The QCD Deconfinement Z_2 Critical Point for $N_f = 2$ Flavors of Staggered Fermions

Reinhold Kaiser

in collaboration with

Owe Philipsen and Alessandro Sciarra

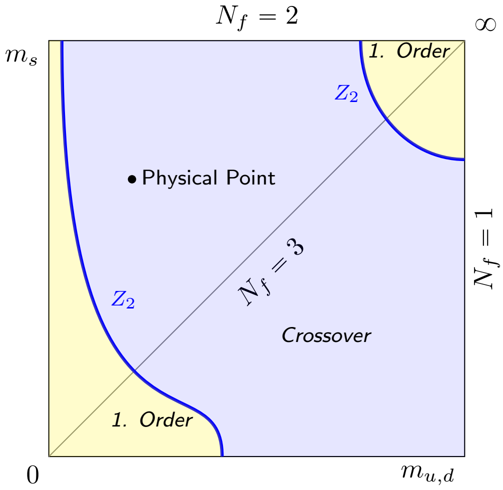
Institute for Theoretical Physics - University of Frankfurt

27.07.2021

The 38th International Symposium on Lattice Field Theory

Outline

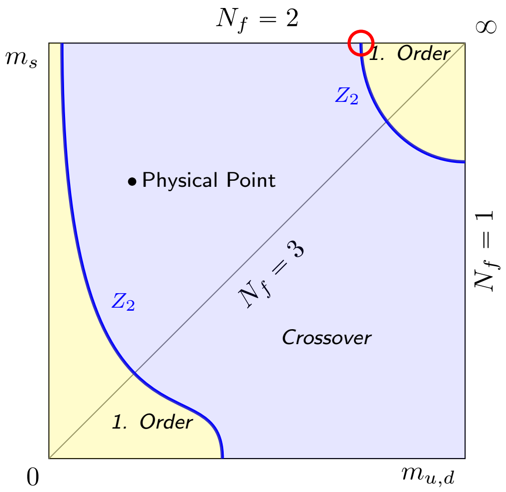
The Columbia Plot and the Deconfinement Transition



- Columbia plot[1]: type of QCD transition at $\mu = 0$
- pure gauge theory: deconfinement due to spontaneous breaking of the Z_3 center symmetry
- dynamical quarks break Z_3 center symmetry explicitly

[1] Brown et al. 1990

The Columbia Plot and the Deconfinement Transition



- Columbia plot[1]: type of QCD transition at $\mu = 0$
- pure gauge theory: deconfinement due to spontaneous breaking of the Z_3 center symmetry
- dynamical quarks break Z_3 center symmetry explicitly

Goal:

Localize the Z_2 critical point for $N_f = 2$ employing the staggered fermion action.

- motivation: first-principles benchmark for effective theories (on the lattice or in continuum)

[1] Brown et al. 1990

Simulation Details

discretization of QCD action:

- Wilson gauge action
- Staggered fermion action

parameter	label	values
am	bare quark mass	$\{0.35, 0.55, 0.75, 0.95, 1.15\}$
N_τ	temporal lattice points	8, $(10$ in progress$)$
N_s	spatial lattice points	$\{32, 40, 48, 56, 64\}$
$\beta(a)$	inverse gauge coupling	2-3 values around β_c

[2]Pinke et al. 2018
[3]Sciarra 2021

Simulation Details

discretization of QCD action:

- Wilson gauge action
- Staggered fermion action

parameter	label	values
am	bare quark mass	$\{0.35, 0.55, 0.75, 0.95, 1.15\}$
N_τ	temporal lattice points	8, (10 in progress)
N_s	spatial lattice points	$\{32, 40, 48, 56, 64\}$
$\beta(a)$	inverse gauge coupling	2-3 values around β_c

observable: Polyakov loop $\langle L \rangle$:

$$L(\boldsymbol{n}) = \mathrm{Tr}\left[\prod_{n_4=0}^{N_\tau-1} U_4(\boldsymbol{n}, n_4)\right]$$

→ approximate order parameter for deconfinement transition

[2]Pinke et al. 2018
[3]Sciarra 2021

Simulation Details

discretization of QCD action:

- Wilson gauge action
- Staggered fermion action

parameter	label	values
am	bare quark mass	$\{0.35, 0.55, 0.75, 0.95, 1.15\}$
N_τ	temporal lattice points	8, (10 in progress)
N_s	spatial lattice points	$\{32, 40, 48, 56, 64\}$
$\beta(a)$	inverse gauge coupling	2-3 values around β_c

observable: Polyakov loop $\langle L \rangle$:

$$L(\boldsymbol{n}) = \mathrm{Tr}\left[\prod_{n_4=0}^{N_\tau-1} U_4(\boldsymbol{n}, n_4)\right]$$

→ approximate order parameter for deconfinement transition

Numerical Tools:

- CL²QCD: LQCD code[2]
- BaHaMAS: run and monitor LQCD simulations[3]
- PLASMA: analyze LQCD data

[2] Pinke et al. 2018
[3] Sciarra 2021

Analysis of the Thermal Transition

- analyze skewness B_3 and kurtosis B_4 of $|\langle L \rangle|$
- standardized moments

$$B_n = \frac{\langle (\bar{X} - X)^n \rangle}{\langle (\bar{X} - X)^2 \rangle^{n/2}}$$

- $B_3(\beta_c) = 0$ determines β_c
- $B_4(\beta_c) \rightarrow$ information on type of transition

infinite volume kurtosis values

Type	1. Order	Z_2 (Ising 3D)	Crossover
$\boldsymbol{B_4(T_c)}$	1	$1.604(1)$[4]	3

[4] Blote, Luijten, and Heringa 1995
[5] Ferrenberg and Swendsen 1989

Analysis of the Thermal Transition

- analyze skewness B_3 and kurtosis B_4 of $|\langle L\rangle|$
- standardized moments

$$B_n = \frac{\langle(\bar{X} - X)^n\rangle}{\langle(\bar{X} - X)^2\rangle^{n/2}}$$

- $B_3(\beta_c) = 0$ determines β_c
- $B_4(\beta_c) \to$ information on type of transition

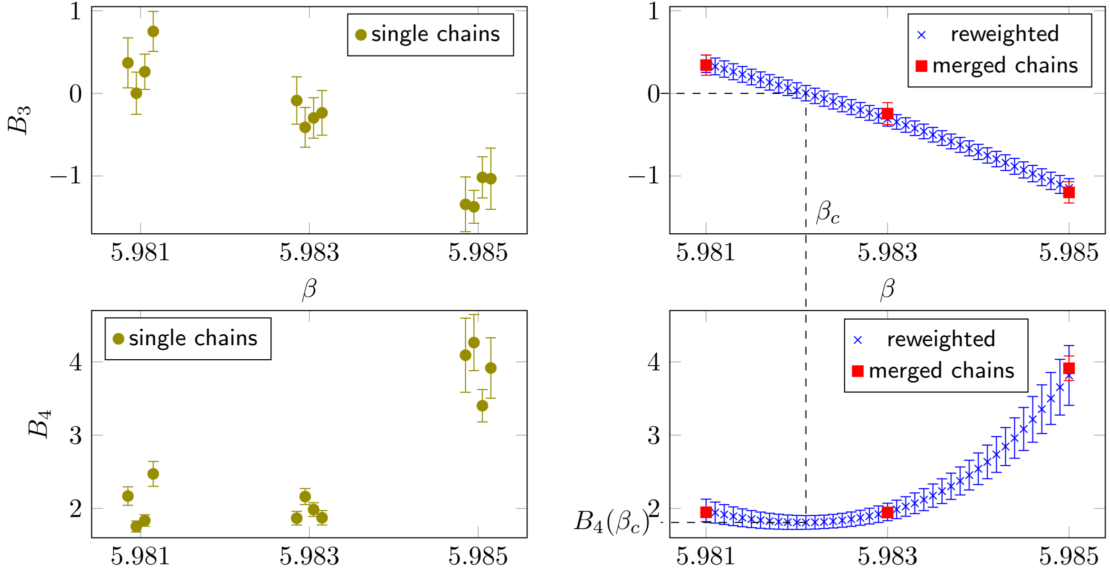
infinite volume kurtosis values

Type	1. Order	Z_2 (Ising 3D)	Crossover
$\boldsymbol{B_4(T_c)}$	1	$1.604(1)$[4]	3

- analyze quantities using jackknife resampling
- reweight with respect to β using multiple histogram method[5]

[4] Blote, Luijten, and Heringa 1995
[5] Ferrenberg and Swendsen 1989

Exemplary Analysis of B_3 and B_4 for $am = 0.55$, $N_s = 56$



Kurtosis Finite Size Scaling Formula

- finite size scaling (FSS) formula for kurtosis of observable[6] $O = c_M \cdot M$

$$B_4(N_s, \beta_c, m) = A + B \cdot x + \mathcal{O}(x^2)$$

scaling variable

$$x = \left(\frac{1}{m} - \frac{1}{m_c}\right) N_s^{1/\nu}$$

critical exponents from Ising 3D universality class[7]

$y_t = 1/\nu$	y_h
1.5870(10)	2.4818(3)

[6] Takeda et al. 2017
[7] Pelissetto and Vicari 2002

Kurtosis Finite Size Scaling Formula

- finite size scaling (FSS) formula for kurtosis of observable[6]
  $O = c_M \cdot M + c_E \cdot E$

$$B_4(N_s, \beta_c, m) = \left(A + B \cdot x + \mathcal{O}(x^2)\right) \times \left(1 + C N_s^{y_t - y_h} + \mathcal{O}\left(N_s^{2(y_t - y_h)}\right)\right)$$

- correction term becomes irrelevant for sufficiently large volumes
- fit kurtosis data to FSS formula to determine m_c

scaling variable

$$x = \left(\frac{1}{m} - \frac{1}{m_c}\right) N_s^{1/\nu}$$

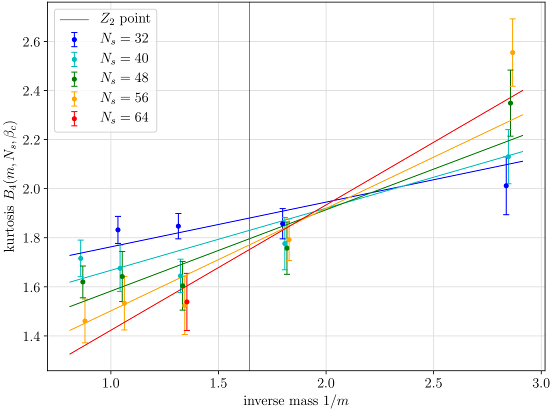
critical exponents from Ising 3D universality class[7]

$y_t = 1/\nu$	y_h
1.5870(10)	2.4818(3)

[6] Takeda et al. 2017
[7] Pelissetto and Vicari 2002

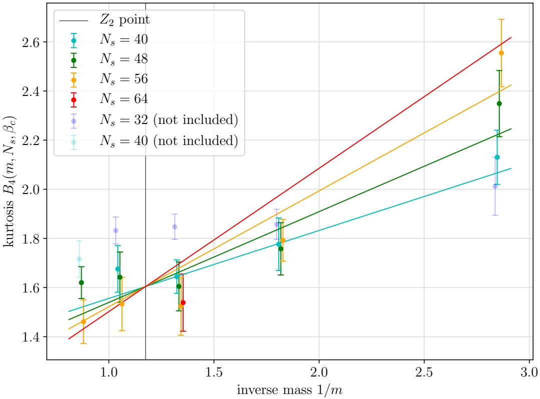
Linear Fits with and without Correction

linear with correction



m_c	a_1	c	ndf	χ^2_{ndf}	Q
0.61(5)	$6.4(7) \cdot 10^{-4}$	3.8(6)	17	1.03	42.0%

linear without correction



m_c	a_1	ndf	χ^2_{ndf}	Q
0.85(5)	$7.9(9) \cdot 10^{-4}$	13	1.13	32.8%

Results for the Z_2 Critical Point

- **linear fit with correction** is best fit
- final critical mass for $N_\tau = 8$:

$$am_c = 0.61(5)$$

- determine pion mass on $N_\tau = 32$, $N_s = 16$ lattice at β_c
- pion mass is not resolved by the lattice
- set the scale using w_0 scale based on Wilson flow[8]

am	β_c	am_π	a {fm}	m_π {GeV}	T_c {MeV}
0.55	5.9821	1.72039(7)	0.0888(10)	3.82(4)	278(3)
0.75	6.0129	1.98121(7)	0.0872(9)	4.48(5)	283(3)

Table: Results from pion mass measurement and scale setting

$m_\pi^{Z_2}$ for various N_τ from Wilson fermions[9]

N_τ	$m_\pi^{Z_2}$ {GeV}
6	5.01(5)
8	4.51(5)
10	4.39(5)

N_f=3 Staggered: talk by Ruben Kara (Thursday 06:30 EDT)

[8] Lüscher 2010
[9] Cuteri et al. 2021

Bibliography I

Blote, H. W. J., E. Luijten, and J. R. Heringa (Nov. 1995). "Ising universality in three dimensions: a Monte Carlo study". In: *Journal of Physics A: Mathematical and General* 28.22, pp. 6289–6313. DOI: 10.1088/0305-4470/28/22/007. URL: https://doi.org/10.1088%2F0305-4470%2F28%2F22%2F007 (cit. on pp. 8, 9).

Brown, Frank R. et al. (Nov. 1990). "On the existence of a phase transition for QCD with three light quarks". In: *Phys. Rev. Lett.* 65 (20), pp. 2491–2494. DOI: 10.1103/PhysRevLett.65.2491. URL: https://link.aps.org/doi/10.1103/PhysRevLett.65.2491 (cit. on pp. 3, 4).

Cuteri, Francesca et al. (Jan. 2021). "Deconfinement critical point of lattice QCD with $N_\mathrm{f} = 2$ Wilson fermions". In: *Phys. Rev. D* 103 (1), p. 014513. DOI: 10.1103/PhysRevD.103.014513. URL: https://link.aps.org/doi/10.1103/PhysRevD.103.014513 (cit. on p. 14).

Ferrenberg, Alan M. and Robert H. Swendsen (Sept. 1989). "Optimized Monte Carlo data analysis". In: *Phys. Rev. Lett.* 63 (12), pp. 1195–1198. DOI: 10.1103/PhysRevLett.63.1195. URL: https://link.aps.org/doi/10.1103/PhysRevLett.63.1195 (cit. on pp. 8, 9).

Lüscher, Martin (Aug. 2010). "Properties and uses of the Wilson flow in lattice QCD". In: *Journal of High Energy Physics* 2010.8. ISSN: 1029-8479. DOI: 10.1007/jhep08(2010)071. URL: http://dx.doi.org/10.1007/JHEP08(2010)071 (cit. on p. 14).

Bibliography II

Pelissetto, Andrea and Ettore Vicari (Oct. 2002). "Critical phenomena and renormalization-group theory". In: *Physics Reports* 368.6, pp. 549–727. ISSN: 0370-1573. DOI: 10.1016/s0370-1573(02)00219-3. URL: http://dx.doi.org/10.1016/S0370-1573(02)00219-3 (cit. on pp. 11, 12).

Pinke, Christopher et al. (Sept. 2018). *CL2QCD*. Version v1.0. DOI: 10.5281/zenodo.5121895. URL: https://doi.org/10.5281/zenodo.5121895 (cit. on pp. 5–7).

Sciarra, Alessandro (Feb. 2021). *BaHaMAS*. Version BaHaMAS-0.4.0. DOI: 10.5281/zenodo.4577425. URL: https://doi.org/10.5281/zenodo.4577425 (cit. on pp. 5–7).

Takeda, Shinji et al. (2017). "Update on Nf=3 finite temperature QCD phase structure with Wilson-Clover fermion action". In: *PoS* LATTICE2016, p. 384. DOI: 10.22323/1.256.0384. arXiv: 1612.05371 [hep-lat] (cit. on pp. 11, 12).