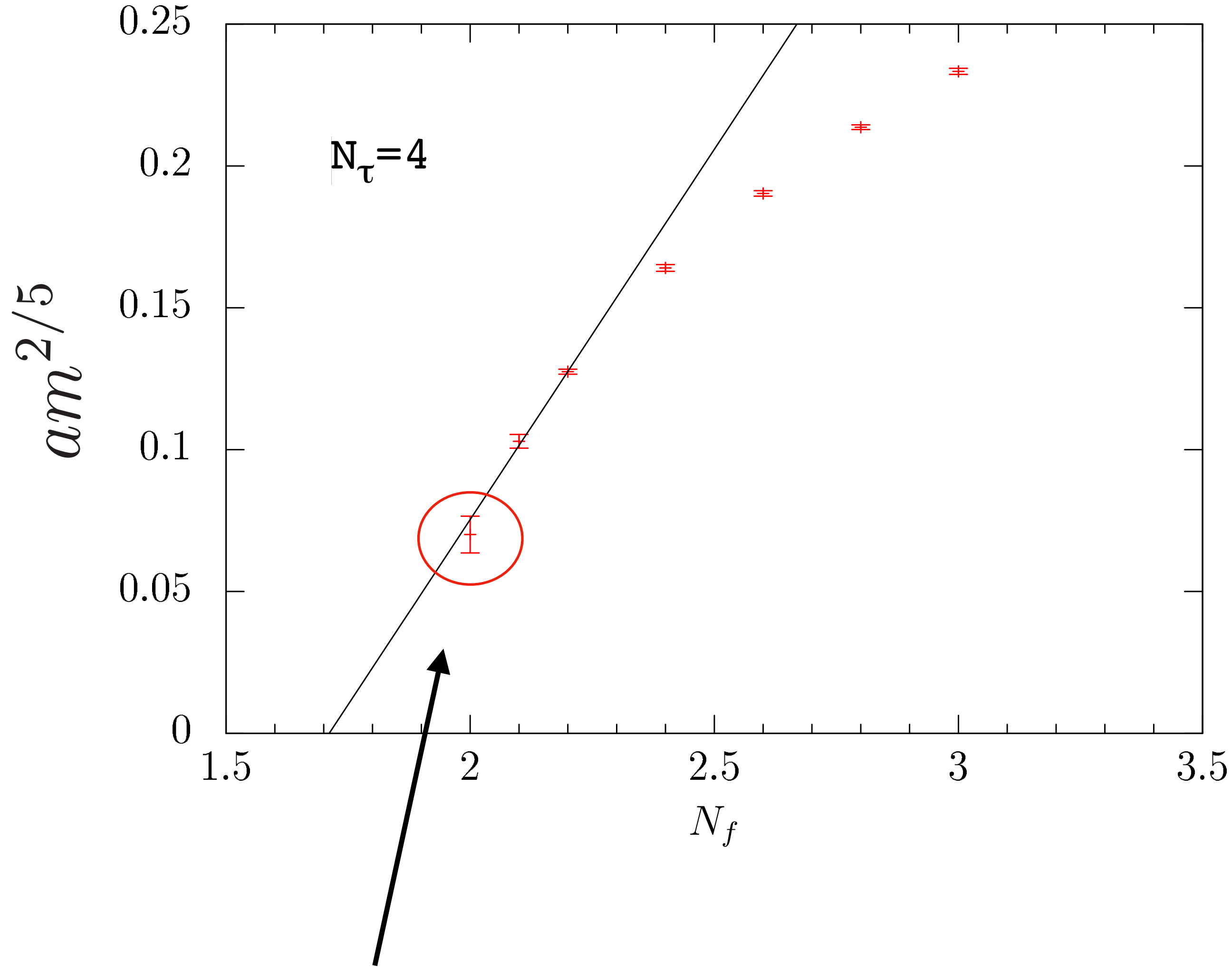
(Pseudo-critical) phase boundary: $B_3 = 0$ 3d manifold

Second-order 3D Ising: $B_4 = 1.604$ 2d chiral critical surface separates 1st order region from crossover

FSS: $B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\,N_\sigma^{1/0.6301}$

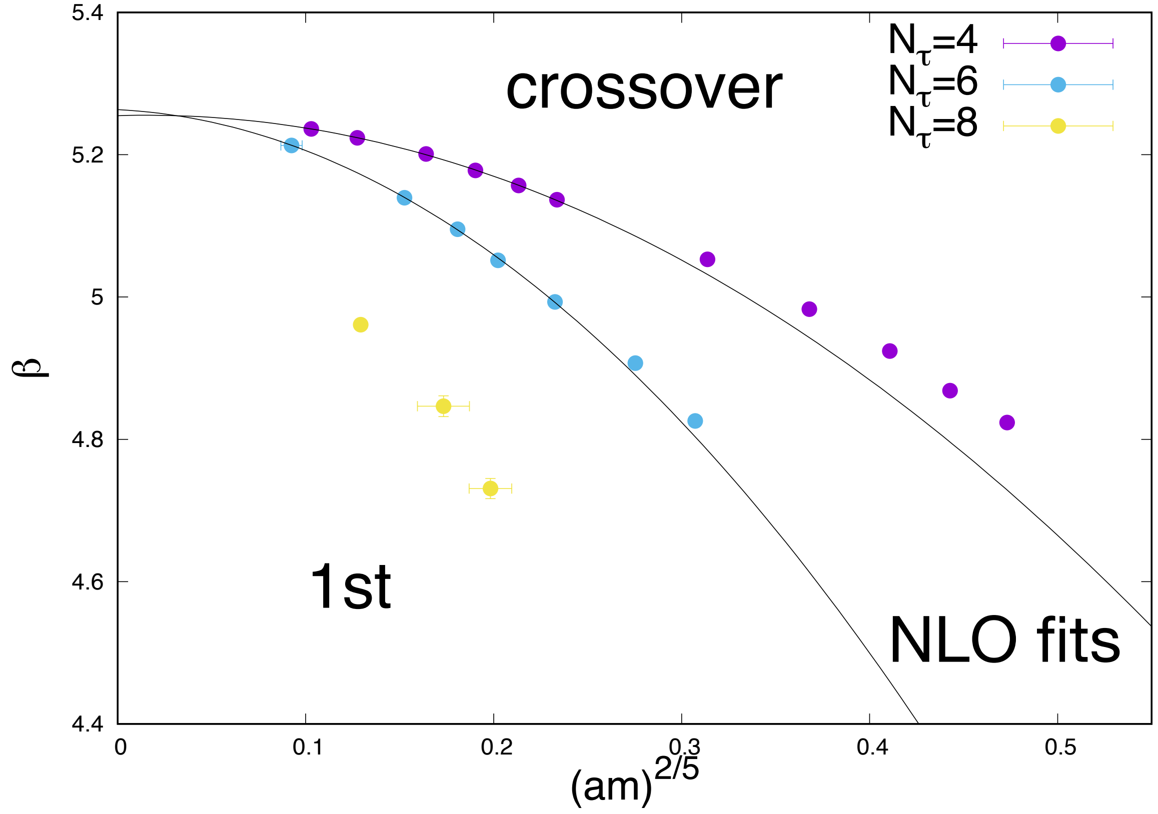
The chiral critical surface in bare parameter space

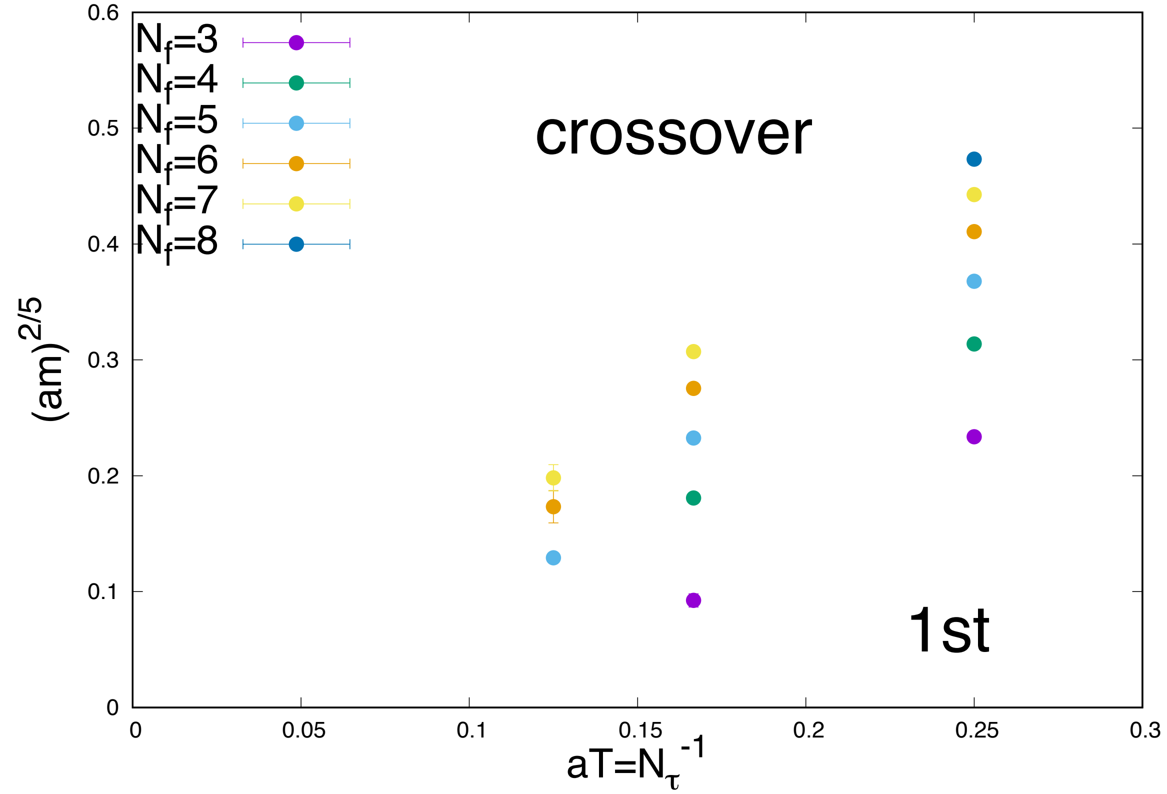




- Tricritical scaling observed!
- Cross checked by tric. scaling from finite imaginary μ [Bonati et al. PRD 14]
- Old question: $m_c/T = 0 \text{ or } \neq 0$? Answered for $N_f = 2$
- New question: will N_f^{tric} slide beyond $N_f = 3$?

Same critical surface plotted in different variables





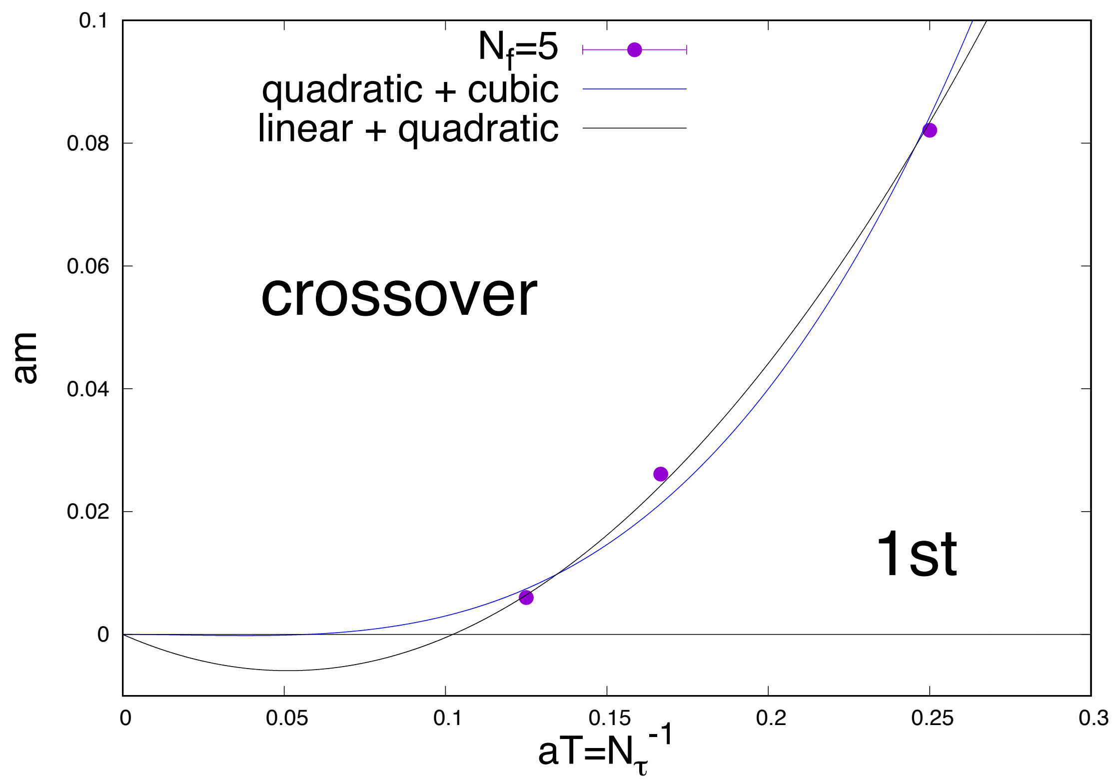
$$\beta_{\rm tric}(N_\tau) = \beta_c(0, N_{\rm f}^{\rm tric}(N_\tau), N_\tau)$$

$\Rightarrow T_{\rm tric}(N_\tau)$ after scale setting

Lines with 3pts appear consistent with NLO scaling

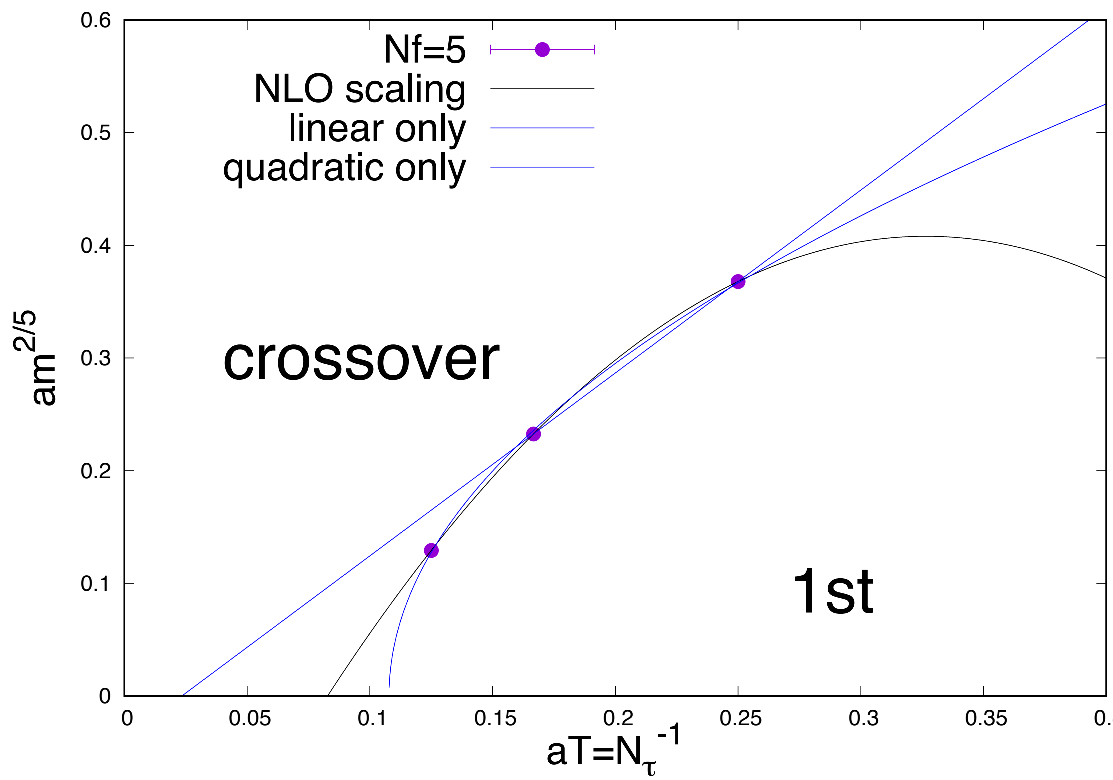
1st order vs. tricritical scaling, example Nf=5

1st: Z(2) line ends in finite continuum bare mass $m_c \Rightarrow am_c = 0$



$$am_c(N_\tau, N_\mathrm{f}) = \tilde{\mathcal{F}}_1(N_\mathrm{f})\; aT + \tilde{\mathcal{F}}_2(N_\mathrm{f})\; (aT)^2 + \mathcal{O}\big((aT)^3\big)$$

Intercept at finite N_τ preferred, tricritical point!



$$\Big(am_c(N_\tau, N_\mathrm{f})\Big)^{2/5} = \mathcal{F}_1(N_\mathrm{f})\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big)$$
$$+\, \mathcal{F}_2(N_\mathrm{f})\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big)^2 + \mathcal{O}\Big(\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big)^3\Big)$$

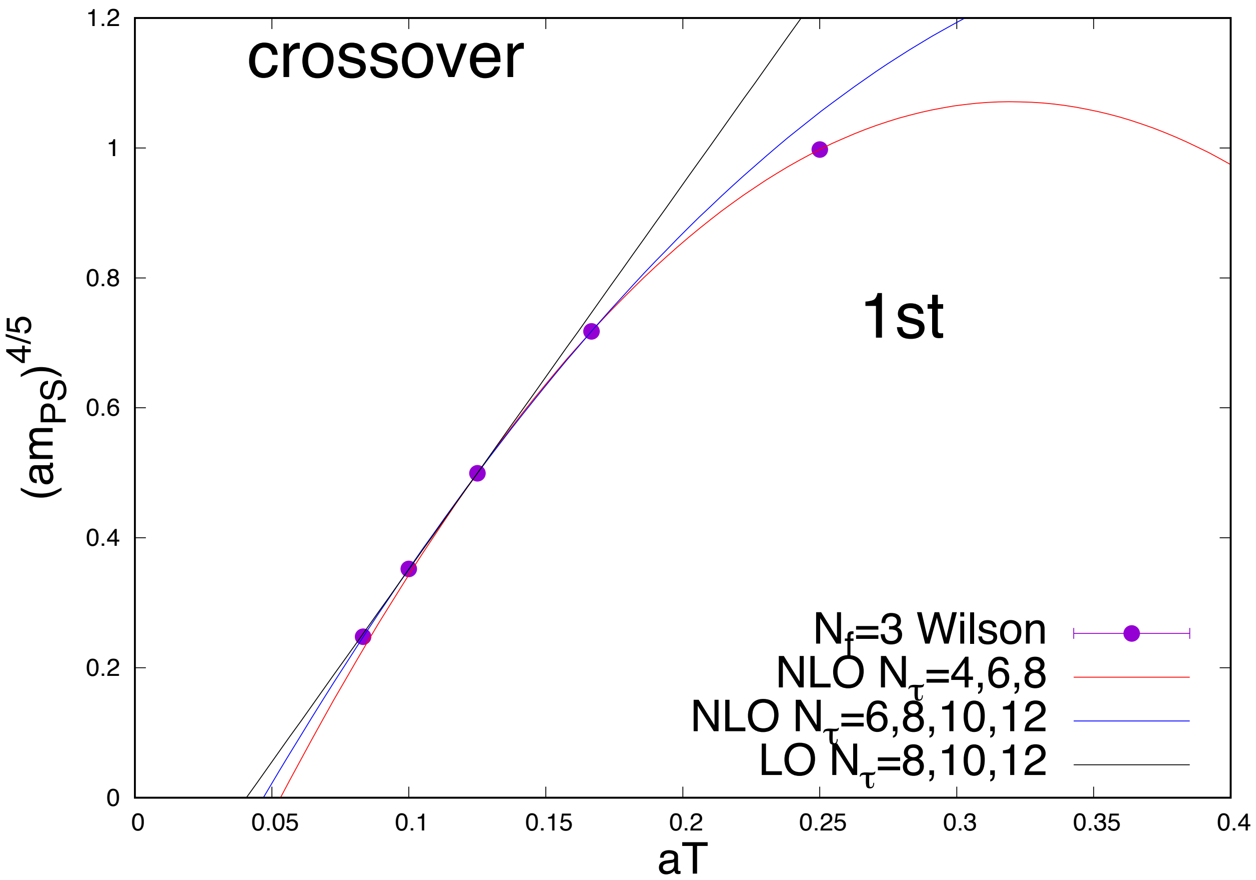
$$\Rightarrow \quad N_f^\mathrm{tric}(N_\tau = 4) \approx 1.8 \qquad N_f^\mathrm{tric}(N_\tau \approx 10) = 5$$

Nf=3 O(a)-improved Wilson fermions

What is the critical pion mass for three flavours?

Long-term effort [Jin et al. PRD 15, Jin et al. PRD 17, Kuramashi et al. PRD 20]

$$N_\tau = 4, 6, 8, 10, 12 \qquad m_{PS}^c \leq 110 \text{ MeV}$$



Re-analysis of published data:

Employ $am_{PS}^2 \propto am_q$

Perfect tricritical scaling!

Nf=3 consistent with second order, as for staggered!

Conclusions

- Could the chiral p.t. in the massless limit be second-order for $N_f \geq 3$?

- Unimproved staggered, $N_f \in [2,6], N_\tau = 4,6,8$:
  All of them consistent with 2nd!

- O(a)-improved Wilson, $N_f = 3, N_\tau = 4,6,8,10,12$:
  Consistent with 2nd!

- Tricritical scaling provides tool
  to confirm or refute this